About the physics of the $X(3872)$ resonance

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ABSTRACT

We construct spectra of decays of the resonance $X(3872)$ with good analytical and unitary properties which allows to define the branching ratio of the $X(3872) \to D^{*0}\bar{D}^0 + c.c.$ decay studying only one more decay, for example, the $X(3872) \to \pi^+\pi^- J/\psi(1S)$ decay, and show that our spectra are effective means of selection of models for the resonance $X(3872)$.

ABSTRACT

Then we discuss the scenario where the $X(3872)$ resonance is the $c\bar{c} = \chi_{c1}(2P)$ charmonium which "sits on" the $D^{*0}\bar{D}^0$ threshold.

We explain the shift of the mass of the $X(3872)$ resonance with respect to the prediction of a potential model for the mass of the $\chi_{c1}(2P)$ charmonium by the contribution of the virtual $D^*\bar{D} + c.c.$ intermediate states into the self energy of the $X(3872)$ resonance.

We suggest a physically clear program of experimental researches for verification of our assumption right now.

Introduction

The $X(3872)$ resonance became the first in discovery of the resonant structures XYZ ($X(3872)$, $Y(4260)$, $Z_b^+(10610)$, $Z_b^+(10650)$, $Z_c^+(3900)$), the interpretations of which as hadron states assumes existence in them at least pair of heavy and pair of light quarks in this or that form.

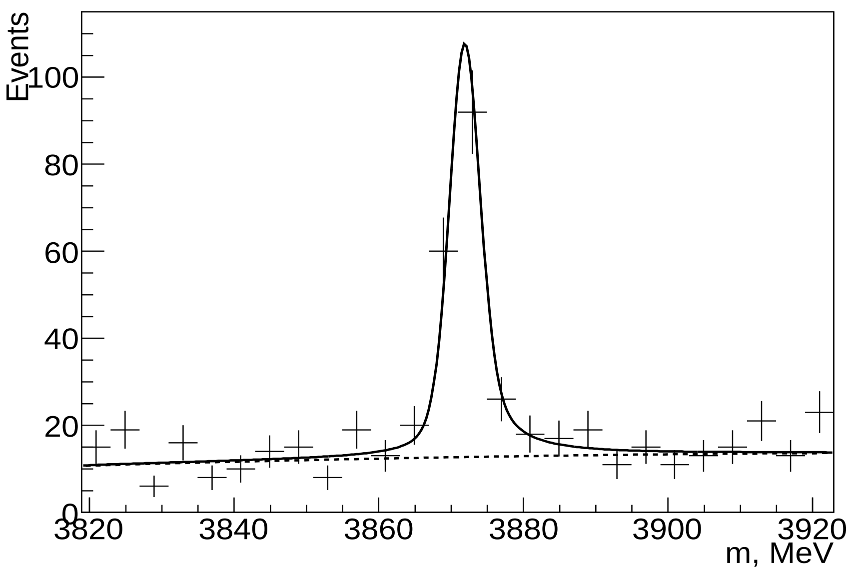
Thousands of articles on this subject already were published in spite of the fact that many properties of new resonant structures are not defined yet and not all possible mechanisms of dynamic generation of these structures are studied, in particular, the role of the anomalous Landau thresholds is not studied.

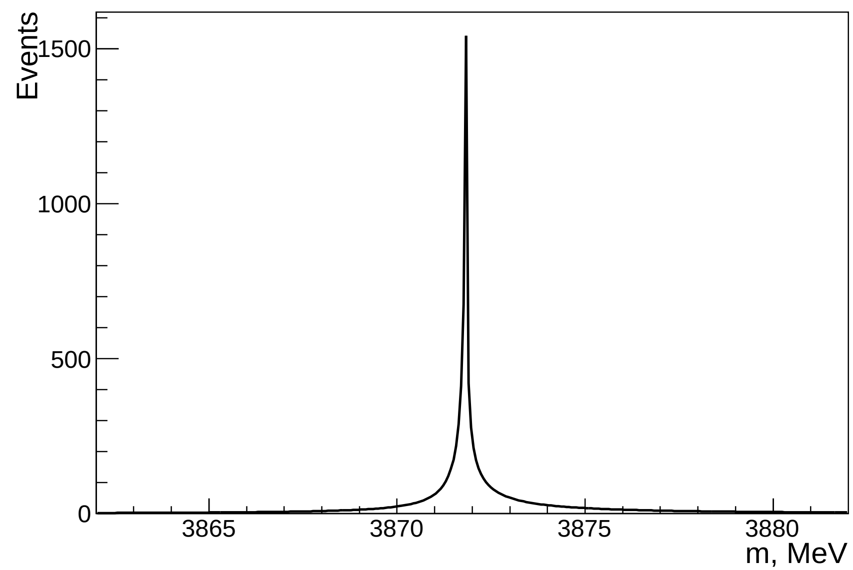
Anyway, this spectroscopy took the central place in physics of hadrons.

Below we give reasons that $X(3872)$, $I^G(J^{PC}) = 0^+(1^{++})$, is the $\chi_{c1}(2P)$ charmonium and suggest a physically clear program of experimental researches for verification of our assumption.

How learn the branching ratio $X(3872) \to D^{*0}\bar{D}^0 + c.c.$

The mass spectrum $\pi^+\pi^- J/\psi(1S)$ looks as the ideal Breit-Wigner one in the $X(3872) \to \pi^+\pi^- J/\psi(1S)$ decay.



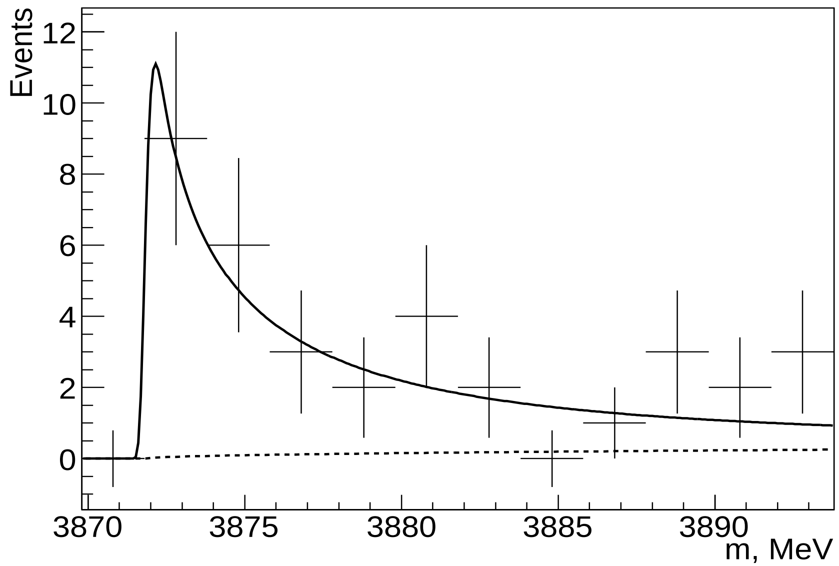


(a) **(b)**

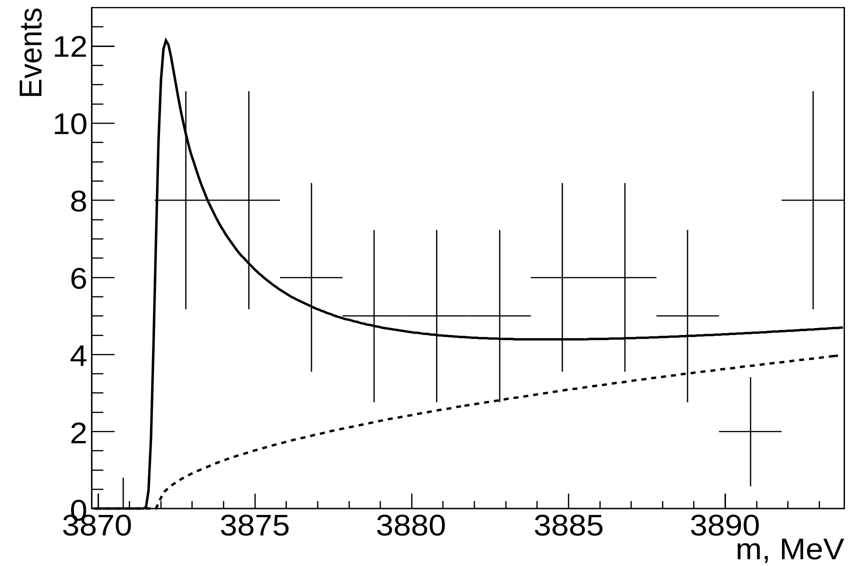
a) The Belle data on the invariant $\pi^+\pi^- J/\psi(1S)$ mass (m) distribution. The solid line is our theoretical one with taking into account the Belle energy resolution.

b) Our undressed theoretical line.

The mass spectrum $D^{*0}\bar{D}^0 + c.c.$



(a)



(b)

The Belle data on the invariant $D^{*0}\bar{D}^0 + c.c.$ mass (m) distribution. The solid line is our theoretical one with taking into account the Belle energy resolution. a) $D^{*0} \to D^0\pi^0$. b) $D^{*0} \to D^0\gamma$.

If structures in the above channels are manifestation of the same resonance, it is possible to define the branching ratio $BR(X(3872) \to D^{*0}\bar{D}^0 + c.c.)$ treating data only of these decay channels.

The mass spectrum $D^{*0}\bar{D}^0 + c.c.$

We believe that the $X(3872)$ is the axial vector, 1^{++}. In this case the S wave dominates in the $X(3872) \to D^{*0}\bar{D}^0 + c.c.$ decay and hence is described by the Lagrangian

$$L(x) = g_A X^\mu \Big(D_\mu(x)\bar{D}(x) + \bar{D}_\mu(x) D(x) \Big).$$

The width of the $X \to D^{*0}\bar{D}^0 + c.c.$ decay

$$\Gamma(X \to D^{*0}\bar{D}^0 + c.c.\,,\, m) = \frac{g_A^2}{8\pi}\frac{\rho(m)}{m}\left(1 + \frac{\mathrm{k}^2}{3m^2_{D^{*0}}}\right) \approx \frac{\mathrm{g}_\mathrm{A}^2}{8\pi}\frac{\rho(\mathrm{m})}{\mathrm{m}}$$

$$\rho(m) = \frac{2|\mathrm{k}|}{m} = \frac{\sqrt{(m^2 - m_+^2)(m^2 - m_-^2)}}{m^2}, m_\pm = m_{D^{*0}} \pm m_{D^0}$$

The mass spectrum $D^{*0}\bar{D}^0 + c.c.$

$$\frac{dBR(X \to D^{*0}\bar{D}^0 + c.c.\,,\, m)}{dm} = 4\frac{1}{\pi}\frac{m^2\Gamma(X \to D^{*0}\bar{D}^0,\, m)}{|D_X(m)|^2}$$

The branching ratio of $X(3872) \to D^{*0}\bar{D}^0 + c.c.$

$$BR(X \to D^{*0}\bar{D}^0 + c.c.) = 4\frac{1}{\pi}\int_{m_+}^{\infty}\frac{m^2\Gamma(X \to D^{*0}\bar{D}^0,\, m)}{|D_X(m)|^2}dm$$

The mass spectra of $X(3872)$ in non-$D^{*0}\bar{D}^0 + c.c.$ chanals

In others $\{i\}$ (non-$D^{*0}\bar{D}^0$) channels the $X(3872)$ state is seen as a narrow resonance that is why we write the mass spectrum in the i channel in the form

$$\frac{dBR(X \to i\,,\, m)}{dm} = 2\frac{1}{\pi}\frac{m_X^2\,\Gamma_i}{|D_X(m)|^2}\,,$$

where Γ_i is the width of the $X(3872) \to i$ decay.

The branching ratio of $X(3872) \to i$

$$BR(X \to i) = 2\frac{1}{\pi}\int_{m_0}^{\infty}\frac{m_X^2\Gamma_i}{|D_X(m)|^2}dm\,,$$

where m_0 is the threshold of the i state.

The inverse propagator $D_X(m)$

$$D_X(m) = m_X^2 - m^2 + Re(\Pi_X(m_X^2)) - \Pi_X(m^2) - \imath m_X \Gamma\,,$$

where $\Gamma = \Sigma\Gamma_i$ is the total width of the $X(3872)$ decay into all non-$D^{*0}\bar{D}^0$ channels.

$$\Pi_X(s) = \frac{g_A^2}{8\pi^2}\left(I^{D^0\bar{D}^{*0}}(s) + I^{D^+D^{*-}}(s)\right), \quad m^2 = s\,.$$

When $m_+ = m_{D^*} + m_D \leq m$,

$$I^{D\bar{D}^*}(m^2) = \left\{ \frac{(m^2 - m_+^2)}{m^2}\frac{m_-}{m_+}\ln\frac{m_{D^{*0}}}{m_{D^0}} \right.$$

$$\left. +\rho(m)\left[\imath\pi + \ln\frac{\sqrt{m^2 - m_-^2} - \sqrt{m^2 - m_+^2}}{\sqrt{m^2 - m_-^2} + \sqrt{m^2 - m_+^2}}\right]\right\},$$

where $m_- = m_{D^*} - m_D\,.$

Unitarity, Fitting Data

Our branching ratios satisfy unitarity

$$1 = BR(X \to D^{*0}\bar{D}^0 + c.c.) + BR(X \to D^{*+}\bar{D}^- + c.c.)$$

$$+ \sum_i BR(X \to i)\,.$$

Fitting the Belle data, we take into account the Belle results that $m_X = 3871.84$ MeV $= m_{D^{*0}} + m_{D^0} = m_+$ and $\Gamma_{X(3872)} < 1.2$ MeV 90%CL that corresponds to $\Gamma < 1.2$ MeV, which controls the width of the $X(3872)$ signal in the $\pi^+\pi^- J/\psi(1S)$ channel and in every non-$D^{*0}\bar{D}^0$ channel.

The results of our fit are in the Table.

Results

TABLE. Γ in MeV, g_A in GeV.

Γ	$1.2_{-0.4}$	mode	$D^{*0}\bar{D}^0 + ..$	$D^{*+}D^- + ..$	$Others$
$\frac{g_A^2}{8\pi}$	1.4^{+5}_{-1}	BR	$0.6^{+0.02}_{-0.1}$	$0.31^{+0.13}_{-0.16}$	$0.1^{+0.3}_{-0.1}$
$\frac{\chi^2}{Ndf}$	45/42	BR_{seen}	$0.3^{+0.1}_{-0.2}$	$0.03^{+0.004}_{-0.02}$	$0.09^{+0.3}_{-0.1}$

BR_{seen} is a branching ratio for $m \leq 3891.84$ MeV.

Our approach can serve as the guide in selection of theoretical models for the $X(3872)$ resonance. Indeed, if 3871.68 MeV $< M_X < 3871.95$ MeV and $\Gamma_{X(3872)} = \Gamma < 1.2$ MeV then for $g_A^2/4\pi < 0.2$ GeV^2 $BR(X \to D^{*0}\bar{D}^0 + c.c.; m \leq 3891.84\,\text{MeV}) < 0.3$.

That is, unknown decays of $X(3872)$ into non-$D^{*0}\bar{D}^0$ states are considerable or dominant.

Why $X(3872)$ is enigmatic

1. The mass of the X resonance is 50 MeV lower than predictions of the most lucky naive potential models for the mass of the $\chi_{c1}(2P)$ resonance, $m_X - m_{\chi_{c1}(2P)} = -\Delta \approx -50$ MeV.
2. $BR(X \to \pi^+\pi^-\pi^0 J/\psi(1S)) \sim BR(X \to \pi^+\pi^- J/\psi(1S))$.

These two dramatic discoveries have generated a stream of the $D^{*0}\bar{D}^0 + D^0\bar{D}^{*0}$ molecular interpretations of the $X(3872)$ resonance.

STOP molecular stream

The point is that the bounding energy is small, $\epsilon_B \lesssim (1 \div 3)$ MeV. That is, the radius of the molecule is large, $r_{X(3872)} \gtrsim (3 \div 5)$ fermi$= (3 \div 5) \cdot 10^{-13}$ cm. As for the charmonium, its radius is less one fermi, $r_{\chi_{c1}(2P)} \approx 0.5$ fermi $= 0.5 \cdot 10^{-13}$ cm.
That is, the molecule volume is $100 \div 1000$ times as large as the charmonium volume, $V_{X(3872)}/V_{\chi_{c1}(2P)} \gtrsim (100 \div 1000)$.

The enthusiasts of the molecular scenario do not discuss this question with rare exception.

STOP molecular stream

How to explain sufficiently abundant inclusive production of the rather extended molecule X(3872) in a hard process

$pp \to X(3872) + anything$

with rapidity in the range 2,5 - 4,5 and transverse momentum in the range 5-20 GeV (R. Aaij et al., Eur. Phys. J. C. 72, 1972 (2012), LHCb Collaboration)? Really.

$$\sigma(pp \to X(3872) + anything) B(X(3872) \to \pi^+\pi^- J/\psi) = 5,4\, nb$$

and

$$\sigma(pp \to \psi(2S) + anything) B(\psi(2S) \to \pi^+\pi^- J/\psi) = 38\, nb.$$

But

STOP molecular stream

$$B(\psi(2S) \to \pi^+\pi^- J/\psi) = 0.34$$

while

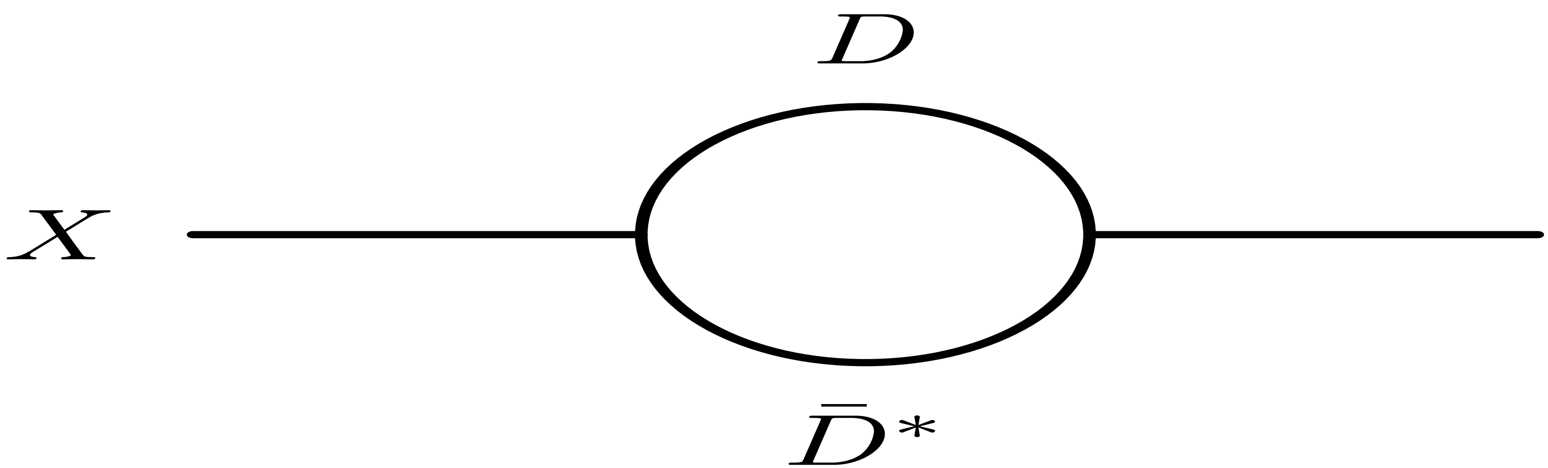
$$0.023 < B(X(3872) \to \pi^+\pi^- J/\psi) < 0.066$$

according to C.-Z. Yuan (Belle Collaboration), arXiv: 0910.3138 [hep-ex], Proceedings of the XXIX PHYSICS IN COLLISION, 2009, Kobe, Japan. So,

$$0.74 < \frac{\sigma(\mathrm{pp} \to \mathrm{X}(3872) + \mathrm{anything})}{\sigma(\mathrm{pp} \to \psi(2\mathrm{S}) + \mathrm{anything})} < 2.1.$$

The extended molecule is produced in hard process as intensively as the compact charmonium. It's a miracle.

How to solve the problem of mass



$$\Pi_X(s) = \frac{g_A^2}{8\pi^2}\left(I^{D^0\bar{D}^{*0}}(s) + I^{D^+D^{*-}}(s)\right)\,,$$

$$I^{D\bar{D}^*}(s) = \int_{m_+^2}^{\Lambda^2} \frac{\sqrt{(s'-m_+^2)(s'-m_-^2)}}{s'(s'-s)}ds' \approx 2\ln\left(\frac{2\Lambda}{m_+}\right)$$

$$-2\cdot\sqrt{\frac{(m_+^2-s)}{s}}\arctan\left(\sqrt{\frac{s}{m_+^2-s}}\right)\,,\quad s < m_+^2\,,$$

$$m_+ = m_{D^*} + m_D\,,\ \ m_+ = m_{D^*} - m_D\,,\ \ m_+^2 \ll \Lambda^2.$$

Mass renormalization

The X(3872) propagator $1/D_X(s)$

$$D_X(s) = m^2_{\chi_{c1}(2P)} - s - \Pi_X(s) - \imath m_X \Gamma, \quad \Gamma < 1.2 \text{ MeV !}$$

$$m^2_{\chi_{c1}(2P)} - m^2_X - \Pi_X(m^2_X) = 0 \Rightarrow$$

$$\Delta\,(2m_X + \Delta) = \Pi_X(m^2_X) \approx \left(g^2_A/8\pi^2\right) 4\ln(2L/m_+)$$

If $m_{\chi_{c1}(2P)} - m_X = \Delta = 50$ MeV, then $g^2_A/8\pi \approx 0.2$ GeV^2 for $L = 10$ GeV and $g^2_A/8\pi \approx 0.1$ GeV^2 for $L = 100$ GeV.

$BR(X \to D^0\bar{D}^{*0} + c.c.) = 0.3$ and 0.2, respectively.

Thus, we expect that unknown decays of $X(3872)$ into non-$D^{*0}\bar{D}^0 + c.c.$ states are considerable or dominant.

Renormalized propagator has the form

$$D_X(s) = m^2_X - s + \Pi_X(m^2_X) - \Pi_X(s) - \imath m_X \Gamma.$$

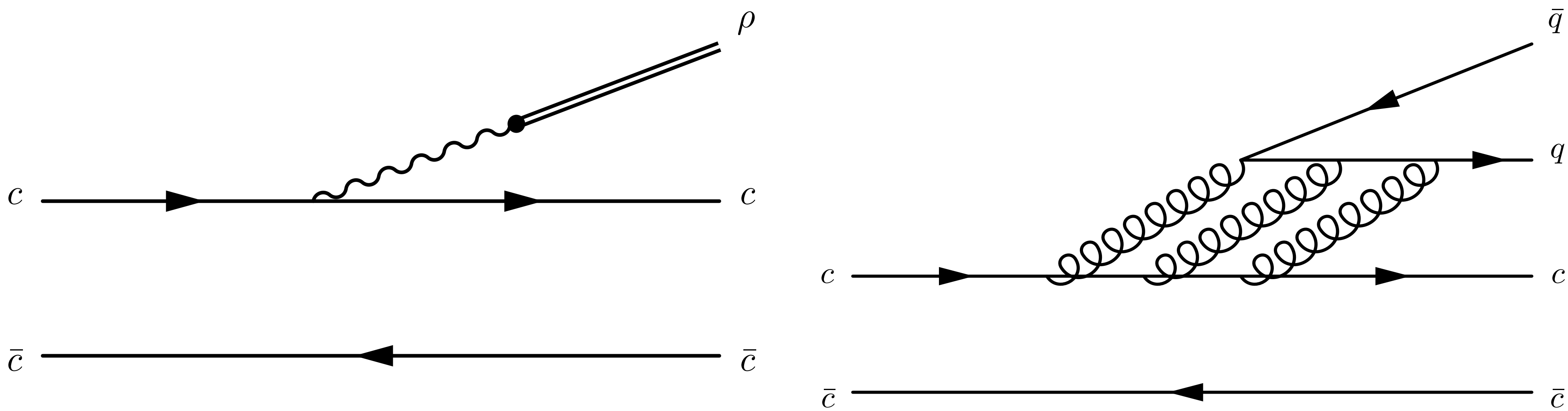
Why the virtual $D^*\bar{D} + c.c.$ states shift the $\chi_{c1}(P)$ mass

The assumption of the determining role of the $D^*\bar{D} + c.c.$ channels in the shift of the mass of the $\chi_{c1}(P)$ meson is based on the following reasoning.

Let us imagine that D and D^* mesons are light, for example, as the K and K^* mesons. Then the width of $X(3872)$ meson is equal $25 \div 50$ MeV for $g_A^2/8\pi = 0.1 \div 0.2\, GeV^2$ that much more than the width of its decay into all non- $D^{*0}\bar{D}^0 + c.c.$ channels, $\Gamma < 1.2$ MeV.

That is, in our case the coupling of the $X(383)$ meson with the $D^*\bar{D} + c.c.$ channels is rather strong.

As for $BR(X \to \rho J/\psi) \sim BR(X \to \omega J/\psi)$



(a) **(b)**

Let us recall

$BR(J/\psi \to \rho\eta') = (1.05 \pm 0.18)10^{-4}$ **and**

$BR(J/\psi \to \omega\eta') = (1.82 \pm 0.21)10^{-4}$.

Note that in the $X(3872)$ case the ω meson is produced on its tail, while the ρ meson is produced on a half. $m_X - m_{J/\psi} = 775$ **MeV**

$X(\chi_{c1}(2P))$ versus $\Upsilon_{b1}(2P)$

Recently, the LHCb Collaboration published a landmark result

R. Aaij et al. (LHCb Collaboration), Nucl. Phys. B 886, 665 (2014).

$$\frac{BR(X \to \gamma\psi(2S))}{BR(X \to \gamma J/\psi)} = C_X \left(\frac{\omega_{\psi(2S)}}{\omega_{J/\psi}}\right)^3 = 2.46 \pm 0.7$$

On the other hand

$$\frac{BR(\chi_{b1}(2P) \to \gamma\Upsilon(2S))}{BR(\chi_{b1}(2P) \to \gamma\Upsilon(1S))} = C_{\chi_{b1}(2P)} \left(\frac{\omega_{\Upsilon(2S)}}{\omega_{\Upsilon(1S)}}\right)^3 = 2.16 \pm 0.28$$

$$C_X = C_{\chi_{c1}(2P)} = 136.78 \pm 38.89$$

$$C_{\chi_{b1}(2P)} = 80 \pm 10.37$$

Note that all versions of the potential model predict

$C_{\chi_{c1}(2P)} \gg 1$ and $C_{\chi_{b1}(2P)} \gg 1$.

Prediction

It is known that

$$BR(\chi_{b1}(2P) \to \omega\Upsilon(1S)) = \left(1.63 \pm^{0.4}_{0.34}\right)\%.$$

After all that has been said here, we are forced to make a predictions.

1. If the one-photon mechanism dominates in the $X(3872) \to \rho J/\psi$ decay then one should expect

$$BR(\chi_{b1}(2P) \to \rho\Upsilon(1S)) \sim (e_b/e_c)^2 \cdot 1.6\%$$

$$= (1/4) \cdot 1.6\% \approx 0.4\% \quad \text{!!!}$$

Where e_c and e_b are the charges of the c and b quarks, respectively.

2. If the three-gluon mechanism dominates in the $X(3872) \to \rho J/\psi$ decay then one should expect

$$BR(\chi_{b1}(2P) \to \rho\Upsilon(1S)) \sim 1.6\% \quad \text{!!!}$$

Conclusion

We believe that discovery of a significant number unknown decays of $X(3872)$ into non-$D^{*0}\bar{D}^0 + c.c.$ states and discovery of the $\chi_{b1}(2P) \to \rho\Upsilon(1S)$ decay could decide destiny of X(3872).

Once more, we discuss the scenario where the $\chi_{c1}(2P)$ charmonium sits on the $D^{*0}\bar{D}^0$ threshold but not a mixing of the giant $D^*\bar{D}$ molecule and the compact $\chi_{c1}(2P)$ charmonium. Note that the mixing of such states requests the special justification.

Thanks

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THANK YOU