Charge asymmetry of high energy bremsstrahlung in the field of a heavy atom

Peter Krachkov

Budker Institute of Nuclear Physics
Novosibirsk State University

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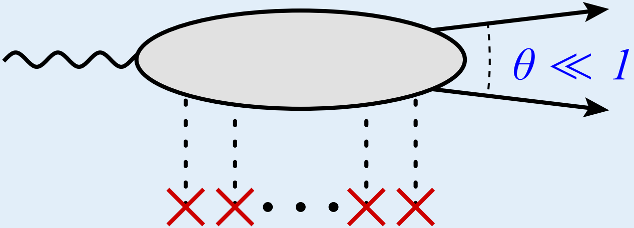
Plan

Introduction

Typical experimental conditions

- High energy $E \gg m$
- Small characteristic angle $\theta \ll 1$
- Large atomic charge number $\Longrightarrow$ $Z \gg 1$



Furry representation

diagram technique

Furry representation allows to take into account exactly the influence of the external field

$$\mathbf{r_1} \xrightarrow{\varepsilon} \mathbf{r_2} \quad \rightarrow \quad G(\mathbf{r_1},\mathbf{r_2}|\varepsilon) = \langle r_2|\frac{1}{\hat{\mathscr{P}}-m+i0}|r_1\rangle$$

$$\mathbf{p} \longrightarrow \mathbf{r} \quad \rightarrow \quad u^{in}_{\mathbf{p}}(\mathbf{r})$$

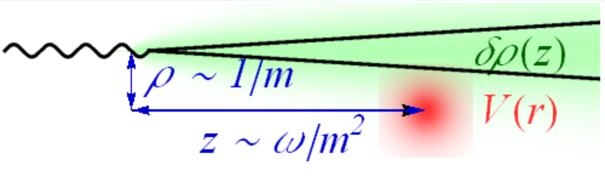
$$G(\mathbf{r_1},\mathbf{r_2}|\varepsilon) = (\hat{\mathscr{P}}+m)D(r_2,r_1|\varepsilon) \qquad \hat{\mathscr{P}} = \gamma^\mu \mathscr{P}_\mu \qquad \mathscr{P}_\mu = (\varepsilon - V(r), i\boldsymbol{\nabla})$$

$$D(r_2,r_1|\varepsilon) = \langle r_2|\frac{1}{\hat{\mathscr{P}}^2-m^2+i0}|r_1\rangle$$

Quasiclassical approximation

The characteristic parameters of the high energy processes

- $\Delta\tau \sim 1/m$ — The virtual pair life time in the comoving frame
- $\Delta t = \Delta\tau\gamma \sim E/m^2$ — The virtual pair life time in the LFR
- $\rho \sim 1/m$ — The loop transverse size in the LFR
- $z = \Delta t \sim E/m^2$ — The loop longitudinal size in the LFR
- $l \sim E\rho \sim E/m \gg 1$ — The angular momentum in the LFR
- $\theta \sim 1/l \sim m/E \ll 1$ — The characteristic angle in the LFR

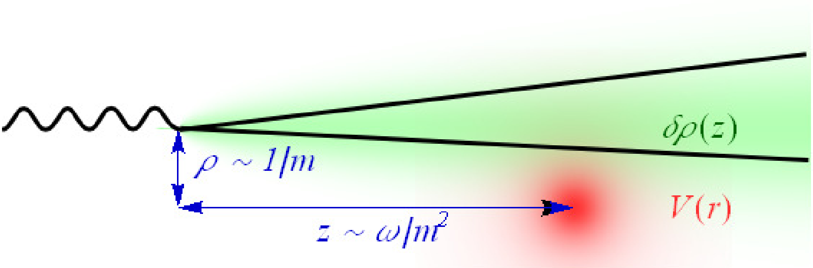


Quasiclassical approximation

The quasiclassical Green's function

$$D(\mathbf{r}_2,\mathbf{r}_1|\varepsilon)=\frac{ie^{ikr}}{4\pi^2 r}\int d^2\mathbf{q}\exp\left[iq^2-ir\int_0^1 dxV(\mathbf{R}_x)\right]\left\{1-\frac{r}{2\varepsilon}\int_0^1 dx\boldsymbol{\alpha}\boldsymbol{\nabla}_1 V(\mathbf{R}_x)\right\}$$

$$\mathbf{R}_x=\mathbf{r}_1+x\mathbf{r}+\mathbf{q}\sqrt{2x\bar{x}r/k}\Longleftarrow \text{quantum fluctuations}$$



Charge asymmetry in high-energy bremsstrahlung

Matrix element

$$M = \int d\mathbf{r}\bar{u}_q^-(\mathbf{r})\mathbf{e}^*\boldsymbol{\gamma}u_p^+(\mathbf{r})e^{-i\mathbf{kr}}$$

Cross section

$$d\sigma = \frac{\alpha\omega q\varepsilon_q}{(2\pi)^4}\,d\Omega_k\,d\Omega_q\,d\omega|M|^2\,,$$

$$d\sigma(p,q,k,\eta) = d\sigma_s(p,q,k,\eta) + d\sigma_a(p,q,k,\eta)$$

$$d\sigma_s(p,q,k,\eta) = \frac{d\sigma(p,q,k,\eta) + d\sigma(p,q,k,-\eta)}{2}$$

$$d\sigma_a(p,q,k,\eta) = \frac{d\sigma(p,q,k,\eta) - d\sigma(p,q,k,-\eta)}{2}$$

Charge asymmetry in high-energy bremsstrahlung

Green's function

$$D(r_2, r_1|\varepsilon) = \langle r_2|\frac{1}{\mathscr{H} + i\boldsymbol{\alpha}\cdot\boldsymbol{\nabla}V(r) + i0}|r_1\rangle$$

$$= \langle r_2|\frac{1}{\mathscr{H}} - \frac{1}{\mathscr{H}}i\boldsymbol{\alpha}\cdot\boldsymbol{\nabla}V(r)\frac{1}{\mathscr{H}} + \frac{1}{\mathscr{H}}i\boldsymbol{\alpha}\cdot\boldsymbol{\nabla}V(r)\frac{1}{\mathscr{H}}i\boldsymbol{\alpha}\cdot\boldsymbol{\nabla}V(r)\frac{1}{\mathscr{H}}|r_1\rangle$$

$$\mathscr{H} = (\varepsilon - V(r))^2 - m^2 + \boldsymbol{\nabla}^2 + i0$$

$$D(r_2, r_1|\varepsilon) = d_0(r_2, r_1) + \boldsymbol{\alpha}\cdot d_1(r_2, r_1) + \boldsymbol{\Sigma}\cdot d_2(r_2, r_1)$$

$$d_0 \sim l_c d_1 \sim l_c^2 d_2 \qquad l_c \sim \varepsilon/\Delta \gg 1$$

Charge asymmetry in high-energy bremsstrahlung

Green's function

$$D(r_2, r_1|\varepsilon) = \langle r_2| \frac{1}{\mathscr{H} + i\boldsymbol{\alpha}\cdot\boldsymbol{\nabla}V(r) + i0}|r_1\rangle$$

$$= \langle r_2|\frac{1}{\mathscr{H}} - \frac{1}{\mathscr{H}}i\boldsymbol{\alpha}\cdot\boldsymbol{\nabla}V(r)\frac{1}{\mathscr{H}} + \frac{1}{\mathscr{H}}i\boldsymbol{\alpha}\cdot\boldsymbol{\nabla}V(r)\frac{1}{\mathscr{H}}i\boldsymbol{\alpha}\cdot\boldsymbol{\nabla}V(r)\frac{1}{\mathscr{H}}|r_1\rangle$$

$$\mathscr{H} = (\varepsilon - V(r))^2 - m^2 + \boldsymbol{\nabla}^2 + i0$$

$$D(r_2, r_1|\varepsilon) = d_0(r_2, r_1) + \boldsymbol{\alpha}\cdot d_1(r_2, r_1) + \boldsymbol{\Sigma}\cdot d_2(r_2, r_1)$$

$$d_0 \sim l_c d_1 \sim l_c^2 d_2 \qquad l_c \sim \varepsilon/\Delta \gg 1$$

$$d_0(r_2, r_1) = \frac{ie^{i\kappa r}}{4\pi^2 r}\int dQ \exp\left[iQ^2 - ir\int_0^1 dx V(R_x)\right]$$
$$\times\left\{1 + \frac{ir^3}{2\kappa}\int_0^1 dx\int_0^x dy(x-y)\boldsymbol{\nabla}_\perp V(R_x)\cdot\boldsymbol{\nabla}_\perp V(R_y)\right\}$$

Charge asymmetry in high-energy bremsstrahlung

Matrix element

$$M = -\delta_{\mu_p\mu_q}(\varepsilon_p\delta_{\lambda\mu_p}+\varepsilon_q\delta_{\lambda\bar{\mu}_p})[N_0(e^*_\lambda,\xi_p p_\perp-\xi_q q_\perp)+N_1(e^*_\lambda,\varepsilon_p\xi_p p_\perp-\varepsilon_q\xi_q q_\perp)]$$
$$-\frac{1}{\sqrt{2}}m\mu_p\delta_{\mu_p\bar{\mu}_q}\delta_{\lambda\mu_p}(\varepsilon_p-\varepsilon_q)[N_0(\xi_p-\xi_q)+N_1(\varepsilon_p\xi_p-\varepsilon_q\xi_q)]$$

$$N_0 = \frac{2i}{\omega m^2\Delta_\perp^2}\int d\boldsymbol{r}\exp[-i\boldsymbol{\Delta}\cdot\boldsymbol{r}-i\chi(\rho)]\,\boldsymbol{\Delta}_\perp\cdot\boldsymbol{\nabla}_\perp V(r)$$

$$N_1 = \frac{1}{\omega m^2\varepsilon_p\varepsilon_q}\int d\boldsymbol{r}\exp[-i\boldsymbol{\Delta}\cdot\boldsymbol{r}-i\chi(\rho)]\int\limits_0^\infty dx\,x\,\boldsymbol{\nabla}_\perp V(\boldsymbol{r}-x\boldsymbol{\nu})\cdot\boldsymbol{\nabla}_\perp V(r)$$

$$\chi(\rho) = \int_{-\infty}^{\infty}V(z,\boldsymbol{\rho})dz \quad \xi_p = \frac{m^2}{m^2+p_\perp^2} \quad \xi_q = \frac{m^2}{m^2+q_\perp^2} \quad \boldsymbol{\Delta} = \boldsymbol{q}+\boldsymbol{k}-\boldsymbol{p}$$

Charge asymmetry in high-energy bremsstrahlung

Square of the matrix element

$$\sum_{\lambda\,\mu_q} |M|^2 = S_0 + S_1 + S_2$$

$$S_0 = \frac{m^2|N_0|^2}{2}\left[\frac{\Delta^2}{m^2}(\varepsilon_p^2+\varepsilon_q^2)\xi_p\xi_q - 2\varepsilon_p\varepsilon_q(\xi_p-\xi_q)^2\right]$$

$$S_1 = \frac{m^2\mathrm{Re}(N_0N_1^*)}{2}\left\{\frac{\Delta^2}{m^2}(\varepsilon_p^2+\varepsilon_q^2)(\varepsilon_p+\varepsilon_q)\xi_p\xi_q\right.$$

$$\left.+\left[(\varepsilon_p^2+\varepsilon_q^2)(\varepsilon_p-\varepsilon_q)-4\varepsilon_p\varepsilon_q(\varepsilon_p\xi_p-\varepsilon_q\xi_q)\right](\xi_p-\xi_q)\right\}$$

$$S_2 = -\mu_p\mathrm{Im}(N_0N_1^*)\,\omega^2(\varepsilon_p+\varepsilon_q)\xi_p\xi_q\,[p_\perp\times q_\perp]\cdot\boldsymbol{\nu}$$

Charge asymmetry in high-energy bremsstrahlung

Coulomb potential

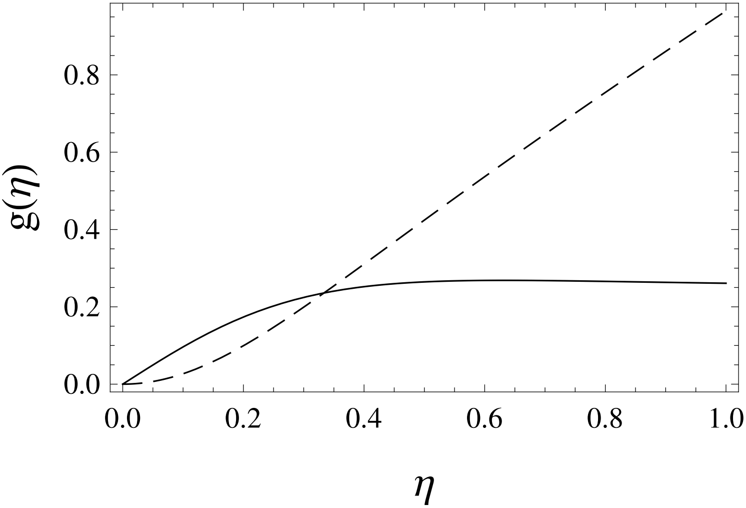
$$r_{scr} \gg r \gg R_{nucl} \qquad V(r) = \frac{Z\alpha}{r}$$

$$N_0 = \frac{8\pi\eta(L\Delta)^{2i\eta}}{\omega m^2\Delta^2}\frac{\Gamma(1-i\eta)}{\Gamma(1+i\eta)} = N_{0B}\frac{\Gamma(1-i\eta)}{\Gamma(1+i\eta)}(L\Delta)^{2i\eta}$$

$$N_1 = \frac{2\pi^2\eta^2(L\Delta)^{2i\eta}}{\omega m^2\varepsilon_p\varepsilon_q\Delta}\frac{\Gamma(1/2-i\eta)}{\Gamma(1/2+i\eta)} = N_{1B}\frac{\Gamma(1/2-i\eta)}{\Gamma(1/2+i\eta)}(L\Delta)^{2i\eta}$$

$$|N_0|^2 = |N_{0B}|^2 = \left(\frac{8\pi\eta}{\omega m^2\Delta^2}\right)^2 \qquad g(\eta) = \eta\frac{\Gamma(1-i\eta)\Gamma(1/2+i\eta)}{\Gamma(1+i\eta)\Gamma(1/2-i\eta)}$$

$$\mathrm{Re}N_0N_1^* = \frac{\pi\mathrm{Re}g(\eta)\Delta}{4\varepsilon_p\varepsilon_q}|N_0|^2 \qquad \mathrm{Im}N_0N_1^* = \frac{\pi\mathrm{Im}g(\eta)\Delta}{4\varepsilon_p\varepsilon_q}|N_0|^2$$



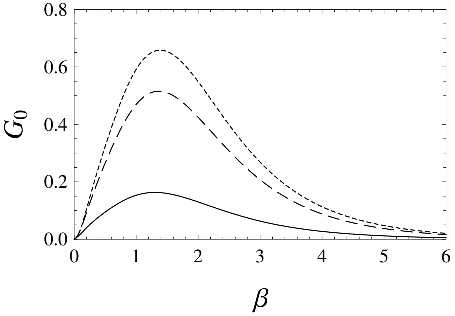
Dependence of $\mathrm{Re}\,g(\eta)$ (solid curve) and $-\mathrm{Im}\,g(\eta)$ (dashed curve) on η.

Charge asymmetry in bremsstrahlung from muons

Accounting of the finite size of the nucleus

$$R_{\text{nuc}} \sim 7\,fm \qquad \lambda_\mu = 1.87\,fm \qquad V_F(\Delta^2) = -\frac{4\pi\eta F(\Delta^2)}{\Delta^2}$$

$$\int \Delta^2(|N_0|^2 - |N_{0B}|^2)d\boldsymbol{\Delta}_\perp = \mp\frac{128\pi^3\eta^2 f(\eta)}{\omega^2 m^4} \quad f(\eta) = \mathrm{Re}\,\psi(1+i\eta) - \psi(1)$$

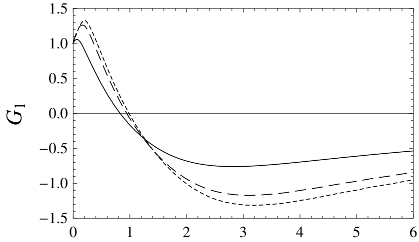


$$F(\Delta^2) = \frac{\Lambda^2}{\Delta^2+\Lambda^2}$$

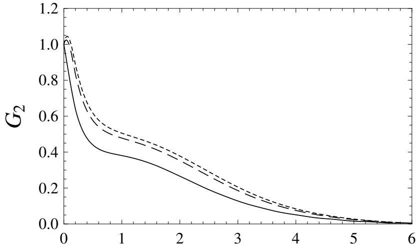
$$\beta = \frac{\Delta}{\Lambda}$$

$$G_0 = |N_0|^2/|N_{0B}|^2 - 1$$

For the potential $V_F(\Delta^2) = -\frac{4\pi\eta F(\Delta^2)}{\Delta^2}$, $F(\Delta^2) = \frac{\Lambda^2}{\Delta^2+\Lambda^2}$



Dependence of $G_1 = \Sigma_R^{-1} \mathrm{Re} N_0 N_1^* / |N_0|^2$ on $\beta = \Delta/\Lambda$ for a few values of η, $\eta = 0.34$ (Ag, solid curve), $\eta = 0.6$ (Pb, dashed curve), и $\eta = 0.67$ (U, dotted curve). $\Sigma_R = \frac{\pi \mathrm{Re}\, g(\eta)\Delta}{4\varepsilon_p \varepsilon_q}$



Dependence of $G_2 = \Sigma_I^{-1} \mathrm{Im} N_0 N_1^* / |N_0|^2$ on $\beta = \Delta/\Lambda$ for a few values of η, $\eta = 0.34$ (Ag, solid curve), $\eta = 0.6$ (Pb, dashed curve), и $\eta = 0.67$ (U, dotted curve). $\Sigma_I = \frac{\pi \mathrm{Im}\, g(\eta)\Delta}{4\varepsilon_p \varepsilon_q}$

Conclusion

- The quasiclassical Green's function and the wave function in the case of arbitraly localaized potential, taking into account, first quasiclassical correction are obtained.
- The charge asimmetry in the process of high energy bremsstrahlung in the field of a heavy atom is investigated.
- Effect of screening and finite size nucleus effect are investigated.
- It is shown that the Coulomb corrections are important.

Thank you for attention!