

# Low-energy hadronic cross sections measurements at BaBar, and implication for the $g-2$ of the muon

---

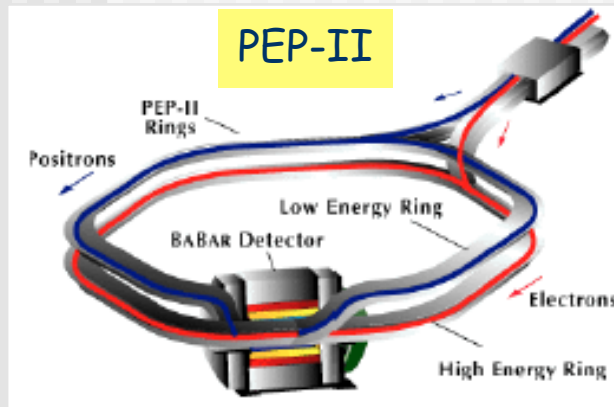
E.Solodov for the BaBar collaboration

Budker INP SB RAS, Novosibirsk, Russia

PHOTON2015, Novosibirsk, Russia

# PEP-II e<sup>+</sup>e<sup>-</sup> collider, Babar detector

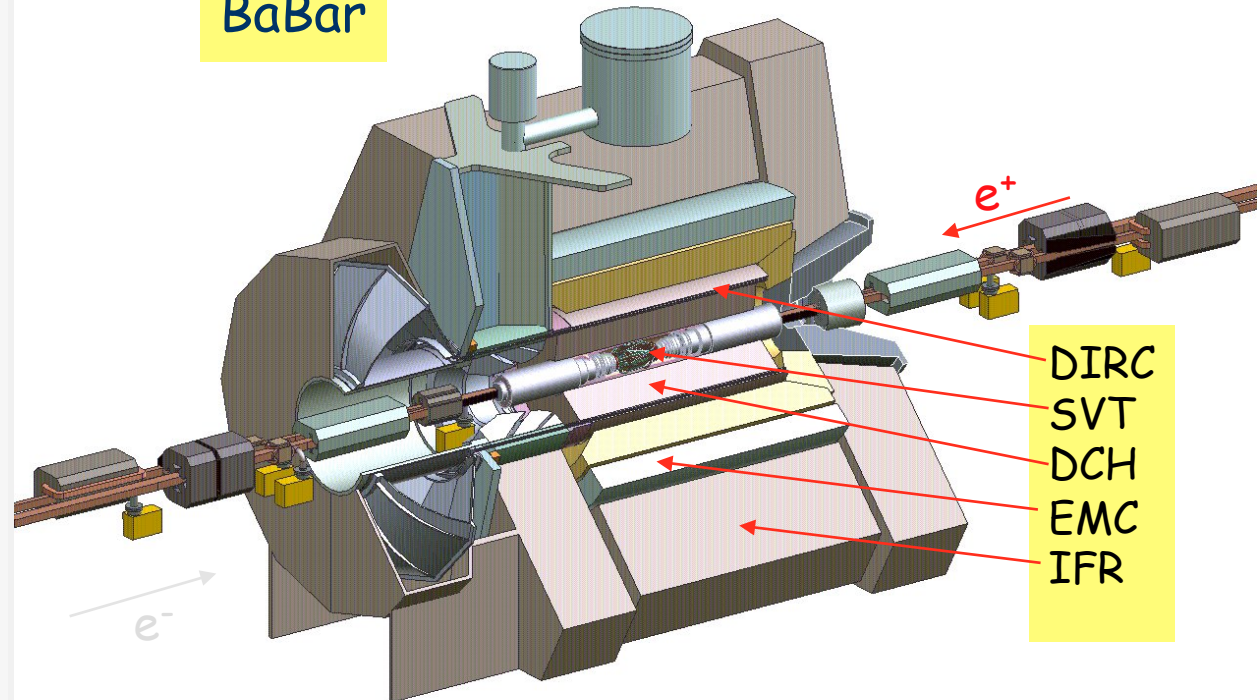
$$E_+ = 3.1 \text{ GeV}, E_- = 9 \text{ GeV}$$



$$E_{CM} = M(Y(4S)) = 10.6 \text{ GeV}$$

2000 - 2008 yrs  
 $\Delta L = 500 \text{ fb}^{-1}$   
 $N(B) = 10^9$

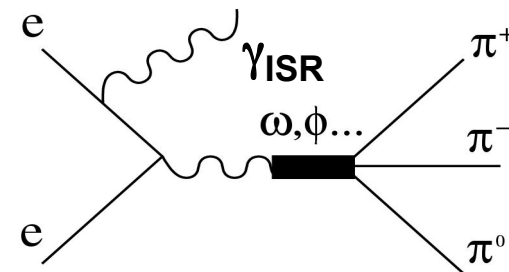
BaBar



$$\frac{d\sigma(s, x)}{dx d(\cos\theta)} = W(s, x, \theta) \cdot \sigma_0(s(1-x)),$$

$$W(s, x, \theta) = \frac{\alpha}{\pi x} \left( \frac{2 - 2x + x^2}{\sin^2 \theta} - \frac{x^2}{2} \right), \quad x = \frac{2E_\gamma}{\sqrt{s}}$$

$\theta$  - photon polar angle in c.m.



# ISR study at BaBar

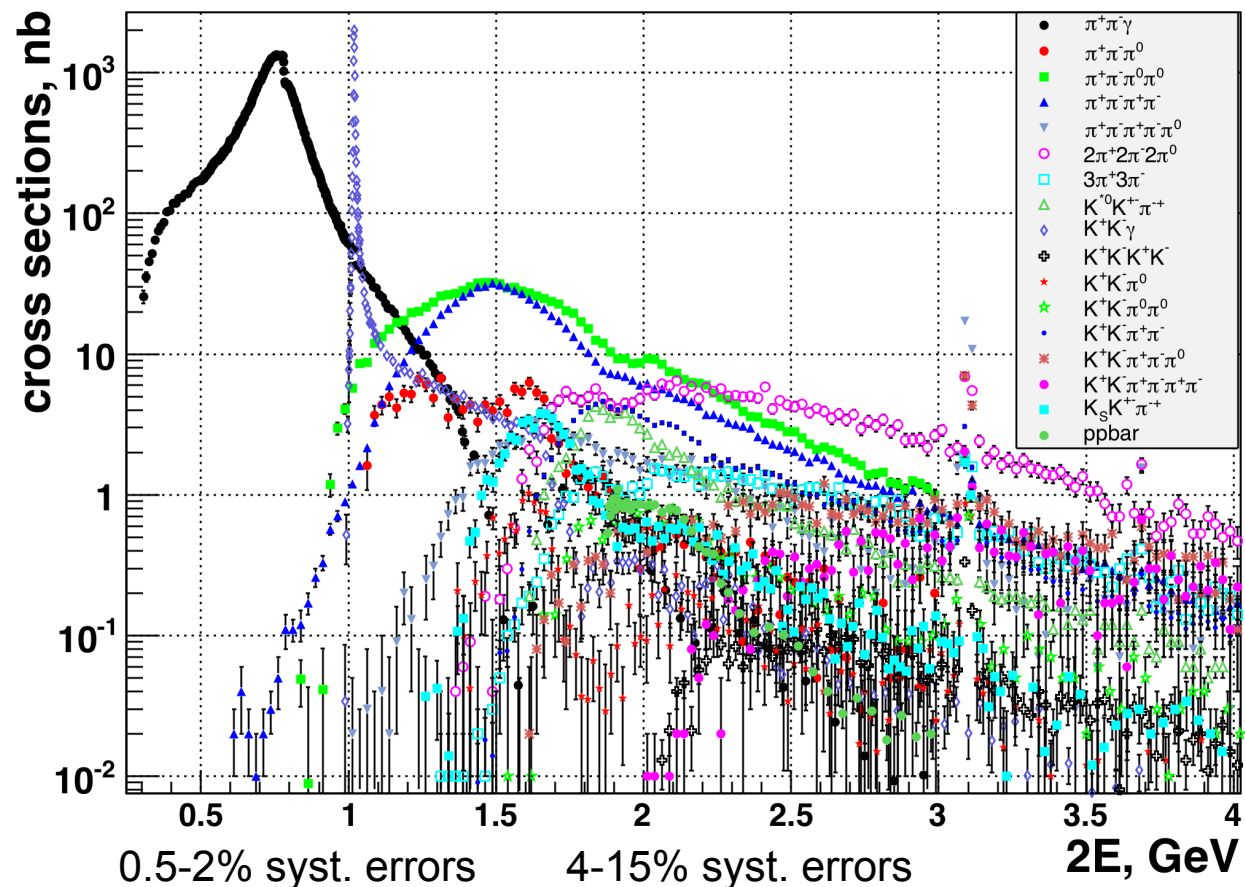
- ISR at BaBar gives competitive (**even dominates!**) statistics
- BaBar has excellent capability for ISR study
- Many major hadronic processes have been studied

published

$e^+e^- \rightarrow \pi^+\pi^-$	PR D 86 (2012) 032013, PR L 103 (2009) 231801
$e^+e^- \rightarrow K^+K^-$	PR D 88, (2013) 032013
$e^+e^- \rightarrow \phi f_0(980)$	PR D 74 (2006) 091103, PR D 76 (2007) 012008
$e^+e^- \rightarrow \pi^+\pi^-\pi^0$	PR D 70 (2004) 072004
$e^+e^- \rightarrow K^+K^-\eta, K^+K^-\pi^0, K_S^0 K^\pm \pi^\mp$	PR D 77 (2008) 092002, PR D 71 (2005) 052001
$e^+e^- \rightarrow 2(\pi^+\pi^-)$	PR D 85 (2012) 112009, PR D 76 (2007) 012008
$e^+e^- \rightarrow K^+K^-\pi^0\pi^0, K^+K^-\pi^+\pi^-, 2(K^+K^-)$	PR D 86 (2012) 012008, PR D 76 (2007) 012008
$e^+e^- \rightarrow K_S^0 K_L^0, K_S^0 K_L^0 \pi^+\pi^-, K_S^0 K_S^0 \pi^+\pi^-, K_S^0 K_S^0 K^+K^-$	PR D 89 (2014) 092002
$e^+e^- \rightarrow 2(\pi^+\pi^-)\pi^0, 2(\pi^+\pi^-)\eta, K^+K^-\pi^+\pi^-\pi^0, K^+K^-\pi^+\pi^-\eta$	PR D 76 (2007) 092005
$e^+e^- \rightarrow 3(\pi^+\pi^-), 2(\pi^+\pi^-\pi^0), 2(\pi^+\pi^-)K^+K^-$	PR D 73 (2006) 052003
$e^+e^- \rightarrow p\bar{p}$ (small $\sqrt{s}$ )	PR D 87 (2013) 092005, PR D 73 (2006) 012005
$e^+e^- \rightarrow p\bar{p}$ (large $\sqrt{s}$ )	PR D 88 (2013) 072009
$e^+e^- \rightarrow \Lambda\bar{\Lambda}, \Lambda\Sigma^0, \Sigma^0\bar{\Sigma}^0$	PR D 76 (2007) 092006
$e^+e^- \rightarrow c\bar{c} \rightarrow \dots$	...

Some reactions are being updated to the full BaBar data with  $\sim 500 \text{ fb}^{-1}$

# BaBar measurements summary



To calculate  $R$  in the energy range 1-2 GeV the processes  $\pi^+\pi^-3\pi^0$ ,  $\pi^+\pi^-4\pi^0$ ,  $K_S K_L$ ,  $K_S K_L \pi^+\pi^-$ ,  $K_S K^+ \pi^-\pi^0$  are under study:  
 $\pi^+\pi^-2\pi^0$  will come soon. Work is in progress for  $K_S K_L \pi^0\pi^0$ ,  $K_S K_L \pi^0$



# BaBar measured: $e^+e^- \rightarrow \pi^+\pi^-$

Motivation: dominance of the  $E < 1\text{GeV}$  region,  
accessed through  $\pi^+\pi^-$

Features of the cross section distribution

- Includes possible FSR
- Dominated by  $\rho(770)$  resonance
- $\rho$ - $\omega$  interference
- Dip at 1.6GeV: interference between  $\rho'$  and  $\rho''$
- Dip at 2.2GeV: higher mass  $\rho$  state

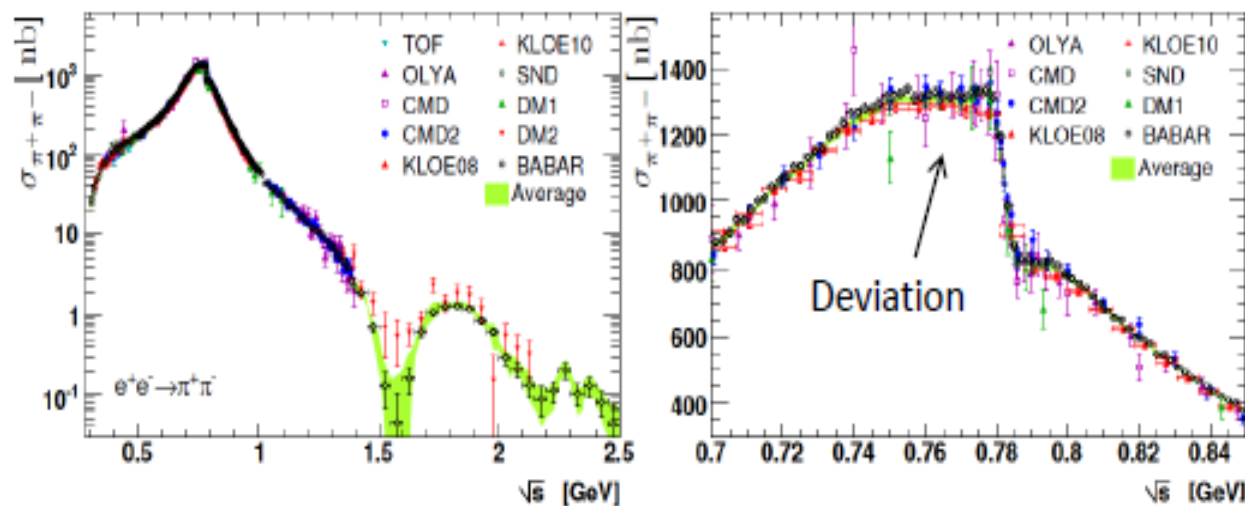
LO Hadr., 0.28-1.8 GeV

Babar

$$a_\mu = (514.1 \pm 3.8) \cdot 10^{-10}$$

all  $e^+e^-$

$$a_\mu = (505.8 \pm 3.0) \cdot 10^{-10}$$



Systematic uncertainties  
at the  $\rho$  region

BABAR: 0.5%

CMD2: 0.8%

SND: 1.5%

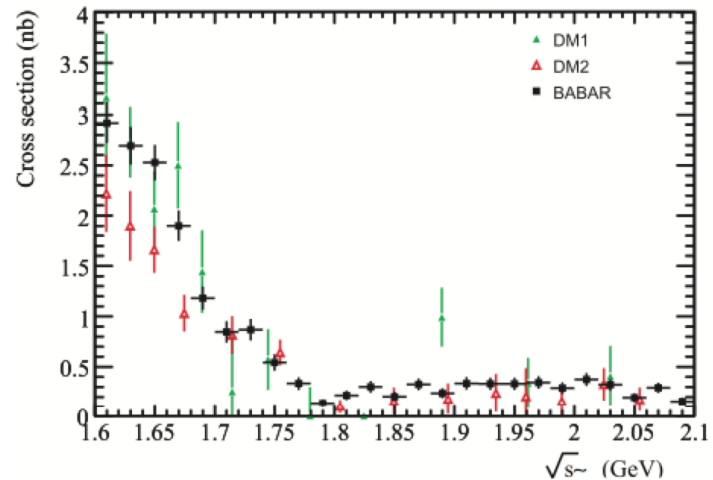
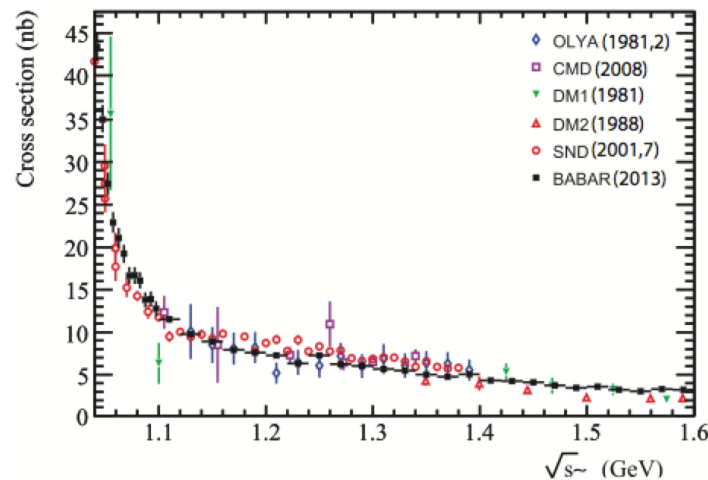
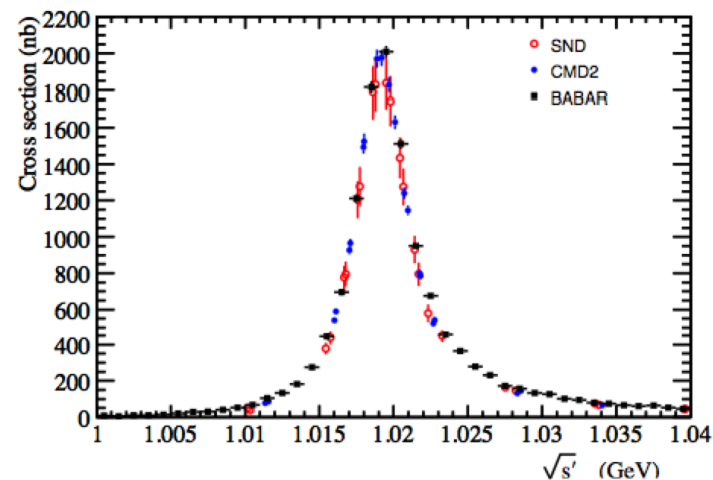
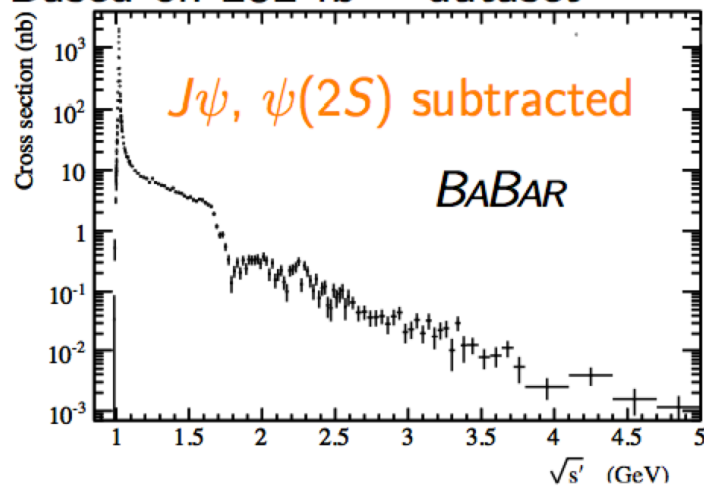
KLOE: 0.8%

# BaBar measured: $e^+e^- \rightarrow K^+K^-$

Published **Phys. Rev. D 88, 032013 (2013)**

Our result is more precise than the current world average

Based on  $232 \text{ fb}^{-1}$  dataset

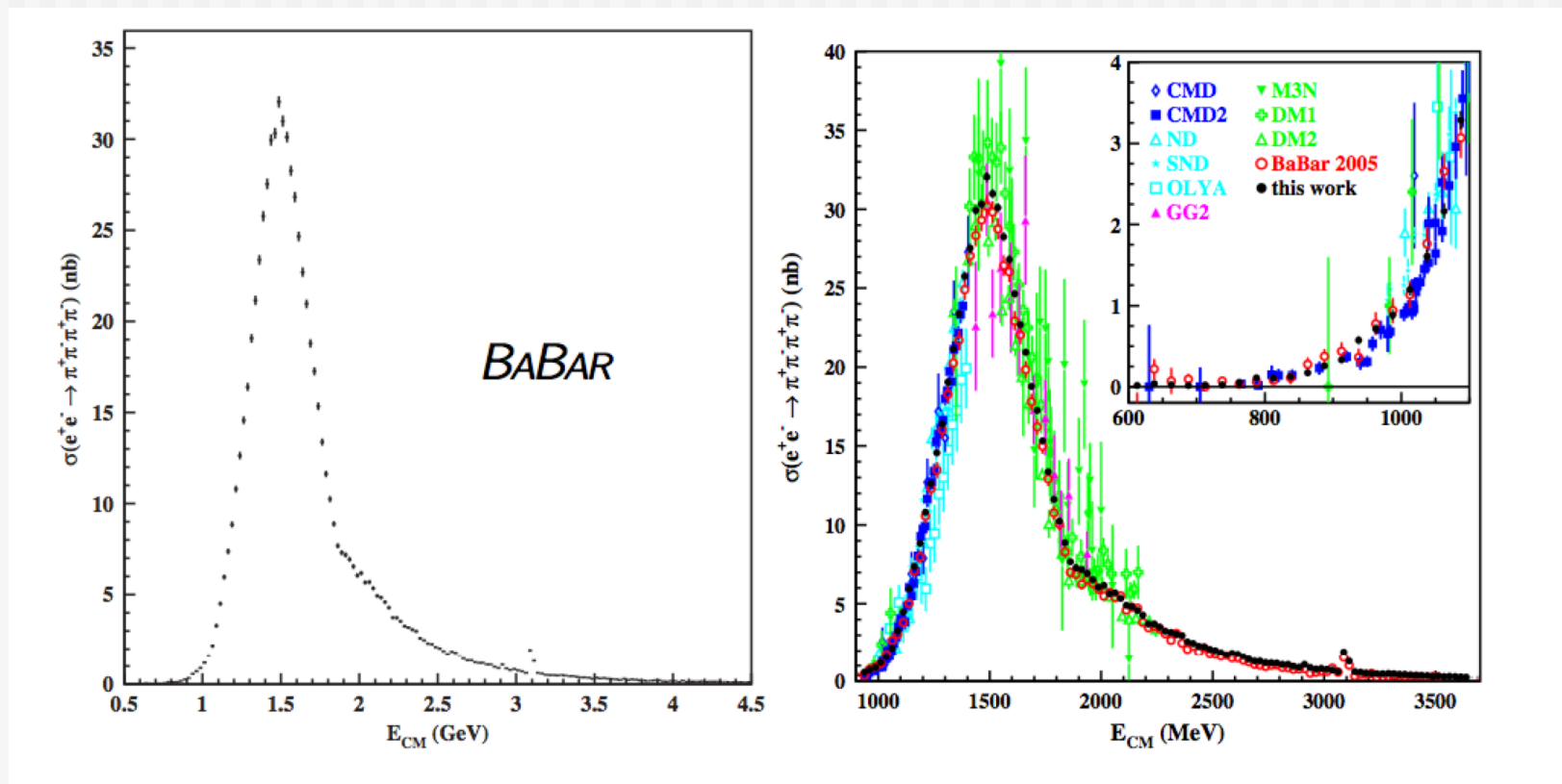


# BaBar updated: $e^+e^- \rightarrow \pi^+\pi^-\pi^+\pi^-$

Published PRD 85 112009 (2012)

Based on 454 fb<sup>-1</sup> dataset (statistical uncertainties are shown)

Our result is more precise than the current world average (<3% systematic error)



# Contribution of missing channels to $a_\mu$

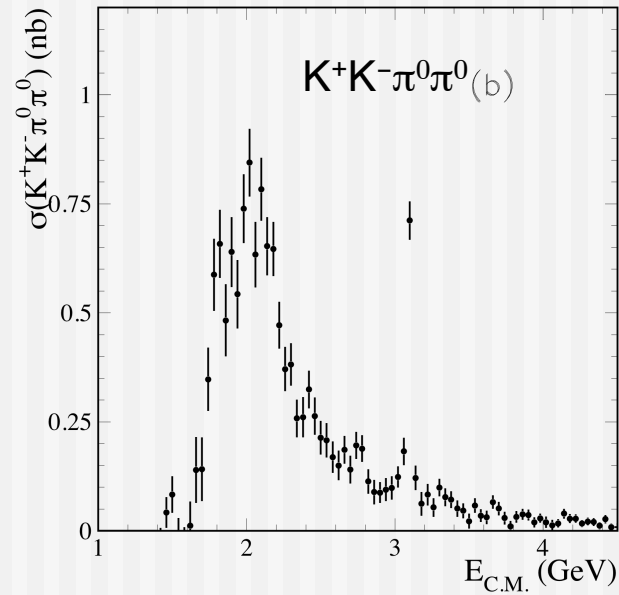
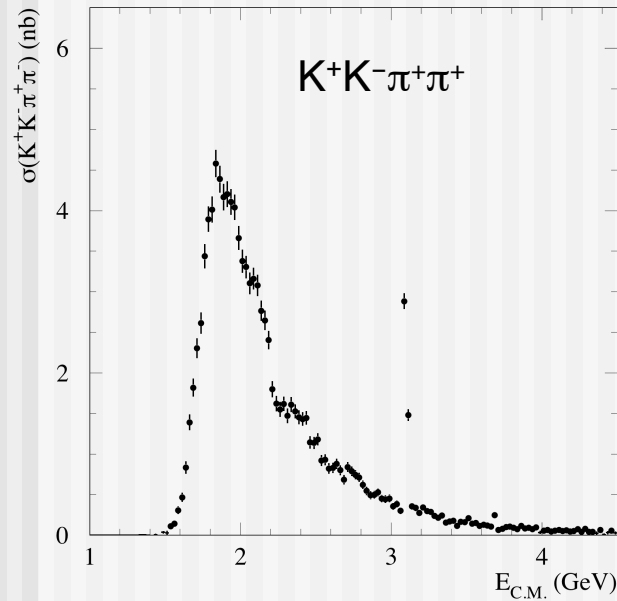
SM-to-experiment comparison [in units  $10^{-10}$  ]

QED	116 584 71.81				$\pm 0.02$
Leading hadronic vacuum polarization (VP)	690.30				$\pm 5.26$
Sub-leading hadronic vacuum polarization	-10.03				$\pm 0.11$
Hadronic light-by-light	11.60				$\pm 3.90$
Weak (incl. 2-loops)	15.32				$\pm 0.18$
Theory	11659179.00				$\pm 6.46$
Experiment	11659208.00				$\pm 6.30$
Exp – theory	29.00				$\pm 9.03$
$a_\mu (\sqrt{s} < 1.8 \text{ GeV})$	$K^+K^-$	$2(\pi^+ \pi^-)$	$3(\pi^+ \pi^-)$	$2(\pi^+ \pi^- \pi^0)$	
without BABAR	$21.63 \pm 0.70$	$13.35 \pm 0.90$	$0.10 \pm 0.10$	$1.42 \pm 0.30$	
with BABAR	$22.95 \pm 0.26$	$13.64 \pm 0.36$	$0.11 \pm 0.02$	$0.89 \pm 0.09$	

Missing channels contribute  $5.98 \pm 0.42$  or  $12.46 \pm 0.76$  if  $\sqrt{s} < 2.0 \text{ GeV}$

Contribution from  $KK\pi$ ,  $KK2\pi$ ,  $2\pi3\pi^0$ ,  $2\pi4\pi^0$ ,  $(7\pi, 8\pi)$ ... added using iso-spin relations (in particular, using measured  $K^+K^-\pi^+\pi^-$  ( $\pi^0\pi^0$ ) channels)

# The cross section comparison – BaBar data

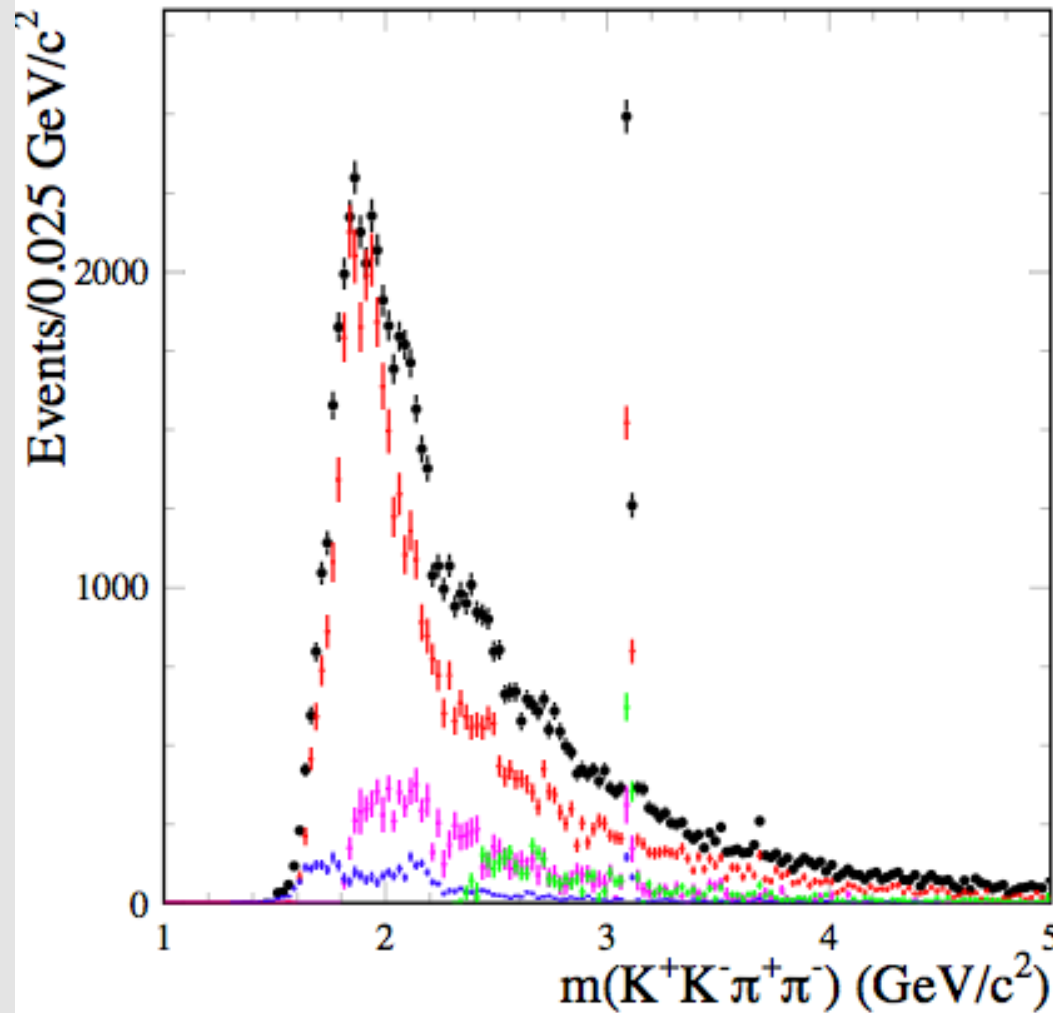


Naively expect:  $N(K^+K^-\pi^+\pi^-) = 2 N(K^+K^-\pi^0\pi^0)$

Many intermediate states break the relation.

Study of intermediate states is important!

# Decomposition of $K^+K^-\pi^+\pi^-$ mass spectrum



$K^+K^-\pi^+\pi^-$

$K^{*0}(892)K\pi$

$K^+K^-\rho(770)$

$\phi\pi^+\pi^-$

$K_2^{*0}(1430)K\pi$

Tables with cross sections  
(corrected for BF) are provided

Phys. Rev. D 86, 012008 (2012)



$$e^+e^- \rightarrow K_S K_L, K_S K_L \pi^+ \pi^-, K_S K_S \pi^+ \pi^- (K^+ K^-)$$

---

We present (with more details) results on the study of the processes:

$$e^+e^- \rightarrow K_S K_L$$

$$e^+e^- \rightarrow K_S K_L \pi^+ \pi^-$$

$$e^+e^- \rightarrow K_S K_S \pi^+ \pi^-$$

$$e^+e^- \rightarrow K_S K_S K^+ K^-$$

Published *Phys. Rev. D* 89, 092002 (2014)

And new (preliminary) results on the process

$$e^+e^- \rightarrow K_S K^{*-} \pi^{*-} \pi^0$$

ready for publication

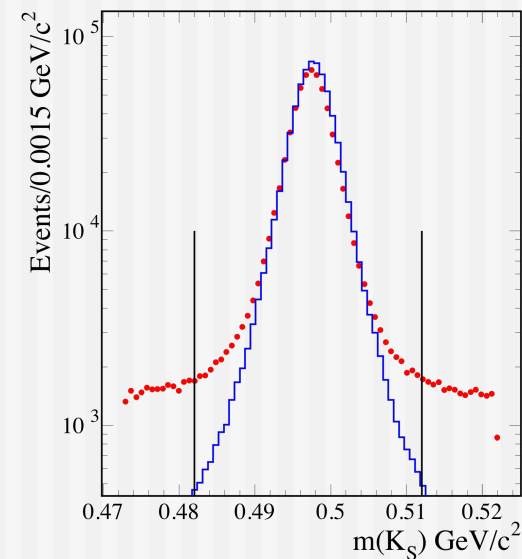
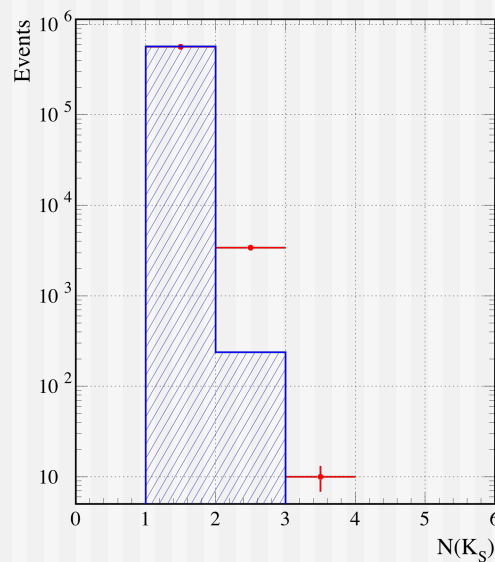
Based on 469 fb<sup>-1</sup> integrated luminosity.

# $K_S$ selection (in $\pi^+\pi^-$ decay)

Loop over all  $K_S$  candidates with ISR photon with  $E_\gamma > 3$  GeV, and select events with:

- Good quality  $K_S$  coming from IP
- No electron ID for either charged track

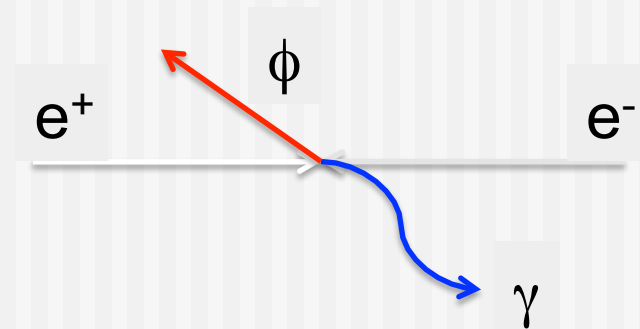
Simulation  
of  $\phi\gamma \rightarrow K_S K_L \gamma$   
compare to data



Dominated by  $\phi\gamma \rightarrow K_S K_L \gamma$  process if require **NO** additional tracks from IP

$$e^+e^- \rightarrow \phi \gamma \rightarrow K_S K_L \gamma \text{ (without } K_L \text{ detection)}$$

$$\begin{aligned} E_0 &= E^+ + E^- \\ \vec{p}_0 &= \vec{p}^+ + \vec{p}^- \\ \vec{p}_\gamma &= n E_\gamma \end{aligned}$$



Assuming  $e^+e^- \rightarrow \phi \gamma$  reaction  
Use  $\phi$  mass to get  $E_{\gamma\text{ISR}}$

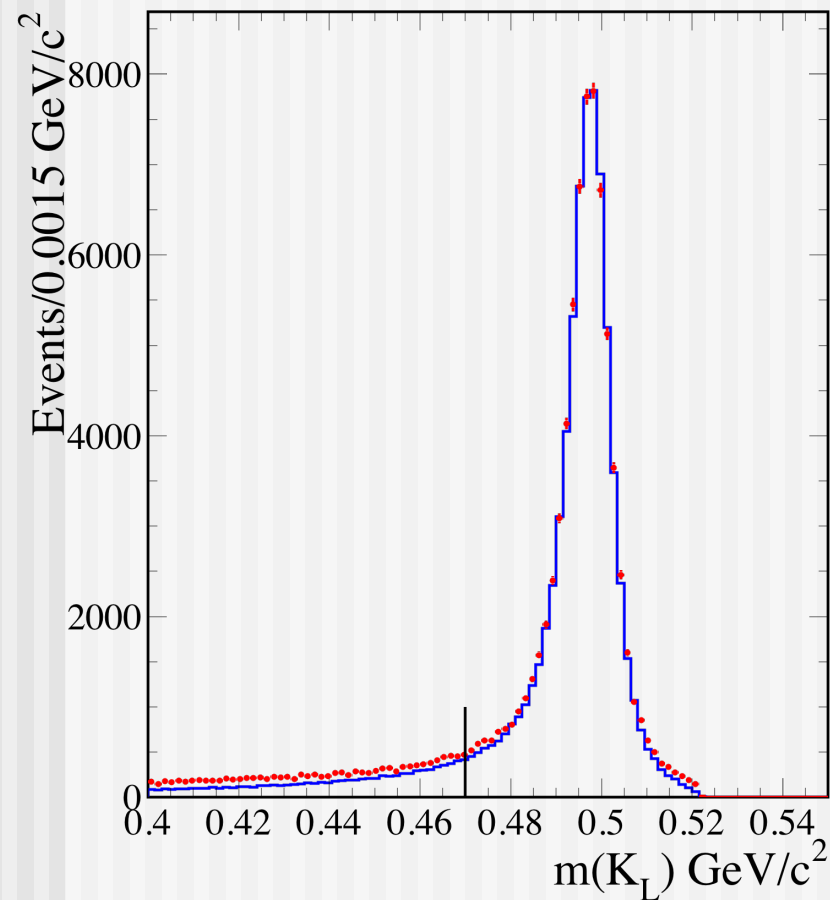
$$E_\gamma^c = \frac{E_0^2 - p_0^2 - m_\phi^2}{2(E_0 - \vec{p}_0 \cdot \vec{n}_\gamma)}$$

Using energy-momentum conservation and detected  $K_S$   
we determine  $K_L$  mass and direction:

$$m^2(K_L) = \left( E^+ + E^- - E_\gamma^c - E_{K_S} \right)^2 - \left( p^+ + p^- - p_\gamma^c - p_{K_S} \right)^2$$

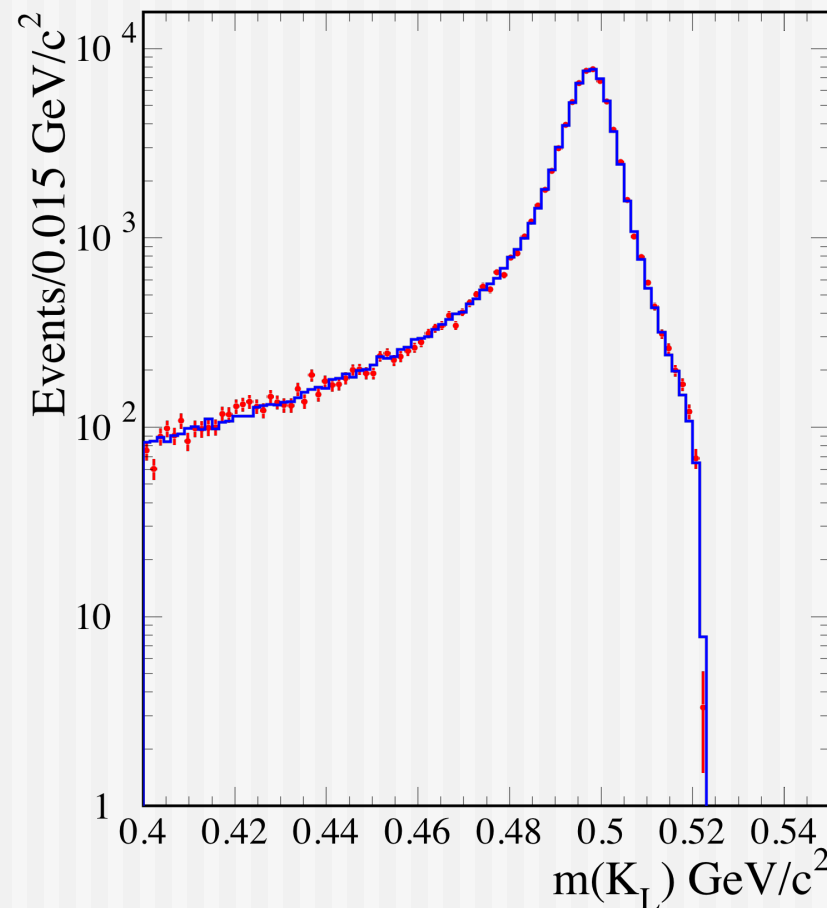
Using this events we can study  $K_L$  detection.

# $K_L$ mass using $\phi$ mass constraint



MC normalized to two bins at peak

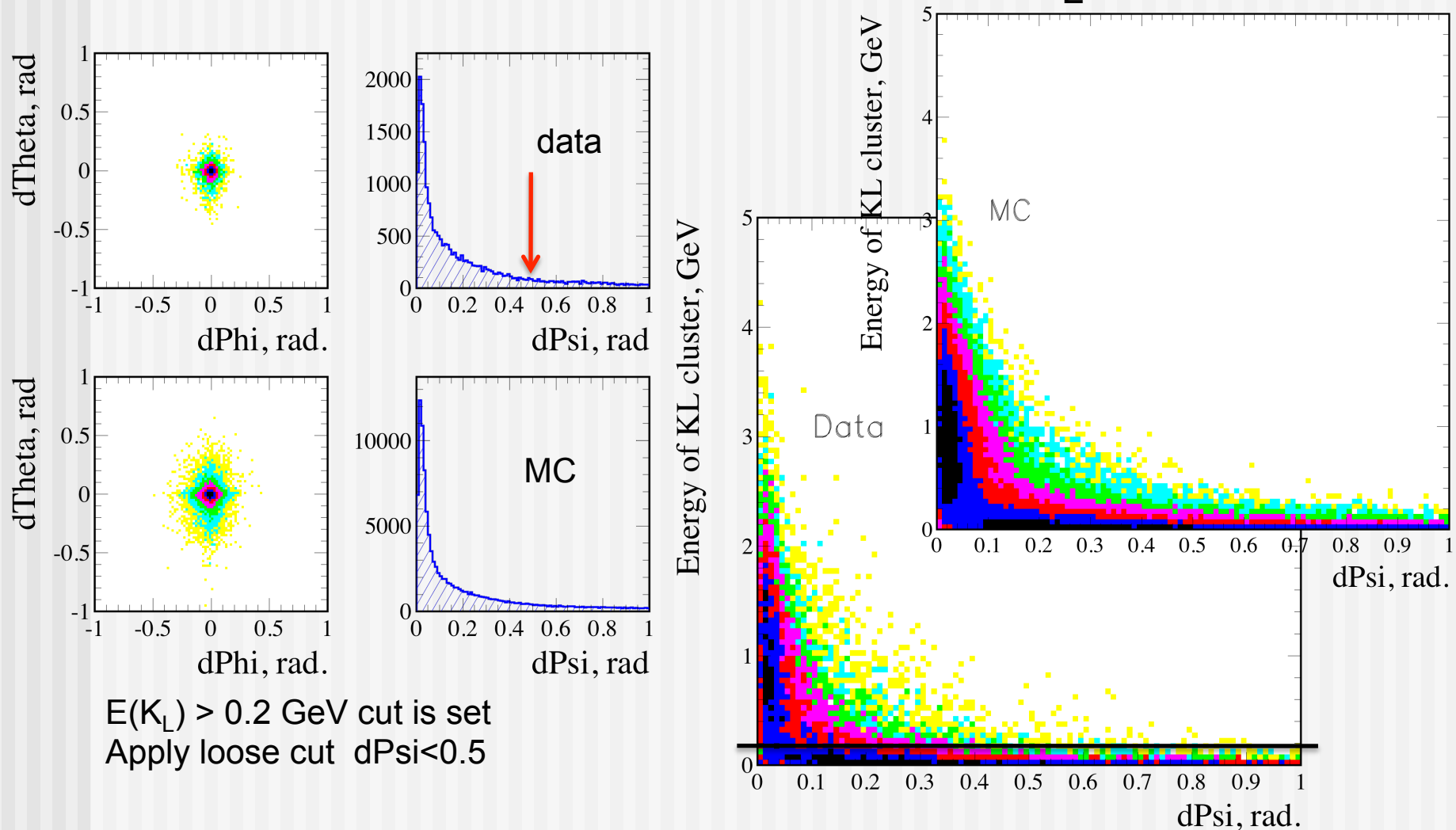
**Very low background!**



After background subtraction (5.6%) we have **81012±285** events (**447434±669** MC). We estimate ~0.5% systematic error for background subtraction uncertainty.

# How $K_L$ cluster in Calorimeter looks like?

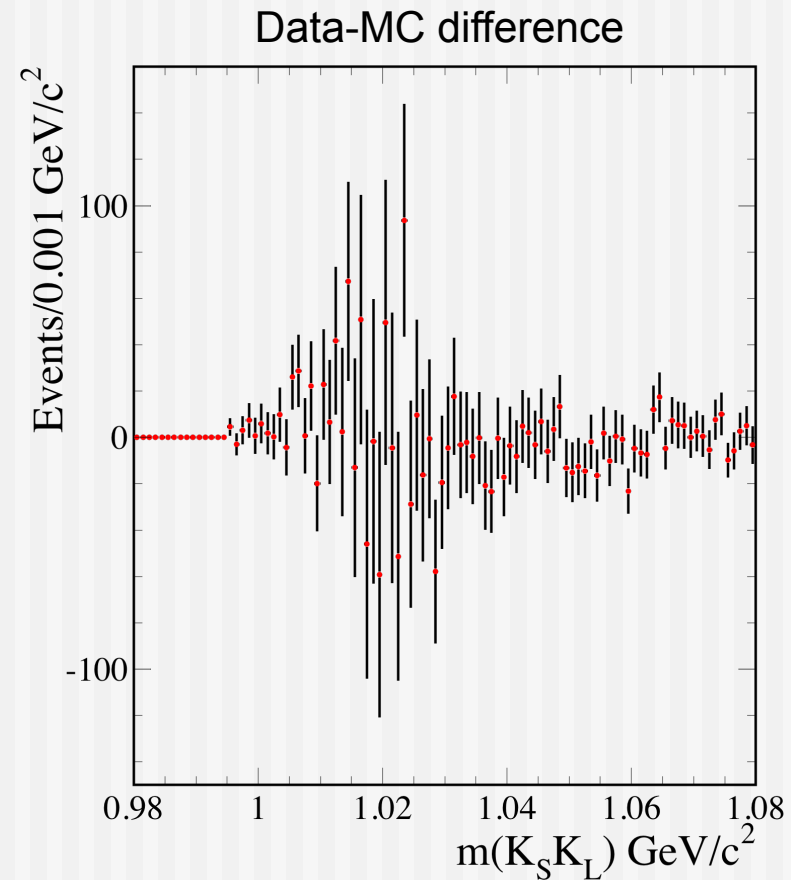
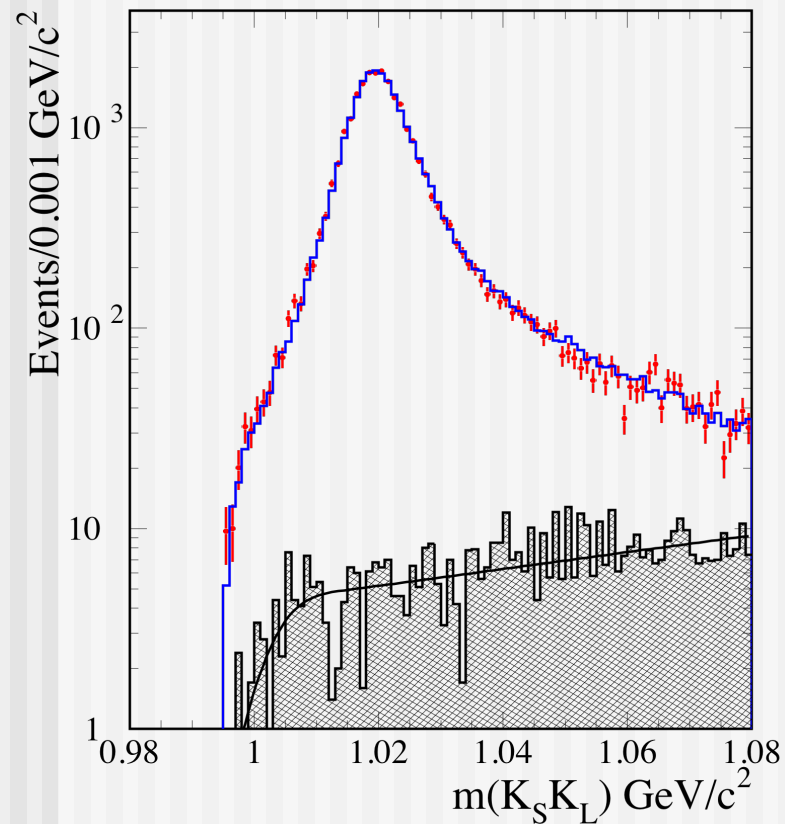
## 1. Search for EMC cluster closest to $K_L$ direction:



$$\text{Data/MC} = 0.9394 \pm 0.0052 \text{ (0.6\%)}$$

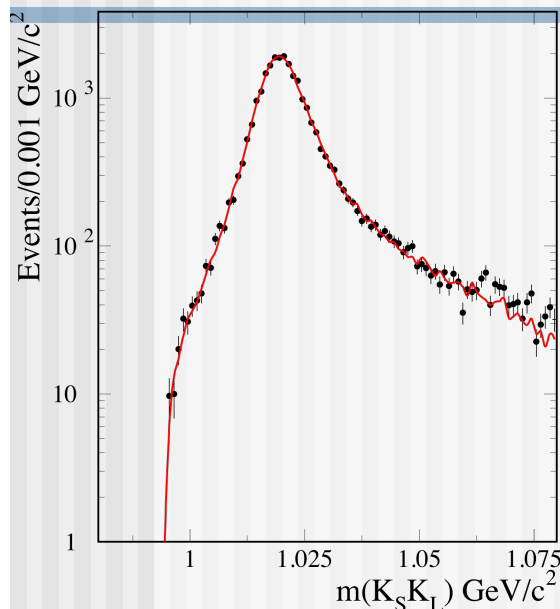
# $\phi$ signal in $e^+e^- \rightarrow K_S K_L$ reaction

Use events with  $\chi^2 < 15$  and reconstructed parameters of  $K_S$  and  $K_L$  to calculate  $m(K_S K_L)$



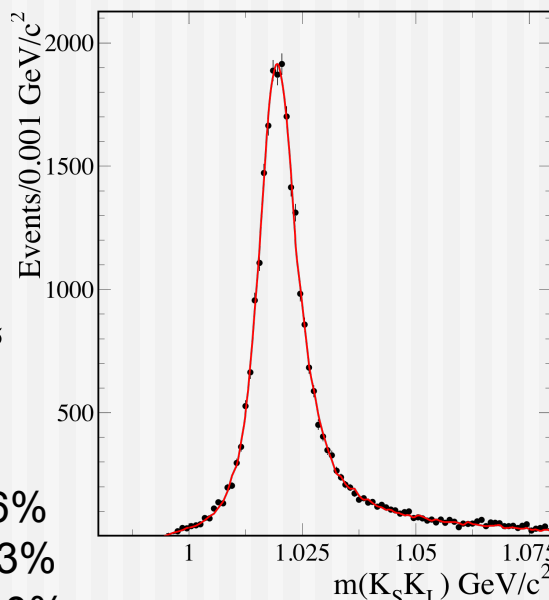


# Fit to $\phi$ parameters



Fit:

$$\begin{aligned}\sigma_0 &= 1409 \pm 33 \pm 42 \pm 15 \text{ nb} \\ m &= 1019.462 \pm 0.042 \pm 0.050 \pm 0.025 \text{ MeV}/c^2 \\ \Gamma_0 &= 4.205 \pm 0.103 \pm 0.050 \pm 0.045 \text{ MeV}\end{aligned}$$



Systematic uncertainty:

$K_L$ efficiency (+ $\chi^2$ )	0.6%
BGFilter efficiency	2.3%
$K_S$ efficiency	1.0%
ISR photon	0.5%
Luminosity	0.5%
Rad.corr.	1.0%
Track overlap	0.6%
Background sub.	0.5%
	<b>2.9%</b>

**BaBar**

$$\begin{aligned}\Gamma_{ee} \cdot B_{KSKL} &= 0.4200 \pm 0.0033 \pm 0.0122 \pm 0.0019 \text{ keV} \\ \Gamma_{ee} &= 1.228 \pm 0.037 \pm 0.014_{(\text{PDG } B_{KSKL})} \text{ keV} \\ B_{ee} \cdot B_{KSKL} &= 0.986 \pm 0.030 \pm 0.009_{(\text{PDG } \Gamma_{KSKL})}\end{aligned}$$

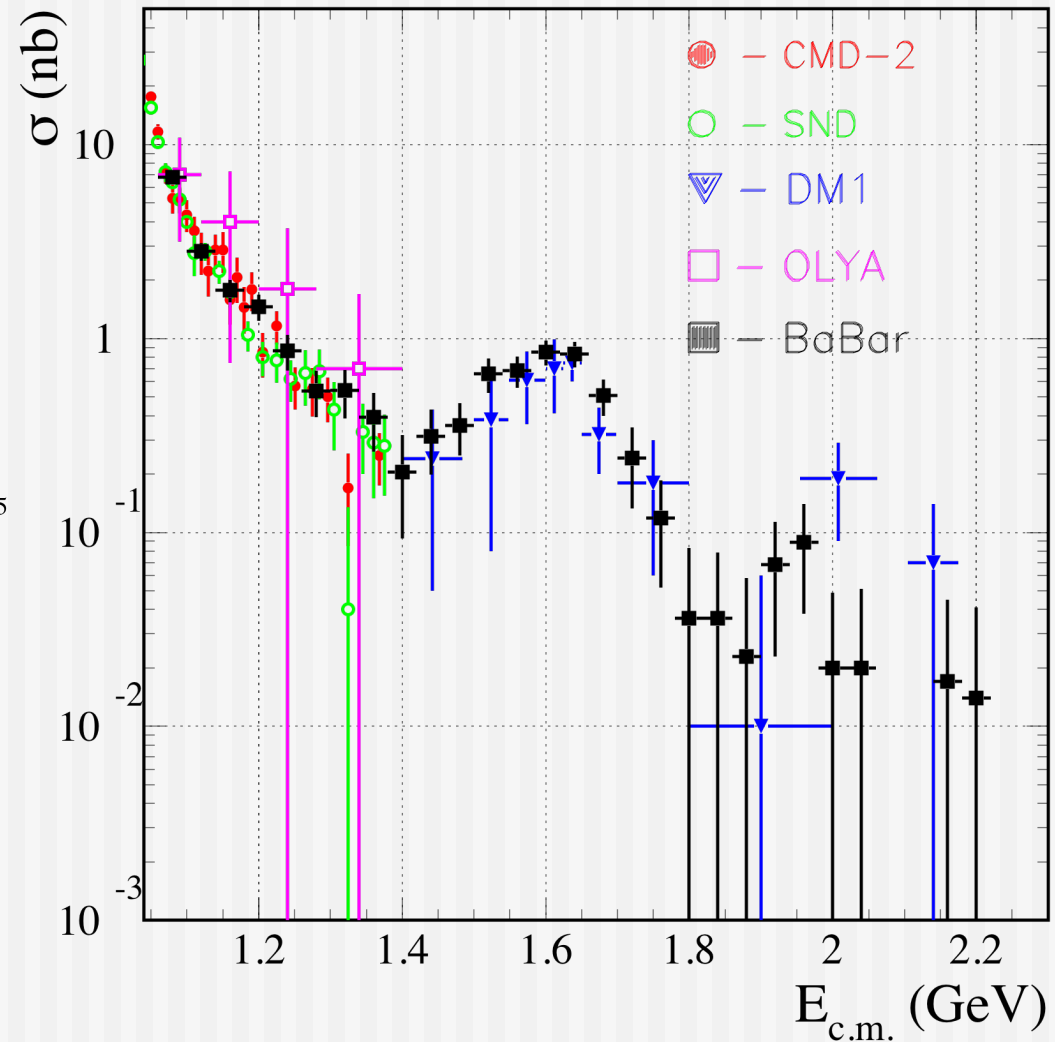
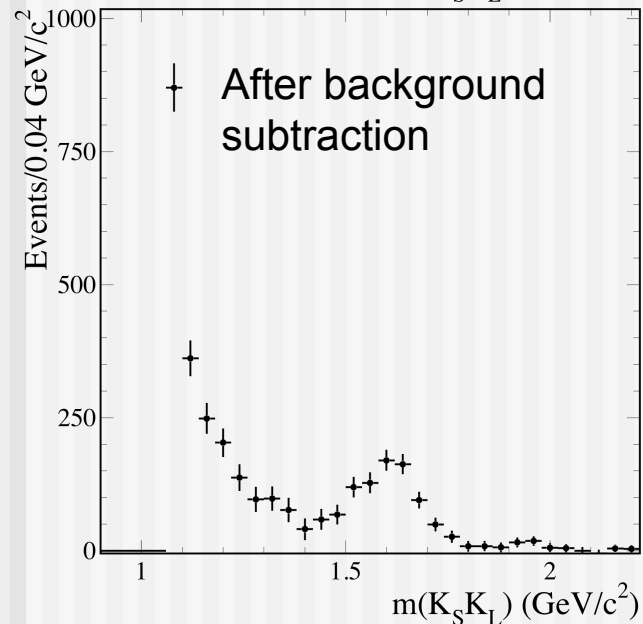
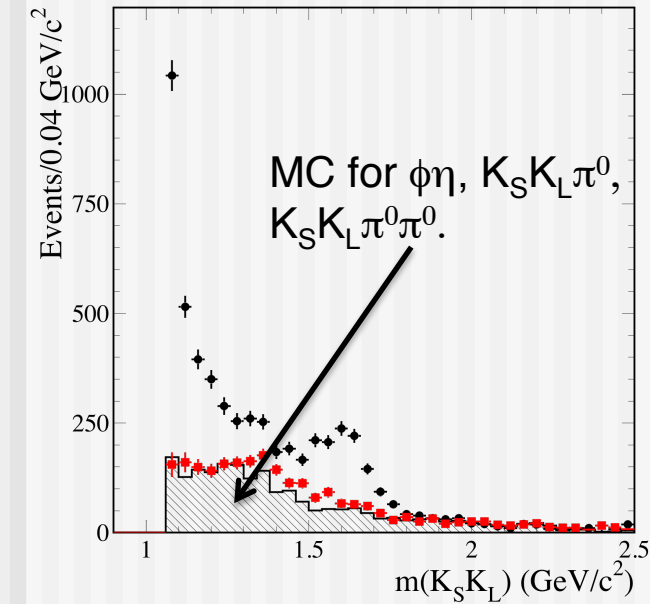
**CMD-2**

$$\begin{aligned}\sigma_0 &= 1376 \pm 6 \pm 23 \text{ nb} \\ m &= 1019.483 \pm 0.011 \pm 0.025 \text{ MeV}/c^2 \\ \Gamma_0 &= 4.280 \pm 0.033 \pm 0.025 \text{ MeV} \\ \Gamma_{ee} &= 1.235 \pm 0.006 \pm 0.022 \text{ keV}\end{aligned}$$

**PDG2010-2012**

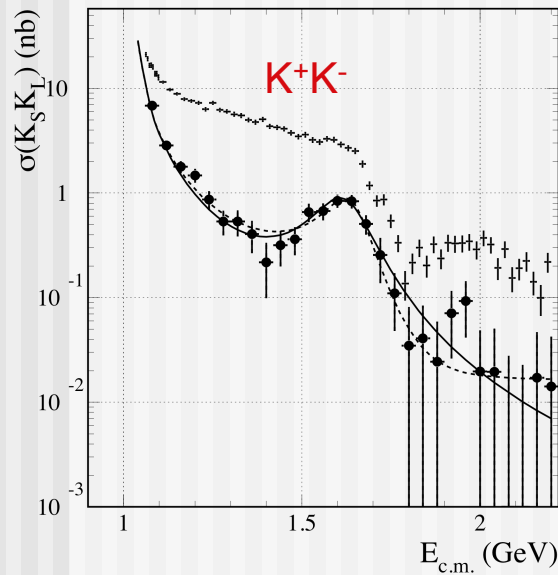
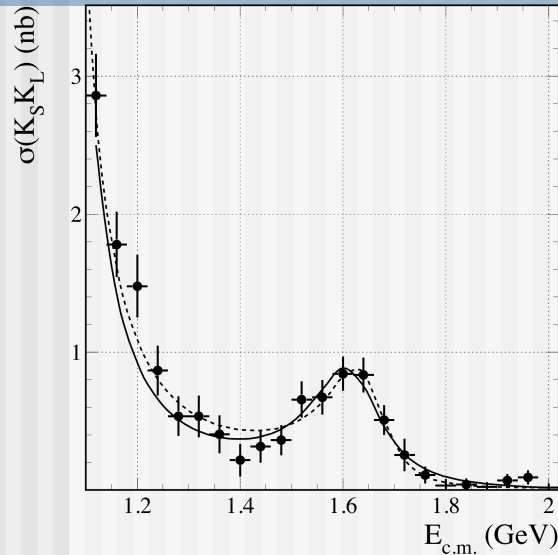
$$\begin{aligned}m &= 1019.455 \pm 0.020 \text{ MeV}/c^2 \\ \Gamma_0 &= 4.26 \pm 0.04 \text{ MeV} \\ \Gamma_{ee} &= 1.27 \pm 0.04 \text{ keV} \\ B_{KSKL} &= 0.342 \pm 0.004 \\ B_{ee} \cdot B_{KSKL} &= 1.006 \pm 0.016\end{aligned}$$

# $e^+e^- \rightarrow K_S K_L$ cross section for $m(K_S K_L) > 1.06$ GeV



Systematic error ~10% (~30% for  $\sigma < 0.3$  nb),  
dominated by background subtraction procedure.

# Is it $\phi(1680)$ ? Fit with single BW



$$\sigma(s) = \frac{P(s)}{s^{5/2}} \left| \frac{A_{\phi(1020)}}{\sqrt{P(m_\phi)}} + \frac{A_X}{\sqrt{P(m_X)}} \cdot e^{i\varphi} + A_{bkg} \right|^2$$

$$P(s) = \left( \left( s/2 \right)^2 - m_{K^0}^2 \right)^{3/2}$$

$$A(s) = \frac{\Gamma(m^2) \cdot m^3 \sqrt{\sigma_0 \cdot m}}{s - m^2 + i\sqrt{s}\Gamma(s)}$$

$$\Gamma(s) = \Gamma \cdot \sum_f B_f \cdot \frac{P_f(s)}{P_f(m_f^2)}$$

$$A_{\phi(1020)} = A_\phi + A_\omega - A_\rho, \quad f = K^* K, \phi\eta, \phi\pi\pi, K_S K_L$$

$$\sigma_0 = 0.46 \pm 0.10 \pm 0.04 \text{ nb}$$

$$m = 1674 \pm 12 \pm 6 \text{ MeV}/c^2$$

$$\Gamma_0 = 165 \pm 38 \pm 70 \text{ MeV}$$

$$\varphi = 3.01 \pm 0.38 - \text{fixed to } \pi$$

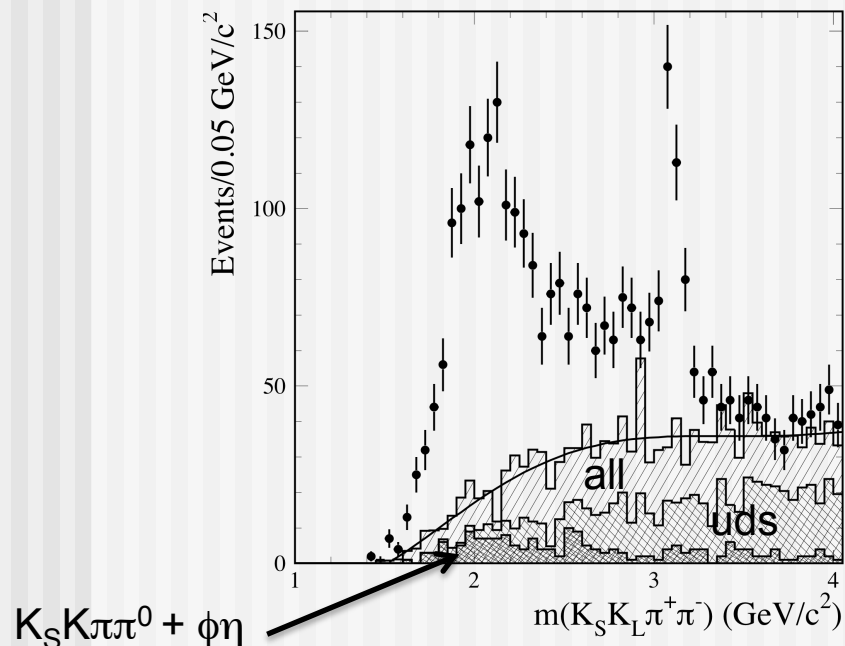
$$\sigma_{bkg} = 0.36 \pm 0.18 \text{ nb}$$

$$\Gamma_{ee} \cdot B_{K_S K_L} = 14.3 \pm 2.4 \pm 1.5 \pm 6.0 \text{ eV}$$

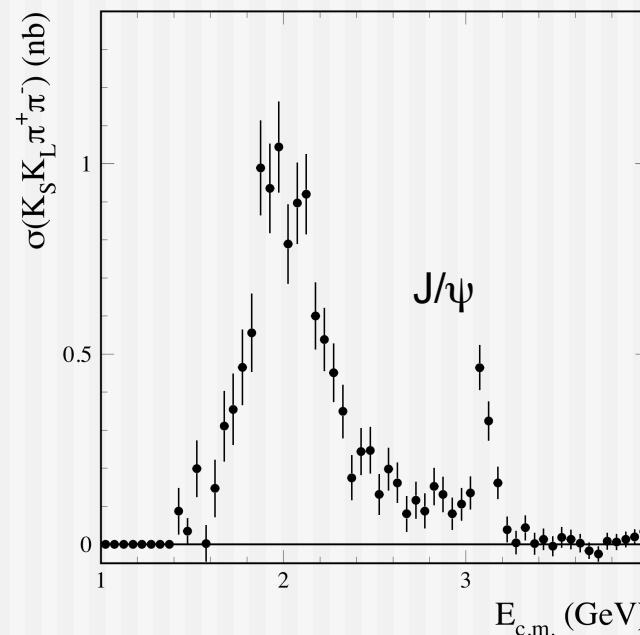
Simultaneous  $K_S K_L$  and  $K^+ K^-$  (and  $\pi\pi$ ) fit is needed to separate  $l=0,1$  states and  $\omega(1420, 1650)$ ,  $\rho(1450, 1700)$  contribution

# $K_S K_L \pi^+ \pi^- \gamma$ event selection

- Select (best)  $K_S$
- Select ISR photon with  $E > 3$  GeV
- Two additional tracks from IP (no kaon ID)
- Cycle over remaining clusters with  $E > 0.2$  GeV –  $K_L$  candidates
- Best  $\chi^2$  for 3C fit ( $K_L$  momentum float)
- $\chi^2 > 100$  and  $|\text{Im}_{\gamma\gamma L} - 0.135| > 0.03$  for the  $K_S K \pi \pi^0 \gamma$  hypothesis

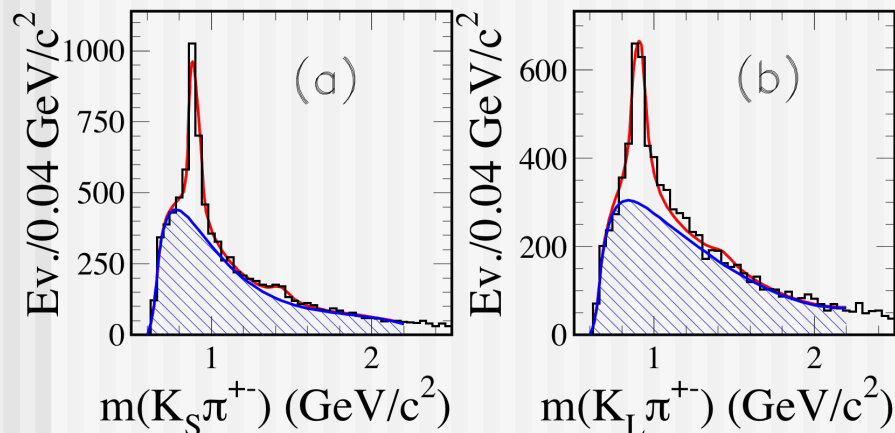


1580 events after  
background subtraction



No other  
measurements exist

# Some mass distributions in $K_S K_L \pi^+ \pi^- \gamma$

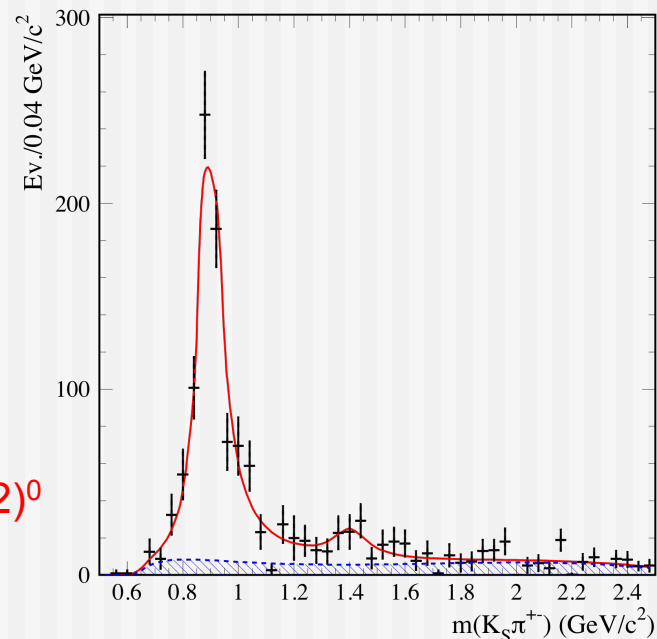
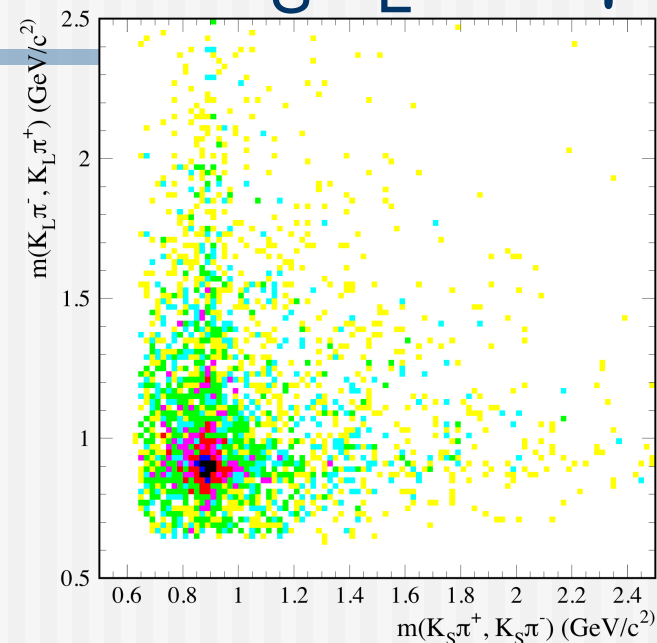


Very clear  $K^*(892)^\pm$  signals with  
 $1322 \pm 70$  for  $K^{*\pm}(K_S \pi)$  and  $1362 \pm 78$  for  $K^{*\pm}(K_L \pi)$   
 Plus  $183 \pm 48$  events for  $K_2(1430)^\pm$

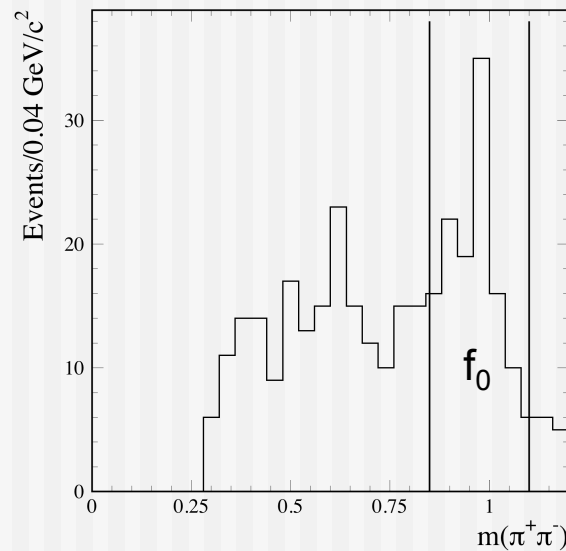
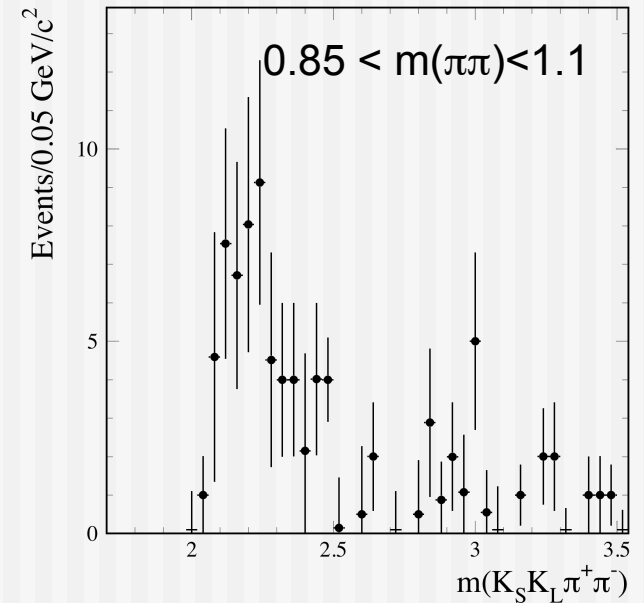
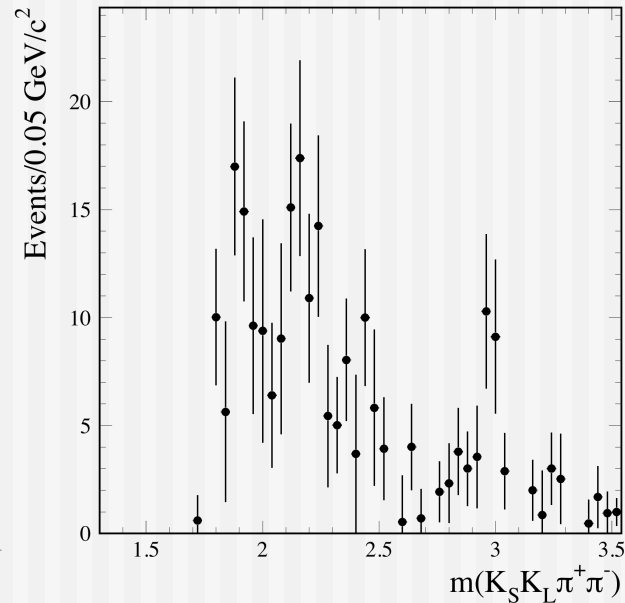
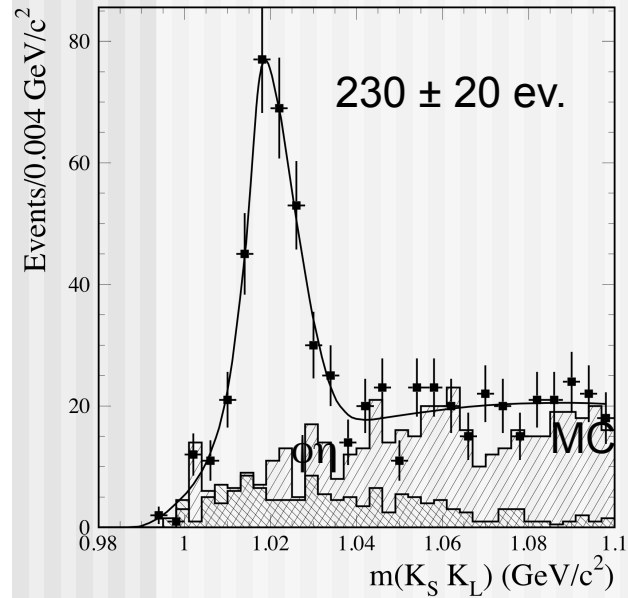
How large is  $K^*(892)^+ K^*(892)^-$  ?  
 Fit slice in  $m(K_L \pi^{+-})$  for number of  $K_S \pi^{+-}$

Very clear signal with  $913 \pm 37$  events (70%)  
 of  $K^*(892)^+ K^*(892)^-$  correlated production!  
 And  $90 \pm 16$  for  $K^*(892)^+ K_2^*(1430)^+$  .

We have negligible contribution from  $K^*(892)^0 \bar{K}^*(892)^0$   
 from our  $K^+ K^- \pi^+ \pi^-$  analysis!



# $\phi(1020)\pi^+\pi^-$ contribution in $K_S K_L \pi^+\pi^- \gamma$



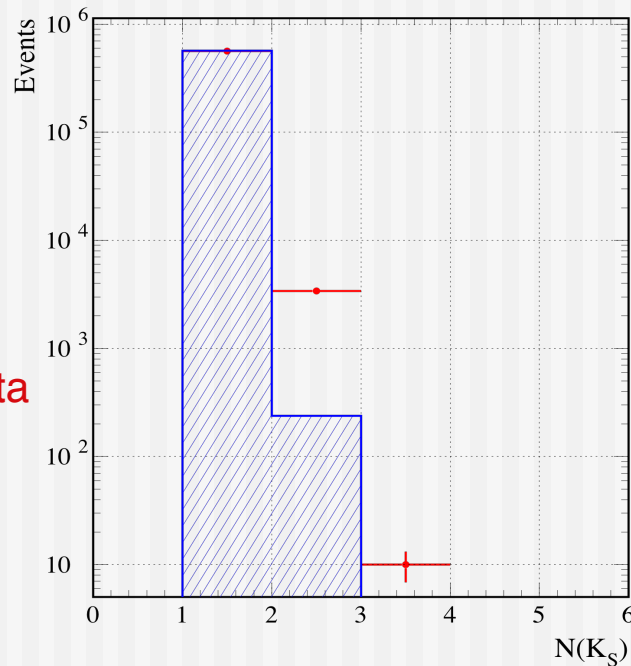
$\phi\pi^+\pi^-$  ( $\phi f_0(980)$ )  
seen as expected in  
agreement with our  
 $K^+K^-\pi^+\pi^-$  study



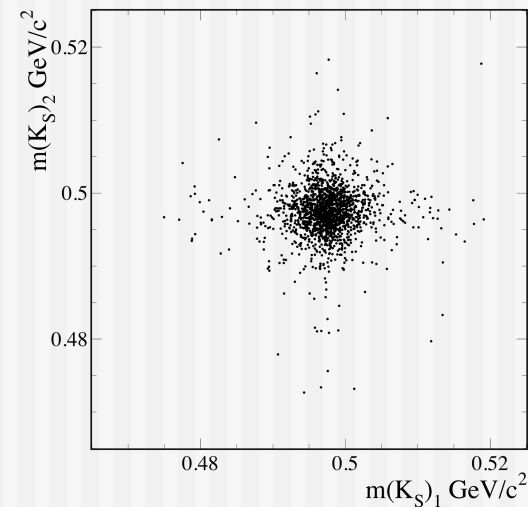
# $K_S K_S \pi^+ \pi^- (K^+ K^-) \gamma$ event selection

- Select 2 (best)  $K_S$
- Select ISR photon with  $E > 3$  GeV
- Two additional tracks from IP with pion or kaon ID
- Best  $\chi^2$  for 4C fit assuming  $K_S K_S \pi^+ \pi^- (K^+ K^-) \gamma$  hypotheses

Simulation  
of  $\phi \gamma \rightarrow K_S K_L \gamma$   
compare to data

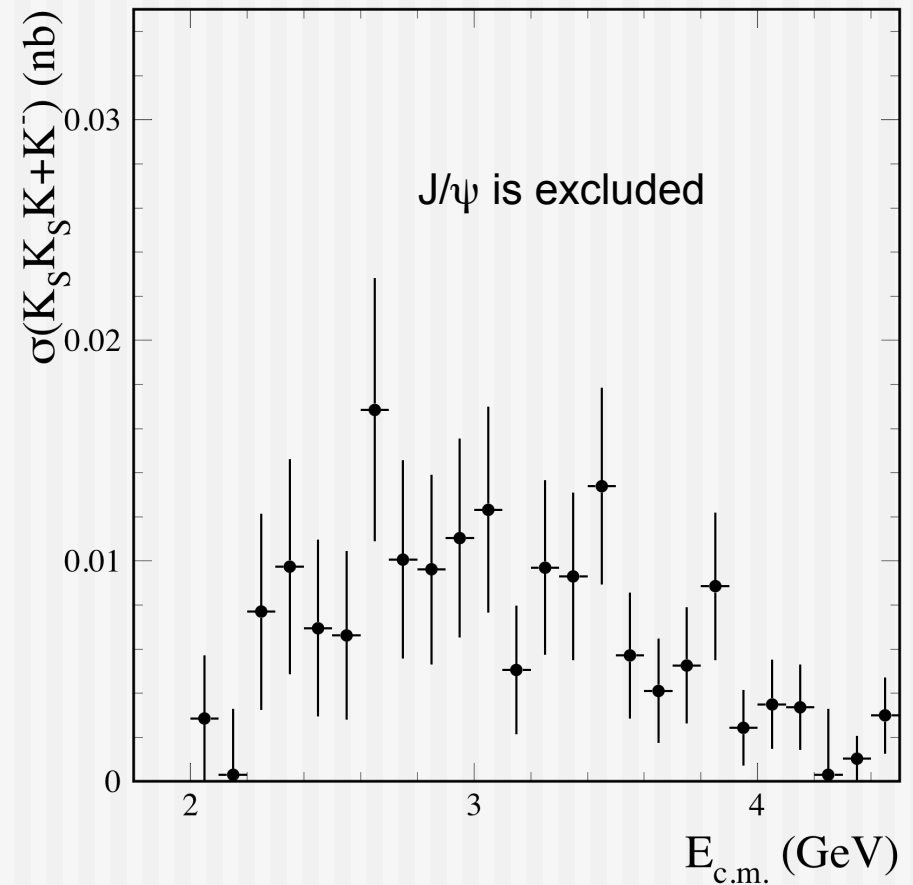
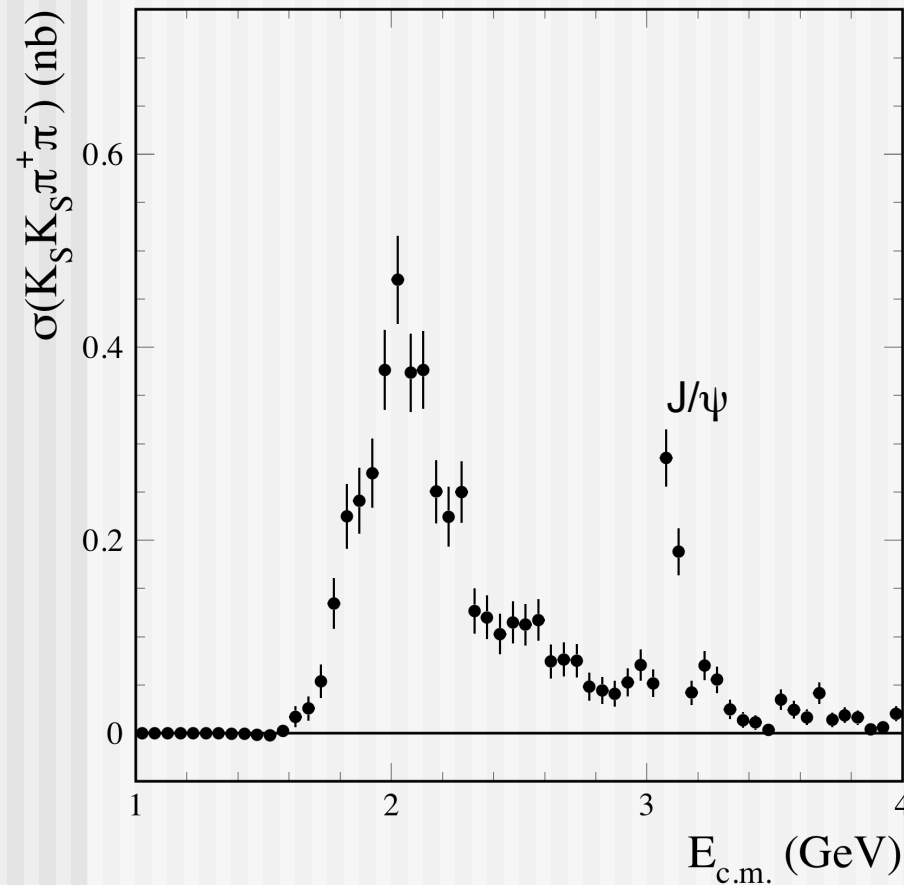


About 3000 ISR events with 2 good  $K_S$



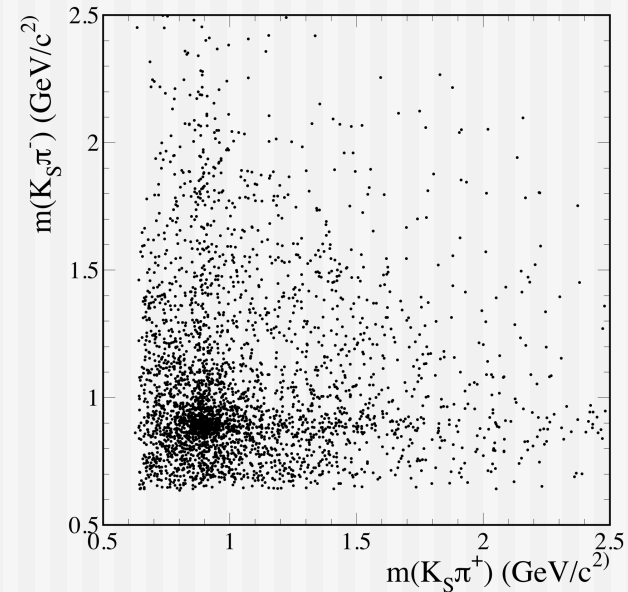
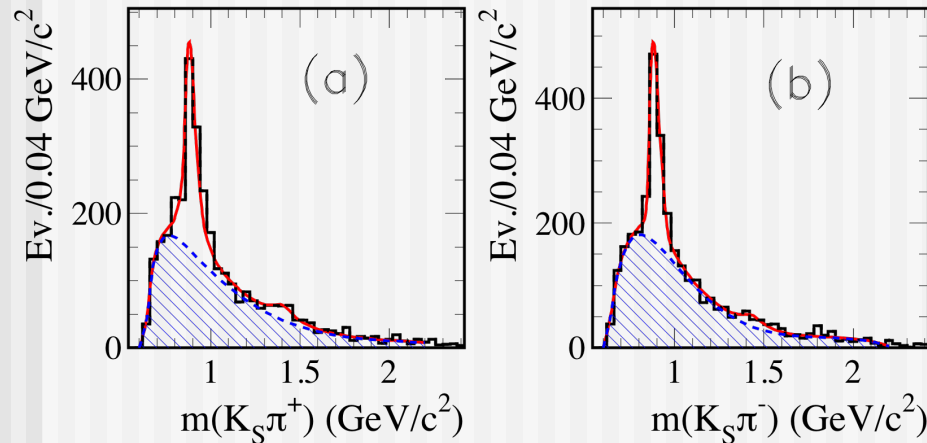
Six tracks with ISR photon – very low background!

# $e^+e^- \rightarrow K_S K_S \pi^+ \pi^- (K^+ K^-)$ cross sections



No other measurements exist

# Some mass distributions in $K_S K_S \pi^+ \pi^-$

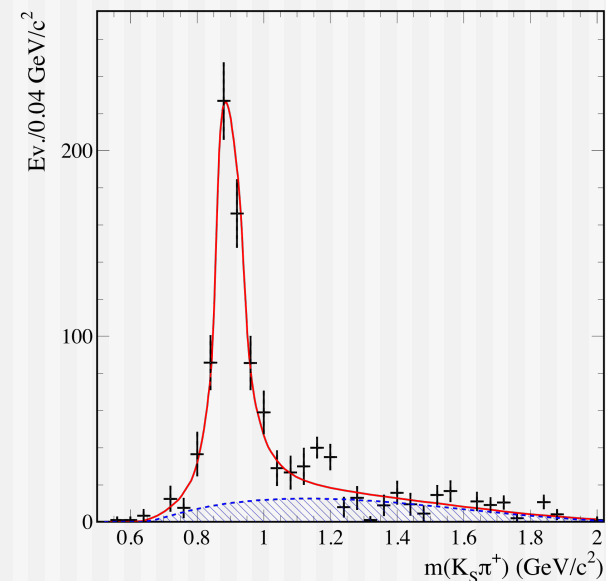


Very clear  $K^*(892)^\pm$  signals with  
 $829 \pm 49$  for  $K^{*+}(K_S \pi^+)$  and  $856 \pm 50$  for  $K^{*-}(K_S \pi^-)$   
 Plus  $116 \pm 40$  ( $70 \pm 34$ ) events for  $K_2(1430)^\pm$

How large is  $K^*(892)^+ K^*(892)^-$  ?  
 Fit slice in  $m(K_S \pi^-)$  for number of  $K_S \pi^+$

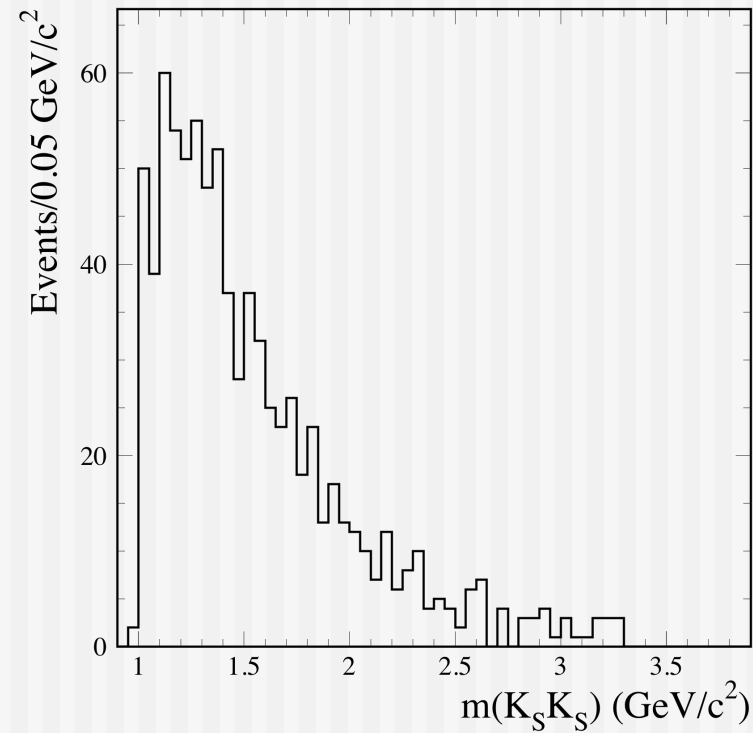
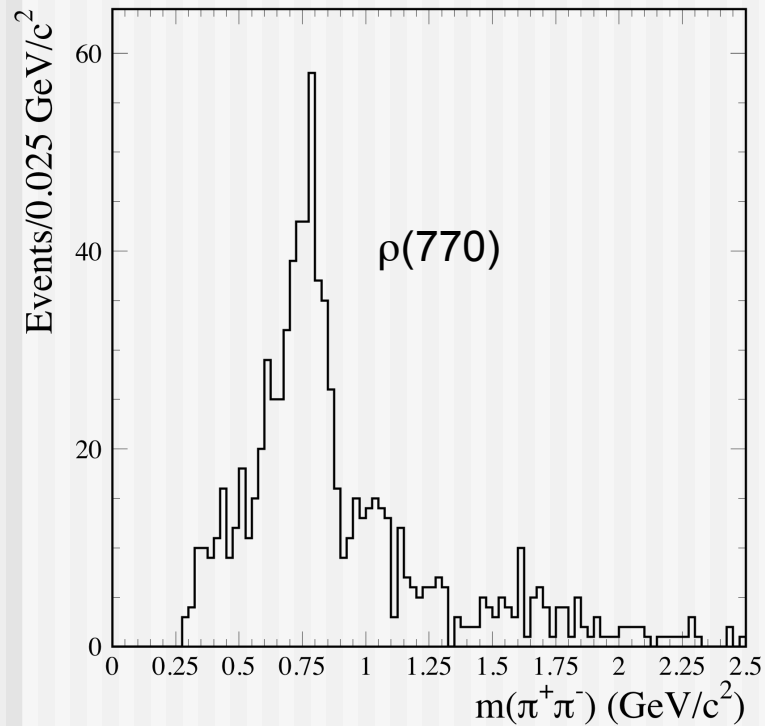
Very clear signal with  $742 \pm 30 \pm 100$  events (50%)  
 of  $K^*(892)^+ K^*(892)^-$  correlated production!

No  $K^*(892)^+ K_2^*(1430)^{-+}$  seen.



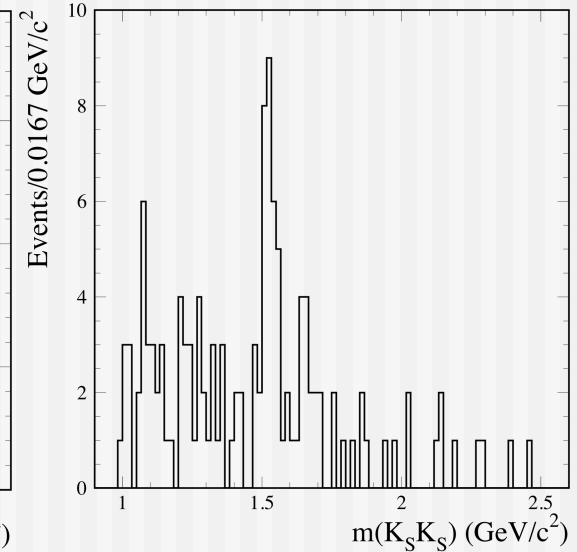
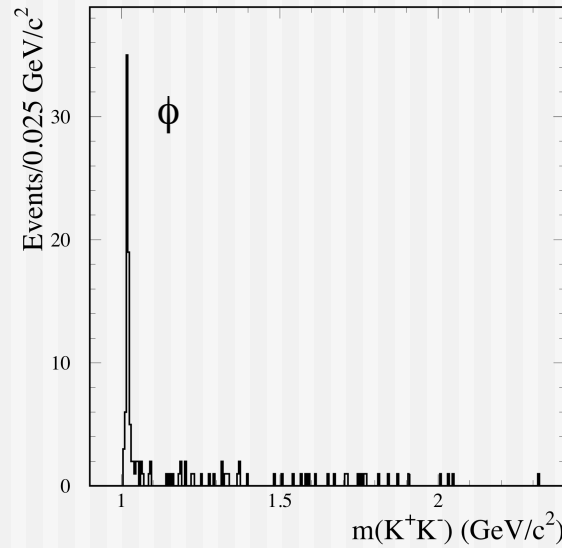
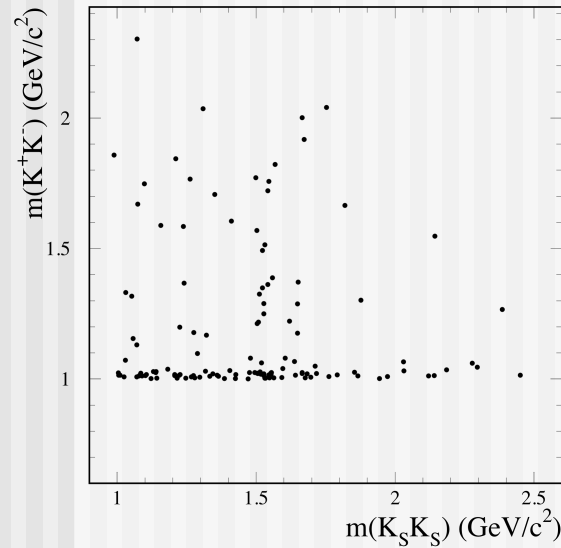
# Some mass distributions in $K_S K_S \pi^+ \pi^-$

If we exclude  $K^*(892)^+ K^*(892)^-$  by  $|m(K_S \pi) - m(K^*)| < 0.15 \text{ GeV}/c^2$  in both combinations:



Plus some number of  $K^*(892) K_S \pi$  events

# Some mass distributions in $K_S K_S K^+ K^-$



$N(K^+ K^- f_2') = 29 \pm 7$  events

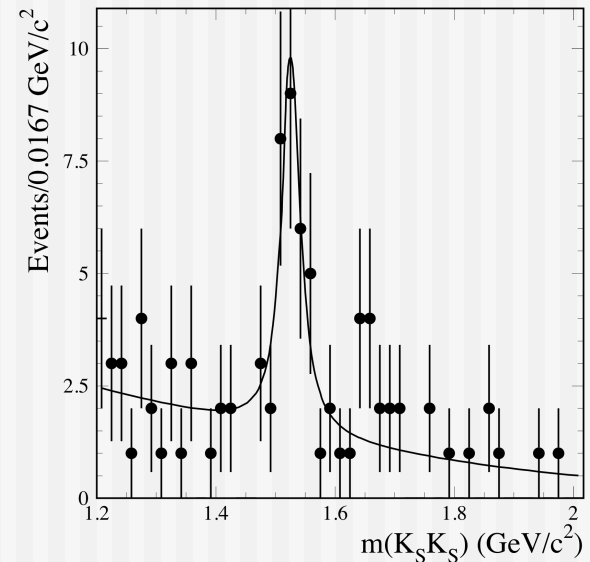
$m(K_S K_S) = 1.526 \pm 0.007$  GeV/c<sup>2</sup>

$\Gamma = 0.037 \pm 0.013$  GeV

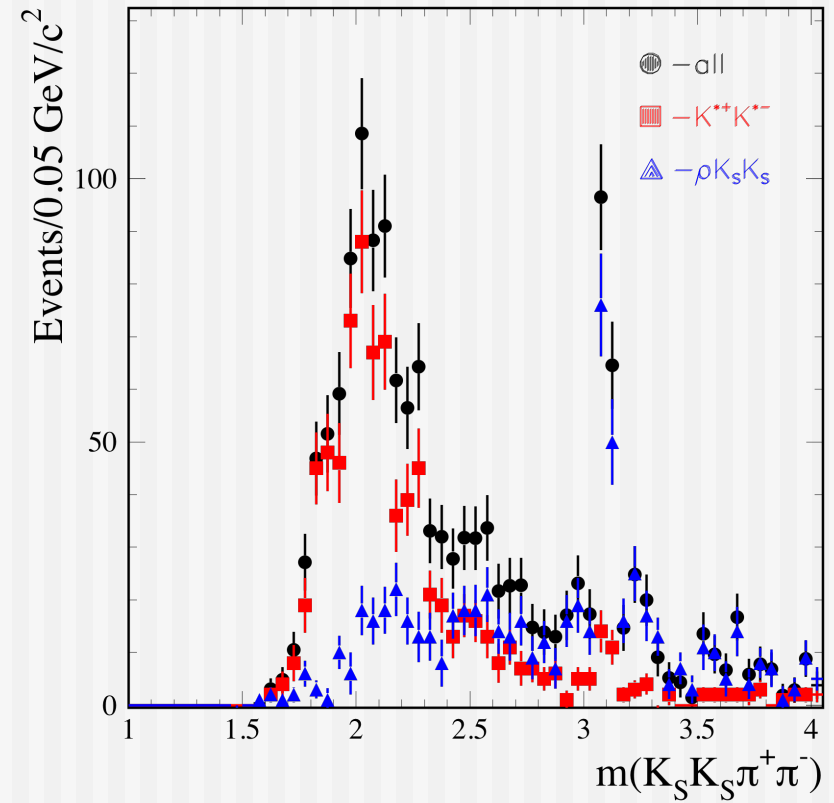
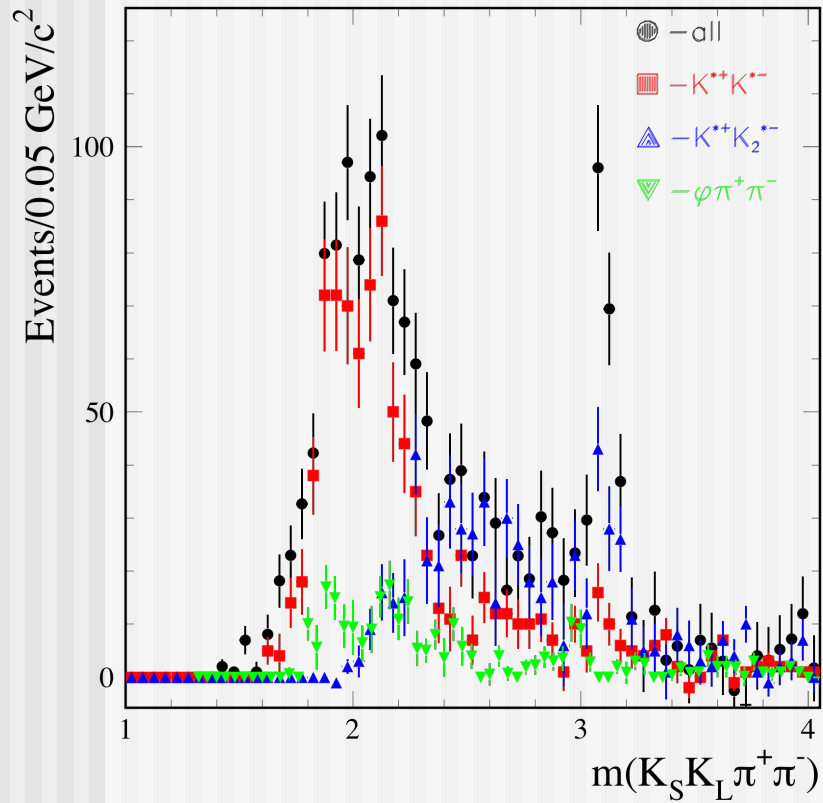
PDG:

$m(f_2') = 1.525 \pm 0.005$  GeV/c<sup>2</sup>

$\Gamma = 0.073 \pm 0.006$  GeV

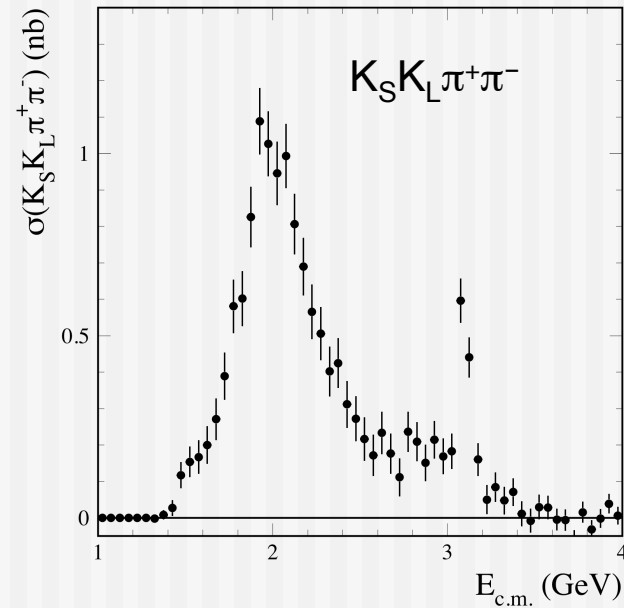
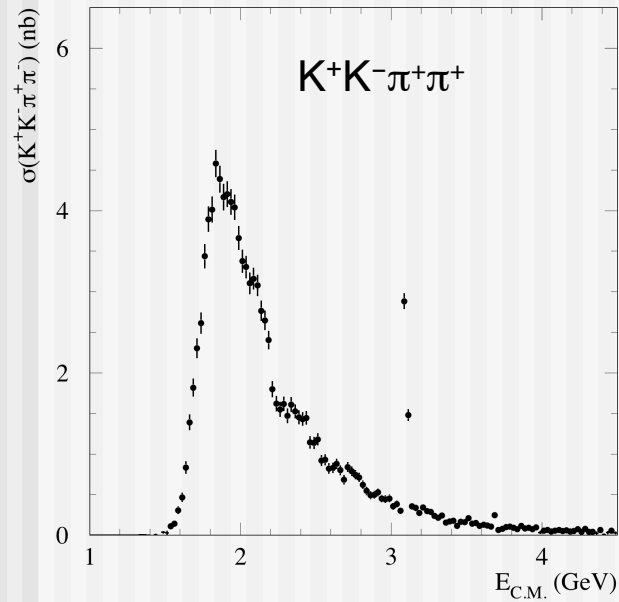


# $K_S K_L \pi^+ \pi^-$ , $K_S K_S \pi^+ \pi^-$ signal decomposition

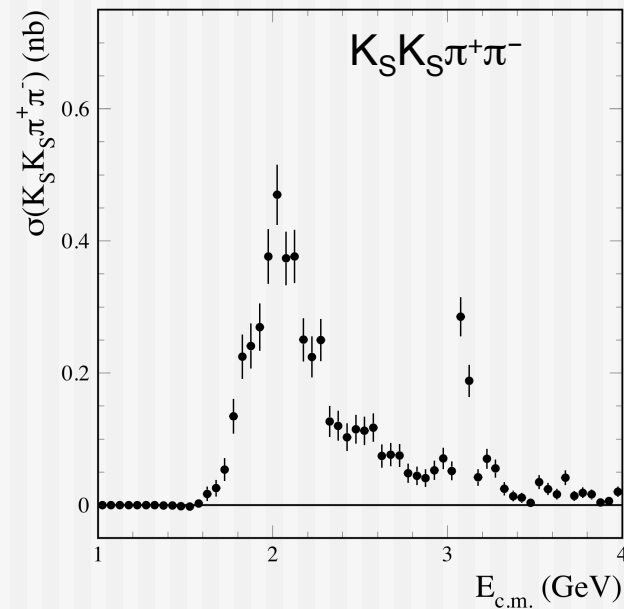
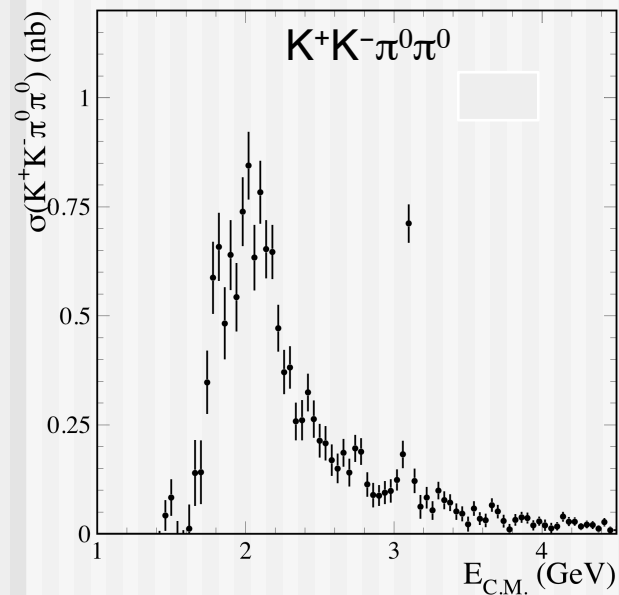




# The cross section comparison – BaBar data



Naively expect:  
 $N(K^+K^-\pi^+\pi^-) =$   
 $2 N(K^+K^-\pi^0\pi^0)$



# Iso-spin relations for $K^+K^-\pi^+\pi^+$ vs. $K^+K^-\pi^0\pi^0$ vs. $K_S K_L \pi^+\pi^-$ vs. $K_S K_S \pi^+\pi^-$

Only  $K^*(892)^+K^*(892)^-$  contribution can be compared using iso-spin relations, and we expect:

$$\begin{aligned} N(K^+K^-\pi^0\pi^0) &= \frac{1}{4} N(K^0\bar{K}^0 \pi^+\pi^-) \\ N(K_S K_L \pi^+\pi^-) &= \frac{1}{2} N(K^0\bar{K}^0 \pi^+\pi^-) \\ N(K_S K_S \pi^+\pi^-) &= N(K_L K_L \pi^+\pi^-) = \frac{1}{4} N(K^0\bar{K}^0 \pi^+\pi^-) \end{aligned}$$

We detect correlated pairs:

$$N(K^+K^-\pi^0\pi^0) = 1750 \pm 60 \quad \text{eff} = 8\%$$

$$N(K_S K_L \pi^+\pi^-) = 2098 \pm 209 \quad \text{eff} = 5\%$$

$$N(K_S K_S \pi^+\pi^-) = 742 \pm 104 \quad \text{eff} = 4.5\%$$

Should be equal numbers after efficiency normalized to 5% and iso-spin correction:

$$2188 \pm 76 \quad \sim \quad 2098 \pm 209 \quad \sim \quad 1648 \pm 232$$

Some tension ( $\sim 2$  sigma)

30%

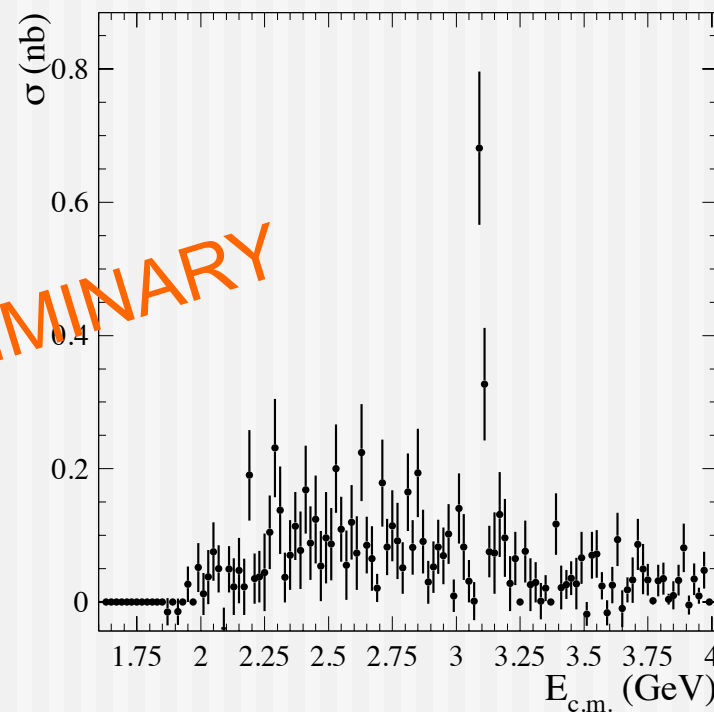
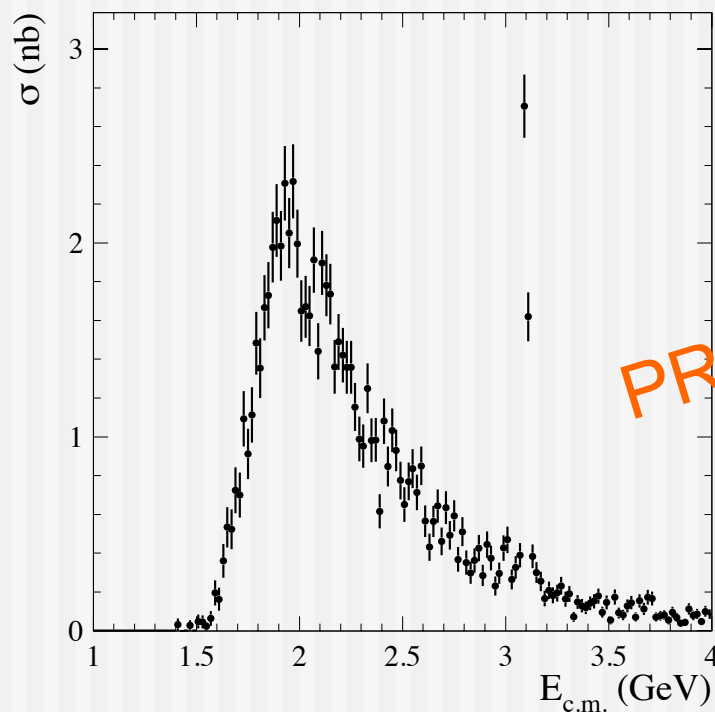
63%

50%

of all events – how the rest are added to the g-2 calculation?

# $K_S K^+ \pi^- \pi^0 (\eta) \gamma$ event selection

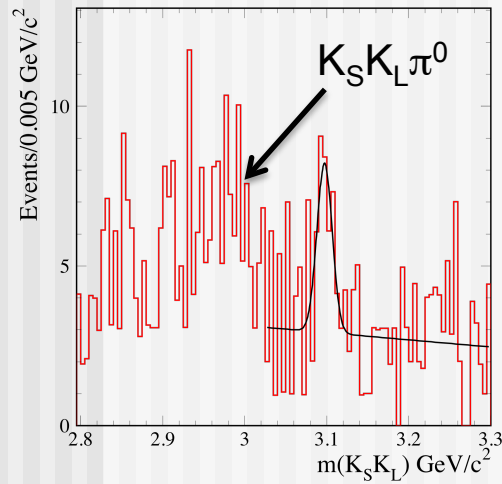
- Select 1 (best)  $K_S$
- Select ISR photon with  $E > 3$  GeV
- Two additional tracks from IP with pion or kaon ID
- Loop over remaining photons in  $\pi^0$  or  $\eta$  mass windows
- Best  $\chi^2$  for 5C fit assuming  $K_S K^+ \pi^- \pi^0 (\eta) \gamma$  hypotheses



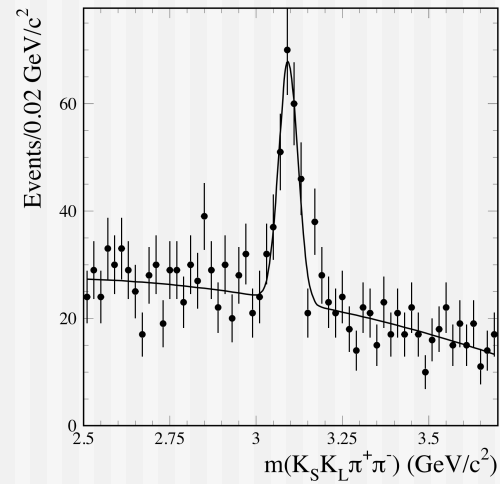
No other measurements exist

# J/ψ region

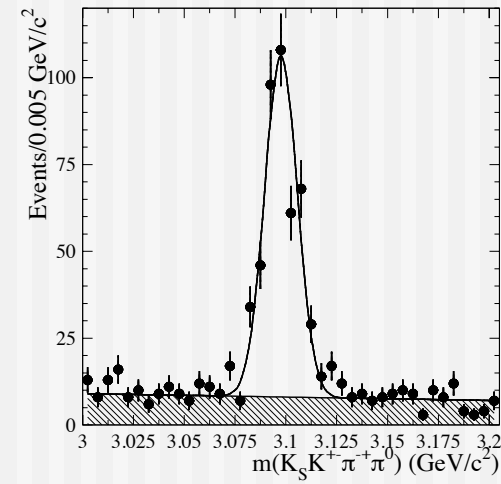
We observe a J/ψ signal in all studied channels



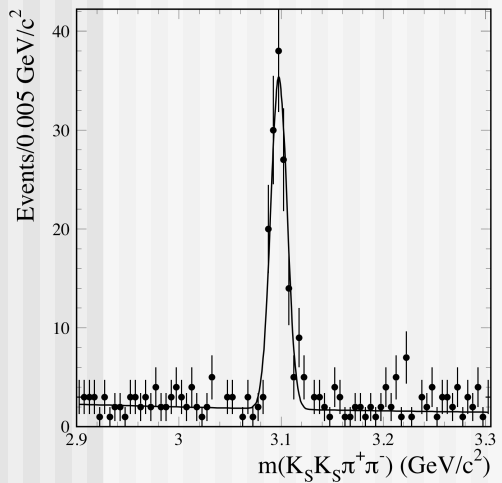
$$N = 24.6 \pm 7.5$$



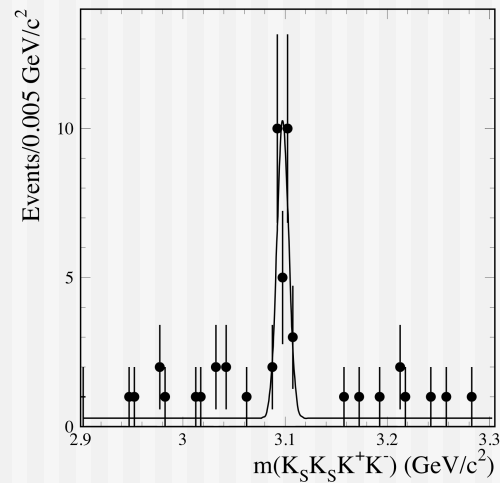
$$N = 154 \pm 19$$



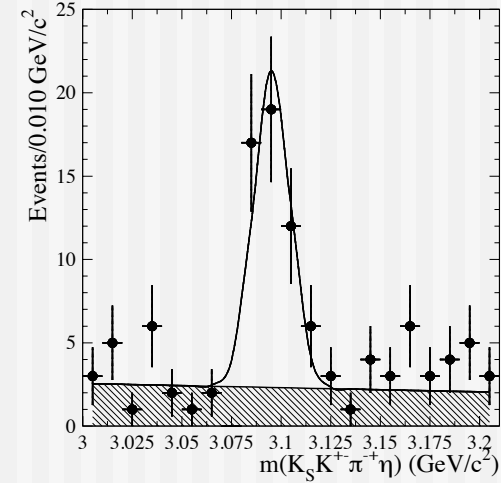
$$N = 393 \pm 23$$



$$N = 248 \pm 27$$



$$N = 28.5 \pm 5.1$$



$$N = 44 \pm 7$$

# J/ψ decay results

Measured Quantity	Measured value (eV)	This work $\Gamma_{ee}$ Br ( $10^{-3}$ ) $\Gamma_{ee} = 5.55 \pm 0.14$ keV	PDG 2014 Br ( $10^{-3}$ )
$\Gamma_{ee} \cdot \text{Br}(J/\psi \rightarrow K_S K_L)$	$1.13 \pm 0.34 \pm 0.11$	$0.20 \pm 0.06 \pm 0.02$	$0.146 \pm 0.026$ S=2.7
$\Gamma_{ee} \cdot \text{Br}(J/\psi \rightarrow K_S K_L \pi^+ \pi^-)$	$20.9 \pm 2.7 \pm 2.1$	$3.7 \pm 0.6 \pm 0.4$	no entry
$\Gamma_{ee} \cdot \text{Br}(J/\psi \rightarrow K_S K_S \pi^+ \pi^-)$	$9.3 \pm 0.9 \pm 0.5$	$1.68 \pm 0.16 \pm 0.08$	no entry
$\Gamma_{ee} \cdot \text{Br}(J/\psi \rightarrow K_S K_S K^+ K^-)$	$2.3 \pm 0.4 \pm 0.1$	$0.42 \pm 0.08 \pm 0.02$	no entry
$\Gamma_{ee} \cdot \text{Br}(J/\psi \rightarrow K_S K_S \phi) \cdot \text{Br}(\phi \rightarrow K^+ K^-)$	$1.6 \pm 0.4 \pm 0.1$	$0.58 \pm 0.14 \pm 0.03$	no entry
$\Gamma_{ee} \cdot \text{Br}(J/\psi \rightarrow f_2' \phi) \cdot \text{Br}(\phi \rightarrow K^+ K^-)$ $\cdot \text{B}(f_2' \rightarrow K_S K_S)$	$0.88 \pm 0.34 \pm 0.04$	$0.45 \pm 0.17 \pm 0.02$	$0.8 \pm 0.4$ S=2.7
$\Gamma_{ee} \cdot \text{Br}(J/\psi \rightarrow K_S K^+ \pi^+ \pi^- \pi^0)$	$31.7 \pm 1.9 \pm 1.8$	$5.7 \pm 0.3 \pm 0.4$	no entry
$\Gamma_{ee} \cdot \text{Br}(J/\psi \rightarrow K_S K^+ \pi^+ \eta)$	$7.3 \pm 1.4 \pm 0.4$	$1.30 \pm 0.25 \pm 0.07$	$2.2 \pm 0.4$

$$\text{B}(J/\psi \rightarrow \phi f_2') = (0.48 \pm 0.18) \cdot 10^{-3} \text{ (MarkII)}$$

$$\text{B}(J/\psi \rightarrow \phi f_2') = (1.23 \pm 0.026 \pm 0.20) \cdot 10^{-3} \text{ (DM2)}$$

We measure:

$$\mathcal{B}_{J/\psi \rightarrow f} \cdot \Gamma_{ee}^{J/\psi} = \frac{N_{J/\psi \rightarrow f} \cdot m_{J/\psi}^2}{6\pi^2 \cdot d\mathcal{L}/dE \cdot \epsilon_f(m_{J/\psi}) \cdot C}$$

# Summary

---

- BaBar continues analysis of collected data and ISR studies in particular
- Most published results for  $e^+e^- \rightarrow \text{hadrons}$  reactions have the best to date accuracy.
- Recently obtained  $e^+e^- \rightarrow K_S K_L \pi^+ \pi^-$ ,  $K_S K_S \pi^+ \pi^-$ ,  $K_S K_S K^+ K^-$ ,  $K_S K^- \pi^+ \pi^0(\eta)$  cross sections were never studied before. Intermediate states study is performed.
- Using these cross sections we can reduce uncertainty in the muon g-2 calculation.
- $J/\psi$  decays to above modes have been measured for the first time.
- Results for  $K_S K_L \pi^0(\pi^0)$  final state should come out soon – it will completely close iso-spin relations problem in the g-2 calculation for the  $KK\pi\pi$  modes.



# SM prediction for muon g-2

ArXiv:1010.4180, arXiv:1105.3149

$$a_{\mu}^{\text{experimental}} = (g-2)/2$$

$$11\,659\,208.9 \pm 6.3 \times 10^{-10} \text{ world average}$$

$$a_{\mu}^{\text{theory}} = a_{\mu}^{\text{QED}} + a_{\mu}^{\text{EW}} + a_{\mu}^{\text{hadron}}$$

QED contribution	11 658 471.808 ± 0.015	Kinoshita & Nio, Aoyama et al
EW contribution	15.4 ± 0.2	Czarnecki et al
NLO hadronic	-9.8 ± 0.1	HLMNT11

## Hadronic contributions

$$\text{LO hadronic} \quad 694.1 \pm 4.3 \times 10^{-10} \text{ HLMNT 11}$$

main channels contribution to precision at  $\sqrt{s} < 1.8 \text{ GeV}$

$\pi^+\pi^-$	505.65 ± 3.09	
$\pi^+\pi^-2\pi^0$	18.62 ± 1.15	
$\pi^+\pi^-\pi^0$	47.38 ± 0.99	(mostly from omega region)
$2\pi^+2\pi^-$	13.64 ± 0.36	(BaBar)
$K^+K^-$	22.95 ± 0.26	(BaBar)

from Isospin relations  $5.98 \pm 0.42$  for not measured  $KK\pi, KK2\pi, 2\pi4\pi^0, 2\pi3\pi^0$

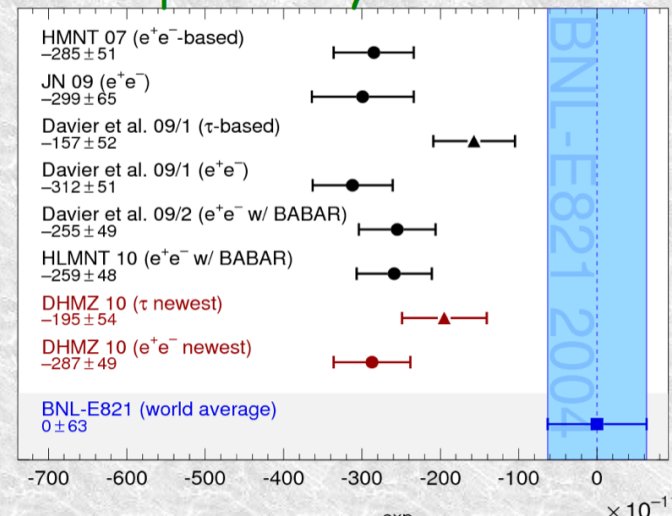
(or  $12.46 \pm 0.76$  for  $\sqrt{s} < 2 \text{ GeV}$ ) (1.5-3σ of total error - crucial in case of isospin violation)

$$\text{Rqcd}_{[2-11.09\text{GeV}]} \quad 41.19 \pm 0.82$$

$$\text{Light-by-light} \quad 10.5 \pm 2.6 \text{ Prades, de Rafael & Vainshtein}$$

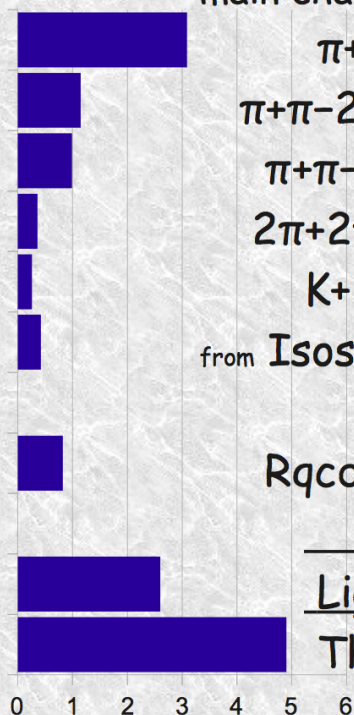
$$\text{Theory TOTAL} \quad \pm 4.9$$

$$\Delta \text{Exp} - \text{Theory} \sim 3.3-3.6\sigma$$

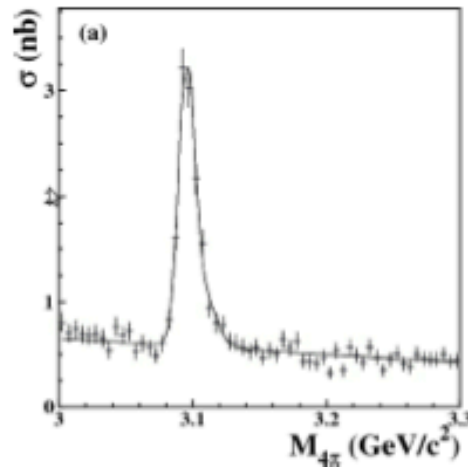


$a_{\mu} - a_{\mu}^{\text{exp}}$   
New g-2 experiments  
at FNAL and J-PARC  
have plans to reduce  
precision to  $1.5 \times 10^{-10}$

From direct integration  
Without model constraints  
 $\delta a_{\mu}$



need more theory, probably with help of  
experimental Transition FormFactors

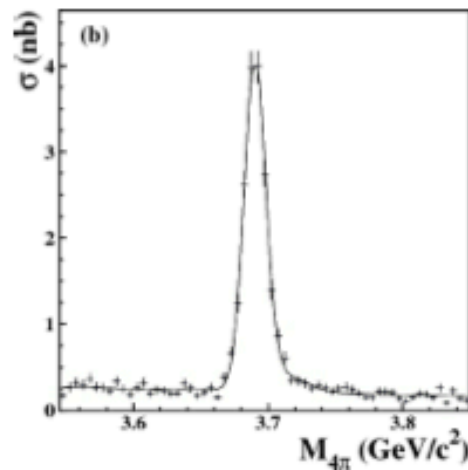


$$\mathcal{B}_{J/\psi \rightarrow 2(\pi^+ \pi^-)} \cdot \sigma_{int}^{J/\psi} = \frac{N(J/\psi \rightarrow 2(\pi^+ \pi^-))}{d\mathcal{L}/dE \cdot \epsilon_{MC}} = (48.9 \pm 2.1_{stat} \pm 1.0_{syst}) \text{ MeV}/c^2 \text{ nb}$$

$$\mathcal{B}_{J/\psi \rightarrow 2(\pi^+ \pi^-)} = (3.67 \pm 0.16_{stat} \pm 0.08_{syst} \pm 0.09_{ext}) \cdot 10^{-3}$$

$$\mathcal{B}_{J/\psi \rightarrow 2(\pi^+ \pi^-)}^{PDG} = (3.55 \pm 0.23) \cdot 10^{-3}$$

→ agrees with PDG, higher in precision



$$\begin{aligned} \mathcal{B}_{\psi(2S) \rightarrow J/\psi \pi^+ \pi^-} \cdot \mathcal{B}_{J/\psi \rightarrow \mu^+ \mu^-} \cdot \sigma_{int}^{\psi(2S)} &= \frac{N(\psi(2S) \rightarrow \pi^+ \pi^- \mu^+ \mu^-)}{d\mathcal{L}/dE \cdot \epsilon_{MC}} \\ &= (84.7 \pm 2.2_{stat} \pm 1.8_{syst}) \text{ MeV}/c^2 \text{ nb} \end{aligned}$$

$$\mathcal{B}_{\psi(2S) \rightarrow J/\psi \pi^+ \pi^-} = 0.354 \pm 0.009_{stat} \pm 0.007_{syst} \pm 0.007_{ext}$$

$$\mathcal{B}_{\psi(2S) \rightarrow J/\psi \pi^+ \pi^-}^{PDG} = 0.336 \pm 0.005$$

$$\mathcal{B}_{\psi(2S) \rightarrow J/\psi \pi^+ \pi^-}^{CLEO} = 0.3504 \pm 0.009_{stat} \pm 0.0007_{syst} \pm 0.0077_{ext}$$

→ agrees with recent CLEO result (PRD 78, 011102 (2008))



# $J/\psi$ region for $K^+K^-\pi^+\pi^-$ , $K^+K^-\pi^0\pi^0$ , $K^+K^-K^+K^-$

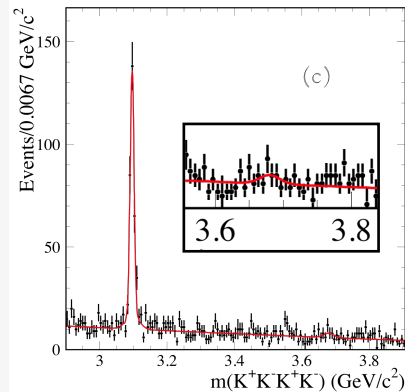
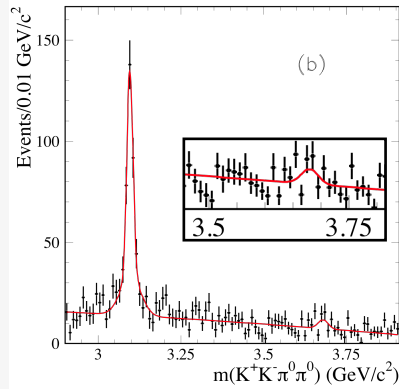
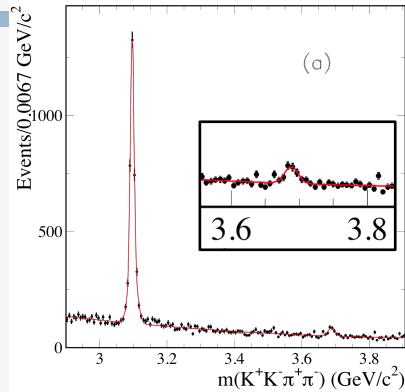


TABLE XIII: Summary of the  $J/\psi$  and  $\psi(2S)$  branching fraction values obtained in this analysis.

Measured Quantity	Measured Value (eV)	$J/\psi$ or $\psi(2S)$ Branching Fraction ( $10^{-3}$ ) This work	PDG2010
$\Gamma_{ee}^{J/\psi} \cdot \mathcal{B}_{J/\psi \rightarrow K^+K^-\pi^+\pi^-}$	$37.94 \pm 0.81 \pm 1.10$	$6.84 \pm 0.15 \pm 0.27$	$6.6 \pm 0.5$
$\Gamma_{ee}^{J/\psi} \cdot \mathcal{B}_{J/\psi \rightarrow K^+K^-\pi^0\pi^0}$	$11.75 \pm 0.81 \pm 0.90$	$2.12 \pm 0.15 \pm 0.18$	$2.45 \pm 0.31$
$\Gamma_{ee}^{J/\psi} \cdot \mathcal{B}_{J/\psi \rightarrow K^+K^-K^+K^-}$	$4.00 \pm 0.33 \pm 0.29$	$0.72 \pm 0.06 \pm 0.05$	$0.76 \pm 0.09$
$\Gamma_{ee}^{J/\psi} \cdot \mathcal{B}_{J/\psi \rightarrow K^{*0}\bar{K}_2^{*0} \cdot \mathcal{B}_{K^{*0} \rightarrow K^+\pi^-} \cdot \mathcal{B}_{\bar{K}_2^{*0} \rightarrow K^-\pi^+}$	$8.59 \pm 0.36 \pm 0.27$	$6.98 \pm 0.29 \pm 0.21$	$6.0 \pm 0.6$
$\Gamma_{ee}^{J/\psi} \cdot \mathcal{B}_{J/\psi \rightarrow K^{*0}\bar{K}_2^{*0} \cdot \mathcal{B}_{K^{*0} \rightarrow K^+\pi^-} \cdot \mathcal{B}_{\bar{K}_2^{*0} \rightarrow K^-\pi^+}$	$0.57 \pm 0.15 \pm 0.03$	$0.23 \pm 0.06 \pm 0.01$	$0.23 \pm 0.07$
$\Gamma_{ee}^{J/\psi} \cdot \mathcal{B}_{J/\psi \rightarrow \phi\pi^+\pi^-} \cdot \mathcal{B}_{\phi \rightarrow K^+K^-}$	$2.19 \pm 0.23 \pm 0.07$	$0.81 \pm 0.08 \pm 0.03$	$0.94 \pm 0.09$
$\Gamma_{ee}^{J/\psi} \cdot \mathcal{B}_{J/\psi \rightarrow \phi\pi^0\pi^0} \cdot \mathcal{B}_{\phi \rightarrow K^+K^-}$	$1.36 \pm 0.27 \pm 0.07$	$0.50 \pm 0.10 \pm 0.03$	$0.56 \pm 0.16$
$\Gamma_{ee}^{J/\psi} \cdot \mathcal{B}_{J/\psi \rightarrow \phi K^+K^-} \cdot \mathcal{B}_{\phi \rightarrow K^+K^-}$	$2.26 \pm 0.26 \pm 0.16$	$1.66 \pm 0.19 \pm 0.12$	$1.83 \pm 0.24^a$
$\Gamma_{ee}^{J/\psi} \cdot \mathcal{B}_{J/\psi \rightarrow \phi f_0} \cdot \mathcal{B}_{\phi \rightarrow K^+K^-} \cdot \mathcal{B}_{f_0 \rightarrow \pi^+\pi^-}$	$0.69 \pm 0.11 \pm 0.05$	$0.25 \pm 0.04 \pm 0.02$	$0.18 \pm 0.04^b$
$\Gamma_{ee}^{J/\psi} \cdot \mathcal{B}_{J/\psi \rightarrow \phi f_0} \cdot \mathcal{B}_{\phi \rightarrow K^+K^-} \cdot \mathcal{B}_{f_0 \rightarrow \pi^0\pi^0}$	$0.48 \pm 0.12 \pm 0.05$	$0.18 \pm 0.04 \pm 0.02$	$0.17 \pm 0.07^c$
$\Gamma_{ee}^{J/\psi} \cdot \mathcal{B}_{J/\psi \rightarrow \phi f_x} \cdot \mathcal{B}_{\phi \rightarrow K^+K^-} \cdot \mathcal{B}_{f_x \rightarrow \pi^+\pi^-}$	$0.74 \pm 0.12 \pm 0.05$	$0.27 \pm 0.04 \pm 0.02$	$0.72 \pm 0.13^d$
$\Gamma_{ee}^{\psi(2S)} \cdot \mathcal{B}_{\psi(2S) \rightarrow K^+K^-\pi^+\pi^-}$	$1.92 \pm 0.30 \pm 0.06$	$0.81 \pm 0.13 \pm 0.03$	$0.75 \pm 0.09$
$\Gamma_{ee}^{\psi(2S)} \cdot \mathcal{B}_{\psi(2S) \rightarrow K^+K^-\pi^0\pi^0}$	$0.60 \pm 0.31 \pm 0.03$	$0.25 \pm 0.13 \pm 0.02$	no entry
$\Gamma_{ee}^{\psi(2S)} \cdot \mathcal{B}_{\psi(2S) \rightarrow K^+K^-K^+K^-}$	$0.22 \pm 0.10 \pm 0.02$	$0.09 \pm 0.04 \pm 0.01$	$0.060 \pm 0.014$
$\Gamma_{ee}^{\psi(2S)} \cdot \mathcal{B}_{\psi(2S) \rightarrow \phi\pi^+\pi^-} \cdot \mathcal{B}_{\phi \rightarrow K^+K^-}$	$0.27 \pm 0.09 \pm 0.02$	$0.23 \pm 0.08 \pm 0.01$	$0.117 \pm 0.029$
$\Gamma_{ee}^{\psi(2S)} \cdot \mathcal{B}_{\psi(2S) \rightarrow \phi f_0} \cdot \mathcal{B}_{\phi \rightarrow K^+K^-} \cdot \mathcal{B}_{f_0 \rightarrow \pi^+\pi^-}$	$0.17 \pm 0.06 \pm 0.02$	$0.15 \pm 0.05 \pm 0.01$	$0.068 \pm 0.024^e$

<sup>a</sup> $\mathcal{B}_{J/\psi \rightarrow \phi \bar{K} K}$  obtained as  $2 \cdot \mathcal{B}_{J/\psi \rightarrow \phi K^+ K^-}$ .

<sup>b</sup>Not corrected for the  $f_0 \rightarrow \pi^0\pi^0$  mode.

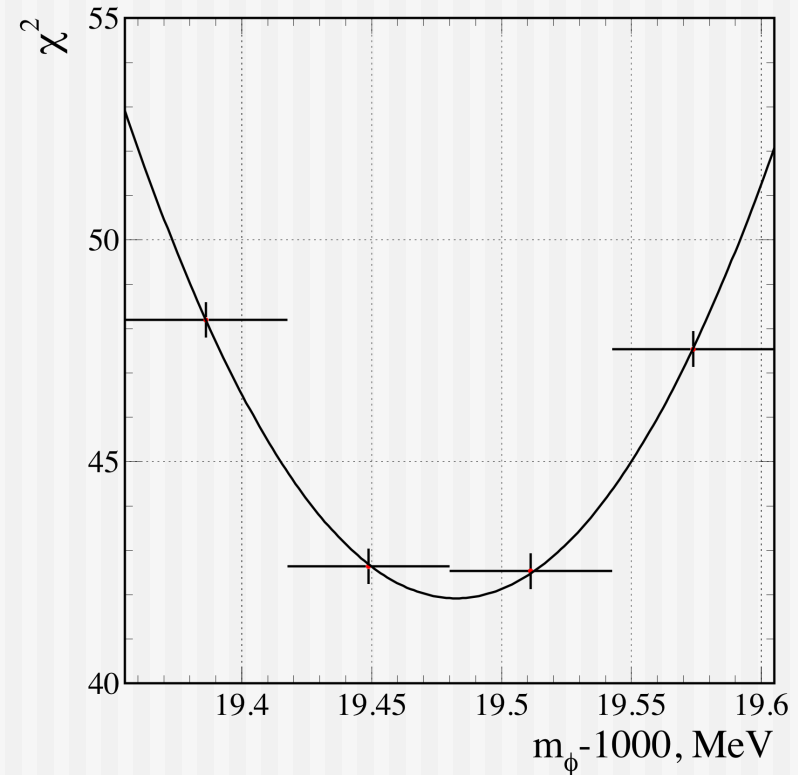
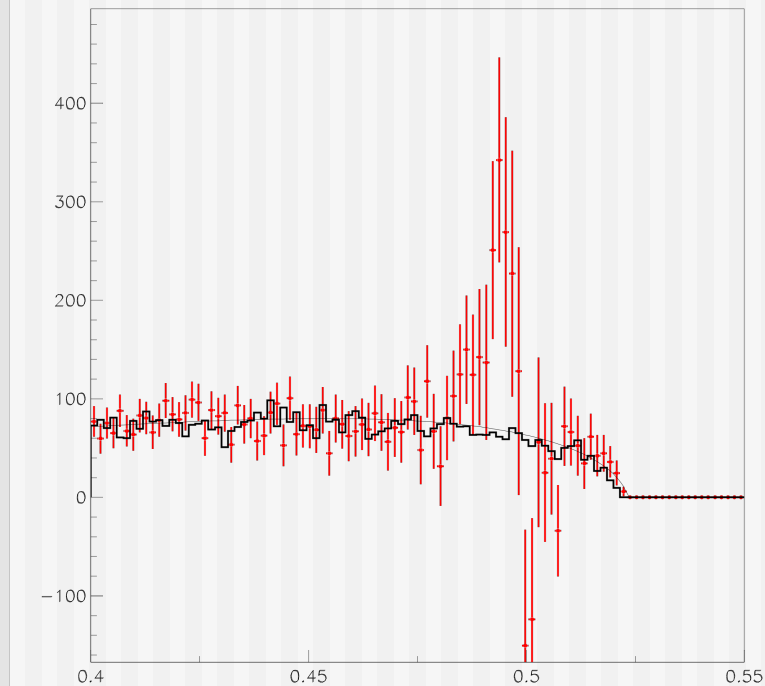
<sup>c</sup>Not corrected for the  $f_0 \rightarrow \pi^+\pi^-$  mode.

<sup>d</sup>We compare our  $\phi f_x$ ,  $f_x \rightarrow \pi^+\pi^-$  mode with  $\phi f_2(1270)$ .

<sup>e</sup> $\mathcal{B}_{\psi(2S) \rightarrow \phi f_0}$ ,  $f_0 \rightarrow \pi^+\pi^-$

Small systematic errors allow BaBar to improve BF for major decay modes.

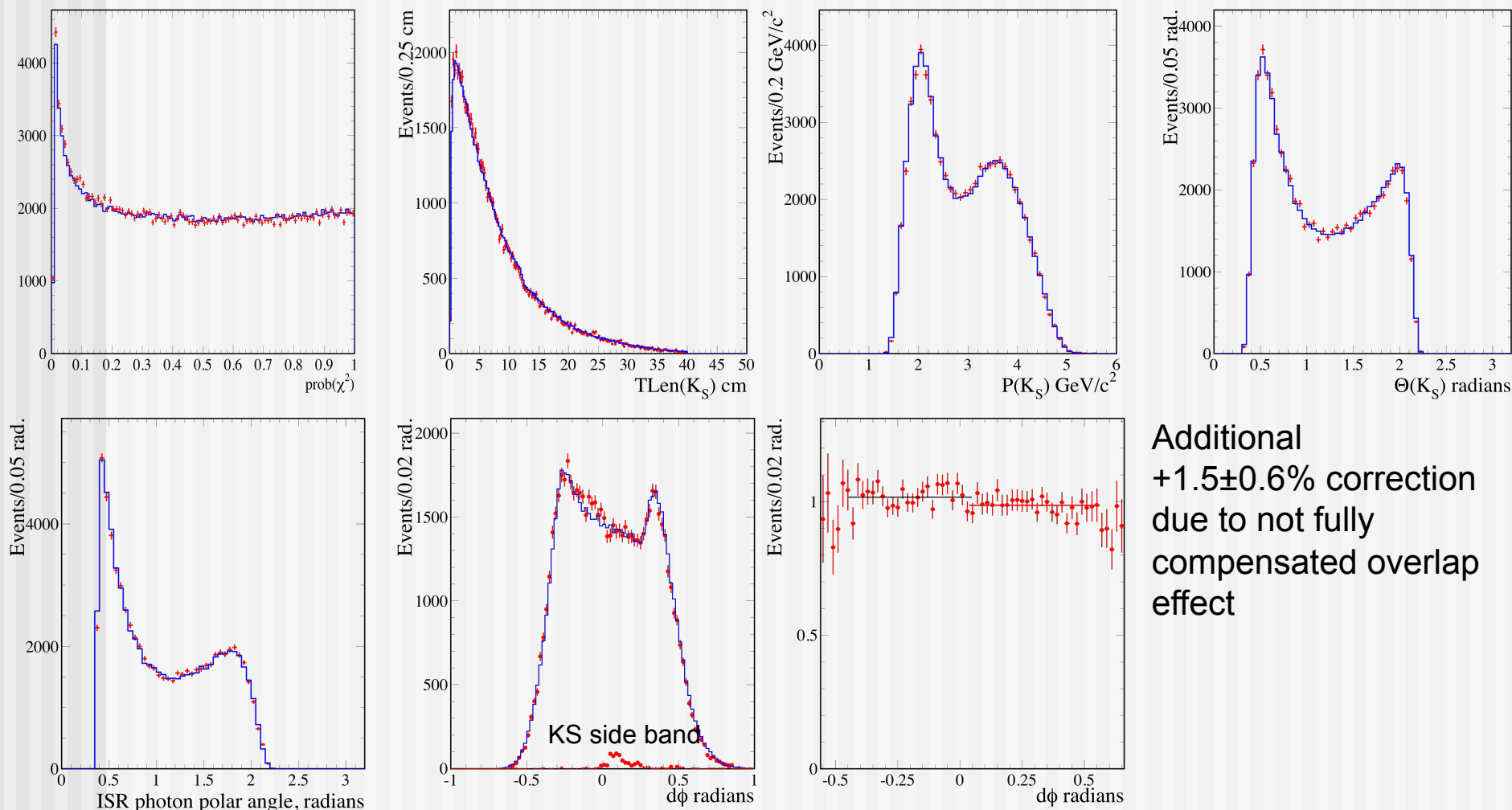
# $\phi(1020)$ mass



In MC we know all inputs and can create a “test”  $m(K_L)$  distribution and compare with data. And the only free parameter is  $\phi(1020)$  mass. By varying  $\phi$  mass we calculate  $\chi^2$  value by fitting data-MC difference with “ARGUS” function. We obtain:

$m_\phi = 1019.483 \pm 0.040 \pm 0.036 \text{ MeV}/c^2$  : 24 keV –  $K^0$  mass uncertainty, 20 keV –  $K_S$  momentum, 18 keV – DCH-EMC mis-alignment.

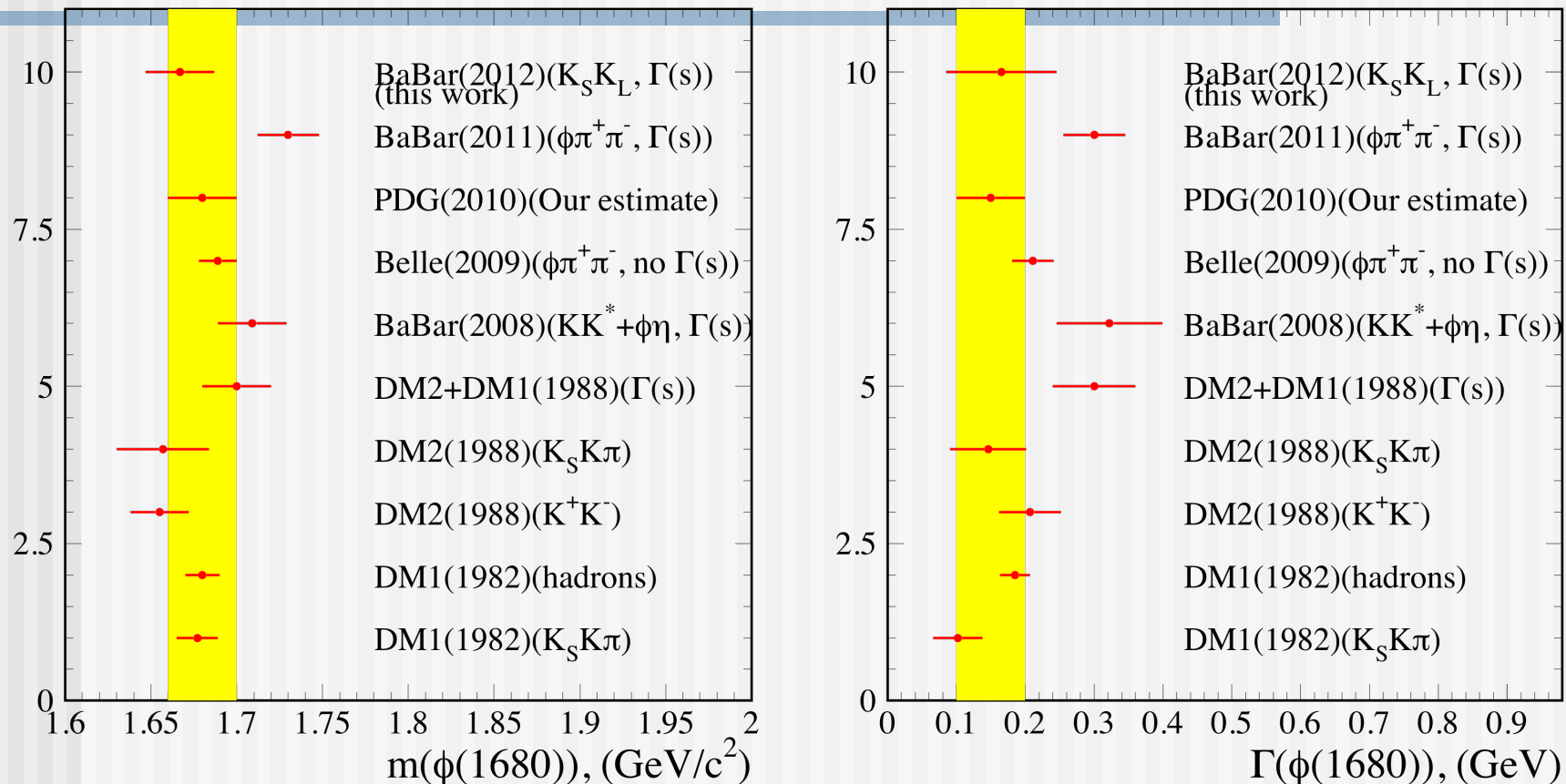
# How other distributions look like



Additional  
+1.5±0.6% correction  
due to not fully  
compensated overlap  
effect

Clean events with small systematic errors - 1% from KS, 0.5% ISR photon, 0.5% background, 0.6% from overlap effect.

# What we know about $\phi(1680)$



Energy dependence significantly increase width.

BaBar has measured  $\phi(1680)$  parameters in major decay modes:

$\phi(1680) \rightarrow K_S K \pi, KK\pi^0 (K^* K), \phi \eta, \phi \pi \pi, K_S K_L$  - still no info in PDG