Branching fractions of $\psi(3770)$, $\psi(4040)$, and $\Upsilon(10580)$ decays to light (non-$D\bar{D}$, non-$D_s\bar{D}_s$, and non-$B\bar{B}$) hadrons.

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Content

- Exclusive decay modes of the $c\bar{c}$ quarkonium

  $$J/\psi(3097)\to\omega\pi^0,\omega\eta,\omega\eta',\rho\pi,\rho\eta,\rho\eta',K^*\bar{K}+c.c.,\phi\eta,\phi\eta',...$$

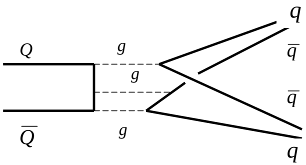
  as well as the decays of the $b\bar{b}$ quarkonium

  $$\Upsilon(1S)\to\rho\pi,\pi^+\pi^-,K\bar{K},...$$

  are crucial for revealing the dynamics of the OZI rule violation.
- Their probabilities are severely suppressed:($B\lesssim 10^{-3}-10^{-4}$).
- The three gluon mechanism: $B_{J/\psi\to 3g}=64.1\%$, $B_{\Upsilon(1S)\to 3g}=81.7\%$ (PDG).

The three gluon mechanism of the OZI rule violation

- The three gluon mechanism of the OZI rule violation OZI:



- The estimate of probability of the three gluon decay:

$$\sum \Gamma_{Q\bar{Q}\to \mathrm{lightquarks}} \sim \Gamma_{Q\bar{Q}\to 3g} = \frac{5}{18\pi}(\pi^2-9)\frac{\alpha_s^3(m^2_{Q\bar{Q}})}{\alpha^2_{e.m.}} \times \left(\frac{2/3}{q_Q}\right)^2 \Gamma_{Q\bar{Q}\to e^+e^-}.$$

- In the frame work of approach based on dispersion relation the suppression of the specific exclusive decay is understood as the compensation of allowed contributions from the loops of mesons with nonzero heavy flavors C ($D\bar{D}, D^*\bar{D} + c.c.$ etc.) or B ($B\bar{B}, B^*\bar{B} + c.c.$ etc.)
- The compensation can violated in case of quarkonia with masses just above the decay thresholds to mesons with nonzero $C \neq 0$ [$\psi(3770)$] or $B \neq 0$ [$\Upsilon(10800)$].
- The purpose of the talk: using refined PDG data to estimate the branching fractions of some exclusive non-$D\bar{D}$, non-$D_s\bar{D}_s$, and non-$B\bar{B}$ decay channels of the mesons $\psi(3770)$, $\psi(4040)$, and $\Upsilon(10800)$ and compare the estimates with existing data.

Imaginary parts of amplitudes via the unitarity relation

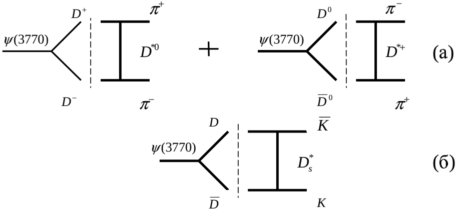
- The amplitude of the decay $\psi(3770) \to M_1M_2$ is approximated by its imaginary part calculated using the unitarity relation:

$$\begin{aligned}
\mathrm{Im}M_{\psi(3770)\to M_1M_2} &= \frac{1}{2(2\pi)^2}\int \frac{d^3q_D}{2E_D}\frac{d^3q_{\bar D}}{2E_{\bar D}}\delta^{(4)}(q_D+q_{\bar D}- \\
&\quad q_{M_1}-q_{M_2})M^*_{D\bar D\to M_1M_2}\times \\
&\quad \times M_{\psi(3770)\to D\bar D}, \\
M_{\psi(3770)\to D\bar D} &= g_{\psi(3770)D\bar D}\epsilon_\mu(q_D-q_{\bar D})_\mu
\end{aligned}$$

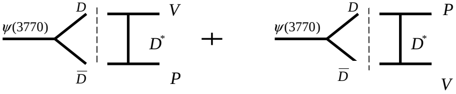
- $M_{D\bar D\to M_1M_2}$ is the amplitude of the decay $D\bar D \to M_1M_2$ calculated in the one meson exchange model.

Diagrams for *PP* and *VP* final states

- The final state PP:



- The final state *VP*:



- $V = \omega, \rho^0, K^*, \bar{K}^*, J/\psi; P = \pi^0, \eta, \eta', K, \bar{K}$.

Amplitudes of reactions 2 → 2

Expressions for amplitudes:

- $D^+D^- \to \pi^+\pi^-$:

$$\begin{aligned} M_{D^+D^-\to\pi^+\pi^-} &= g^2_{D^*D\pi^+}(q_{\pi^+}+q_{D^+}, q_{\pi^-}+q_{D^-}) \\ &\times \frac{\exp[\lambda_{D^*}(t-m^2_{D^{*0}})]}{m^2_{D^{*0}}-t}, \end{aligned}$$

- $D^+D^- \to \omega\pi^0$:

$$\begin{aligned} M_{D^+D^-\to\omega\pi^0} &= 2g_{D^*D\omega}g_{D^*D\pi^0}\varepsilon_{\mu\nu\lambda\sigma}(q_\omega)_\mu \times \\ &\omega_\nu(q_{\pi^0})_\lambda(q_{D^-})_\sigma \times \\ &\frac{\exp[\lambda_{D^*}(t-m^2_{D^{*+}})]}{m^2_{D^{*+}}-t}. \end{aligned}$$

Imaginary parts of the OZI suppressed coupling constants

- Imaginary parts of coupling constants of $\psi(3770)$ allowing for the threshold proximity:

$$\mathrm{Im} g_{\psi(3770)\to\pi^+\pi^-} \approx -4g^2_{D^*D\pi^+} r_{\mp} \exp(-s\lambda_{D^*}/2),$$
$$\mathrm{Im} g_{\psi(3770)\to\omega\pi^0} \approx 4g_{D^*D\omega} g_{D^*D\pi^0} r_{\mp} \exp(-s\lambda_{D^*}/2),$$

- Notation:

$$r_{\mp} = \frac{g_{\psi(3770)DD}}{6\pi m^3_{\psi(3770)}} \times (q^3_{D^+D^-} \mp q^3_{D^0\bar{D}^0}).$$

The sign is +(-) in case of conservation (nonconservation) of the isospin in the decay. Reminder: $I^G = ?^?$ until year of 2000, $I^G = 0^-$ after 2000.

Coupling constants of mesons with $C \neq 0$

- Coupling constants which can be found from existing data:

$$|g_{\psi(3770)D\bar{D}}| = \left[\frac{6\pi m^2_{\psi(3770)}\Gamma_{\psi(3770)}}{q^3_{D^+D^-} + q^3_{D^0\bar{D}^0}}\right]^{1/2} = 13.4,$$

$$|g_{D^*D\pi^+}| = \left[\frac{6\pi\Gamma_{D^{*\pm}}BR_{D^{*\pm}\to D\pi}}{q^3_{D^0\pi^\pm} + \frac{1}{2}q^3_{D^\pm\pi^0}}\right]^{1/2} = 9.1.$$

Coupling constants of mesons with $C \neq 0$

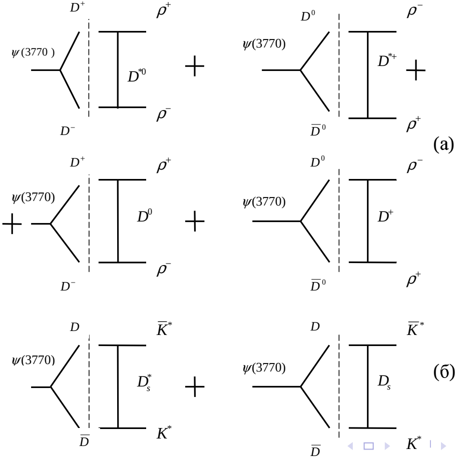
$g_{D^*D\omega}$ cannot be found directly from the data. Instead the model estimates are applied:

- The quark model (c, s quarks as spectators): $g_{D^*D\omega} \approx g_{K^*K\omega} \approx \frac{1}{2} g_{\omega\rho\pi} = 7.2\ \mathrm{GeV}^{-1}$
- The quark model relations for estimation of branching ratios of the decays $\psi(3770) \to \omega\eta$, $\omega\eta'$, $\rho\pi$, $\rho\eta$, $\rho\eta'$, $K\bar{K}$, $K^*\bar{K}$:

$$\begin{aligned} g_{D^*D\eta} &= -\sqrt{\frac{2}{3}} g_{D^*D\pi^0} = \sqrt{2} g_{D^*D\eta'}, \\ g_{D^{*0}D^0\omega} &= -g_{D^{*0}D^0\rho} = g_{D^{*+}D^-\rho}, \\ g_{D_s^{*+}D^+K^0} &= g_{D^{*+}D^0\pi^+} \end{aligned}$$

The two vector meson final states $VV = \rho^+\rho^-, K^*\bar{K}^*$

- Diagrams for the decays $\psi(3770) \to \rho^+\rho^-, K^*\bar{K}^*$



The decay amplitude into pair of vector mesons

- Decay amplitude:

$$\begin{aligned} M(V \to V_1 V_2) &= \frac{1}{2} g_1(\epsilon^{(V)}, q_1 - q_2)(\epsilon^{(V_1)}, \epsilon^{(V_2)}) \\ &+ g_2(\epsilon^{(V_2)}, q_1)(\epsilon^{(V)}, \epsilon^{(V_1)}) + \\ &g_3(\epsilon^{(V_1)}, q_2)(\epsilon^{(V)}, \epsilon^{(V_2)}) + \\ &\frac{1}{2} g_4(\epsilon^{(V)}, q_1 - q_2)(\epsilon^{(V_1)}, q_2) \times \\ &(\epsilon^{(V_2)}), q_1). \end{aligned}$$

The widths of the decay $\psi(3770)\to\rho^+\rho^-$

- The widths of the decay $\psi(3770)\to\rho^+\rho^-$:

$$\Gamma_{\psi(3770)\to\rho^+\rho^-}(s) = \frac{q^3_{\rho\rho}}{24\pi s}\left\{2\left[|g_1|^2+\left(|g_2|^2+|g_3|^2\right)\frac{s}{m^2_\rho}\right]+\right.$$
$$\left.|g_1+(g_2-g_3)\frac{s^{1/2}}{m_\rho}+Gq^2_{\rho\rho}|^2\right\},$$
$$G = \frac{1}{m^2_\rho}\left[2g_1+\frac{2s^{1/2}(g_2-g_3)}{(s^{1/2}+2m_\rho)}+g_4 s\right].$$

- Expressions for the amplitudes of non-$B\bar{B}$ decays of $\Upsilon(10580)$ are obtained with the help of evident replacements.

Some details

- The slopes λ of the form factor in the $2\to 2$ amplitude are estimated as follows:

$$\lambda_D\approx\lambda_{D_s}\approx\lambda_{D^*}\approx\lambda_{D_s^*}=0.27\text{ GeV}^{-2}\sim 1/m^2_{D^*},$$

$$\lambda_B\approx\lambda_{B_s}\approx\lambda_{B^*}\approx\lambda_{B_s^*}=0.04\text{ GeV}^{-2}\sim 1/m^2_{B^*}$$

- The suppression factor of each specific channel

$$\left[e^{-\lambda_{D,B}m^2_{\psi(3770),\Upsilon(4S)}}\right]_{\text{form factor}}\times\left(\frac{1}{4}\right)_{\text{absorption}}$$

- Absorption in specific channel means the enhancement of the net contribution of other non-$D\bar{D}$ and non- $B\bar{B}$ states.

$\psi(3770)$

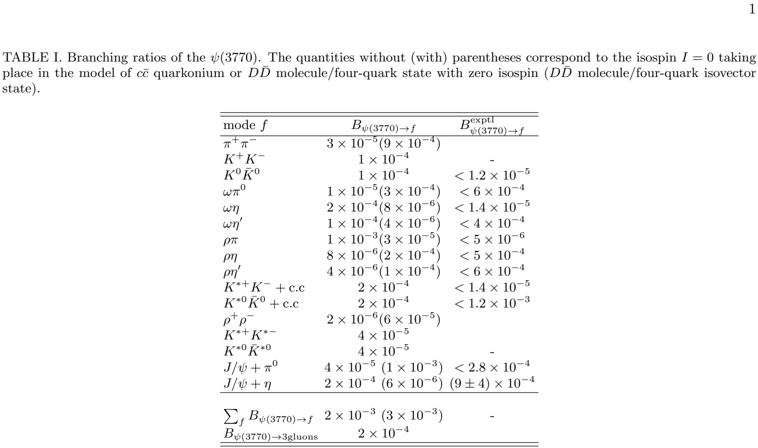


TABLE I. Branching ratios of the $\psi(3770)$. The quantities without (with) parentheses correspond to the isospin $I = 0$ taking place in the model of $c\bar{c}$ quarkonium or $D\bar{D}$ molecule/four-quark state with zero isospin ($D\bar{D}$ molecule/four-quark isovector state).

mode f	$B_{\psi(3770)\to f}$	$B^{\rm exptl}_{\psi(3770)\to f}$
$\pi^+\pi^-$	$3\times10^{-5}(9\times10^{-4})$	
K^+K^-	1×10^{-4}	-
$K^0\bar{K}^0$	1×10^{-4}	$< 1.2\times10^{-5}$
$\omega\pi^0$	$1\times10^{-5}(3\times10^{-4})$	$< 6\times10^{-4}$
$\omega\eta$	$2\times10^{-4}(8\times10^{-6})$	$< 1.4\times10^{-5}$
$\omega\eta'$	$1\times10^{-4}(4\times10^{-6})$	$< 4\times10^{-4}$
$\rho\pi$	$1\times10^{-3}(3\times10^{-5})$	$< 5\times10^{-6}$
$\rho\eta$	$8\times10^{-6}(2\times10^{-4})$	$< 5\times10^{-4}$
$\rho\eta'$	$4\times10^{-6}(1\times10^{-4})$	$< 6\times10^{-4}$
$K^{*+}K^- + \text{c.c}$	2×10^{-4}	$< 1.4\times10^{-5}$
$K^{*0}\bar{K}^0 + \text{c.c}$	2×10^{-4}	$< 1.2\times10^{-3}$
$\rho^+\rho^-$	$2\times10^{-6}(6\times10^{-5})$	
$K^{*+}K^{*-}$	4×10^{-5}	
$K^{*0}\bar{K}^{*0}$	4×10^{-5}	-
$J/\psi+\pi^0$	$4\times10^{-5}\ (1\times10^{-3})$	$< 2.8\times10^{-4}$
$J/\psi+\eta$	$2\times10^{-4}\ (6\times10^{-6})$	$(9\pm4)\times10^{-4}$
$\sum_f B_{\psi(3770)\to f}$	$2\times10^{-3}\ (3\times10^{-3})$	-
$B_{\psi(3770)\to 3\text{gluons}}$	2×10^{-4}	

$\Upsilon(10580)$

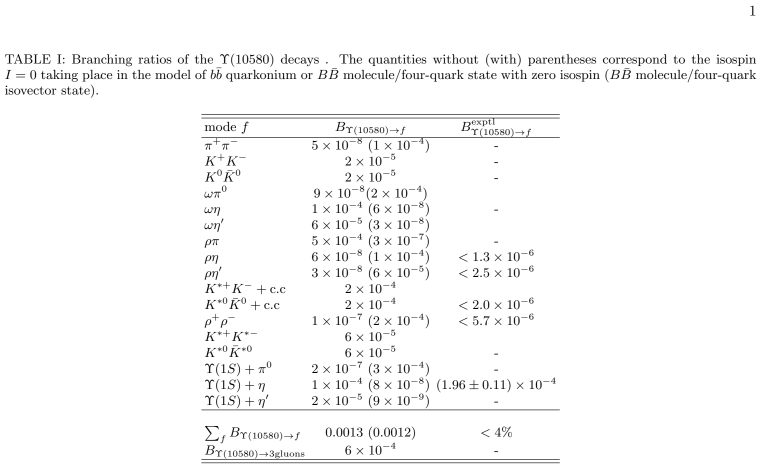


TABLE I: Branching ratios of the $\Upsilon(10580)$ decays . The quantities without (with) parentheses correspond to the isospin $I=0$ taking place in the model of $b\bar{b}$ quarkonium or $B\bar{B}$ molecule/four-quark state with zero isospin ($B\bar{B}$ molecule/four-quark isovector state).

mode f	$B_{\Upsilon(10580)\to f}$	$B^{\text{exptl}}_{\Upsilon(10580)\to f}$
$\pi^+\pi^-$	5×10^{-8} (1×10^{-4})	-
K^+K^-	2×10^{-5}	-
$K^0\bar{K}^0$	2×10^{-5}	-
$\omega\pi^0$	$9\times10^{-8}(2\times10^{-4})$	
$\omega\eta$	1×10^{-4} (6×10^{-8})	-
$\omega\eta'$	6×10^{-5} (3×10^{-8})	
$\rho\pi$	5×10^{-4} (3×10^{-7})	-
$\rho\eta$	6×10^{-8} (1×10^{-4})	$<1.3\times10^{-6}$
$\rho\eta'$	3×10^{-8} (6×10^{-5})	$<2.5\times10^{-6}$
$K^{*+}K^-+\text{c.c}$	2×10^{-4}	
$K^{*0}\bar{K}^0+\text{c.c}$	2×10^{-4}	$<2.0\times10^{-6}$
$\rho^+\rho^-$	1×10^{-7} (2×10^{-4})	$<5.7\times10^{-6}$
$K^{*+}K^{*-}$	6×10^{-5}	
$K^{*0}\bar{K}^{*0}$	6×10^{-5}	-
$\Upsilon(1S)+\pi^0$	2×10^{-7} (3×10^{-4})	-
$\Upsilon(1S)+\eta$	1×10^{-4} (8×10^{-8})	$(1.96\pm0.11)\times10^{-4}$
$\Upsilon(1S)+\eta'$	2×10^{-5} (9×10^{-9})	-
$\sum_f B_{\Upsilon(10580)\to f}$	0.0013 (0.0012)	$<4\%$
$B_{\Upsilon(10580)\to 3\text{gluons}}$	6×10^{-4}	-

OZI violating decays of $\psi(4040)$

- Besides $D\bar{D}$, the following processes are possible:

  $$\psi(4040)\to D^*\bar{D}+\mathrm{c.c}, D^*\bar{D}^*\to PP, VP, VV.$$

  Little can be said about estimation of their branching fractions without invoking numerous model assumptions.
- $\psi(4040)\to D_s^+D_s^-$. CLEO-c: $\sigma_{e^+e^-\to D_s^+D_s^-}\sim 0.1$ nb at $\sqrt{s}=4040$ MeV.

The $\psi(4040) \to D_s^+ D_s^-$ coupling

- The partial width $\Gamma_{\psi(4040)\to D_s^+ D_s^-}$ as estimated from CLEO-c and PDG data:

$$\Gamma_{\psi(4040)\to D_s^+ D_s^-} = \frac{\left(m_{\psi(4040)\to D_s^+ D_s^-}\Gamma_{\psi(4040)\to D_s^+ D_s^-}\right)^2}{12\pi\Gamma_{\psi(4040)\to e^+e^-}} \times \sigma_{e^+e^-\to\psi(4040)\to D_s^+ D_s^-} \sim 8\times 10^{-4}\mathrm{GeV};$$

- $|g_{\psi(4040)D_s^+ D_s^-}| = \left(\frac{6\pi\Gamma_{\psi(4040)\to D_s^+ D_s^-} m^2_{\psi(4040)}}{q^3_{D_s}}\right)^{1/2} \sim 2.$

Branching fractions of $\psi(4040) \to \varphi\eta, \varphi\eta'$

Necessary couplings are estimated in quark model:

$$g_{D_s^* D\eta} = -\sqrt{\frac{2}{3}} g_{K^* K\pi^0},\; g_{D_s^* D\eta'} = \frac{2}{\sqrt{3}} g_{K^* K\pi^0},\; g_{\varphi D^* D} = \frac{1}{\sqrt{2}} g_{\omega\rho\pi}.$$

Branching fractions as estimated using the slope in the exchange form factor $\lambda_{D_s^*} \sim 1/m^2_{D_s^*}$:

- $B_{\psi(4040)\to\varphi\eta} \sim 3 \times 10^{-6}$,
- $B_{\psi(4040)\to\varphi\eta'} \sim 6 \times 10^{-6}$.

- The estimate of the inclusive p-wave annihilation cross section $P\bar{P}\to X$, $P\bar{P}\equiv P^+P^-+P^0\bar{P}^0(P=D,B)$ using the unitarity relation is

$$\sum_X B_{V\to P\bar{P}\to X} \sim \frac{m_P^2 v_P^2}{48\pi} B_{V\to P^0\bar{P}^0}\times\sigma_P(P^0\bar{P}^0\to X)\times \sum_X\left[1+(-1)^{I_X+I_V}|c_{P^\pm}|^2\left(\frac{v_{P^\pm}}{v_{P^0}}\right)^3\right]^2 .$$

- The "wrong" isospin contribution $(-1)^{I_X+I_V}=-1$ is suppressed:

$$r_P=\left[\frac{1-|c_{P^\pm}|^2(v_{P^\pm}/v_{P^0})^3}{1+|c_{P^\pm}|^2(v_{P^\pm}/v_{P^0})^3}\right]^2=0.04(0.0008)$$

for $P=D(B)$, respectively.

Allowed bounds on p-wave annihilation cross section

- The data admit $\sum_X B_{\psi(3700)\to X}$ up to 10%, $\sum_X B_{\Upsilon(10580)\to X} < 4\%$. Then, for example,

$$\begin{aligned}\sum_X B_{\psi(3700)\to D^+D^-+D^0\bar{D}^0\to X} &= 1\%,\\ \sum_X B_{\Upsilon(10580)\to B^+B^-+B^0\bar{B}^0\to X} &= 1\%\end{aligned}$$

corresponds to

$$\begin{aligned}\sigma_P(D^0\bar{D}^0\to X) &\sim 1.5\mu\text{b},\\ \sigma_P(B^0\bar{B}^0\to X) &\sim 0.6\mu\text{b}.\end{aligned}$$

Coulomb corrections in D^+D^- system

PDG data on $\psi(3770)$ and $\Upsilon(10580)$ permit one to make conclusions on electromagnetic corrections $|c_{P^\pm}|^2$.

- $\psi(3770)$.

$$r_D^{\rm expt} \equiv \frac{B_{\psi(3770)\to D^+D^-}}{B_{\psi(3770)\to D^0\bar{D}^0}} = 0.778 \pm 0.108.$$

Compare it with

$$r_D^{\rm theor} \equiv \left(\frac{q_{D^+}}{q_{D^0}}\right)^3 = 0.692 \pm 0.002.$$

Hence $|c_{D^\pm}^{\rm expt}|^2 = r_D^{\rm expt}/r_D^{\rm theor} = 1.139 \pm 0.156$. This does not contradict to point-like electromagnetic correction

$$|c_{D^\pm}^{\rm theor}|^2 \approx 1 + \frac{\alpha\pi}{2v_{D^\pm}} = 1.086 \pm 0.001.$$

Coulomb corrections in B^+B^- system

- $\Upsilon(10580)$.

$$r_B^{\rm expt}\equiv\frac{B_{\Upsilon(10580)\to B^+B^-}}{B_{\Upsilon(10580)\to B^0\bar{B}^0}}=1.053\pm0.018.$$

Compare it with

$$r_B^{\rm theor}\equiv\left(\frac{q_{B^+}}{q_{B^0}}\right)^3=1.047\pm0.002.$$

Hence $|c_{B^\pm}^{\rm expt}|^2=r_B^{\rm expt}/r_B^{\rm theor}=1.006\pm0.017$. The resulting point-like electromagnetic correction is

$$|c_{B^\pm}^{\rm theor}|^2\approx1+\frac{\alpha\pi}{2v_{B^\pm}}=1.182\pm0.038.$$

Coulomb corrections in B^+B^- system. Discussion

- The point-like Coulomb corrections in $B\bar{B}$ system is appreciable. This results in

$$|c_{B^\pm}|^2 \equiv \left|\frac{g_{\Upsilon(10580)\to B^+B^-}}{g_{\Upsilon(10580)\to B^0\bar{B}^0}}\right|^2 = 0.851 \pm 0.031.$$

- Interpretation is unclear. Is it the consequence of the internal structure of flavored mesons $B^\pm$, or isospin symmetry breaking for coupling constants?
- In any case, $|c_{B^\pm}|^2 \neq 1$ inevitably results in increasing of the estimated branching fractions of decays of $\Upsilon(10580)$ to light hadrons with "wrong" isospin by more than order of magnitude.

Conclusion

- Despite the long story of searches of the exclusive decays of heavy just-above-open-flavor threshold quarkonia $\psi(3770)$ and $\Upsilon(10580) \equiv \Upsilon(4S)$ etc. to light hadrons the necessary branching fractions are in fact unknown.
- Obtaining the firm information about such decays could permit both to elucidate the issues of the structure and nuclear interactions of mesons with nonzero heavy flavors and to specify the dynamics of violation of approximate hadronic symmetries in the decays of heavy quarkonia.

Thank You!