# Remarks on resonances

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# What is a resonance ?

The $S$–matrix is characterized by its analytic structure (in $s$).

$\rightarrow$ branch points (and the corresponding cuts)

$\longrightarrow$ at each channel opening for $s > s^{\rm thres}$: right hand cut

$\longrightarrow$ in the crossed channels for $s < s^{\rm thres}$: left hand cut

$\longrightarrow$ inside the un-physical sheet (see below)

$\rightarrow$ poles on the physical sheet: bound states

$\longrightarrow$ only for real $s < s^{\rm thres}_{\rm min}$ (no other singul. allowed here)

$\rightarrow$ poles on the un-physical sheet (closest to the physical one)

$\longrightarrow$ for real $s < s^{\rm thres}_{\rm min}$: virtual state

$\longrightarrow$ for complex $s$: resonance

# For bound states

Weinberg PR 130 (1963) 776

Expand in terms of non–interacting quark and meson states

$$|\Psi\rangle = \begin{pmatrix} \lambda|\psi_0\rangle \\ \chi(\mathbf{p})|h_1 h_2\rangle \end{pmatrix} ,$$

here $|\psi_0\rangle$ = elementary state and $|h_1 h_2\rangle$ = two–hadron cont., then

$\lambda^2$ equals probability to find the bare state in the physical state

$\rightarrow$ $\lambda^2$ is the quantity of interest!

After some algebra we get for the residue at the pole

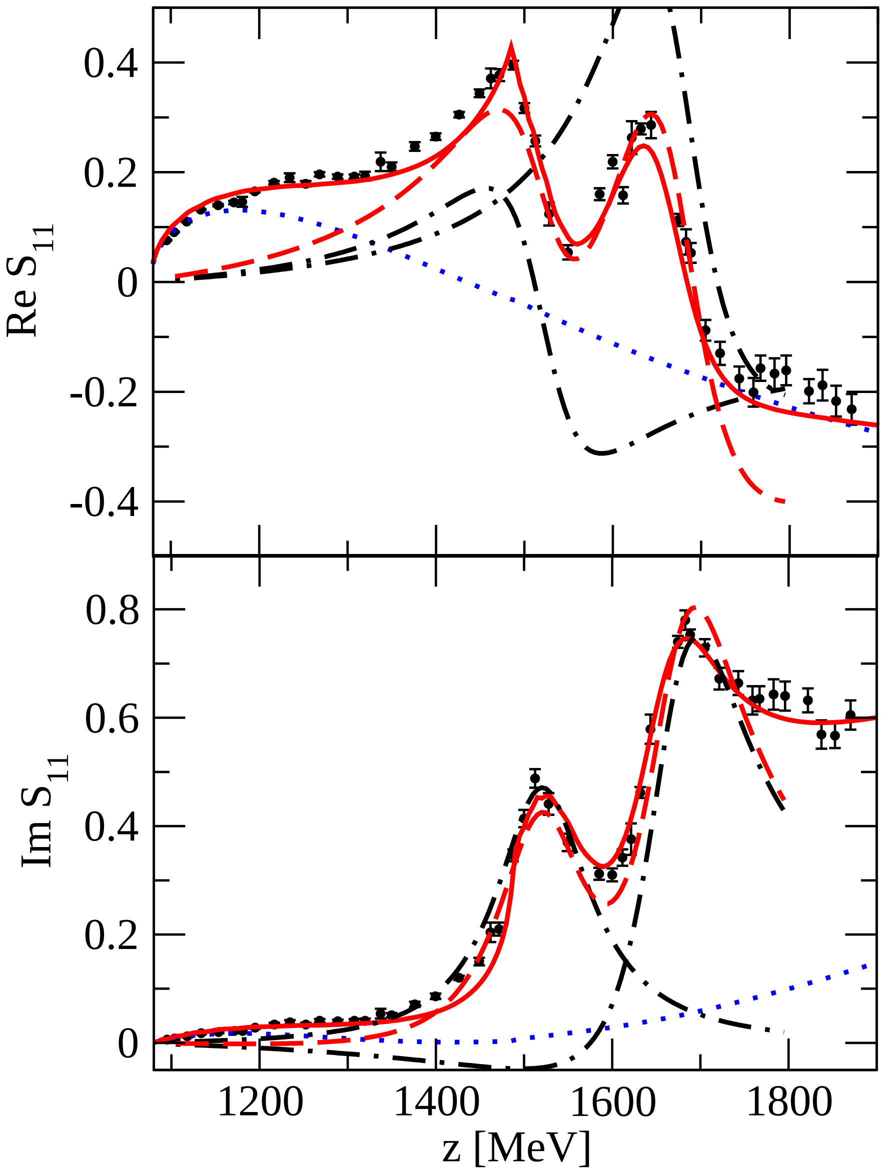
$$\boxed{g_{\rm eff}^2 = 2(1-\lambda^2)\sqrt{\epsilon/m} \le 2\sqrt{\epsilon/m}}$$

For bound state low E amplitude fixed in $hh$ channel!

Picture not changed by far away threshold  Baru et al. PLB586 (2004) 53

Equivalent to, e.g.,  Morgan NPA543 (1992) 63; Törnqvist PRD51 (1995) 5312

# More on Resonances




A resonance is uniquely and unambiguously characterized by its pole position and residues

$$BR_{\mathrm{pole}}(i) = \frac{|\mathrm{res}_i|^2 \sigma_i}{2\,|m_{\mathrm{pole}}|\,(\Gamma_{\mathrm{pole}}/2)}$$

with $\sigma_i$ =phase space channel i

used e.g. for $f_0(500) \to \gamma\gamma$

Thus, naively one may write

$$T_{ij} = -\sum_r \frac{\mathrm{res}_\mathrm{i}^\mathrm{r}\,\mathrm{res}_\mathrm{j}^\mathrm{r}}{s - s_r}$$

which is a sum of Breit-Wigners

But, in general this is wrong!

# Reason I: Unitarity

For one channel only one has: $\text{Im}(T) = \sigma|T|^2$
where $\sigma = \sqrt{1 - 4m^2/s}$ is the phase space. Then for

$$T = -\frac{\text{res}_{(1)}{}^2}{s - M_1^2 + iM_1\Gamma_1} - \frac{\text{res}_{(2)}{}^2}{s - M_2^2 + iM_1\Gamma_2}$$

we get using $\sigma\,\text{res}_{(\text{i})}{}^2 = M_i\Gamma_i$ (implies $\text{res}_{(i)}$ real)

$$\text{Im}(T) = \frac{\text{res}_{(1)}{}^2\Gamma_1\text{M}_1}{(s - M_1^2)^2 + M_1^2\Gamma_1^2} + \frac{\text{res}_{(2)}{}^2\Gamma_2\text{M}_2}{(s - M_2^2)^2 + M_2^2\Gamma_2^2}$$

$$\sigma|T|^2 = \frac{\text{res}_{(1)}{}^2\Gamma_1\text{M}_1}{(s - M_1^2)^2 + M_1^2\Gamma_1^2} + \frac{\text{res}_{(2)}{}^2\Gamma_2\text{M}_2}{(s - M_2^2)^2 + M_2^2\Gamma_2^2}$$

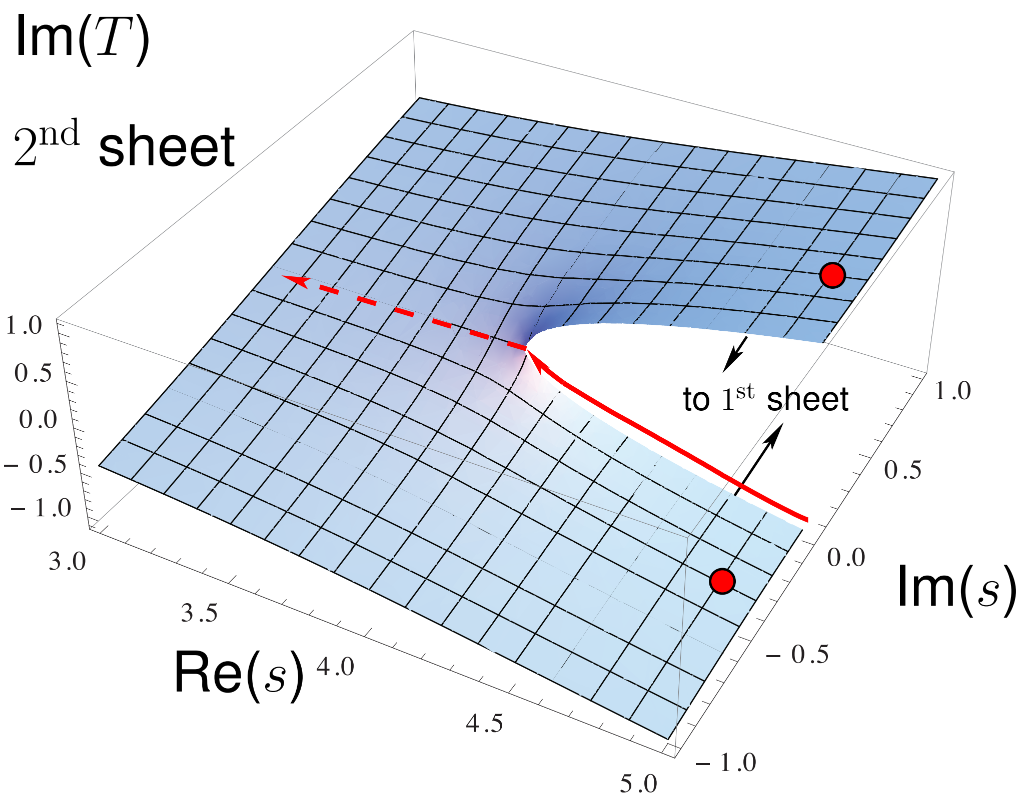
$$+2\sigma\text{Re}\left(\frac{\text{res}_{(1)}{}^2}{s - M_1^2 + iM_1\Gamma_1}\frac{\text{res}_{(2)}{}^2}{s - M_2^2 - iM_1\Gamma_2}\right)$$

Interference term violates unitarity!

# Reason II: Analyticity

→ For real $s < s_{\rm min}^{\rm thres}$, $S$ is real → Branchpoint at $s = s^{\rm thres}$

→ $S(s^*) = S^*(s) \longrightarrow$ pole at $s$ implies pole at $s^*$




For narrow resonances:

In resonance region:
only lower pole matters

At threshold:
both poles important!

For broad resonances:

always both important

Keep track of the cuts!

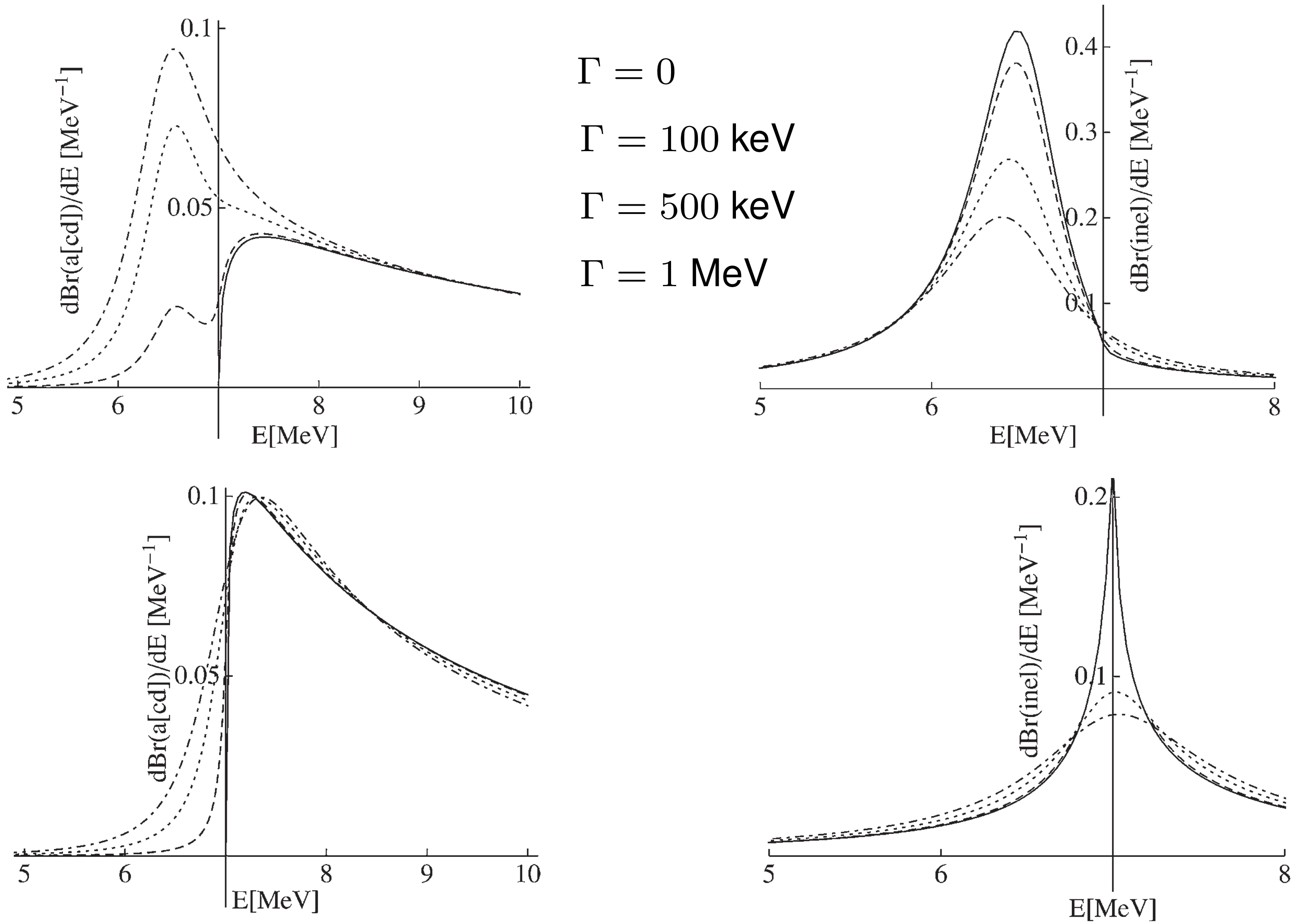
# Lineshapes of near threshold states

Braaten & Lu PRD76 (2007) 094028; C.H. et al. PRD81(2010)094028

**Direct channel** — **Inelastic channel**

bound state:

virtual state:

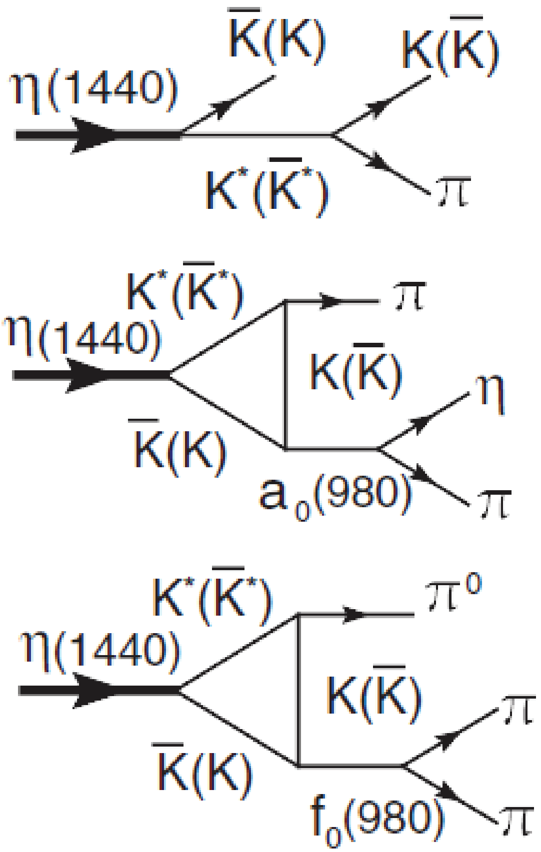


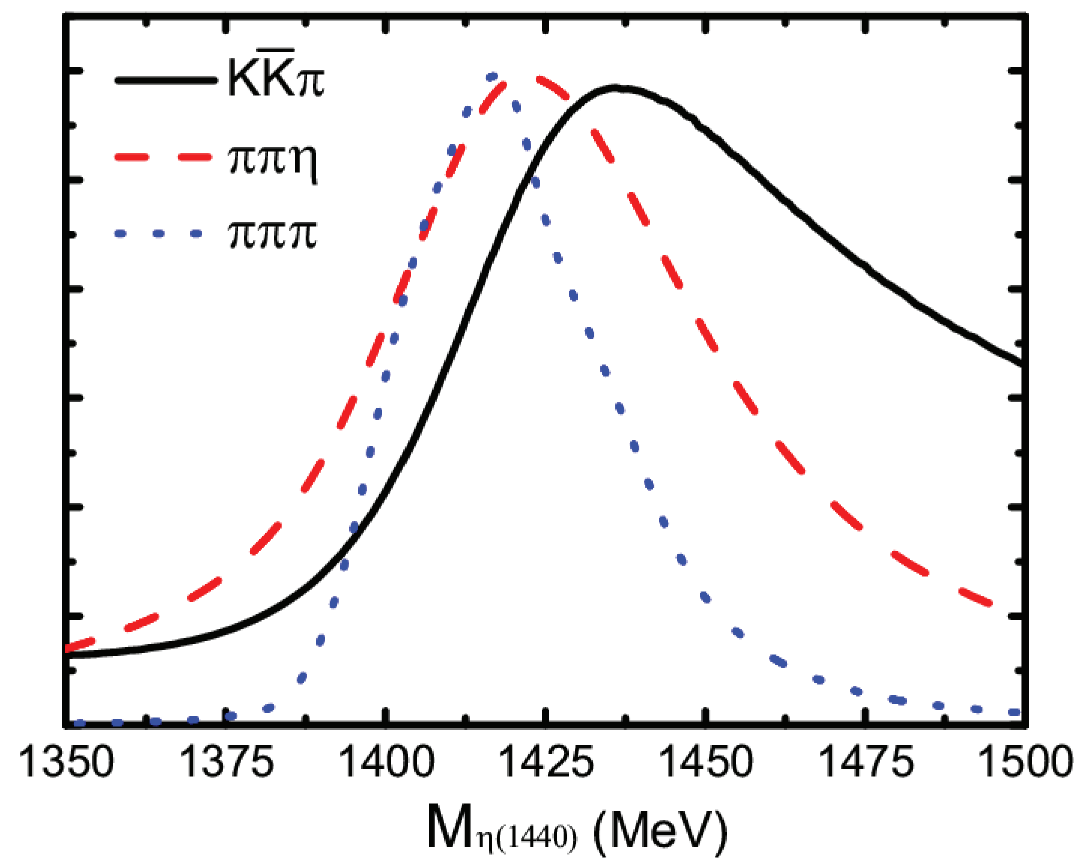

**Only molecules can appear as virtual states!**

# Only poles are physical

Line shapes and peak positions are channel dependent

Only pole–locations are physical!

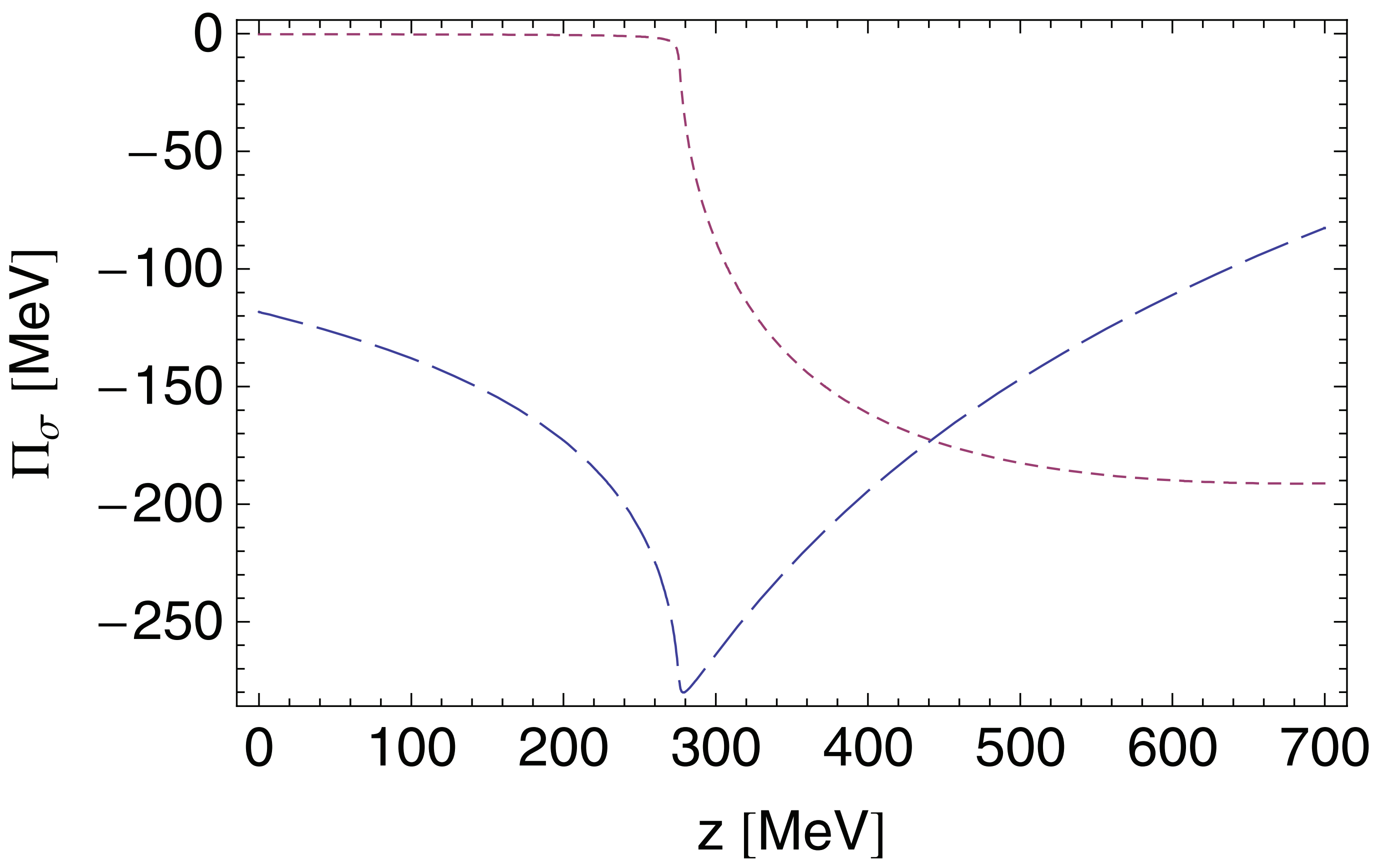



J. J. Wu et al., PRL **108**, 081803 (2012)

# Crossing a threshold

M. Döring et al. NPA 829(2009)170

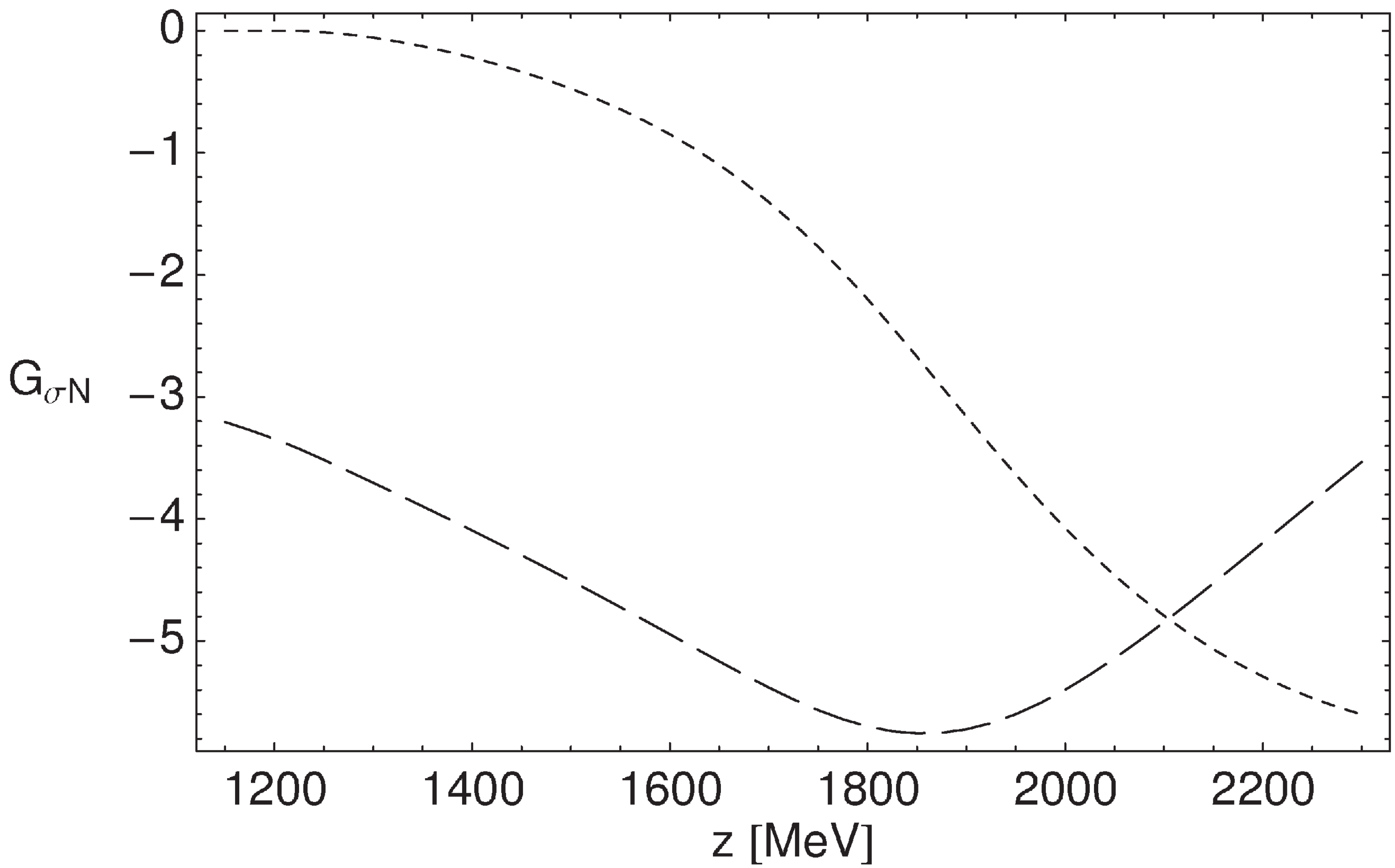



→ The amplitude is non–analytic in $s$ at the threshold

→ The imaginary part rises as $\sqrt{1 - s^{\text{thres}}/s}^{(2L+1)}$
very steep for $s$–waves; no cusp for $L > 0$

# ... with unstable constituents

M. Döring et al. NPA 829(2009)170




→ Non–analyticity in $s$ is moved into complex plane

→ The shorter the life time the weaker the structure

# Summary

Physical states show up as poles in the $S$–matrix

Poles appear as

→ on the physical sheet (bound states)

→ or on the unphysical sheets (virtual states/resonances)

In addition there are

→ branch points (on the real axis or in the unphysical sheets)

→ triangle singularities ...

To understand QCD in the non–perturbative regime

its singularity structure needs to be mapped out and understood