Design Optimization of a Helical Plasma Thruster

Alexei D. Beklemishev[1,2,a)]

[1] *Budker Institute of Nuclear Physics, Novosibirsk, Russia.*
[2] *Novosibirsk State University, Novosibirsk, Russia.*

[a)] Corresponding author: bekl@bk.ru

Abstract. The Helical Plasma Thruster [1] is intended to work as the main megawatt-class thruster for interplanetary missions or inter-orbital tug shuttles. Its plasma accelerator channel has a strong axial magnetic field for wall insulation, ending with a magnetic nozzle. These features, as well as the ability to change the exhaust velocity and thrust in flight, are similar to that of the well-known VASIMR engine [2]. However, the plasma acceleration method of rotating plasmas in helically corrugated field is original. It is not relying on the cyclotron (or other) resonances and can thus work with ion mixtures and even with partially ionized plasmas. However, the plasma rotation needs biased electrodes, so that erosion might be a problem. This paper describes an optimized design of the thruster for protection of electrodes. There is also a possibility for effective reduction of the engine volume.

INTRODUCTION

To produce thrust, any electro-jet rocket engine should provide axial acceleration of plasma ions by somehow transferring momentum from the engine parts to the propellant plasma. The momentum flux to the engine parts can be in the form of particle pressure, acting on the engine walls, in the form of the magnetic field pressure, acting on the field coils or magnets , and in the form of the electric field tension, acting on biased electrodes or antennas. It is quite obvious that the electromagnetic momentum transfer types are preferable, since the particle momentum flux is inevitably associated with the particle energy flux, and thus with energy losses, wall heating and erosion. The use of the electric field tension looks most straightforward since it allows direct energy supply to the plasma. Indeed, most plasma thrusters use it in combination with transverse magnetic field to limit the axial electron mobility and increase the sustainable axial electric field for ion acceleration. However, the axial magnetic field would be very beneficial for plasma-wall insulation, especially in high-power high-thrust engines. It would also lead to limitation of the axial electric tension due to high parallel electron conductivity, so that the electric tension becomes inefficient.

Momentum transfer of the magnetic type boils down to diamagnetic forces expelling plasma with finite transverse pressure from an expanding magnetic field. The primary examples of such setup are the magnetic mirror and the magnetic nozzle. Stationary magnetic field produces no work, so that functioning of a magnetic nozzle relies on plasma to be heated. An efficient two-stage heating scheme was proposed and developed at Ad Astra Rocket Co. by the VASIMR team. The first heating stage is relatively weak and is based on a helicon discharge for ionization of the propellant gas by hot electrons. The pressure and temperature of this plasma are relatively low. The second heating stage is based on single-pass cyclotron heating of ions [3] within the expanding field of the magnetic nozzle. The electrons remain relatively cold while the transverse energy of ions is directly transformed into the axial kinetic energy of the exhaust by the magnetic nozzle. Significant work on plasma detachment from the axial magnetic nozzle was also done in connection with the VASIMR project [4].

Design of the Helical Plasma Thruster [1] is based on axial acceleration of rotating magnetized plasmas in magnetic field with helical corrugation, similar to principles incorporated into SMOLA open trap [5, 6]. The idea is that the propellant ionization zone can be placed into the local magnetic well, so that initial ions are trapped. The $E\times B$ plasma rotation is provided by an applied radial electric field. Then, from the rotating plasma viewpoint, the magnetic wells of the helically corrugated field look like axially moving mirror traps. Specific shaping of the corrugation can allow continuous acceleration of trapped plasma ions along the magnetic field by diamagnetic forces. At the end the accelerated propellant is expelled through the expanding field of a magnetic nozzle. By features of the acceleration principle the Helical Plasma Thruster may operate at high energy densities but requires a rather high axial magnetic field, which places it in the same class as the VASIMR rocket engine [2]. It also allows in-flight variability of specific impulse, power and thrust. This paper describes an optimized design of the thruster with detached plasma for reduced erosion. Scheme of the optimized discharge is shown in Fig.1. Due to spiral shape of the ion drifts the discharge occupies just a fraction of the flux-tube volume. In particular, in the optimized regime, plasma in the magnetic nuzzle will occupy just a small fraction of the flux, so that this bulky part of the engine can be designed much smaller.

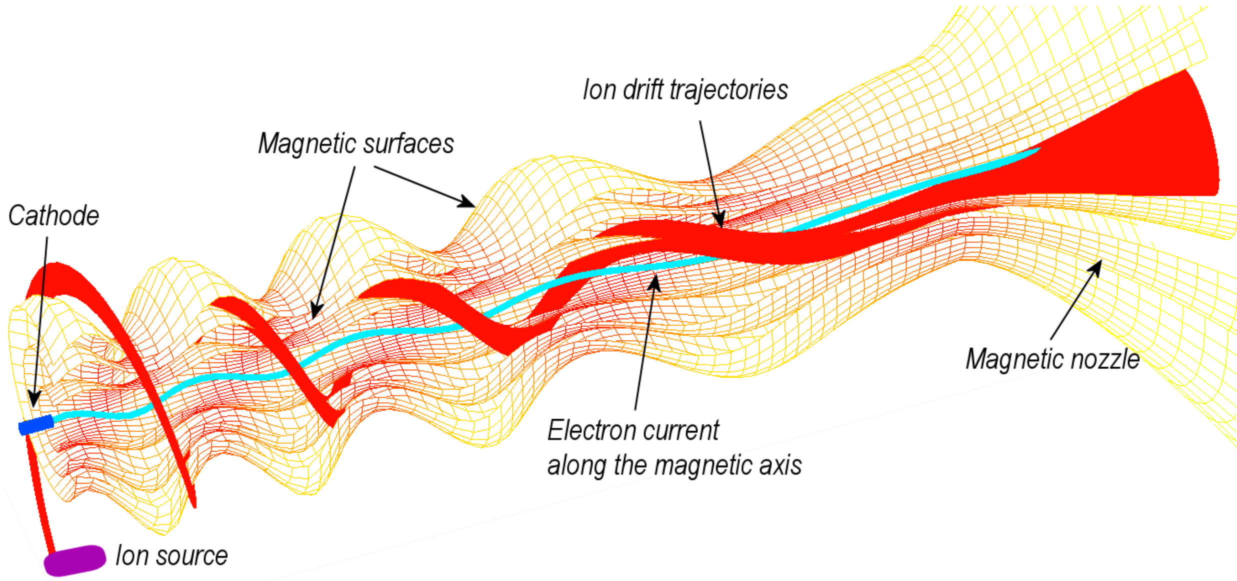

FIGURE 1. Scheme of the plasma drifts within the helical plasma thruster in the optimized regime (optimized length and period of the helix, negative biasing of the axis, optimal ratio of the biasing to the guiding magnetic field.)

MODEL OF THE DISCHARGE

An ion born in crossed fields is immediately accelerated by the electric field, obtains transverse energy and moves along a cycloidal-like trajectory. Its height in radial direction is smaller than the system radius, for otherwise the ion would end up on the wall even without further radial drift. On the far side of cycloid the ion kinetic energy is due to its displacement by two larmor radii ρ in the radial electric field E

$$mv^2/2 = 2\rho qE, \quad \rho = v/\Omega, \tag{1}$$

where Ω is the cyclotron frequency. Then

$$\rho = \frac{4}{m}\frac{qE}{\Omega^2}, \tag{2}$$

and the minimum required radius of the system (containing $N > 2$ larmor radii) satisfies

$$r_{\min} > \frac{2c}{B}\sqrt{UN\frac{m}{q}} \approx 2\times 10^{-4} B^{-1}\sqrt{UNA}. \tag{3}$$

Here B is the average magnetic field, U is the applied biasing potential in Volts, m/q is the charge-to-mass ratio of the propellant ions, radius is in meters, the magnetic field is in Tesla, and A is the atomic mass number of single-charge ions. Thus, transverse confinement of ions requires significant magnetic field and heavy propellants are usable in superconducting devices with fields of order of a few Tesla.

For continuous acceleration of ions it is necessary for them to be trapped in local magnetic wells of the helically corrugated field. To formulate the trapping condition it is useful to go into the rotating reference frame, where the $E\times B$ drift of the ion vanishes, but the well gets axial velocity V due to its helical structure:

$$V = h(r,z)\frac{\omega_E}{2\pi} = h(r,z)\frac{cE}{2\pi rB} = \Omega\rho\frac{h(r,z)}{8\pi r}. \tag{4}$$

Here $h(r,z)$ is the period of the helical corrugation of the field along a field line (which itself may be helical, so that the periods may have to be subtracted.) Since the helical periods of the magnetic axis and of the magnetic coils are identical, the period of the magnetic wells along the axis is infinite, $h(0,z) \to \infty$. Fortunately, the field lines on other flux surfaces can be (and generally are) inclined to the direction of the helix [7]. By the flux surfaces in this context we understand magnetic surfaces with constant electrostatic potential (rather than with constant pressure as in the theory of equilibrium.) Thus the $E \times B$ drifts occur on flux surfaces. Then each flux surface can be characterized by $\alpha = 2\pi r/h(r,z)$, which is the inclination of the magnetic field to the direction of the magnetic axis.

Let us return to the trapping condition. Assuming that the freshly born ion has a relatively small axial velocity in the rest frame (which is true for single-atom gases), its parallel velocity with respect to the moving magnetic well is $-V$. Then the trapping occurs if

$$\frac{m}{2}V^2 \leq \mu\tilde{B} = \frac{m}{2}\rho^2\Omega^2\frac{\tilde{B}}{B}, \quad \to \quad \frac{\tilde{B}}{B} \geq \frac{h^2}{64\pi^2 r^2}, \tag{5}$$

where $\tilde{B}/B$ is the corrugation amplitude. Trapping is, of course, impossible on the magnetic axis itself. However, the small numerical factor makes this condition quite realizable over most of the plasma radius. Note that the trapping condition is independent of the applied potential, so that variability of acceleration regimes can be achieved with the same magnetic system.

It is not immediately clear where the energy needed for sustained acceleration comes from, as the resulting kinetic energy of the ion in the rest frame exceeds its initial energy. The answer is in the work of the electric field upon the ion charge, as the ion drifts in radius during the process of axial acceleration. This radial drift can be interpreted as the magnetic-gradient drift due to azimuthal gradient of the field strength of the helical magnetic structure:

$$v_r = \frac{\rho^2}{2}\nabla_\phi\Omega = \frac{\mu\nabla_\phi B}{m\Omega} = \frac{h}{2\pi r}\frac{\mu\nabla_\| B}{m\Omega}. \tag{6}$$

Thus, the radial drift velocity v_r is proportional to the axial acceleration due to diamagnetic force $a_\| = \mu\nabla_\| B/m$. If we now try to compare the work of the accelerating force and the work of the electric field, we will get an identity $\mu\nabla_\| B \cdot Vdt \equiv qE_r \cdot v_r dt$. The energy conservation law is fulfilled, as expected, but the ion drift trajectory acquires radial extension. Also note that the mean transverse energy of the ion along the drift trajectory changes approximately proportional to the local value of B, i.e., in the absence of scattering the ion would keep moving along some kind of curved cycloid. Example of the ion drift trajectory is presented in Fig.1.

One can design the helical structure with an increasing period $h(r,z)$ along the drift trajectory in order to keep the ion in the accelerating phase of the field for as long as needed. Let us find the synchronization condition in approximation of small larmor radius. In the maximum acceleration phase $\nabla_\| B \approx \pi\tilde{B}/h(z)$, so that

$$a_\| = \ddot{z} = \pi\mu\tilde{B}/mh(z). \tag{7}$$

This can be integrated in time after multiplication by $\dot{z}$ to yield

$$\dot{z}^2 = 2\pi\mu\tilde{B}/m \cdot \int^z \frac{dz}{h(z)} = V^2(h). \tag{8}$$

Here the last equality comes from the synchronization requirement $\dot{z} = V$, where V is given by Eq.(4) and is itself dependent on h. Solving this for h we find

$$h(z) = \left(h_0^3 + H^2 z\right)^{1/3}, \quad H = 8\pi r\sqrt{3\pi\tilde{B}/2B}. \tag{9}$$

Taking into account the trapping condition, (5), we note that $H \geq 2.17h$, so that the rate of increase of $h(z)$ is sufficiently fast and it doubles in less than one and a half periods: we get $h = 2h_0$ at $z \approx 1.5h_0$ for the minimum $\tilde{B}$ satisfying the trapping requirement. The exhaust velocity for accelerator of length L can be estimated as

$$V_{ex} \approx V(h(L)) = V(h_0)\left(1 + \frac{H^2 L}{h_0^3}\right)^{1/3}. \tag{10}$$

Note that the above estimates are rather rough, since the drift trajectory in reality extends in radius, while $h = h(r)$, $\tilde{B} = \tilde{B}(r)$, etc., so that the synchronization condition should be optimized numerically for each particular design of the magnetic system. Also note that the diamagnetic force and hence the acceleration rate is really limited by the maximum gradient of the magnetic field along a field line rather than by the corrugation depth and the period as in above estimates, where the field modulation is assumed sinusoidal. It means that fields with sharp gradients are preferable, but, of course, they are harder to generate.

In practice it will be difficult to place all ions exactly into the maximum acceleration phase. Thus the magnetic system should be designed with some slack, providing significant phase volume for trapped ions. This means that the synchronization condition, Eq.(9), should be regarded as realizing the upper limit of the acceleration tempo. The optimum value of H for good performance should be a factor of 2 lower than that.

OPTIMIZATION AND ANALYSIS

As shown above, the accelerated ions acquire energy from the electric field during radial drift, i.e., the drift direction of a positive ion is toward the negative flux surface. Placing the cathode on axis (as in Fig.1) is clearly preferable from the viewpoint of suppressed erosion, since it will cause the radial pinch of ions, while the opposite biasing would cause radial expansion. There are two other factors in favor of the negative bias on axis: 1) there is a natural increase of the helix period toward the magnetic axis that is consistent with synchronization requirement for the ion drift; 2) the plasma exhaust in this case would occur near the axis of the magnetic nozzle, which makes it more focused in the required direction and increases the power efficiency of the engine.

The maximum energy that an ion can reach during acceleration determines the specific impulse of the engine. If the drift trajectory goes from the outer ion source at the anode potential to the inner magnetic axis biased by the cathode, the energy is equal to the total biasing potential. However, if the trajectory reaches the axis before the exit, the length of the accelerator channel can be judged too long, while if it is too short, only a part of the applied potential could be absorbed by ions. We cannot change the channel length in flight, so that using the biasing potential as a handle to variation of the specific impulse has limited applicability. However, if the magnetic field can be varied as well, the situation becomes much better.

Suppose that we want the radial drift velocity, given by Eq.(6), to cover exactly the device radius r_d during the time of axial acceleration over the device length, then $r_d/v_r \propto L/V$. Considering the scaling of involved velocities with E and B we get $V/v_r \propto B^2/U = const.$, i.e., in order to change the biasing potential (and the exhaust energy) U and stay in the optimum discharge regime, one should change the guiding magnetic field accordingly:

$$B \propto \sqrt{U}. \tag{11}$$

If the optimum discharge configuration can be ensured, a lot of the magnetic field volume becomes unnecessary, as the ions do not enter it. This can be used for weight reduction of the system. Furthermore, some areas of the discharge, in particular close to the ion source, can be placed entirely out of the main solenoid. The discharge there can fit into a separate spiral drift channel.

As a last remark it should be noted that the proposed scheme of ion acceleration (that is coupled to the drift concentration of the ion beam on the magnetic axis) may be very useful for fuel injection into linear open traps for fusion.

ACKNOWLEDGMENT

This work has been supported by Russian Science Foundation (project N 14-50-00080).

REFERENCES

[1] A. D. Beklemishev, Physics of Plasmas **22**, 103506 (2015).
[2] F. R. Chang-Diaz, Sci. Am. **283**, 90 (2000).
[3] A.V. Arefiev, B.N. Breizman, Physics of Plasmas **11**, pp. 2942-2949 (2004).
[4] A.V. Arefiev, B.N. Breizman, Physics of Plasmas **12**, 043504 (2005).
[5] A. D. Beklemishev, Fusion Science & Technology **63** (1T), 355 (2013).
[6] V. V. Postupaev, A. V. Sudnikov, A. D. Beklemishev, I. A. Ivanov, Fusion Eng. and Design, **106**, 29 (2016).
[7] A.I. Morozov, L.S. Soloviev “The structure of magnetic fields,” in Reviews of Plasma Physics 2, Consultants Bureau, New York (1966).