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To learn production of the scalar and tensor mesons in $\gamma\gamma^*(Q^2) \to \eta\pi^0$ reaction

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Introduction

This work is the development of N.N. Achasov and G.N. Shestakov, Phys. Rev. D 81, 094029 (2010), where the high-statistical Belle data on the $\gamma\gamma \to \eta\pi^0$ cross-section were analyzed.

We will analyze the Belle data together with the KLOE data on the $\phi \to \eta\pi^0\gamma$ decay and predict the $\gamma^*(Q^2)\gamma \to \eta\pi^0$ cross-section.

References:

S. Uehara *et al.* (Belle Collaboration) Phys. Rev. D 80, 032001 (2009).

A. Aloisio *et al.* (KLOE Collaboration) Phys. Lett. B 536, 209 (2002).

Light scalar mesons

• Nonet of light scalar mesons: $a_0(980)$, $f_0(980)$, $\sigma(600), \kappa(800)$

• Were discovered ~ 50 years ago and became hard problem for the naive quark model from the outset

• Elucidation of their nature can shed light on confinement and the chiral symmetry realization way in the low energy region

• Perturbation theory and sum rules don't work

• The $\sigma(600)$, $a_0(980)$, and $f_0(980)$ are studied in $\phi \to S\gamma$ decays, $\pi\pi$ scattering, $\gamma\gamma \to \pi\pi$, $\eta\pi^0$ and other processes

Light scalars in $\gamma\gamma$ collisions

Let's $S = \sigma(600),\ a_0(980),\ f_0(980)$ **and** $T = a_2(1320),\ f_2(1270)$**.**

In the $q\bar{q}$ **model** $\Gamma_{S\to\gamma\gamma}$ **are originated from direct** $S\gamma\gamma$ **coupling. From the experimental results**

$\Gamma_{f_2(1270)\to\gamma\gamma} \approx 3$ **keV**, $\Gamma_{a_2(1320)\to\gamma\gamma} \approx 1$ **keV**

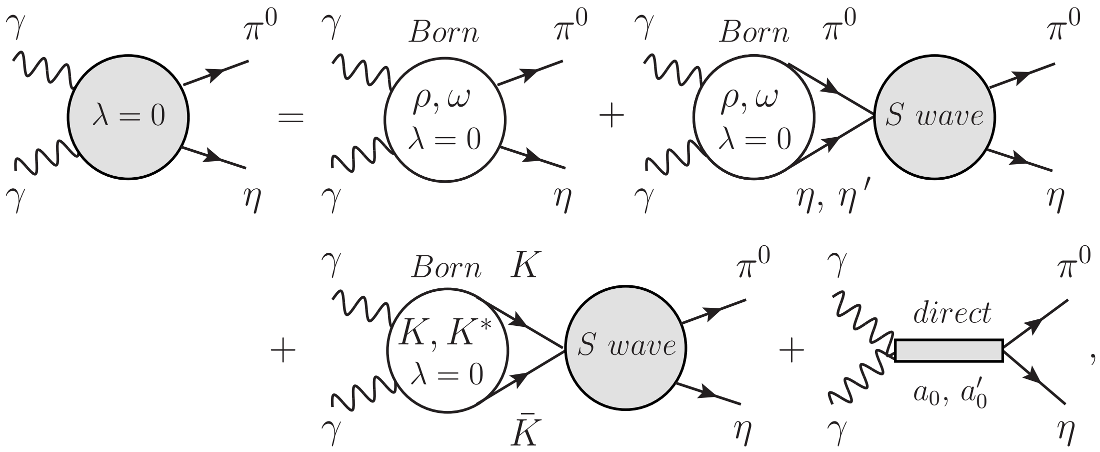
it was found $\Gamma_{f_0(980)\to\gamma\gamma} \geq 3.4$ **keV**, $\Gamma_{a_0(980)\to\gamma\gamma} \geq 1.3$ **keV.**

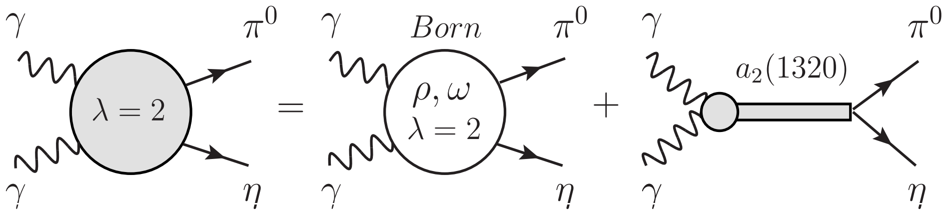
Four quark model: $\Gamma_{f_0(980)\to\gamma\gamma} \sim \Gamma_{a_0(980)\to\gamma\gamma} \sim 0.27$ **keV**
(Achasov, Devyanin, Shestakov, 1982)
These widths are caused by rescatterings:
$f_0 \to K^+K^- + \pi^+\pi^- \to \gamma\gamma$, $a_0 \to K^+K^- + \eta\pi^0 + \eta'\pi^0 \to \gamma\gamma$

The $\gamma\gamma \to \eta\pi^0$ data description

$$\sigma(\gamma\gamma \to \eta\pi^0) = \sigma_0 + \sigma_2$$

$$\sigma_\lambda = \frac{\rho_{\pi\eta}(s)}{64\pi s} \int |M_\lambda|^2 d\cos\theta \; ; \; \rho_{\pi\eta}(s) = \sqrt{\left(1 - \frac{(m_\eta + m_\pi)^2}{s}\right)\left(1 - \frac{(m_\eta - m_\pi)^2}{s}\right)}$$





$$
\begin{aligned}
M_0(\gamma\gamma\to\pi^0\eta;s,\theta) &= M_0^{\mathbf{Born}\,V}(\gamma\gamma\to\pi^0\eta;s,\theta) \\
&+\widetilde{I}^{V}_{\pi^0\eta}(s)\,T_{\pi^0\eta\to\pi^0\eta}(s)+\widetilde{I}^{V}_{\pi^0\eta'}(s)\,T_{\pi^0\eta'\to\pi^0\eta}(s) \\
&+\left(\widetilde{I}^{K^{*+}}_{K^+K^-}(s)-\widetilde{I}^{K^{*0}}_{K^0\bar{K}^0}(s)+\widetilde{I}^{K^+}_{K^+K^-}(s,x_1)\right) \\
&\times T_{K^+K^-\to\pi^0\eta}(s)+M^{\mathbf{direct}}_{\mathbf{res}}(s),
\end{aligned}
\tag{1}
$$

$$
\begin{aligned}
M_2(\gamma\gamma\to\pi^0\eta;s,\theta) &= M_2{}^{\mathbf{Born}\,V}(\gamma\gamma\to\pi^0\eta;s,\theta) \\
&+80\pi d^2_{20}(\theta)M_{\gamma\gamma\to a_2(1320)\to\pi^0\eta}(s),
\end{aligned}
\tag{2}
$$

$d^2_{20}(\theta)=(\sqrt{6}/4)\sin^2\theta$

$$T_{\eta\pi^0\to\eta\pi^0} = \frac{e^{2i\delta_B^{\eta\pi}} - 1}{2i\rho_{\eta\pi}(m)} + e^{2i\delta_B^{\eta\pi}} \sum_{S,S'} \frac{g_{S\eta\pi^0}\, G^{-1}_{SS'}\, g_{S'\eta\pi^0}}{16\pi}$$

$$G_{SS'}(m) = \begin{pmatrix} D_{a_0'}(m) & -\Pi_{a_0a_0'}(m) \\ -\Pi_{a_0a_0'}(m) & D_{a_0}(m) \end{pmatrix}$$

$$T_{ab\to cd} = e^{i\delta_B^{ab}} e^{i\delta_B^{cd}} T^{res}_{ab\to cd}$$

$$T^{res}_{ab\to cd} = \sum_{S,S'} \frac{g_{Sab}\, G^{-1}_{SS'}\, g_{S'cd}}{16\pi}$$

$$S, S' = a_0, a_0'$$

Kaon form factor

$$M_{00}^{Born\,K^+}(s', x_1) = 4\pi\alpha\frac{1-\rho_K^2(s)}{\rho_K(s)}\left(\ln\frac{1+\rho_K(s)}{1-\rho_K(s)} - \ln\frac{1+\frac{\rho_K(s)}{1+2x_1^2/s}}{1-\frac{\rho_K(s)}{1+2x_1^2/s}}\right)$$

$$\tilde{I}_{K^+K^-}^{K^+}(s, x_1) = \frac{s}{\pi}\int_{4m_K^2}^{\infty}\frac{\rho_K(s')M_{00}^{Born\,K^+}(s', x_1)}{s'(s'-s-i\epsilon)}$$

New data description: we get rid of kaon formfactor.

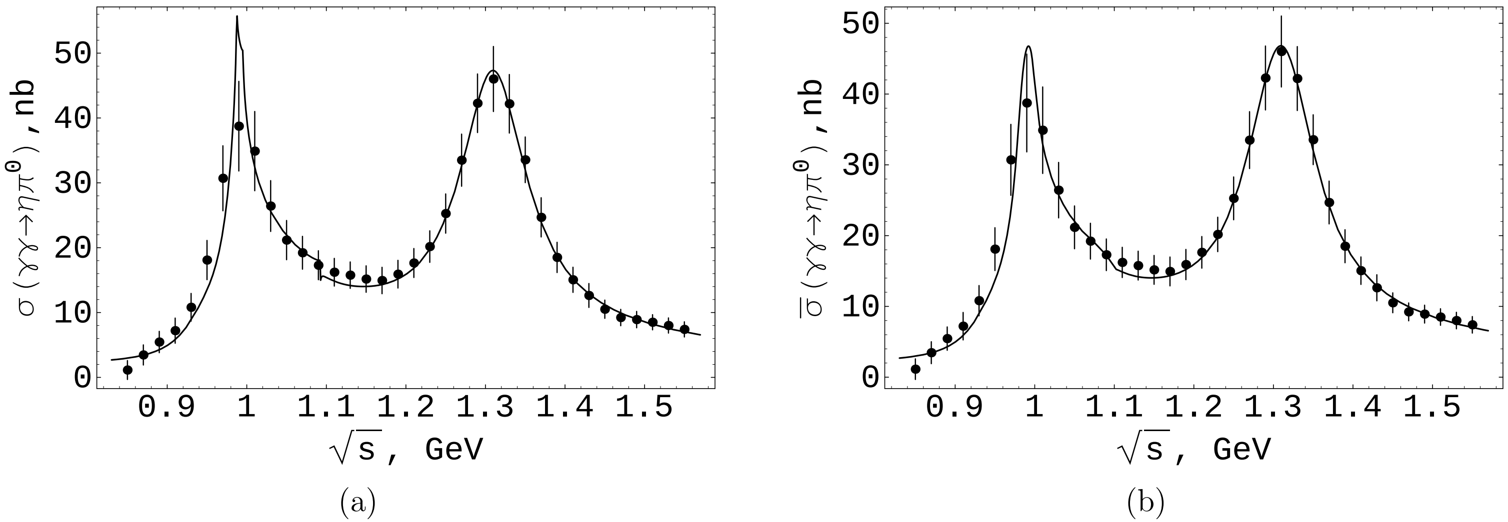


Рис. 1: The $\gamma\gamma \to \eta\pi^0$ cross-section, "as is" (a) and averaged taking into account 20 MeV bins (b). Points are the Belle data.

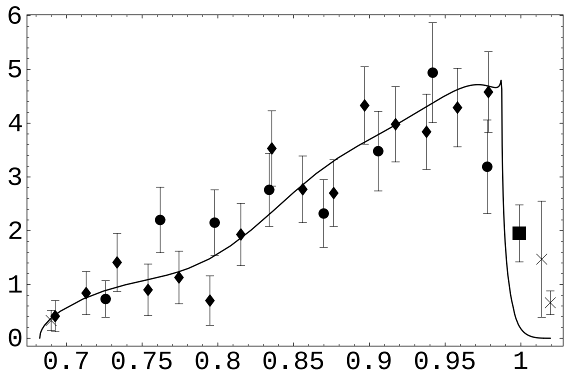


Рис. 2: The $\eta\pi^0$ mass spectrum in $\phi \to \eta\pi^0\gamma$ decay. Points represent the KLOE data.

The $\gamma\gamma \to \eta\pi^0$ and $\phi \to \eta\pi^0\gamma$ description

The invariant mass spectrum of $\eta\pi^0$ in $a_0 \to \eta\pi^0$

$$\frac{dN_{\eta\pi^0}}{dm} \sim \frac{2m^2}{\pi}\frac{\Gamma(a_0 \to \eta\pi^0, m)}{|D_{a_0}(m)|^2}. \qquad (3)$$

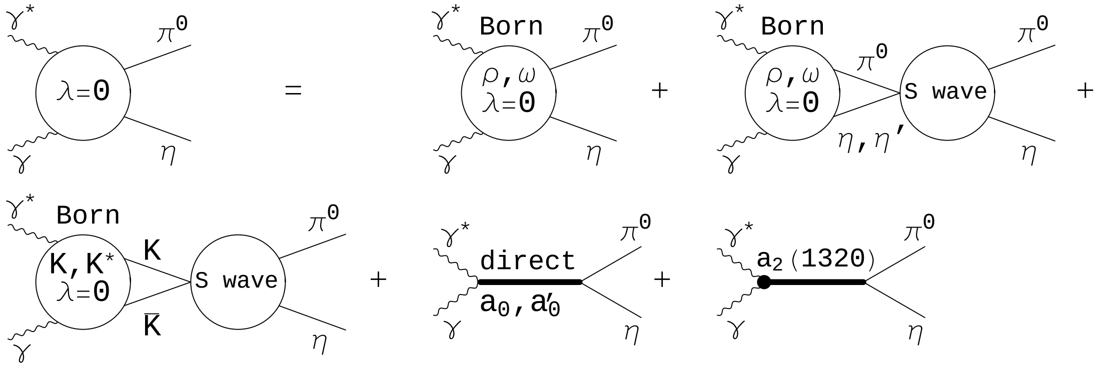
The width averaged over the resonance mass distribution

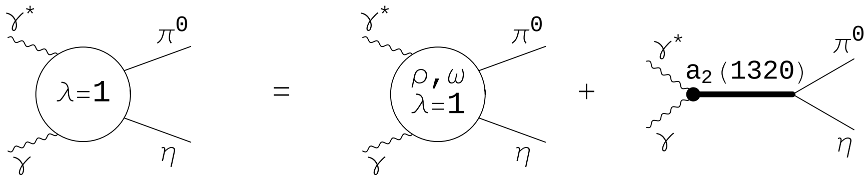
$$\langle\Gamma_{a_0\to\gamma\gamma}\rangle_{\eta\pi^0} = \int_{0.9\,\mathbf{GeV}}^{1.1\,\mathbf{GeV}} \frac{s}{4\pi^2}\sigma_0^{\mathbf{res}}(\gamma\gamma \to \pi^0\eta; s)d\sqrt{s} \qquad (4)$$

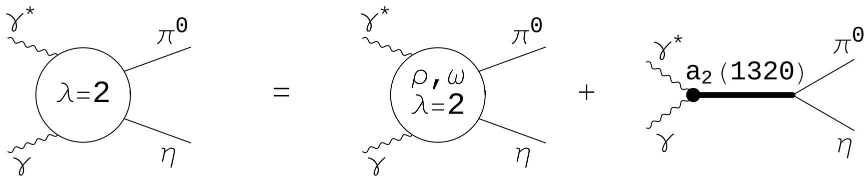
Fit	**1**	**2**	**3**	**4**
m_{a_0}, **MeV**	993.86	994.50	995.34	988.69
$g^2_{a_0K^+K^-}/4\pi$, **GeV^2**	0.60	0.47	1.19	1.10
$g^2_{a_0\eta\pi}/4\pi$, **GeV^2**	0.60	0.76	1.09	1.04
$g^{(0)}_{a_0\gamma\gamma}$, 10^{-3} **GeV^{-1}**	1.8	2.7810	4.5919	3.2520
$\Gamma^{(0)}_{a_0\to\gamma\gamma}$, **keV**	0.063	0.151	0.414	0.203
$< \Gamma^{direct}_{a_0\to\gamma\gamma} >_{\eta\pi^0}$, **keV**	0.019	0.031	0.030	0.024
$< \Gamma_{a_0\to(K\bar{K}+\eta\pi^0+\eta'\pi^0)\to\gamma\gamma} >_{\eta\pi^0}$, **keV**	0.126	0.113	0.141	0.129
$< \Gamma_{a_0\to(K\bar{K}+\eta\pi^0+\eta'\pi^0+direct)\to\gamma\gamma} >_{\eta\pi^0}$, **keV**	0.225	0.236	0.273	0.243
$\Gamma_{a_0}(m_{a_0})$, **MeV**	116.76	140.81	218.65	186.75
$\Gamma^{eff}_{a_0}$, **MeV**	34.6	57.24	38.76	44.76

$m_{a_0'}$, **MeV**	1400	1400	1300	1500
$g^2_{a_0' K^+K^-}/4\pi$, **GeV**^{2}	0.21	0.005	0.07	0.44
$g^2_{a_0'\eta\pi}/4\pi$, **GeV**^{2}	0.77	0.18	0.52	0.63
$g^2_{a_0'\eta'\pi}/4\pi$, **GeV**^{2}	1.80	4.12	2.15	2.64
$\Gamma^{(0)}_{a_0'\to\gamma\gamma}(m_{a_0'})$, **keV**	1.65	5.38	2.96	5.21
$\Gamma_{a_0'}(m_{a_0'})$, **MeV**	330.91	399.4	271.0	453.40
$\chi^2_{\gamma\gamma}$ / 36 **points**	12.4	4.8	5.3	6.7
χ^2_{sp} / 24 **points**	24.5	24.7	24.1	24.3
$(\chi^2_{\gamma\gamma}+\chi^2_{sp})$/**n.d.f.**	36.9/46	29.5/46	29.4/46	31.0/46

Non-zero $k_2^2 = -Q^2$







The effective Lagrangian of the $a_2 \to V(1)V(2)$ transition (N.N. Achasov and V.A. Karnakov, Z. Phys. C 30, 141 (1986))

$$L = g_{a_2 V(1) V(2)} T_{\mu\nu} F^{V(1)}_{\mu\sigma} F^{V(2)}_{\nu\sigma}\,,$$

$$F^{V(i)}_{\mu\sigma} = \partial_\mu V(i)_\sigma - \partial_\sigma V(i)_\mu\,;\; i = 1, 2$$

$\{V(1), V(2)\} = \{\rho, \omega\}, \{\rho', \omega'\}, \{\rho'', \omega''\}$

$$M_2(\gamma^*\gamma \to a_2(1320) \to \pi^0\eta; s, Q, \theta) = A(s, Q)\sin^2\theta$$

$$M_1(\gamma^*\gamma \to a_2(1320) \to \pi^0\eta; s, Q, \theta) = -\sqrt{2}A(s, Q)\sqrt{\frac{Q^2}{s}}\sin\theta\cos\theta$$

$$M_0(\gamma^*\gamma \to a_2(1320) \to \pi^0\eta; s, Q, \theta) = -A(s, Q)\frac{Q^2}{s}(\cos^2\theta - \frac{1}{3})$$

$$A(s, Q) = 20\pi F_{a_2}(Q)\sqrt{\frac{6s\Gamma_{a_2\to\gamma\gamma}(s)\Gamma_{a_2\to\eta\pi^0}(s)}{\rho_{\eta\pi^0}(s)}}\frac{1}{D_{a_2}(s)}(1 + \frac{Q^2}{s})$$

QCD: At $Q \to \infty$ $M_0 \sim 1/Q^2$, $M_1 \sim 1/Q^3$, $M_0 \sim 1/Q^4$. **It is reached if** $F_{a_2}(Q) \sim 1/Q^4$.

V.L. Chernyak and A.R. Zhitnitsky, Phys. Rep. 112, 173 (1984).
V.N. Baier and A.G. Grozin, Fiz. Elem. Chast. At. Yad. 16, 5 (1985) [Sov. J. Part. Nucl. 16, 1 (1985)].

Let's $F_{a_2}(Q) = \widetilde{F}_{a_2}(Q)/\widetilde{F}_{a_2}(0)$,

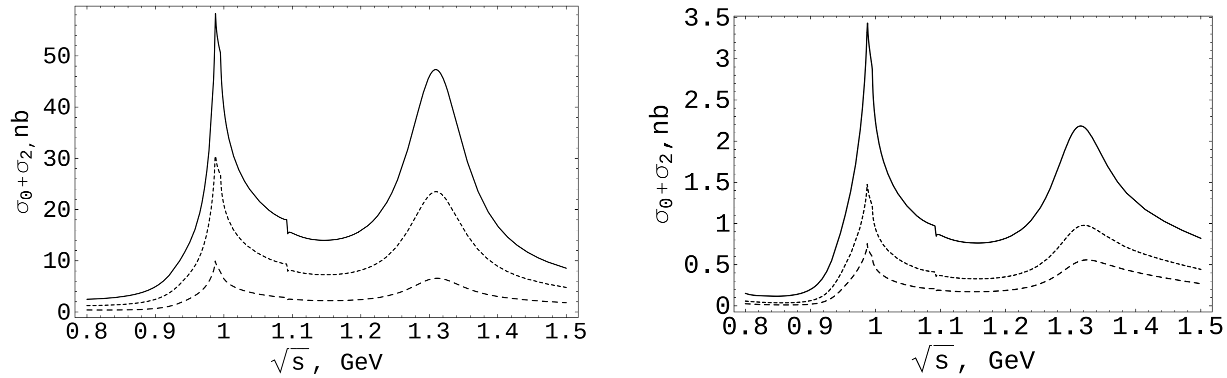
$$\widetilde{F}_{a_2}(Q) = \frac{g_{a_2\rho\omega}}{f_\rho f_\omega}(\frac{1}{1+Q^2/m_\rho^2}+\frac{1}{1+Q^2/m_\omega^2})+\frac{g_{a_2\rho'\omega'}}{f_{\rho'} f_{\omega'}}(\frac{1}{1+Q^2/m_{\rho'}^2}+\frac{1}{1+Q^2/m_{\omega'}^2})$$

$$+\frac{g_{a_2\rho''\omega''}}{f_{\rho''} f_{\omega''}}(\frac{1}{1+Q^2/m_{\rho''}^2}+\frac{1}{1+Q^2/m_{\omega''}^2}) = \frac{g_{a_2\rho\omega}}{f_\rho f_\omega}(\frac{1}{1+Q^2/m_\rho^2}+\frac{1}{1+Q^2/m_\omega^2}$$

$$+a(\frac{1}{1+Q^2/m_{\rho'}^2} + \frac{1}{1+Q^2/m_{\omega'}^2}) + b(\frac{1}{1+Q^2/m_{\rho''}^2} + \frac{1}{1+Q^2/m_{\omega''}^2}))$$

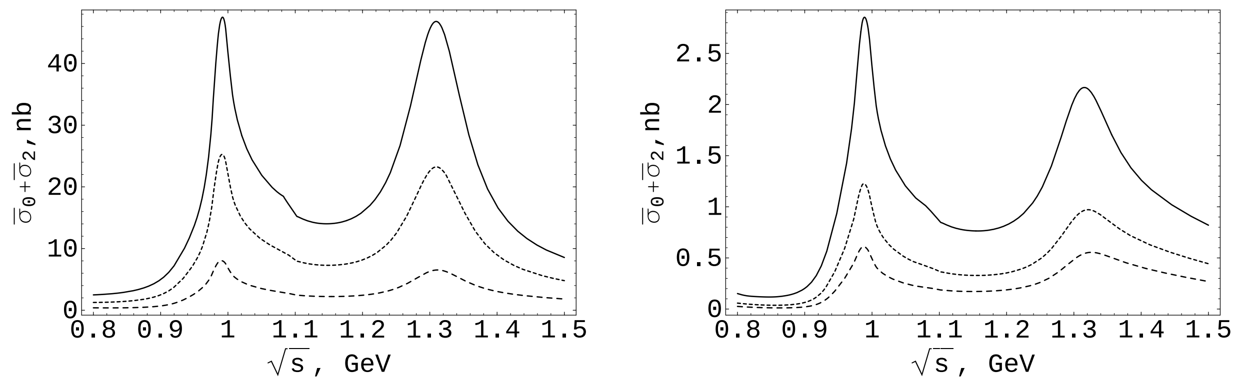
The condition $m_\rho^2+m_\omega^2+a(m_{\rho'}^2+m_{\omega'}^2)+b(m_{\rho''}^2+m_{\omega''}^2) = 0$ **leads to asymptotics, based on QCD.**

The loop function $\tilde{I}^{K^+}_{K^+K^-}(s,Q) \to 8\alpha \ln\frac{Q^2}{m_K^2}$ **at** $Q \to \infty$**.**



(a) (b)

Рис. 3: The cross-section $\sigma_0(\gamma\gamma^*(Q^2) \to \eta\pi^0, s) + \sigma_2(\gamma\gamma^*(Q^2) \to \eta\pi^0, s)$, $|\cos\theta| < 0.8$, for Fit1 parameters, $a = -0.082$ and $b = -0.15$. a) Solid line - $Q^2 = 0$, point line - $Q^2 = 0.25$ GeV^2, dashed line - $Q^2 = 1$ GeV^2; b) Solid line - $Q^2 = 2.25$ GeV^2, point line - $Q^2 = 4$ GeV^2, dashed line - $Q^2 = 6.25$ GeV^2.



(a) (b)

Рис. 4: The same cross-section averaged over bins.

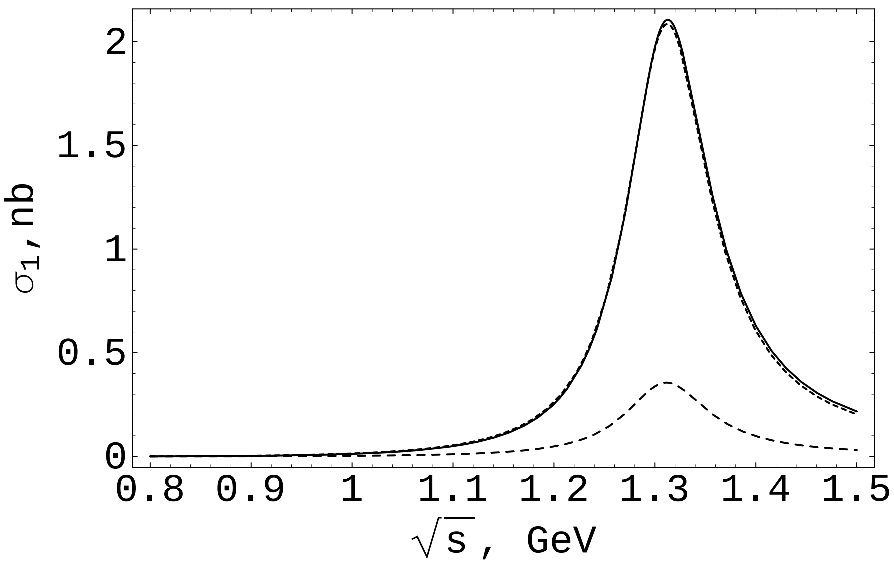
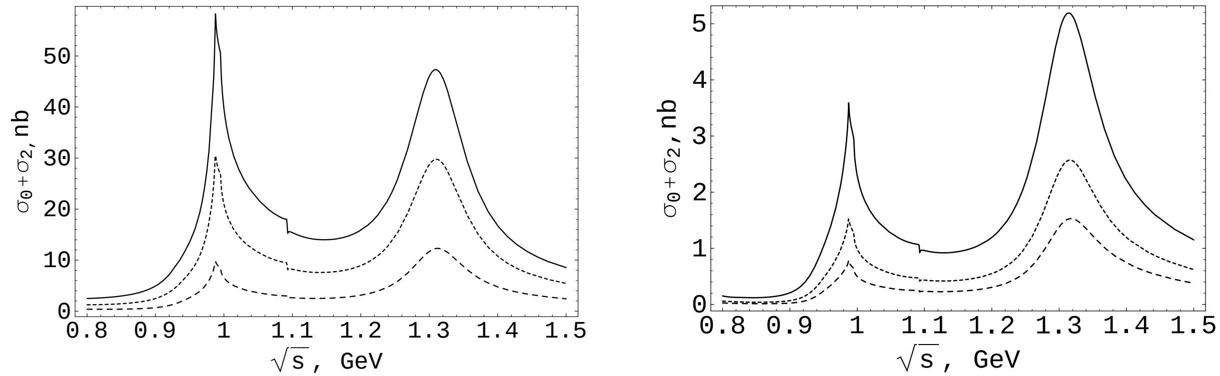
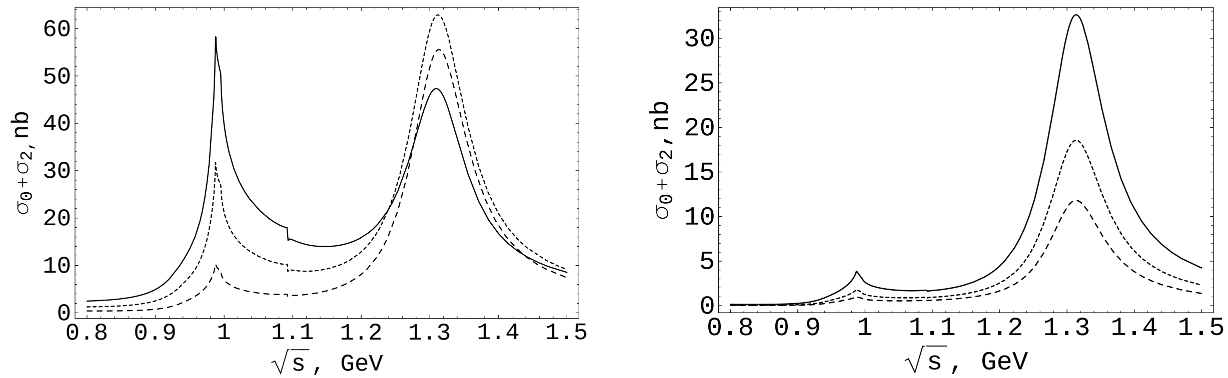


Рис. 5: $\sigma_1(\gamma\gamma^*(Q^2) \to \eta\pi^0, s)$, $|\cos\theta| < 0.8$ for the same parameters. Solid line - $Q^2 = 0.25$ GeV^2, point line - $Q^2 = 1$ GeV^2, dashed line - $Q^2 = 6.25$ GeV^2.



(a) (b)

Рис. 6: The $\sigma_0(\gamma\gamma^*(Q^2) \to \eta\pi^0, s) + \sigma_2(\gamma\gamma^*(Q^2) \to \eta\pi^0, s)$, for Fit 1, $a = 3.835$ and $b = -3$.



(a) (b)

Рис. 7: The $\sigma_0(\gamma\gamma^*(Q^2) \to \eta\pi^0, s) + \sigma_2(\gamma\gamma^*(Q^2) \to \eta\pi^0, s)$, $|\cos\theta| < 0.8$, for Fit 1, $a = -4.413$ and $b = 3$.

Conclusion

1. Belle data Данные on the $\gamma\gamma \to \eta\pi^0$ where analyzed simultaneously with the KLOE data on the $\phi \to \eta\pi^0\gamma$ decay without kaon formfactor, $G_{K^+}(t,u) = 1$. The data supports scenario based on the four-quark model of $a_0(980)$.
2. The prediction of the $\sigma(\gamma\gamma^*(Q^2) \to \eta\pi^0)$ is presented, different variants are considered.
3. The QCD-based asymptotics of $\sigma(\gamma\gamma^*(Q^2) \to \eta\pi^0)$ is reached after taking into account vector excitations.