# Theory of electromagnetic wave generation via a beam-plasma antenna

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## Introduction

It is experimentally found that during the propagation of an electron beam throught a thin plasma column immersed in external magnetic field the powerful electromagnetic radiation is observed. What is the mechanism? How does the efficiency of radiation depend on plasma parameters?

## Formulation of the Problem

- An electron beam (density $n_b$, speed $v_b$ and the relativistic factor $\gamma_b$) excites an unstable longitudinal wave.
- The frequency and growth rate of this wave are
$$\frac{\omega_b}{\omega_p} = 1 - \frac{(n_b/n_0)^{1/3}}{2^{4/3}\gamma_b}, \quad \frac{\Gamma}{\omega_p} = \frac{\sqrt{3}\,(n_b/n_0)^{1/3}}{2^{4/3}\gamma_b}.$$
- The electric field of the wave: $E_x(t,x) = E_0\cos(k_\parallel z - \omega t)$, where $E_0 = \gamma_b^3\Gamma^2 v_b$; Its wave vector: $k_\parallel = \frac{\omega}{v_b}$
- If such a wave scatters on harmonic density perturbation ($n = n_0 + \delta n\cos qz$) it can radiate EM waves with frecuency $\omega$ and wave vector
$$\mathcal{K}_\parallel = k_\parallel(1-\mathcal{Q}), \quad \mathcal{Q} = q/k_\parallel,$$
$$\mathcal{K}_\perp = \sqrt{1-\mathcal{K}_\parallel^2}.$$
- Generation is possible when
$$1 - \hat{v}_b < \mathcal{Q} < 1 + \hat{v}_b$$
- The direction of the radiation
$$\theta = \arctan\left(\sqrt{\frac{v_b^2}{(1-\mathcal{Q})^2} - 1}\right)$$

## The Particular Case

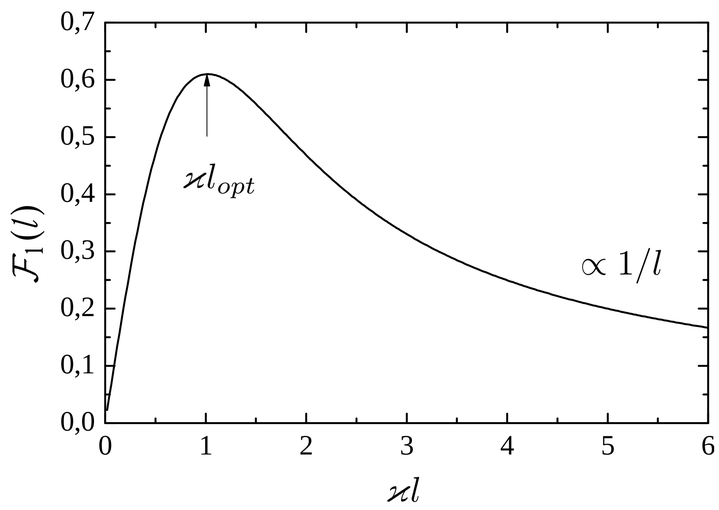


Fig. 1 : The factor $\mathcal{F}_1(l)$ as a function of plasma thickness for $\omega = 0.94$.

- When the modulation period coincides with the wavelength of the beam-driven mode ($\mathcal{Q} = 1$) the radiation angle $\theta = 90°$.
- Dispersion relations of eigenmodes: $\varkappa_1^2 = \eta$ and $\varkappa_2^2 = (\varepsilon^2 - g^2)/\varepsilon$.
- $F_2 = 0$, this mode has X-polarization, can propagate inside the plasma but cannot interact with the longitudinal current.
- The first mode has O-polarization and penetrates into the plasma only to the skin-depth ($\varkappa_1 = i\varkappa$).
- The radiation power depends on $l$ as
$$\mathcal{F}_1(l) = \frac{\sinh^2(\varkappa l)}{\varkappa l\left[\omega^2 + \sinh^2(\varkappa l)\right]},$$
- Efficiency of such radiation can be raised to the level of 5–10%.

## Cylindrical Antenna

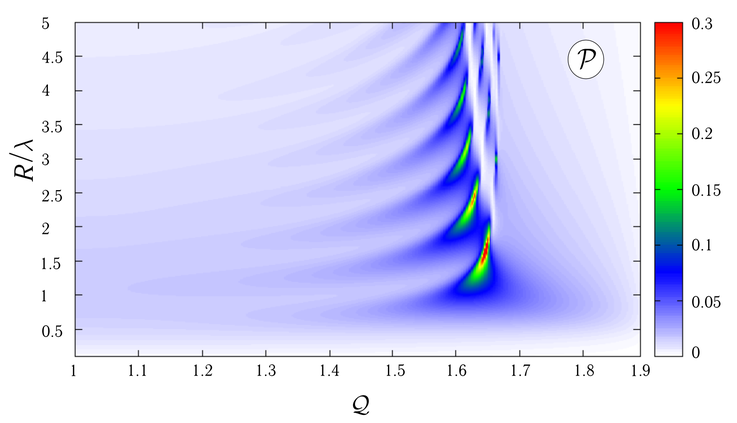


Fig. 2 : Radiation efficiency as a function of the modulation period and radius of plasma column for the cylindrical antenna.

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## Plane Beam-Plasma Antenna

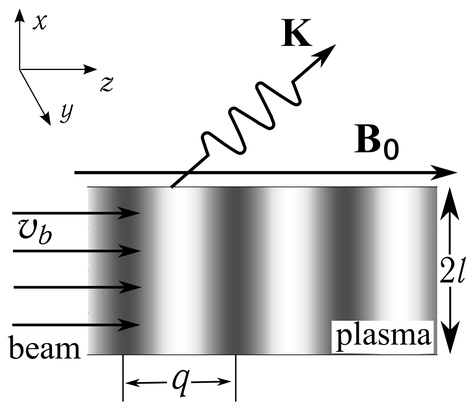



Fig. 3 : Geometry of the problem.

Inside the plasma, electromagnetic fields $\mathbf{E} = \mathbf{E}(x)\exp(i\mathcal{K}_\parallel z - i\omega t) +$ c.c. obey

$$\frac{\partial^2 E_z}{\partial x^2} + a_1 E_z - a_2\frac{\partial E_y}{\partial x} = -\frac{i\mathcal{J}}{\omega}\left(1 - \mathcal{K}_\parallel^2/\varepsilon\right),$$
$$\frac{\partial^2 E_y}{\partial x^2} + a_3 E_y + a_4\frac{\partial E_z}{\partial x} = 0,$$

where

$$a_1 = \eta\left(1 - \mathcal{K}_\parallel^2/\varepsilon\right), \quad a_2 = \mathcal{K}_\parallel g/\varepsilon,$$
$$a_3 = \varepsilon - \mathcal{K}_\parallel^2 - \frac{g^2}{\varepsilon - \mathcal{K}_\parallel^2}, \quad a_4 = \frac{\mathcal{K}_\parallel g}{\varepsilon - \mathcal{K}_\parallel^2},$$

the current amplitude $\mathcal{J} = j_0/(en_0c) = i\delta n E_0(z)/(4\omega)$ and

$$\varepsilon = 1 - \frac{1}{\omega^2 - \Omega_e^2}, \quad g = \frac{\Omega_e/\omega}{\omega^2 - \Omega_e^2}, \quad \eta = 1 - \frac{1}{\omega^2}.$$

Inside the plasma the solution of the system:

$$E_z = b_1\left(C_1 e^{i\varkappa_1 x} + C_2 e^{-i\varkappa_1 x}\right) + b_2\left(C_3 e^{i\varkappa_2 x} - C_4 e^{-i\varkappa_2 x}\right) - \frac{i\mathcal{J}}{\eta\omega},$$
$$E_y = b_3\left(C_1 e^{i\varkappa_1 x} - C_2 e^{-i\varkappa_1 x}\right) + b_4\left(C_3 e^{i\varkappa_2 x} + C_4 e^{-i\varkappa_2 x}\right),$$

where

$$\varkappa_{1,2}^2 = \frac{a_1 + a_3 + a_2 a_4 \mp \sqrt{(a_1 + a_3 + a_2 a_4)^2 - 4a_1 a_3}}{2},$$

$$b_1 = a_3 - \varkappa_1^2, \quad b_2 = i\varkappa_2 a_2,$$
$$b_3 = -i\varkappa_1 a_4, \quad b_4 = a_1 - \varkappa_2^2.$$

In vacuum, EM waves

$$E_z = C_5 e^{i\mathcal{K}_\perp(x-l)}, \qquad E_z = C_7 e^{-i\mathcal{K}_\perp(x+l)},$$
$$E_y = C_6 e^{i\mathcal{K}_\perp(x-l)}, \qquad E_y = C_8 e^{-i\mathcal{K}_\perp(x+l)}.$$

The energy flux density

$$\mathcal{S} = \frac{S_x}{n_0 m_e c^3} = 2\left[\frac{|C_5|^2}{\mathcal{K}_\perp} + \mathcal{K}_\perp |C_6|^2\right].$$

The full radiation power $P_{rad}$ in units of beam power $P_b$

$$\mathcal{P} = \frac{P_{rad}}{P_b} = \frac{\omega}{(\gamma_b - 1)n_b v_b l}\int_0^{L_z}\mathcal{S}\,dz,$$

where $L_z$ is the length of plasma. After substituting constants and the current

$$\mathcal{P} = \frac{\delta n^2 \mathcal{F}_1(l)}{8(\gamma_b - 1)n_b v_b\sqrt{1-\omega^2}}\int_0^{L_z} E_0^2\,dz,$$

where

$$\mathcal{F}_1(l) = \frac{\mathcal{K}_\perp\omega(F_1 + F_2)}{l(1-\omega^2)^{3/2}}.$$

$F_1$ and $F_2$ are contributions from different plasma eigenmodes:

$$F_1 = \left|\frac{b_5\sin(\varkappa_1 l) + G b_6\sin(\varkappa_2 l)}{Z}\right|^2,$$
$$F_2 = \left|\frac{b_3\sin(\varkappa_1 l) + G b_4\sin(\varkappa_2 l)}{Z}\right|^2,$$

where

$$Z = b_1\cos(\varkappa_1 l) + i\mathcal{K}_\perp b_5\sin(\varkappa_1 l) + G\left(b_2\cos(\varkappa_2 l) + i\mathcal{K}_\perp b_6\sin(\varkappa_2 l)\right),$$
$$G = -\frac{b_3}{b_4}\left(\frac{\varkappa_1\cos(\varkappa_1 l) - i\mathcal{K}_\perp\sin(\varkappa_1 l)}{\varkappa_2\cos(\varkappa_2 l) - i\mathcal{K}_\perp\sin(\varkappa_2 l)}\right),$$
$$b_5 = -\varkappa_1 b_1 - i\mathcal{K}_\parallel b_3(\varepsilon - \mathcal{K}_\parallel^2 - \varkappa_1^2)/g,$$
$$b_6 = -\varkappa_2 b_2 - i\mathcal{K}_\parallel b_4(\varepsilon - \mathcal{K}_\parallel^2 - \varkappa_2^2)/g.$$

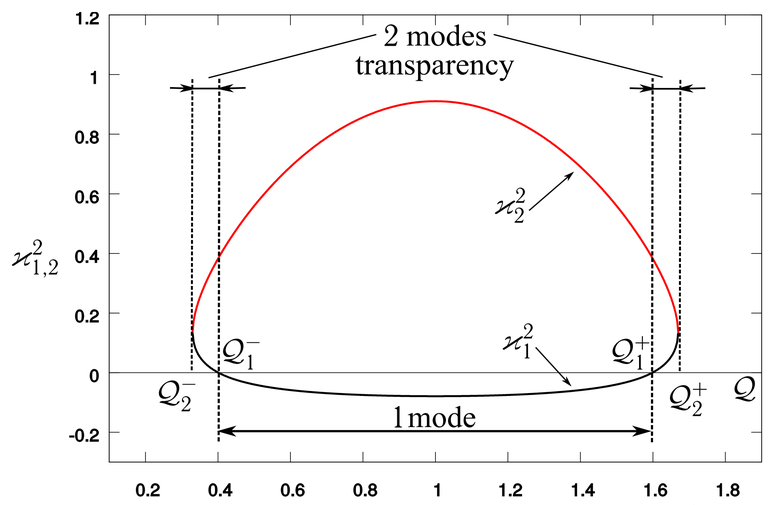
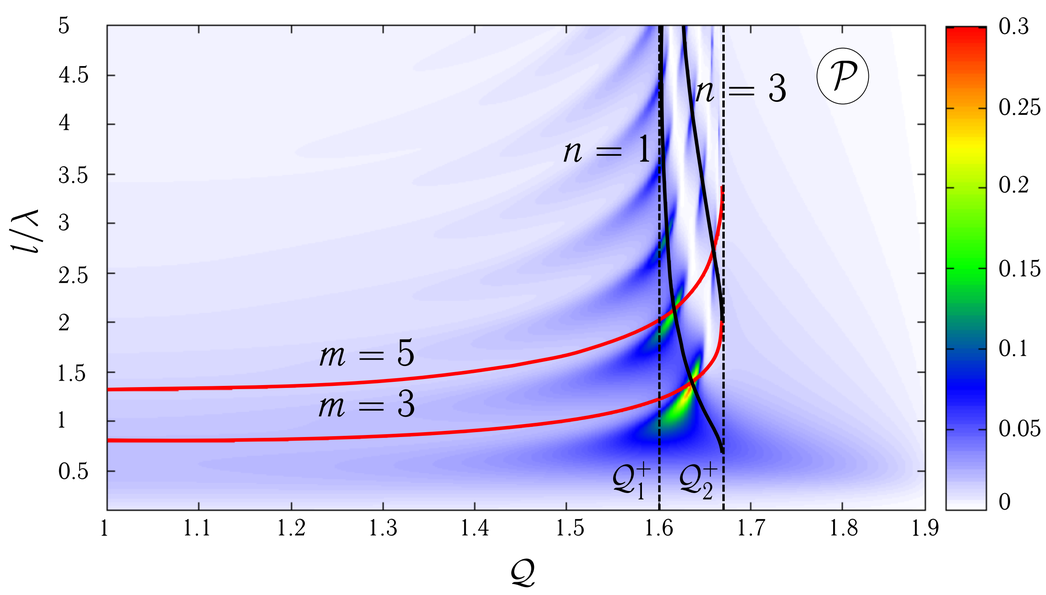



Fig. 4 : Window of plasma transparency to the generated EM waves ($v_b = 0.9c$, $\delta n = 0.1n_0$, $n_b = 0.01n_0$, $\Omega = 0.9\omega_p$).

- The region of trancparency for both modes is bounded by
$$\varkappa_1^2 = 0, \qquad \varkappa_1^2 = \varkappa_2^2.$$
- Solutions for $\mathcal{Q}$:
$$\mathcal{Q}_1^\pm = 1 \pm \hat{v}_b\sqrt{\varepsilon + g},$$
$$\mathcal{Q}_2^\pm = 1 \pm \hat{v}_b\sqrt{\varepsilon + g\xi},$$
where
$$\xi = -\frac{g(\eta + \varepsilon) + 2\sqrt{\varepsilon\eta(g^2 - (\eta - \varepsilon)^2)}}{(\eta - \varepsilon)^2}$$
- Both plasma modes penetrate into the plasma in regions $\mathcal{Q}_1^+ < \mathcal{Q} < \mathcal{Q}_2^+$ and $\mathcal{Q}_2^- < \mathcal{Q} < \mathcal{Q}_1^-$
- These modes have X-polarization with different angles to the magnetic field.

(a)




(b)

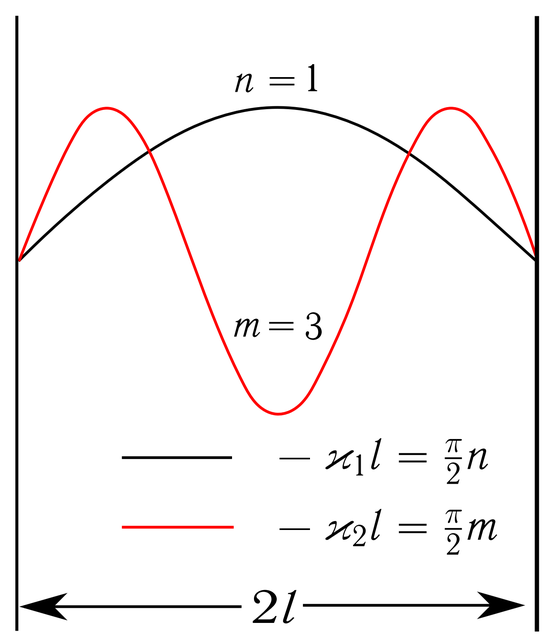



Fig. 5 : (a)Radiation efficiency as a function of the modulation period and plasma thickness for the plane antenna (for the parameters $n_b = 0.01n_0$, $v_b = 0.9c$, $\Omega_e = 0.9\omega_p$, $\delta n = 0.1n_0$); (b) Transverse structure of electric fields for plasma eigenmodes at the point of the global maximum of $\mathcal{P}$.

Fig. 5(a) shows the relative power of EM radiation for the constant electric field amplitude ($\int \sim E_0^2 L_z$), where the radiating plasma region is characterized by the length $L_z = v_b/\Gamma$ required for beam trapping. The conditions for local maxima on Fig.5(a) are $\varkappa_1 l = \pi n/2$(black) and $\varkappa_2 l = \pi m/2$(red) for odd $n$ and $m$. This is due to the fact that work of the uniform current under the field of plasma wave becomes maximal when we integrate over the half-wavelength plasma and the minimum when the plasma width is raised to the wavelength.

## Summary

- The theory of EM emission generated in a thin magnetized plasma with the longitudinal density modulation under the injection of an electron beam has been formulated in terms of plasma antenna.
- It has been predicted that, at certain emission angles, plasma becomes transparent to radiation and the whole plasma volume may be involved in generation of EM waves.
- The relative power remains enough high even for relatively thick plasma ($\sim 10 - 15\%$).
- The proposed method can be generalized to the turbulent regime in which random fluctuations of plasma density are represented by a set of periodic perturbations of the type
$$\delta n \sim \sum_q n_q e^{iq\,r}$$