

Low x evolution equation for quadrupole operator

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Introduce the **light cone vectors** n_1 and n_2

$$n_1 = (1, 0, 0, 1), \quad n_2 = \frac{1}{2}(1, 0, 0, -1), \quad n_1^+ = n_2^- = n_1 n_2 = 1$$

For any p define p^\pm

$$p^+ = p n_2 = \frac{1}{2}(p^0 + p^3), \quad p^- = p n_1 = p^0 - p^3,$$

$$p^2 = 2p^+ p^- - \vec{p}^2;$$

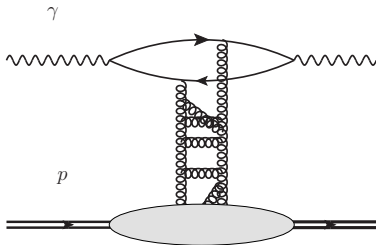
The **scalar products**:

$$p = p^+ n_1 + p^- n_2 + p_\perp, \quad (pk) = p^\mu k_\mu = p^+ k^- + p^- k^+ - \vec{p} \vec{k}.$$

Wilson line describing interaction with **external field** b_η^- made of **slow** gluons with $p^+ < e^\eta$

$$U_{\vec{z}}^\eta = P e^{ig \int_{-\infty}^{+\infty} dz^+ b_\eta^-(z^+, \vec{z})}, \quad b_\eta^- = \int \frac{d^4 p}{(2\pi)^4} e^{-ipz} b^-(p) \theta(e^\eta - p^+).$$

Dipole picture $s \gg Q^2 \gg \Lambda_{QCD}^2$



$$\sigma_{\gamma^*}(s, Q^2) = \int d^2\mathbf{r} |\Psi_{\gamma^*}(\mathbf{r}, Q^2)|^2 \sigma_{dip}(\mathbf{r}, s), \quad \sigma_{dip}(\mathbf{r}, s) = 2 \int d\mathbf{b} \left(1 - \frac{1}{N_c} F(\mathbf{b}, \mathbf{r}, s)\right)$$

$\mathbf{r} = \mathbf{r}_1 - \mathbf{r}_2$ — dipole size, $\mathbf{b} = \frac{1}{2}(\mathbf{r}_1 + \mathbf{r}_2)$ — impact parameter, $F = \text{tr}(U_1 U_2^\dagger)$, — dipole Green function,
 $U_i = U_i^\eta$ — Wilson lines, describing fast moving quarks interacting with the target.

η — rapidity divide, gluons with $p^+ > e^\eta$ belong to photon wavefunction, gluons with $p^+ < e^\eta$ belong to Wilson lines, describing the field of the target.

$tr(U_1 U_2^\dagger)$ obeys the LO **Balitsky-Kovchegov** evolution equation

$$\frac{\partial tr(U_1 U_2^\dagger)}{\partial \eta} = \frac{\alpha_s}{2\pi^2} \int d\vec{r}_4 \frac{\vec{r}_{12}^2}{\vec{r}_{14}^2 \vec{r}_{42}^2} \left[tr(U_1 U_4^\dagger) tr(U_4 U_2^\dagger) - N_c tr(U_1 U_2^\dagger) \right].$$

LO equation was obtained in 1996-99, NLO — in 2007-2010 (Balitsky and Chirilli).

Shock wave

For a **fast** moving particle with the velocity $-\beta$ and the field strength tensor $\mathbb{F}(x^+, x^-, \vec{x})$ in **its rest frame**, in the **observer's frame** the field will look like

$$\mathfrak{F}^{-i}(y^+, y^-, \vec{y}) = \lambda \mathbb{F}^{-i}(\lambda y^+, \frac{1}{\lambda} y^-, \vec{y}) \rightarrow \delta(y^+) \mathfrak{F}^i(\vec{y}),$$

$$\mathfrak{F}^{-i} \gg \mathfrak{F}^{\dots}$$

in the **Regge limit** $\lambda \rightarrow +\infty$, $\lambda = \sqrt{\frac{1+\beta}{1-\beta}}$.

Therefore the natural choice for the gauge is $b^{i,+} = 0$,

b^- is the solution of the equations

$$\frac{\partial b^-}{\partial y^i} = \delta(y^+) \mathfrak{F}^i(\vec{y}), \text{ i.e.}$$

$$b^\mu(y) = \delta(y^+) B(\vec{y}) n_2^\mu$$

It is the **shock-wave** field.

Propagator in the shock wave background

Choose the gluon field \mathcal{A} in the gauge $\mathcal{A}n_2 = 0$ as a sum of external classical b and quantum A .

$$\mathcal{A} = A + b, \quad b^\mu(x) = \delta(x^+) B(\vec{x}) n_2^\mu.$$

The A - b interaction lagrangian has only one vertex

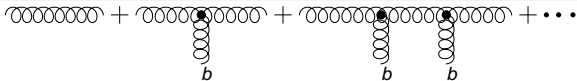
$$\mathcal{L}_i = \frac{g}{2} f^{acb} (b^-)^c g_\perp^{\alpha\beta} \left[A_\alpha^a \overleftrightarrow{\frac{\partial}{\partial x^-}} A_\beta^b \right].$$

The free propagator $G_0^{\mu\nu}(x^+, p^+, \vec{p}) =$

$$= \frac{-d_0^{\mu\nu}(p^+, p_\perp)}{2p^+} e^{-i\frac{\vec{p}_\perp^2 x^+}{2p^+}} (\theta(x^+)\theta(p^+) - \theta(-x^+)\theta(-p^+)) + n_2^\mu n_2^\nu \dots,$$

$$d_0^{\mu\nu}(p) = g_\perp^{\mu\nu} - \frac{p_\perp^\mu n_2^\nu + p_\perp^\nu n_2^\mu}{p^+} - \frac{n_2^\mu n_2^\nu \vec{p}^2}{(p^+)^2}.$$

Propagator in the shock-wave background



Sum the diagrams

- b does not depend on x^- , hence the conservation of p^+ ,
- $b \sim \delta(x^+)$, hence $e^{-i\vec{p}^2(x_1^+ - x_2^+)/2p^+} \rightarrow 1$ in every internal vertex,
- $g_{\perp}^{\mu\nu} d_{0\nu\rho} g_{\perp}^{\rho\sigma} = g_{\perp}^{\mu\sigma}$, hence no dependence on $\vec{p} \implies$ conservation of \vec{x} in every internal vertex

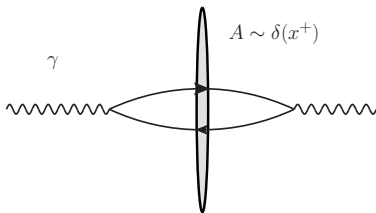
Propagator in the **shock-wave** background:

$$G_{\mu\nu}(x, y)|_{x^+ > 0 > y^+} = 2iA^\mu(x) \int d^4z \delta(z^+) F^{+i}(z) \frac{U_{\vec{z}}}{\frac{\partial}{\partial z^-}} F^{+i}(z) A^\nu(y).$$

where the interaction with b is through Wilson line

$$U_{\vec{z}} = P e^{ig \int_{y^+}^{x^+} dz^+ b^-(z^+, \vec{z})}.$$

Dipole picture



Color field of a **fast** moving particle $A^- \sim \delta(z^+)A^\eta(z_\perp)$
 $A^\eta(z_\perp)$ contains slow components with rapidities $< \eta$

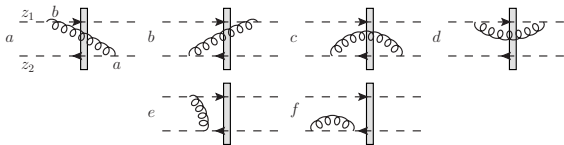
Quark propagator in such an external field $G(x, y) \sim U^\eta(z_\perp)$

DIS matrix element contains a **Wilson loop = color dipole operator** $U_{12}^\eta = \text{tr}(U^\eta(z_{1\perp})U^{\eta\dagger}(z_{2\perp}))$. Balitsky 1996

Balitsky derivation of the BK equation

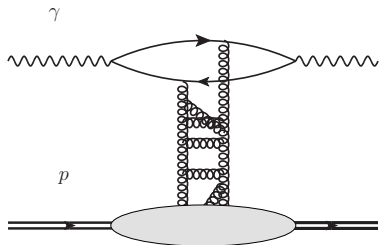
To derive the evolution equation we have to change $\eta \rightarrow \eta + \Delta\eta$ and integrate over the fields with the rapidities in the strip $\Delta\eta$

$$U_{12}^{\eta+\Delta\eta} = U_{12}^{\eta} + \frac{\langle 0 | T(U_{12}^{\Delta\eta} e^{i \int \mathcal{L}(z) dz}) | 0 \rangle}{\langle 0 | T(e^{i \int \mathcal{L}(z) dz}) | 0 \rangle}.$$

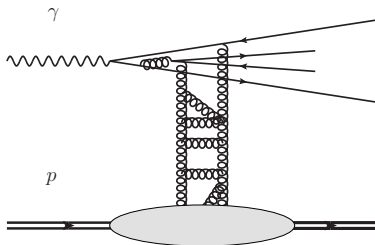


$$\frac{\partial U_{12}^{\eta}}{\partial \eta} = \frac{\alpha_s}{2\pi^2} \int d\vec{z}_4 \frac{\vec{z}_{12}^2}{\vec{z}_{14}^2 \vec{z}_{42}^2} [U_{14}^{\eta} U_{42}^{\eta} - N_c U_{12}^{\eta}].$$

Motivation



Dipole picture,
BK equation for dipole
 $tr(U_1 U_2^\dagger)$



Evolution equation for
quadrupole operator
 $tr(U_1 U_2^\dagger U_3 U_4^\dagger)$

LO Evolution equation for quadrupole

$$\text{tr}(U_i U_j^\dagger \dots U_k U_l^\dagger) \equiv \mathbf{U}_{ij^\dagger \dots kl^\dagger},$$

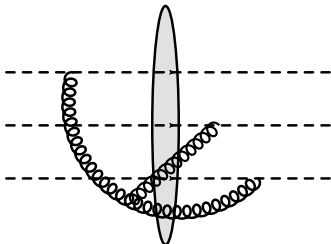
Jalilian-Marian Kovchegov Dumitru 2004, 2010

Dominguez Mueller Munier Xiao 2011

$$\begin{aligned} \frac{\partial \mathbf{U}_{12^\dagger 34^\dagger}}{\partial \eta} &= \frac{\alpha_s}{4\pi^2} \int d\vec{r}_0 \left\{ \frac{\vec{r}_{14}^2}{\vec{r}_{10}^2 \vec{r}_{40}^2} (\mathbf{U}_{10^\dagger} \mathbf{U}_{02^\dagger 34^\dagger} + \mathbf{U}_{4^\dagger 0} \mathbf{U}_{12^\dagger 30^\dagger} - (0 \rightarrow 1)) \right. \\ &+ \frac{\vec{r}_{12}^2}{\vec{r}_{10}^2 \vec{r}_{20}^2} (\mathbf{U}_{10^\dagger} \mathbf{U}_{02^\dagger 34^\dagger} + \mathbf{U}_{2^\dagger 0} \mathbf{U}_{10^\dagger 34^\dagger} - (0 \rightarrow 1)) \\ &- \frac{\vec{r}_{24}^2}{2\vec{r}_{20}^2 \vec{r}_{40}^2} (\mathbf{U}_{10^\dagger} \mathbf{U}_{02^\dagger 34^\dagger} + \mathbf{U}_{30^\dagger} \mathbf{U}_{04^\dagger 12^\dagger} - (0 \rightarrow 4)) \\ &- \frac{\vec{r}_{13}^2}{2\vec{r}_{10}^2 \vec{r}_{30}^2} (\mathbf{U}_{4^\dagger 0} \mathbf{U}_{12^\dagger 30^\dagger} + \mathbf{U}_{2^\dagger 0} \mathbf{U}_{34^\dagger 10^\dagger} - (0 \rightarrow 1)) \\ &\left. + (1 \leftrightarrow 3, 2 \leftrightarrow 4) \right\}. \end{aligned}$$

NLO evolution of 1 and 2 Wilson lines with open indices from
Balitsky and Chirilli 2013

NLO evolution of 3 Wilson lines Grabovsky 2013



$$\langle K_{NLO} \otimes \mathbf{U}_{12^{\dagger}34^{\dagger}} \rangle = \frac{\alpha_s^2}{8\pi^4} \int d\vec{r}_0 d\vec{r}_5 (\mathbf{G}_s + \mathbf{G}_a) + \frac{\alpha_s^2}{8\pi^3} \int d\vec{r}_0 (\mathbf{G}_\beta + \mathbf{G}),$$

NLO corrections: symmetric part

$$\mathbf{G}_S = \mathbf{G}_{S1} + n_f \mathbf{G}_q + \mathbf{G}_{S2} + (1 \leftrightarrow 3, 2 \leftrightarrow 4).$$

$$\mathbf{G}_{S1} = (\{\mathbf{U}_{0\uparrow 34\uparrow 15\uparrow 02\uparrow 5} - \mathbf{U}_{5\uparrow 0} \mathbf{U}_{2\uparrow 5} \mathbf{U}_{0\uparrow 34\uparrow 1} - (5 \rightarrow 0)\} + (5 \leftrightarrow 0)) (L_{12} + L_{32} - L_{13}) \\ + (\{\mathbf{U}_{0\uparrow 15\uparrow 02\uparrow 34\uparrow 5} - \mathbf{U}_{0\uparrow 5} \mathbf{U}_{5\uparrow 1} \mathbf{U}_{2\uparrow 34\uparrow 0} - (5 \rightarrow 0)\} + (5 \leftrightarrow 0)) (L_{12} + L_{14} - L_{42}),$$

$$\mathbf{G}_q = (\{\frac{\mathbf{U}_{0\uparrow 34\uparrow 12\uparrow 5} + \mathbf{U}_{2\uparrow 34\uparrow 15\uparrow 0}}{N_c} - \frac{\mathbf{U}_{0\uparrow 5} \mathbf{U}_{2\uparrow 34\uparrow 1}}{N_c^2} - \mathbf{U}_{2\uparrow 5} \mathbf{U}_{0\uparrow 34\uparrow 1} - (5 \rightarrow 0)\} + (5 \leftrightarrow 0)) \\ \times \frac{1}{2} (L_{12}^q + L_{32}^q - L_{13}^q) + \frac{1}{2} (L_{12}^q + L_{14}^q - L_{42}^q) \\ \times (\{\frac{\mathbf{U}_{0\uparrow 12\uparrow 34\uparrow 5} + \mathbf{U}_{2\uparrow 34\uparrow 15\uparrow 0}}{N_c} - \frac{\mathbf{U}_{0\uparrow 5} \mathbf{U}_{2\uparrow 34\uparrow 1}}{N_c^2} - \mathbf{U}_{5\uparrow 1} \mathbf{U}_{2\uparrow 34\uparrow 0} - (5 \rightarrow 0)\} + (5 \leftrightarrow 0)),$$

$$2\mathbf{G}_{S2} = (\mathbf{U}_{0\uparrow 15\uparrow 02\uparrow 34\uparrow 5} - \mathbf{U}_{0\uparrow 5} \mathbf{U}_{5\uparrow 1} \mathbf{U}_{2\uparrow 34\uparrow 0} + (5 \leftrightarrow 0)) (M_2^{14} + M_4^{12} + (5 \leftrightarrow 0)) \\ + (\mathbf{U}_{0\uparrow 34\uparrow 15\uparrow 02\uparrow 5} - \mathbf{U}_{5\uparrow 0} \mathbf{U}_{2\uparrow 5} \mathbf{U}_{0\uparrow 34\uparrow 1} + (5 \leftrightarrow 0)) (M_1^{23} + M_3^{21} + (5 \leftrightarrow 0)) \\ + (\mathbf{U}_{0\uparrow 34\uparrow 52\uparrow 05\uparrow 1} - \mathbf{U}_{0\uparrow 1} \mathbf{U}_{2\uparrow 5} \mathbf{U}_{4\uparrow 05\uparrow 3} + (5 \leftrightarrow 0)) (M_1^{34} - M_1^{24} + M_2^{43} - M_2^{13} + (5 \leftrightarrow 0)) \\ + (\mathbf{U}_{0\uparrow 35\uparrow 02\uparrow 54\uparrow 1} - \mathbf{U}_{0\uparrow 3} \mathbf{U}_{2\uparrow 5} \mathbf{U}_{4\uparrow 15\uparrow 0} + (5 \leftrightarrow 0)) (M_3^{14} - M_3^{24} + M_2^{41} - M_2^{31} + (5 \leftrightarrow 0))$$



NLO corrections: antisymmetric part

$$\mathbf{G}_a = \mathbf{G}_{a1} + \mathbf{G}_{a2} + \mathbf{G}_{a3}.$$

$$\begin{aligned}\mathbf{G}_{a1} = & (\mathbf{U}_{0\uparrow 1} \mathbf{U}_{2\uparrow 5} \mathbf{U}_{4\uparrow 05\uparrow 3} + \mathbf{U}_{0\uparrow 34\uparrow 52\uparrow 05\uparrow 1} - (5 \leftrightarrow 0)) (M_2^{31} - M_2^{34} - M_1^{42} + M_1^{43}) \\ & + (\mathbf{U}_{0\uparrow 3} \mathbf{U}_{2\uparrow 5} \mathbf{U}_{4\uparrow 15\uparrow 0} + \mathbf{U}_{0\uparrow 35\uparrow 02\uparrow 54\uparrow 1} - (5 \leftrightarrow 0)) (M_2^{13} - M_2^{14} - M_3^{42} + M_3^{41}) \\ & + (1 \leftrightarrow 3, 2 \leftrightarrow 4).\end{aligned}$$

$$\begin{aligned}\mathbf{G}_{a2} = & \frac{1}{2} (\mathbf{U}_{0\uparrow 34\uparrow 15\uparrow 02\uparrow 5} - (5 \leftrightarrow 0)) (\tilde{L}_{13} + 2M_{21} - 2M_{23} - M_1^{23} + M_3^{21} - (5 \leftrightarrow 0)) \\ & + \frac{1}{2} (\mathbf{U}_{0\uparrow 15\uparrow 02\uparrow 34\uparrow 5} - (5 \leftrightarrow 0)) (\tilde{L}_{42} - 2M_{12} + 2M_{14} + M_2^{14} - M_4^{12} - (5 \leftrightarrow 0)) \\ & + (1 \leftrightarrow 3, 2 \leftrightarrow 4).\end{aligned}$$

$$\begin{aligned}\mathbf{G}_{a3} = & \frac{1}{2} (\mathbf{U}_{0\uparrow 5} \mathbf{U}_{5\uparrow 1} \mathbf{U}_{2\uparrow 34\uparrow 0} - (5 \leftrightarrow 0)) (\tilde{L}_{12} + \tilde{L}_{14} - 2M_{24} + M_2^{14} + M_4^{12} - (5 \leftrightarrow 0)) \\ & + \frac{1}{2} (\mathbf{U}_{5\uparrow 0} \mathbf{U}_{2\uparrow 5} \mathbf{U}_{0\uparrow 34\uparrow 1} - (5 \leftrightarrow 0)) (\tilde{L}_{21} + \tilde{L}_{23} - 2M_{13} + M_1^{23} + M_3^{21} - (5 \leftrightarrow 0)) \\ & + (1 \leftrightarrow 3, 2 \leftrightarrow 4).\end{aligned}$$

NLO corrections

Pomeron contribution $L_{12}(0 \leftrightarrow 5) = L_{12}$

$$L_{12} = \left[\frac{1}{\vec{r}_{01}^2 \vec{r}_{25}^2 - \vec{r}_{02}^2 \vec{r}_{15}^2} \left(-\frac{\vec{r}_{12}^4}{8} \left(\frac{1}{\vec{r}_{01}^2 \vec{r}_{25}^2} + \frac{1}{\vec{r}_{02}^2 \vec{r}_{15}^2} \right) + \frac{\vec{r}_{12}^2}{\vec{r}_{05}^2} - \frac{\vec{r}_{02}^2 \vec{r}_{15}^2 + \vec{r}_{01}^2 \vec{r}_{25}^2}{4\vec{r}_{05}^4} \right) + \frac{\vec{r}_{12}^2}{8\vec{r}_{05}^2} \left(\frac{1}{\vec{r}_{02}^2 \vec{r}_{15}^2} - \frac{1}{\vec{r}_{01}^2 \vec{r}_{25}^2} \right) \right] \ln \left(\frac{\vec{r}_{01}^2 \vec{r}_{25}^2}{\vec{r}_{15}^2 \vec{r}_{02}^2} \right) + \frac{1}{2\vec{r}_{05}^4}.$$

2-point contribution to odderon $\tilde{L}_{12}(0 \leftrightarrow 5) = -\tilde{L}_{12}$

$$\tilde{L}_{12} = \frac{\vec{r}_{12}^2}{8} \left[\frac{\vec{r}_{12}^2}{\vec{r}_{01}^2 \vec{r}_{02}^2 \vec{r}_{15}^2 \vec{r}_{25}^2} - \frac{1}{\vec{r}_{01}^2 \vec{r}_{05}^2 \vec{r}_{25}^2} - \frac{1}{\vec{r}_{02}^2 \vec{r}_{05}^2 \vec{r}_{15}^2} \right] \ln \left(\frac{\vec{r}_{01}^2 \vec{r}_{25}^2}{\vec{r}_{15}^2 \vec{r}_{02}^2} \right).$$

Nonconformal structure

$$M_2^{13} = \left(\frac{\vec{r}_{12}^2 \vec{r}_{23}^2}{\vec{r}_{01}^2 \vec{r}_{02}^2 \vec{r}_{25}^2 \vec{r}_{35}^2} - \frac{\vec{r}_{15}^2 \vec{r}_{23}^2}{\vec{r}_{01}^2 \vec{r}_{05}^2 \vec{r}_{25}^2 \vec{r}_{35}^2} - \frac{\vec{r}_{03}^2 \vec{r}_{12}^2}{\vec{r}_{01}^2 \vec{r}_{02}^2 \vec{r}_{05}^2 \vec{r}_{35}^2} + \frac{\vec{r}_{13}^2}{\vec{r}_{01}^2 \vec{r}_{05}^2 \vec{r}_{35}^2} \right) \times \frac{1}{4} \ln \left(\frac{\vec{r}_{02}^2}{\vec{r}_{25}^2} \right).$$

NLO corrections: β -functional contribution

$$\begin{aligned}
 \mathbf{G}_\beta &= \frac{\vec{r}_{14}^2}{\vec{r}_{10}^2 \vec{r}_{40}^2} M_{14}^\beta (\mathbf{U}_{10^\dagger} \mathbf{U}_{02^\dagger 34^\dagger} + \mathbf{U}_{4^\dagger 0} \mathbf{U}_{12^\dagger 30^\dagger} - (0 \rightarrow 1)) \\
 &+ \frac{\vec{r}_{12}^2}{\vec{r}_{10}^2 \vec{r}_{20}^2} M_{12}^\beta (\mathbf{U}_{10^\dagger} \mathbf{U}_{02^\dagger 34^\dagger} + \mathbf{U}_{2^\dagger 0} \mathbf{U}_{10^\dagger 34^\dagger} - (0 \rightarrow 1)) \\
 &- \frac{\vec{r}_{24}^2}{2\vec{r}_{20}^2 \vec{r}_{40}^2} M_{24}^\beta (\mathbf{U}_{10^\dagger} \mathbf{U}_{02^\dagger 34^\dagger} + \mathbf{U}_{30^\dagger} \mathbf{U}_{04^\dagger 12^\dagger} - (0 \rightarrow 4)) \\
 &- \frac{\vec{r}_{13}^2}{2\vec{r}_{10}^2 \vec{r}_{30}^2} M_{13}^\beta (\mathbf{U}_{4^\dagger 0} \mathbf{U}_{12^\dagger 30^\dagger} + \mathbf{U}_{2^\dagger 0} \mathbf{U}_{34^\dagger 10^\dagger} - (0 \rightarrow 1)) \\
 &+(1 \leftrightarrow 3, 2 \leftrightarrow 4).
 \end{aligned}$$

$$M_{12}^\beta = \frac{N_c \beta}{2} \left\{ \ln \left(\frac{\vec{r}_{12}^2}{\tilde{\mu}^2} \right) + \frac{\vec{r}_{01}^2 \vec{r}_{02}^2}{\vec{r}_{12}^2} \ln \left(\frac{\vec{r}_{02}^2}{\vec{r}_{01}^2} \right) \left(\frac{1}{\vec{r}_{02}^2} - \frac{1}{\vec{r}_{01}^2} \right) \right\}.$$

$$\beta = \left(\frac{11}{3} - \frac{2}{3} \frac{n_f}{N_c} \right), \quad \beta \ln \frac{1}{\tilde{\mu}^2} = \left(\frac{11}{3} - \frac{2}{3} \frac{n_f}{N_c} \right) \ln \left(\frac{\mu^2}{4e^{2\psi(1)}} \right) + \frac{67}{9} - \frac{\pi^2}{3} - \frac{10}{9} \frac{n_f}{N_c}$$

NLO corrections: 1gluon contribution

$$\mathbf{G} = \mathbf{G}_1 + \mathbf{G}_0.$$

$$\begin{aligned} \mathbf{G}_0 = & \frac{N_c}{4} (\mathbf{U}_{4\uparrow 1} \mathbf{U}_{2\uparrow 3} - \mathbf{U}_{4\uparrow 3} \mathbf{U}_{2\uparrow 1}) \left\{ \left(\frac{\vec{r}_{14}^2}{\vec{r}_{10}^2 \vec{r}_{40}^2} + \frac{\vec{r}_{23}^2}{\vec{r}_{20}^2 \vec{r}_{30}^2} - \frac{\vec{r}_{13}^2}{\vec{r}_{10}^2 \vec{r}_{30}^2} - \frac{\vec{r}_{24}^2}{\vec{r}_{20}^2 \vec{r}_{40}^2} \right) \right. \\ & \times \ln \left(\frac{\vec{r}_{10}^2}{\vec{r}_{12}^2} \right) \ln \left(\frac{\vec{r}_{20}^2}{\vec{r}_{12}^2} \right) + \left(\frac{\vec{r}_{13}^2}{\vec{r}_{10}^2 \vec{r}_{30}^2} + \frac{\vec{r}_{24}^2}{\vec{r}_{20}^2 \vec{r}_{40}^2} - \frac{\vec{r}_{34}^2}{\vec{r}_{30}^2 \vec{r}_{40}^2} - \frac{\vec{r}_{12}^2}{\vec{r}_{20}^2 \vec{r}_{10}^2} \right) \\ & \times \ln \left(\frac{\vec{r}_{10}^2}{\vec{r}_{14}^2} \right) \ln \left(\frac{\vec{r}_{40}^2}{\vec{r}_{14}^2} \right) + \left(\ln \left(\frac{\vec{r}_{20}^2}{\vec{r}_{24}^2} \right) \ln \left(\frac{\vec{r}_{40}^2}{\vec{r}_{24}^2} \right) + \ln \left(\frac{\vec{r}_{10}^2}{\vec{r}_{13}^2} \right) \ln \left(\frac{\vec{r}_{30}^2}{\vec{r}_{13}^2} \right) \right) \\ & \left. \times \left(\frac{\vec{r}_{12}^2}{\vec{r}_{10}^2 \vec{r}_{20}^2} - \frac{\vec{r}_{14}^2}{\vec{r}_{10}^2 \vec{r}_{40}^2} \right) \right\} + (1 \leftrightarrow 3, 2 \leftrightarrow 4). \end{aligned}$$

$$\begin{aligned} \mathbf{G} = & \frac{\vec{r}_{12}^2}{\vec{r}_{10}^2 \vec{r}_{20}^2} \ln \left(\frac{\vec{r}_{10}^2}{\vec{r}_{12}^2} \right) \ln \left(\frac{\vec{r}_{20}^2}{\vec{r}_{12}^2} \right) \left\{ \frac{N_c}{2} (2N_c \mathbf{U}_{2\uparrow 34\uparrow 1} - \mathbf{U}_{0\uparrow 1} \mathbf{U}_{2\uparrow 34\uparrow 0} - \mathbf{U}_{2\uparrow 0} \mathbf{U}_{4\uparrow 10\uparrow 3}) \right. \\ & \left. + (\mathbf{U}_{2\uparrow 10\uparrow 34\uparrow 0} - \mathbf{U}_{2\uparrow 0} \mathbf{U}_{4\uparrow 3} \mathbf{U}_{0\uparrow 1} - (0 \rightarrow 1)) \right\} \\ & + \frac{\vec{r}_{14}^2}{\vec{r}_{10}^2 \vec{r}_{40}^2} \ln \left(\frac{\vec{r}_{10}^2}{\vec{r}_{14}^2} \right) \ln \left(\frac{\vec{r}_{40}^2}{\vec{r}_{14}^2} \right) \left\{ \frac{N_c}{2} (2N_c \mathbf{U}_{2\uparrow 34\uparrow 1} - \mathbf{U}_{0\uparrow 1} \mathbf{U}_{2\uparrow 34\uparrow 0} - \mathbf{U}_{4\uparrow 0} \mathbf{U}_{2\uparrow 30\uparrow 1}) \right. \\ & \left. + (\mathbf{U}_{2\uparrow 30\uparrow 14\uparrow 0} - \mathbf{U}_{4\uparrow 0} \mathbf{U}_{2\uparrow 3} \mathbf{U}_{0\uparrow 1} - (0 \rightarrow 1)) \right\} \end{aligned}$$

NLO corrections: 1gluon contribution

$$\begin{aligned}
 & + \frac{1}{2} \left(\frac{\vec{r}_{13}^2}{\vec{r}_{10}^2 \vec{r}_{30}^2} \ln \frac{\vec{r}_{10}^2}{\vec{r}_{13}^2} \ln \frac{\vec{r}_{30}^2}{\vec{r}_{13}^2} + \frac{\vec{r}_{24}^2}{\vec{r}_{20}^2 \vec{r}_{40}^2} \ln \frac{\vec{r}_{20}^2}{\vec{r}_{24}^2} \ln \frac{\vec{r}_{40}^2}{\vec{r}_{24}^2} \right) \\
 & \times \{ (\mathbf{U}_{4\uparrow 0} \mathbf{U}_{2\uparrow 1} + \mathbf{U}_{4\uparrow 1} \mathbf{U}_{2\uparrow 0}) \mathbf{U}_{0\uparrow 3} - \mathbf{U}_{2\uparrow 0 4\uparrow 1 0\uparrow 3} - \mathbf{U}_{2\uparrow 0 4\uparrow 3 0\uparrow 1} - (0 \rightarrow 3) \} \\
 & + \{ \mathbf{U}_{2\uparrow 0} \mathbf{U}_{4\uparrow 1} \mathbf{U}_{0\uparrow 3} - \mathbf{U}_{2\uparrow 0} \mathbf{U}_{0\uparrow 1} \mathbf{U}_{34\uparrow} + \mathbf{U}_{2\uparrow 1 0\uparrow 3 4\uparrow 0} - \mathbf{U}_{0\uparrow 3 2\uparrow 0 4\uparrow 1} \} \\
 & \times \frac{1}{2\vec{r}_{20}^2} \left(\frac{\vec{r}_{23}^2}{\vec{r}_{30}^2} - \frac{\vec{r}_{12}^2}{\vec{r}_{10}^2} \right) \ln \frac{\vec{r}_{10}^2}{\vec{r}_{13}^2} \ln \frac{\vec{r}_{30}^2}{\vec{r}_{13}^2} + \frac{1}{2\vec{r}_{10}^2} \left(\frac{\vec{r}_{14}^2}{\vec{r}_{40}^2} - \frac{\vec{r}_{12}^2}{\vec{r}_{20}^2} \right) \ln \frac{\vec{r}_{20}^2}{\vec{r}_{24}^2} \ln \frac{\vec{r}_{40}^2}{\vec{r}_{24}^2} \\
 & \times \{ \mathbf{U}_{2\uparrow 3} \mathbf{U}_{4\uparrow 0} \mathbf{U}_{0\uparrow 1} - \mathbf{U}_{2\uparrow 0} \mathbf{U}_{0\uparrow 1} \mathbf{U}_{34\uparrow} + \mathbf{U}_{2\uparrow 1 0\uparrow 3 4\uparrow 0} - \mathbf{U}_{0\uparrow 1 4\uparrow 0 2\uparrow 3} \} \\
 & + \{ \mathbf{U}_{4\uparrow 0} \mathbf{U}_{2\uparrow 1} \mathbf{U}_{0\uparrow 3} - N_c \mathbf{U}_{2\uparrow 0} \mathbf{U}_{4\uparrow 1 0\uparrow 3} + \mathbf{U}_{2\uparrow 3 4\uparrow 1} - \mathbf{U}_{2\uparrow 0 4\uparrow 3 0\uparrow 1} \} \\
 & \times \frac{1}{2\vec{r}_{30}^2} \left(\frac{\vec{r}_{23}^2}{\vec{r}_{20}^2} - \frac{\vec{r}_{13}^2}{\vec{r}_{10}^2} \right) \ln \frac{\vec{r}_{10}^2}{\vec{r}_{12}^2} \ln \frac{\vec{r}_{20}^2}{\vec{r}_{12}^2} + \frac{1}{2\vec{r}_{40}^2} \left(\frac{\vec{r}_{14}^2}{\vec{r}_{10}^2} - \frac{\vec{r}_{24}^2}{\vec{r}_{20}^2} \right) \ln \frac{\vec{r}_{10}^2}{\vec{r}_{12}^2} \ln \frac{\vec{r}_{20}^2}{\vec{r}_{12}^2} \\
 & \times \{ \mathbf{U}_{4\uparrow 0} \mathbf{U}_{2\uparrow 1} \mathbf{U}_{0\uparrow 3} - N_c \mathbf{U}_{0\uparrow 1} \mathbf{U}_{2\uparrow 3 4\uparrow 0} + \mathbf{U}_{2\uparrow 3 4\uparrow 1} - \mathbf{U}_{2\uparrow 0 4\uparrow 3 0\uparrow 1} \} \\
 & + \{ \mathbf{U}_{2\uparrow 0} \mathbf{U}_{4\uparrow 1} \mathbf{U}_{0\uparrow 3} - N_c \mathbf{U}_{4\uparrow 0} \mathbf{U}_{12\uparrow 3 0\uparrow} + \mathbf{U}_{2\uparrow 3 4\uparrow 1} - \mathbf{U}_{2\uparrow 0 4\uparrow 1 0\uparrow 3} \} \\
 & \times \frac{1}{2\vec{r}_{30}^2} \left(\frac{\vec{r}_{34}^2}{\vec{r}_{40}^2} - \frac{\vec{r}_{13}^2}{\vec{r}_{10}^2} \right) \ln \frac{\vec{r}_{10}^2}{\vec{r}_{14}^2} \ln \frac{\vec{r}_{40}^2}{\vec{r}_{14}^2} + \frac{1}{2\vec{r}_{20}^2} \left(\frac{\vec{r}_{12}^2}{\vec{r}_{10}^2} - \frac{\vec{r}_{24}^2}{\vec{r}_{40}^2} \right) \ln \frac{\vec{r}_{10}^2}{\vec{r}_{14}^2} \ln \frac{\vec{r}_{40}^2}{\vec{r}_{14}^2} \\
 & \times \{ \mathbf{U}_{2\uparrow 0} \mathbf{U}_{4\uparrow 1} \mathbf{U}_{0\uparrow 3} - N_c \mathbf{U}_{0\uparrow 1} \mathbf{U}_{02\uparrow 3 4\uparrow} + \mathbf{U}_{2\uparrow 3 4\uparrow 1} - \mathbf{U}_{2\uparrow 0 4\uparrow 1 0\uparrow 3} \} + (1 \leftrightarrow 3, 2 \leftrightarrow 4).
 \end{aligned}$$

Results

- The nonlinear NLO low- x evolution equation for a quadrupole Green function.
- The nonlinear NLO low- x evolution equation for a double dipole Green function.
- Transformation of the NLO equations to the quasi-conformal form.

Thank you for your attention