Application of the shockwave formalism to the production of $q\bar{q}g$ jets in diffractive DIS

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[RB, A.Grabovsky, L.Szymanowski, S.Wallon (arXiv:1405.7676)]
[Published in JHEP 409 (2014) 026]

Overview

1. DGLAP regime vs BFKL regime
2. Diffractive DIS
   - Rapidity gap events at HERA
   - Collinear factorization approach
   - k_T-factorization approach : two exchanged gluons
   - Confrontation of the two approaches with HERA data
3. Diffractive production of jets : our approach
   - $q\bar{q}$ production
   - $q\bar{q}g$ production
   - Linear approximation : 2 and 3 exchanged gluons
4. Conclusion

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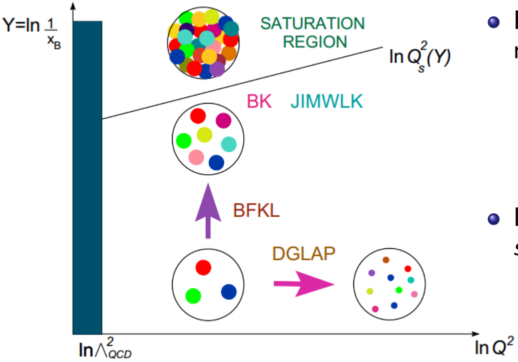
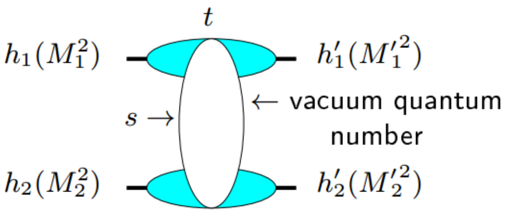
Two regimes of perturbative QCD



- DGLAP dynamics : $Q^2 \to \infty$, moderate x_B
  - Governed by collinear dynamics
  - Resummation of Q^2 logs : $(\alpha_s \ln Q^2)^n$, $\alpha_s(\alpha_s \ln Q^2)^n$...
- BFKL dynamics (Regge limit) $s \gg Q^2 \gg \Lambda_{QCD}$ $(x_B \ll 1)$
  - Governed by soft dynamics
  - Resummation of $\frac{1}{x_B} \sim s$ logs : $(\alpha_s \ln s)^n$, $\alpha_s(\alpha_s \ln s)^n$...

BFKL dynamics



$$\sigma = \frac{1}{s}\mathrm{Im}\mathcal{A} \sim s^{\alpha(0)-1}$$

$$\alpha(0) - 1 = C\alpha_s$$

$$C > 0$$

$\Rightarrow$ Violation of the Froissart bound

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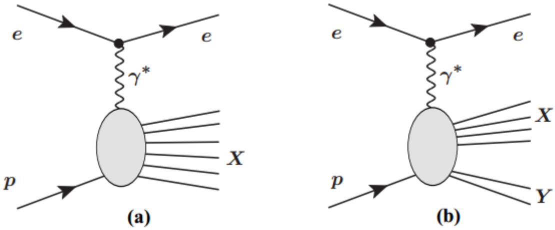
Diffractive DIS

Rapidity gap events at HERA

Experiments at HERA : about 10% of scattering events reveal a rapidity gap



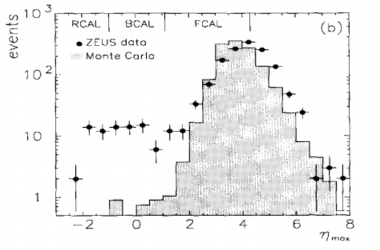
DIS events

DDIS events

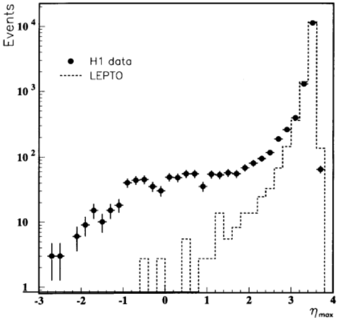
Diffractive DIS

Rapidity gap events at HERA

Experiments at HERA : about 10% of events reveal a rapidity gap



ZEUS, 1993



H1,1994

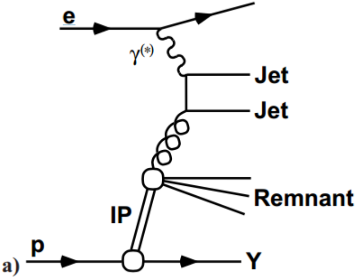
Diffractive DIS

Theoretical approaches for DDIS using pQCD

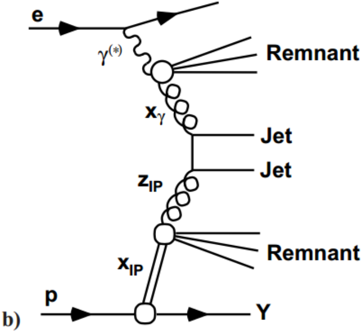
- Collinear factorization approach
  - Relies on QCD factorization theorem, using a hard scale such as the virtuality Q^2 of the incoming photon
  - One needs to introduce a diffractive distribution function for partons *within a pomeron*
- k_T factorization approach for two exchanged gluons
  - low-x QCD approach : $s \gg Q^2 \gg \Lambda_{QCD}$
  - The pomeron is described as a two-gluon color-singlet state

Theoretical approaches for DDIS using pQCD

Collinear factorization approach



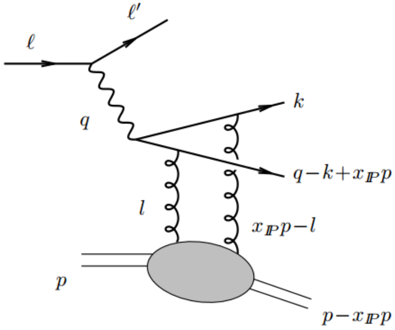
Direct



Resolved

Theoretical approaches for DDIS using pQCD

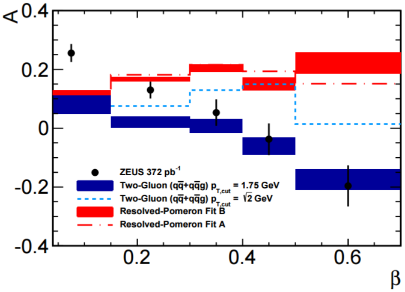
k_T-factorization approach : two gluon exchange



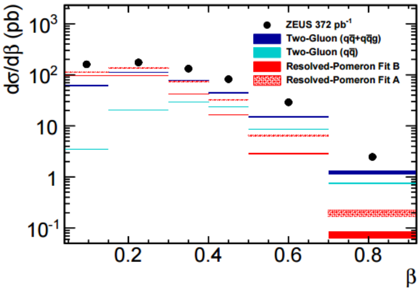
Bartels, Ivanov, Jung, Lotter, Wüsthoff
Braun and Ivanov developed a similar model in collinear factorization

Theoretical approaches for DDIS using pQCD

Confrontation of the two approaches with HERA data



ZEUS collaboration, 2015



ZEUS collaboration, 2015

Overview

Assumptions

- Regge limit : $s \gg Q^2 \gg \Lambda_{QCD}$
- No approximation for the outgoing gluon, contrary to e.g. :
  - Collinear approximation [Wüsthoff, 1995]
  - Soft approximation [Bartels, Jung, Wüsthoff, 1999]
- Lightcone coordinates and lightcone gauge :

$$\begin{aligned} p^+ &= n_2 \cdot p = \frac{1}{2}\left(p^0 + p^3\right), \\ p^- &= n_1 \cdot p = p^0 - p^3 \\ n_2 \cdot \mathcal{A} &= 0 \\ n_2^2 &= 0 \end{aligned}$$

- Shockwave (Wilson lines) approach [Balitsky, 1995]

The shockwave approach

One decomposes the gluon field $\mathcal{A}$ into an internal field and an external field :

$$\mathcal{A}^{\mu} = A^{\mu} + b^{\mu}$$

The internal one contains the gluons with rapidity $p^{+} > e^{\eta} = \sigma$ and the external one contains the gluons with rapidity $p^{+} < \sigma$. One writes :

$$b^{\mu}\left(z\right) = \delta\left(z^{+}\right) B\left(\vec{z}\right) n_{2}^{\mu}$$

Intuitively, large boost λ along the $+$ direction :

$$b^{+}\left(x^{+}, x^{-}, \vec{x}\right) \rightarrow \frac{1}{\lambda} b^{+}\left(\lambda x^{+}, \frac{1}{\lambda} x^{-}, \vec{x}\right)$$

$$b^{-}\left(x^{+}, x^{-}, \vec{x}\right) \rightarrow \lambda b^{-}\left(\lambda x^{+}, \frac{1}{\lambda} x^{-}, \vec{x}\right)$$

$$b^{i}\left(x^{+}, x^{-}, \vec{x}\right) \rightarrow b^{i}\left(\lambda x^{+}, \frac{1}{\lambda} x^{-}, \vec{x}\right)$$

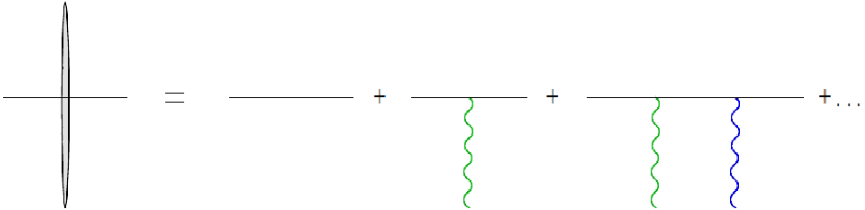
Propagator through a shockwave

$$G\left(z_2,\, z_0\right) = -\int d^4 z_1 \theta\left(z_2^+\right)\delta\left(z_1^+\right)\theta\left(-z_0^+\right) G\left(z_2 - z_1\right)\gamma^+ G\left(z_1 - z_0\right) U_1$$

Wilson lines :

$$U_i = U_{\vec{z}_i} = U\left(\vec{z}_i, \eta\right) = P\exp\left[ig\int_{-\infty}^{+\infty} b_\eta^-\left(z_i^+, \vec{z}_i\right) dz_i^+\right]$$

$$U_i = 1 + ig\int_{-\infty}^{+\infty} b_\eta^-(z_i^+, \vec{z}_i) dz_i^+ + (ig)^2\int_{-\infty}^{+\infty} b_\eta^-(z_i^+, \vec{z}_i) b_\eta^-(z_j^+, \vec{z}_j)\theta\left(z_{ji}^+\right) dz_i^+ dz_j^+$$



The BK equation

Dipole operator

$$\mathbf{U}_{12} = \frac{1}{N_c} \mathrm{Tr}\left(U_1 U_2^\dagger\right) - 1$$

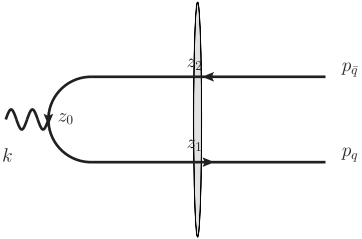
BK equation [Balitsky, 1995] [Kovchegov, 1999]

$$\frac{d\mathbf{U}_{12}}{d\ln\sigma} = \frac{\alpha_s N_c}{2\pi^2} \int d\vec{z}_3 \frac{\vec{z}_{12}^{\,2}}{\vec{z}_{13}^{\,2}\vec{z}_{23}^{\,2}} \left[\mathbf{U}_{13} + \mathbf{U}_{32} - \mathbf{U}_{12} - \mathbf{U}_{13}\mathbf{U}_{32}\right]$$

Non-linear term : saturation

$q\bar{q}$ production

Matrix element for EM current



$q\bar{q}$ production

$$M_0^\alpha = \int d\vec{z}_1 d\vec{z}_2 F(\vec{z}_1, \vec{z}_2)^\alpha N_c \mathbf{U}_{12}$$

$q\bar{q}$ production

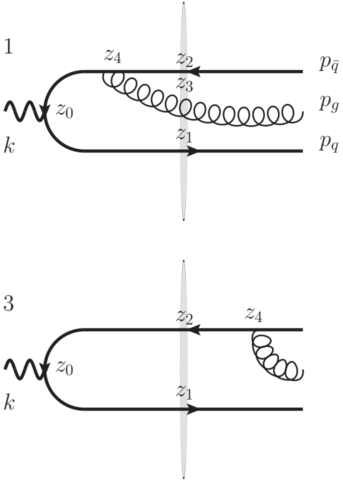
We recover the well-known results :

$$F\left(\vec{z}_1,\vec{z}_2\right)^{\alpha}\,\varepsilon_{L\alpha} \quad = \quad \theta(p_q^+)\,\theta(p_{\bar{q}}^+)\,\frac{\delta\left(k^+ - p_q^+ - p_{\bar{q}}^+\right)}{(2\pi)^2}\,e^{-i\vec{p}_q\cdot\vec{z}_1 - i\vec{p}_{\bar{q}}\cdot\vec{z}_2} \times(-2i)\delta_{\lambda_q,-\lambda_{\bar{q}}}\,x_q x_{\bar{q}}\,Q\,K_0\left(Q\sqrt{x_q x_{\bar{q}}\vec{z}_{12}^{\,2}}\right)$$

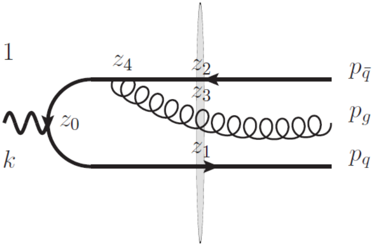
and

$$F\left(\vec{z}_1,\vec{z}_2\right)^{j}\,\varepsilon_{Tj} \quad = \quad \theta(p_q^+)\,\theta(p_{\bar{q}}^+)\,\frac{\delta\left(k^+ - p_q^+ - p_{\bar{q}}^+\right)}{(2\pi)^2}\,e^{-i\vec{p}_q\cdot\vec{z}_1 - i\vec{p}_{\bar{q}}\cdot\vec{z}_2}\,\delta_{\lambda_q,-\lambda_{\bar{q}}} \times\left(x_q - x_{\bar{q}} + s\lambda_q\right)\frac{\vec{z}_{12}\cdot\vec{\varepsilon}_T}{\vec{z}_{12}^{\,2}}\,Q\sqrt{x_q x_{\bar{q}}\vec{z}_{12}^{\,2}}K_1\left(Q\sqrt{x_q x_{\bar{q}}\vec{z}_{12}^{\,2}}\right).$$

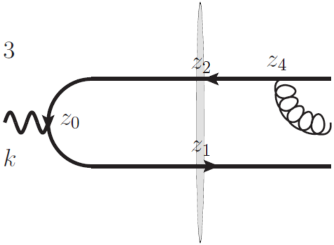
$q\bar{q}g$ production



$q\bar{q}g$ production



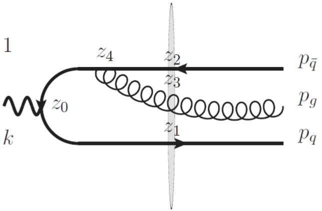
First kind



Second kind

$$M^\alpha = \frac{N_c^2}{2}\int d\vec{z}_1 d\vec{z}_2 d\vec{z}_3 F_1\left(\vec{z}_1, \vec{z}_2, \vec{z}_3\right)^\alpha \left(\mathbf{U}_{13} + \mathbf{U}_{32} - \mathbf{U}_{12} + \mathbf{U}_{13}\mathbf{U}_{32}\right) + \int d\vec{z}_1 d\vec{z}_2\, F_2\left(\vec{z}_1, \vec{z}_2\right)^\alpha \left(N_c^2 - 1\right) \mathbf{U}_{12}$$

$q\bar{q}g$ production : first kind



$q\bar{q}g$ production : first kind

$q\bar{q}g$ production : first kind

Result for a longitudinal photon

$$
\begin{aligned}
& F_{1L}(\vec{z}_1, \vec{z}_2, \vec{z}_3) \\
= \quad & \delta(k^+ - p_g^+ - p_q^+ - p_{\bar{q}}^+)\theta(p_g^+ - \sigma) 2Qg \frac{e^{-i\vec{p}_q\cdot\vec{z}_1 - i\vec{p}_{\bar{q}}\cdot\vec{z}_2 - i\vec{p}_g\cdot\vec{z}_3}}{\pi\sqrt{2p_g^+}} K_0(QZ_{123}) \\
\times \quad & \delta_{\lambda_q, -\lambda_{\bar{q}}} \left\{ (x_{\bar{q}} + x_g \delta_{-s_g \lambda_q}) x_q \frac{\vec{z}_{32} \cdot \vec{\varepsilon}_g^{\,*}}{\vec{z}_{32}^{\,2}} - (x_q + x_g \delta_{-s_g \lambda_{\bar{q}}}) x_{\bar{q}} \frac{\vec{z}_{31} \cdot \vec{\varepsilon}_g^{\,*}}{\vec{z}_{31}^{\,2}} \right\}
\end{aligned}
$$

$Z_{123} = \sqrt{x_q x_{\bar{q}} \vec{z}_{12}^{\,2} + x_q x_g \vec{z}_{13}^{\,2} + x_{\bar{q}} x_g \vec{z}_{23}^{\,2}}$

$q\bar{q}g$ production : first kind

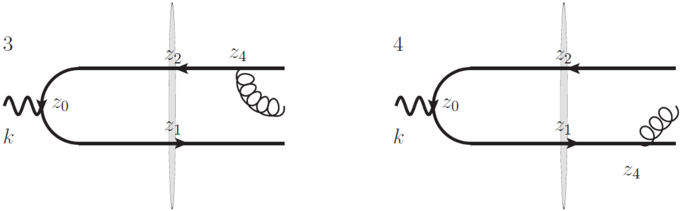
Result for a transverse photon

$$F_{1T}\left(\vec{z}_1,\vec{z}_2,\vec{z}_3\right) = 2igQ\delta(k^+ - p_g^+ - p_q^+ - p_{\bar{q}}^+)\theta(p_g^+ - \sigma)\frac{e^{-i\vec{p}_q\cdot\vec{z}_1 - i\vec{p}_{\bar{q}}\cdot\vec{z}_2 - i\vec{p}_g\cdot\vec{z}_3}}{\pi Z_{123}\sqrt{2p_g^+}}$$

$$\times \quad \delta_{\lambda_q,-\lambda_{\bar{q}}} K_1(QZ_{123}) \left\{ -\frac{\left(\vec{z}_{23}\cdot\vec{\varepsilon}_g^{\,*}\right)\left(\vec{z}_{13}\cdot\vec{\varepsilon}_T\right)}{\vec{z}_{23}^{\,2}} x_q\left(x_q - \delta_{s\lambda_{\bar{q}}}\right)\left(x_{\bar{q}} + x_g\delta_{-s_g\lambda_q}\right)\right.$$

$$\left. -\frac{\left(\vec{z}_{23}\cdot\vec{\varepsilon}_g^{\,*}\right)\left(\vec{z}_{23}\cdot\vec{\varepsilon}_T\right)}{\vec{z}_{23}^{\,2}} x_q x_{\bar{q}}\left(x_{\bar{q}} + x_g\delta_{-s_g\lambda_q} - \delta_{s\lambda_q}\right)\right\} - (q \leftrightarrow \bar{q})$$

$q\bar{q}g$ production : second kind



$q\bar{q}$ production : second kind

$q\bar{q}g$ production : second kind

Result for a longitudinal photon

$$\tilde{F}_{2L}(\vec{z}_1, \vec{z}_2) = 4ig\, Q\, \theta(p_g^+ - \sigma)\delta(k^+ - p_g^+ - p_q^+ - p_{\bar{q}}^+)\frac{e^{-i\vec{p}_q\cdot\vec{z}_1 - i\vec{p}_{\bar{q}}\cdot\vec{z}_2}}{\sqrt{2p_g^+}}$$

$$\times\delta_{\lambda_q,-\lambda_{\bar{q}}}\frac{x_q\,(x_g + x_{\bar{q}})\left(\delta_{-s_g\lambda_q}x_g + x_{\bar{q}}\right)}{x_{\bar{q}}\,x_g}\frac{\vec{P}_{\bar{q}}\cdot\vec{\varepsilon}_g^{\,*}}{\vec{P}_{\bar{q}}^2}\,e^{-i\vec{p}_g\cdot\vec{z}_2}K_0(QZ_{122}) - (q \leftrightarrow \bar{q})$$

$Z_{122} = \sqrt{x_q\,(1 - x_q)\,\vec{z}_{12}^{\,2}}$

$q\bar{q}g$ production : second kind

Result for a transverse photon

$$\tilde{F}_{2T}(\vec{z}_1,\vec{z}_2) = -4g\,\theta(p_g^+ - \sigma)\,\delta(k^+ - p_g^+ - p_q^+ - p_{\bar{q}}^+)\frac{e^{-i\vec{p}_q\cdot\vec{z}_1 - i\vec{p}_{\bar{q}}\cdot\vec{z}_2}}{\sqrt{2p_g^+}}\delta_{\lambda_q,-\lambda_{\bar{q}}}$$

$$\times\frac{\left(\delta_{\lambda_{\bar{q}}s} - x_q\right)\left(\delta_{-s_g\lambda_q}x_g + x_{\bar{q}}\right)}{x_{\bar{q}}\,x_g}\frac{\vec{P}_{\bar{q}}\cdot\vec{\varepsilon}_g^{\,*}}{\vec{P}_{\bar{q}}^2}\frac{\vec{z}_{12}\cdot\vec{\varepsilon}_T}{\vec{z}_{12}^2}\,Q\,Z_{122}K_1(QZ_{122})e^{-i\vec{p}_g\cdot\vec{z}_2} - (q\leftrightarrow\bar{q})$$

$Z_{122} = \sqrt{x_q\,(1-x_q)\,\vec{z}_{12}^{\,2}}$

Linear approximation

Back to the general expression for the matrix element M^α :

$$
\begin{aligned}
M^\alpha &= \frac{N_c^2}{2}\int d\vec{z}_1 d\vec{z}_2 d\vec{z}_3 F_1\left(\vec{z}_1,\vec{z}_2,\vec{z}_3\right)^\alpha \left(\mathbf{U}_{13}+\mathbf{U}_{32}-\mathbf{U}_{12}+\mathbf{U}_{13}\mathbf{U}_{32}\right) \\
&+ \int d\vec{z}_1 d\vec{z}_2\, F_2\left(\vec{z}_1,\vec{z}_2\right)^\alpha \left(N_c^2-1\right)\mathbf{U}_{12}
\end{aligned}
$$

For 2 or 3 gluon exchange, one can linearize this expression by neglecting the $\mathbf{U}_{13}\mathbf{U}_{32}$ term.

Linear approximation

After linearization one gets :

$$M^{\alpha} \overset{g^3}{=} \frac{1}{2} \int d\vec{z}_1 d\vec{z}_2 \mathbf{U}_{12} \left\{ \tilde{F}_1 \left(\vec{z}_1, \vec{z}_2\right)^{\alpha} + \left(N_c^2 - 1\right) \tilde{F}_2 \left(\vec{z}_1, \vec{z}_2\right)^{\alpha} \right\}$$

$$\tilde{F}_1 \left(\vec{z}_1, \vec{z}_2\right)^{\alpha} = \int d\vec{z}_3 \left[N_c^2 F_1 \left(\vec{z}_1, \vec{z}_3, \vec{z}_2\right)^{\alpha} + N_c^2 F_1 \left(\vec{z}_3, \vec{z}_2, \vec{z}_1\right)^{\alpha} - F_1 \left(\vec{z}_1, \vec{z}_2, \vec{z}_3\right)^{\alpha} \right]$$

$\Rightarrow$ One has to integrate the previously derived expressions

Linear approximation

No analytical expression for most of the integrals. BUT :

- For null transverse momenta $\vec{p}_q$, $\vec{p}_{\bar{q}}$ and $\vec{p}_g$ they can be performed analytically

- In any case, they can be reduced to convergent integrals over a real parameter in [0,1] so a numerical calculation can be done.

Overview

Conclusion

Further analysis from our results

- The calculation of virtual correction for the cross section is still to be performed (Work in progress)
- A lot of phenomenology can be done from HERA data :
  - From $q\bar{q}g$ production to jet production
  - Is our approach better than the 2-gluon approximation for H1 and ZEUS data?
- The same calculation can be done again for massive quarks
- One could adapt those results for the study of hard diffractive events in ultraperipheral collisions at LHC

Backup slides

Results in momentum space

First kind, longitudinal photon

$$F_{1L}\left(\vec{p}_1,\vec{p}_2,\vec{p}_3\right)=\delta(k^+-p_g^+-p_q^+-p_{\bar{q}}^+)\delta\left(\vec{p}_{1q}+\vec{p}_{2\bar{q}}+\vec{p}_{3g}\right)\theta(p_g^+-\sigma)$$
$$\times\frac{\delta_{\lambda_q,-\lambda_{\bar{q}}}}{\sqrt{2p_g^+}}\frac{4iQ\,g\left(x_q+x_g\delta_{-s_g\lambda_{\bar{q}}}\right)\left((\vec{p}_{2\bar{q}}\cdot\vec{\varepsilon}_g^{\,*})x_q+\left(\vec{p}_{1q}\cdot\vec{\varepsilon}_g^{\,*}\right)(1-x_{\bar{q}})\right)}{(1-x_{\bar{q}})\,x_g x_q\left(Q^2+\frac{\vec{p}_{2\bar{q}}^{\,2}}{x_{\bar{q}}(1-x_{\bar{q}})}\right)\left(Q^2+\frac{\vec{p}_{1q}^{\,2}}{x_q}+\frac{\vec{p}_{2\bar{q}}^{\,2}}{x_{\bar{q}}}+\frac{\vec{p}_{3g}^{\,2}}{x_g}\right)}-(q\leftrightarrow\bar{q}),$$

Results in momentum space

Second kind, longitudinal photon

$$\tilde{F}_{2L}\left(\vec{p}_1,\vec{p}_2\right) = \theta(p_g^+ - \sigma)\delta(k^+ - p_g^+ - p_q^+ - p_{\bar{q}}^+)\delta(\vec{p}_{1q} + \vec{p}_{2\bar{q}} - \vec{p}_g)$$
$$\times \frac{4igQ}{\sqrt{2p_g^+}}\delta_{\lambda_q,-\lambda_{\bar{q}}}\frac{x_{\bar{q}} + \delta_{-s_g\lambda_q}x_g}{x_{\bar{q}}x_g}\frac{2\pi}{Q^2 + \frac{\vec{p}_{1q}^{\,2}}{x_q\left(1-x_q\right)}}\frac{\vec{P}_{\bar{q}}\cdot\vec{\varepsilon}_g^{\,*}}{\vec{P}_{\bar{q}}^2} - (q \leftrightarrow \bar{q})\,.$$

Results in momentum space

First kind, transverse photon

$$F_{1T}\left(\vec{p}_1,\vec{p}_2,\vec{p}_3\right)=\frac{2ig}{\sqrt{2p_g^+}}\frac{\delta(k^+-p_g^+-p_q^+-p_{\bar{q}}^+)\delta\left(\vec{p}_{1q}+\vec{p}_{2\bar{q}}+\vec{p}_{3g}\right)\theta(p_g^+-\sigma)\delta_{-\lambda_{\bar{q}}\lambda_q}}{Q^2\left(1-x_q\right)\left(\frac{\vec{p}_{1q}{}^2}{x_q}+\frac{\vec{p}_{2\bar{q}}{}^2}{x_{\bar{q}}}+\frac{\vec{p}_{3g}{}^2}{x_g}+Q^2\right)}\left\{\delta_{ss_g}\delta_{s\lambda_q}\right.$$

$$\left.+2(\vec{p}_{1q}\cdot\vec{\varepsilon}_T)((\vec{p}_{2\bar{q}}\cdot\vec{\varepsilon}_g{}^*)\left(x_{\bar{q}}+x_g\right)+(\vec{p}_{1q}\cdot\vec{\varepsilon}_g{}^*)\,x_{\bar{q}})\frac{\left(x_q-\delta_{s\lambda_{\bar{q}}}\right)\left(x_g\,\delta_{-s_g\lambda_q}+x_{\bar{q}}\right)}{\left(1-x_q\right)x_q x_{\bar{q}} x_g\left(Q^2+\frac{\vec{p}_{1q}{}^2}{(1-x_q)x_q}\right)}\right\}-(q\leftrightarrow\bar{q}).$$

Results in momentum space

Second kind, transverse photon

$$\tilde{F}_{2T}\left(\vec{p}_1,\vec{p}_2\right) = -\,\theta(p_g^+ - \sigma)\,\delta(k^+ - p_g^+ - p_q^+ - p_{\bar{q}}^+)\delta(\vec{p}_{1q} + \vec{p}_{2\bar{q}} - \vec{p}_g)\frac{\delta_{\lambda_q,-\lambda_{\bar{q}}}}{\sqrt{2p_g^+}}$$
$$\times 4g\,\frac{\left(\delta_{\lambda_{\bar{q}}s} - x_q\right)\left(\delta_{-s_g\lambda_q}x_g + x_{\bar{q}}\right)}{x_{\bar{q}}x_g}\,\frac{2\pi i\left(\vec{p}_{1q}\cdot\vec{\varepsilon}_T\right)}{x_q\left(1-x_q\right)Q^2 + \vec{p}_{1q}^{\,2}}\,\frac{\vec{P}_{\bar{q}}\cdot\vec{\varepsilon}_g^{\,*}}{\vec{P}_{\bar{q}}^2} - \left(q\leftrightarrow\bar{q}\right)$$

Linear approximation

Example of an integral for linear approximation calculations

$$\int d\vec{z}_3 e^{-i\vec{p}_g\cdot\vec{z}_3}\frac{\vec{z}_{32}}{\vec{z}_{32}^{\,2}}K_0\left(QZ_{123}\right)$$
$$= -\frac{\pi e^{-i\vec{p}_g\cdot\vec{z}_2}}{(1-x_g)x_g}\int_0^1 d\alpha e^{\alpha\frac{ix_q\left(\vec{z}_{21}\cdot\vec{p}_g\right)}{x_{\bar{q}}+x_q}}\left(\frac{i\vec{p}_g Z_{q\bar{q}g}}{Q_g\left(\alpha\right)}K_1\left(Q_g\left(\alpha\right)Z_{q\bar{q}g}\right)+x_g x_q\vec{z}_{21}K_0\left(Q_g\left(\alpha\right)Z_{q\bar{q}g}\right)\right)$$

$$\int d\vec{z}_3\frac{\vec{z}_{32}}{\vec{z}_{32}^{\,2}}K_0\left(QZ_{123}\right) = -\frac{2\pi}{x_g x_q Q}\frac{\vec{z}_{21}}{\vec{z}_{21}^{\,2}}\left(Z_{q\bar{q}}K_1\left(QZ_{q\bar{q}}\right)-Z_{122}K_1\left(QZ_{122}\right)\right)$$