Improved Plasma Confinement at High Beta

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Abstract. A new efficient method of confinement is proposed for use in linear traps. The plasma equilibrium in a linear trap at high β can evolve into a kind of diamagnetic "bubble" with a very small field inside. If the vacuum magnetic field of the trap has a quasi-uniform stretch near its minimum, the roughly cylindrical "bubble" with non-paraxial ends will occupy just this stretch. The "bubble" radius is determined by the balance of particle and energy fluxes, while the force balance corresponds to $\beta \approx 1$ at any radius. The effective mirror ratio of the trap in this diamagnetic regime can become very large, stifling the axial losses, but the cross-field transport inside it increases. The total confinement time can be found from solution of the system of equilibrium and transport equations and is shown to be $\tau_E \approx \sqrt{\tau_\parallel \tau_\perp}$. This means that the diamagnetic confinement at high β allows construction of relatively short linear traps as fusion reactors.

INTRODUCTION

This paper describes a novel concept of efficient high-beta plasma confinement that is half-way between the FRC and the linear gas-dynamic trap (GDT) [1]. It certainly improves on the standard GDT confinement by offering a greatly reduced reactor size at the cost of a more tricky MHD stabilization. The energy confinement in the diamagnetic "bubbles" will be probably worse than that in FRCs, but initial estimates suggest that a less stringent maintenance of equilibrium and stability will be required.

Currently, the main drawback of linear gas-dynamic traps as fusion reactors is in the geometry: while it is easy to construct an axially symmetric tube-like reactor, it has to be very long and thin. The reasons are as follows: the fusion power is proportional to the plasma volume and squared density, n, while the lost power is proportional to the plasma cross-section in mirror throats and density. The plasma occupies a magnetic flux-tube, so that cross-sections of the mirror throats and of the active zone are related as the ratio of the magnetic fields, i.e., the mirror ratio $R = B_m/B_0$. As a result, $Q_{DT} \propto nRL$. Now the maximum density as well as the mirror ratio are related to the maximum attainable confining magnetic field. Indeed, in the paraxial approximation the equilibrium is limited by β: $n \propto \beta B_v^2$, where B_v is the confining (vacuum) field in the active zone, while the magnetic field within the plasma is reduced as $B_0 = B_v \sqrt{1-\beta}$. Thus

$$Q_{DT} \propto LB_vB_m \frac{\beta}{\sqrt{1-\beta}}. \tag{1}$$

It follows that both the mirror field, B_m, and the confining field, B_v, should be chosen as high as technically possible, while the plasma radius can be made small as long as transverse losses stay less than axial. This last requirement is actually determining the length-to-radius ratio of the optimized reactor, $L/a \gg 1$, and its fusion power, W_F. The pure gas-dynamic scheme leads thus to $L > 5km$ and $W_F > 10GW$, which is clearly unacceptable. Still, the successful GDT design with sloshing ions may be used for a neutron driver of nuclear waste burner or a hybrid reactor [2].

Improvement of efficiency of mirror plugs may be in theory sufficient for construction of a gas-dynamic fusion reactor with reasonable length and power. This approach can be based on the multiple-mirror scheme as in the GDMT project [3], or on the active-helical-mirror scheme that will be tested by SMOLA device [4]. Still, in order to pretend to burn advanced fuels with low reactivity, the open traps need to increase the volume of reacting plasma without increasing axial losses, and this opportunity is offered by the very interesting β-dependent factor in Eq.(1). As $\beta \to 1$ the effective mirror ratio of the trap starts to grow rapidly due to diamagnetic radial expansion of the flux-tubes (see

Fig.1). This effect has been noticed at least 50 years ago for theta pinches, but now it can become a game-changer for the gas-dynamic confinement due to linear scaling of $Q_{DT} \propto R \propto 1/\sqrt{1-\beta}$. The increase in fusion efficiency due to β can be translated into a corresponding decrease in L, i.e., a really compact fusion reactor based on a linear trap may become possible.

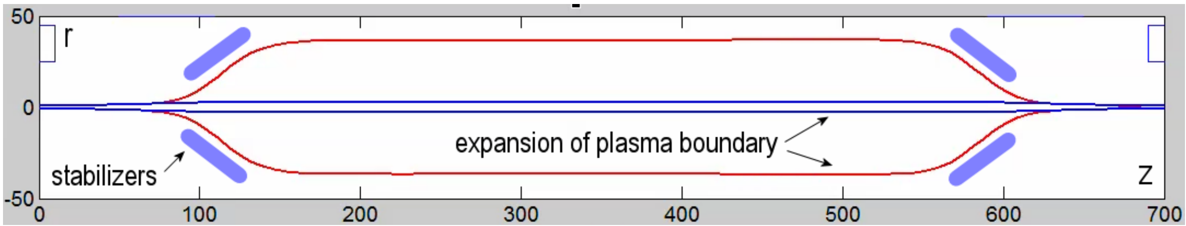

FIGURE 1. Expansion of flux tubes at high β leads to corresponding increase in the effective mirror ratio of a linear trap. If there is a quasi-uniform patch of the vacuum field at the bottom of the magnetic well, the resulting "bubble" will be roughly cylindrical. The plasma boundary at cylinder ends needs stabilization.

For realization of the $\beta \to 1$ limit we should address some difficult questions. How large the $1/\sqrt{1-\beta}$-factor can be made in realistic equilibria? Even if it is large, it may not be as beneficial as expected. While the magnetic flux is expelled from the plasma core, the transverse transport is also bound to increase up to infinity as the ions become unmagnetized. Will the gain in axial confinement be sufficient to justify the increased radial diffusion? The existence of an equilibrium state does not guarantee that it can be realized in experiment. It should be made stable at least to the ideal MHD modes. This is also very tricky at high β, as the predicted limit of ballooning stability in linear traps may be significantly lower than 1 [5].

EQUILIBRIUM

Our aims here are very specific: to study the high-β limit of equilibrium in long and thin axially symmetric traps. It appears that this particular limit typically results in significant growth of the initially thin plasma in radius. That expansion is axially localized around the minimum of the vacuum magnetic field.

The transverse pressure of plasma in a linear trap can be approximated as $p_\perp \approx p_\perp(\psi, B)$, where ψ is the flux function labeling magnetic surfaces, and B is the local magnetic field strength. Using the minimum of the magnetic field on a field line $B_0(\psi)$ for normalization of field, it can also be rewritten as $p_\perp = p_\perp(\psi, R)$, where $R(\psi, \ell) = B/B_0$ is the local mirror ratio of the magnetic field. The equation of paraxial equilibrium looks like $B_v^2 = B^2 + 8\pi p_\perp$ where $B_v(\vec{r})$ is the confining vacuum magnetic field. Let's divide it by the square of the minimum vacuum magnetic field on a field line, B_{v0}^2, and normalize the pressure by its value at the field minimum. We get

$$R_v^2 = (1-\beta) R^2 + \beta P(R), \tag{2}$$

where $R_v(\psi, \ell) = B_v/B_{v0}$ is the mirror ratio of the vacuum field, $\beta(\psi) = 8\pi p_\perp(\psi, 1)/B_{v0}^2$ is the β-value at the minimum of the field on a given field line, while $P(\psi, R) = p_\perp(\psi, R)/p_\perp(\psi, 1)$ is the normalized profile of pressure along the field line. This equation should be solved for $R(\psi, \ell)$ with restriction $R > 1$, while we are particularly interested in the case $1-\beta \ll 1$. Let's assume that we are dealing with a typical mirror with a monotonically growing field from its center. Then the left-hand side grows with ℓ, $\partial R_v^2/\partial\ell \geq 0$, and to find a solution for all ℓ we should have a growing right-hand side too, then the solubility condition becomes

$$\partial P(R)/\partial R^2 \geqslant -(1-\beta)/\beta. \tag{3}$$

In linear traps there are always some areas, where the pressure derivative along the field line is negative, since the pressure should be higher inside of the trap than in the mirror throats. Looking at Eq.(3), one can see that such decreases of pressure are restricted, and at $\beta \to 1$ they are entirely prohibited by the paraxial equilibrium. According to Kotelnikov [6] the equilibrium solutions of the paraxial equilibrium equations can be piecewise continuous, while the points of discontinuity can be interpreted as non-paraxial areas. At high β the function $R(\ell)$ becomes discontinuous. It is comprised of two (or more) continuous intervals: the "bubble" branch, where condition (3) is satisfied at low $R \sim 1$, and the outer branch, where the same condition is satisfied at large R only.

In the most typical quasi-isotropic case the function $P(R)$ is monotonously decreasing from 1 to 0, and one can approximate the pressure profile by parabola: $P(R) \approx 1 - \delta^2 \left(R^2 - 1\right)^2$. Then the equilibrium equation becomes quadratic and has a positive solution for $R^2 - 1$ only if $D = (1-\beta)^2 - 4\delta^2\beta\left(R_v^2(\ell) - 1\right) \geqslant 0$. It follows that the solution is discontinuous, and the length to discontinuity is defined by

$$R_v^2(\ell_d) = 1 + \frac{(1-\beta)^2}{4\delta^2\beta}. \tag{4}$$

One can see that $R_v(\ell_d) \to 1$ with $\beta \to 1$, i.e., the "bubble" branch of solution collapses to the bottom of the magnetic well. This behavior is different in presence of sloshing ions. Then the "bubble" branch is finite-length, $R_v^2(\ell_d) \to P_r > 1$, where P_r is the maximum of the curve $P(R)$ [7]. However, formation of non-paraxial ends of a "bubble" will soon cause the pressure anisotropy to relax, so that $P_r \to 1$ and the quasi-isotropic limit will be restored.

The function $R_v(\ell)$ is extremely important for shaping the equilibrium. It describes the form of the magnetic well of the vacuum field of the trap along field lines. In particular, one can design a linear trap with a finite-length patch of uniform field at the well bottom. Then the branch of equilibrium that exists only at $R_v = 1$ becomes extended into a cylinder. The "bubble" length can thus be prescribed via the form of the vacuum field. The bubble edges at high β will coincide with the ends of the uniform-field patch, so that we will be able to place there some equipment for MHD stabilization. Indeed, in the cylindrical case the interchange source term is finite only at the ends of the cylinder, while in the middle uniform patch the plasma is marginally stable. Thus, by increasing length we can add plasma without worsening stability. Furthermore, by placing localized stabilizers directly at the ends of the cylinder it should be theoretically possible to suppress even the edge ballooning modes.

TRANSPORT

Let's try to describe an axisymmetric steady-state equilibrium taking into account diffusion of the external magnetic field into the cylindrical "bubble". If everything is stationary, this means that there is a steady flux of plasma from the inside, $F_\perp$, that is closed by axial plasma losses, $F_\parallel$. The flux continuity equation is

$$[rF_\perp]' + rF_\parallel = 0, \tag{5}$$

that can be rewritten in terms of $\beta(r)$ as[7]

$$\left[\frac{\beta\beta' r}{1-\beta}\right]' = \lambda^{-2} r\beta\sqrt{1-\beta}. \tag{6}$$

Here the characteristic linear scale $\lambda = \sqrt{D_\perp \tau_\parallel/2}$ can be interpreted as the skin depth of the magnetic field by the time of the axial plasma outflow from the vacuum field of the trap. For gas-dynamic traps it is normally very small. Qualitatively, solution for the "developed bubble" looks as follows: over most of the radius $\beta \approx 1$, while the transition layer from $\beta \approx 1$ to $\beta \approx 0$ (the boundary) has the characteristic radial scale $\lambda \ll r$. This structure can be successfully described in the slab approximation, i.e., we set $r \approx a = const$, $\lambda^{-1}y/a = f$, and introduce the normalized radial coordinate $x = (r-a)/\lambda$.

Equation (6) can be rewritten as a system

$$f_x' = \beta\sqrt{1-\beta}, \quad \beta_x' = f(1-\beta)/\beta, \tag{7}$$

with boundary conditions $\beta(\infty) = 0$, $f(-\infty) = f_0$, where f_0 is the normalized source of ions. It can be partially integrated:

$$f^2 = \frac{2}{15}\left[8 - \sqrt{1-\beta}\left(8 + 4\beta + 3\beta^2\right)\right], \tag{8}$$

so that $f_0 = 4/\sqrt{15} \approx 1.03$. Substituting Eq.(8) into the second line of Eq.(7), we get the radial structure as

$$\beta' = -\frac{4}{\sqrt{15}}\frac{1-\beta}{\beta}\sqrt{1 - \sqrt{1-\beta}\left(1 + \frac{\beta}{2} + \frac{3\beta^2}{8}\right)}. \tag{9}$$

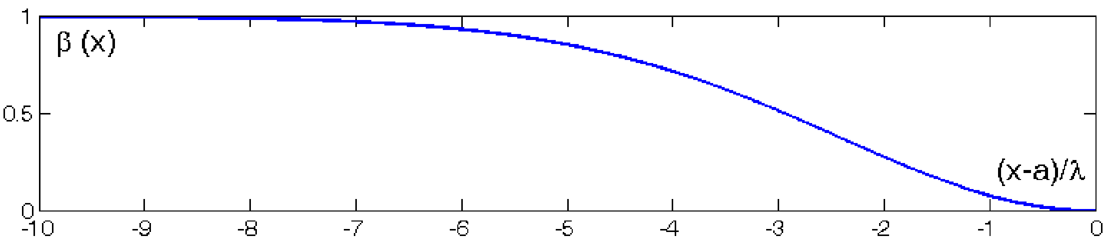

FIGURE 2. The radial structure of the boundary layer of the "bubble" in the MHD slab-transport model. There is vacuum ($\beta = 0$) beyond the $r = a$ surface, and the low-field interior ($\beta \approx 1$) to the left.

The plasma boundary is quite "rigid", i.e., there is no pressure at all beyond $r = a$, see Fig.2.

The particle confinement time is the ion content in the "bubble" divided by the flux of particles that are lost from it in a stationary state. The flux can be found using f_0, so that

$$\tau_n = \frac{\pi a^2 L n}{\Phi} = \frac{a}{\lambda f_0}\tau_\parallel \approx \sqrt{2\tau_\perp \tau_\parallel}, \tag{10}$$

where $\tau_\parallel$ is the axial confinement time in the vacuum field, and $\tau_\perp$ is the diffusion time over the full "bubble" radius. If the axial electron recycling is limited by the properly designed expanders, the energy loss is proportional to the particle loss [8], so that $\tau_E \approx 3\tau_n/8 \approx \sqrt{\tau_\perp \tau_\parallel}$.

The whole process can be approximately described as follows. Deep within the "bubble" the radial diffusion dominates, while the axial loss is vanishingly small. In fact the ions may not be magnetized inside of the "developed bubble" at all, having almost straight trajectories. All of the radial confinement is concentrated in the relatively thin boundary layer of width $\sim 6\lambda$. However, due to finite magnetic field within the boundary layer, the effective mirror ratio is also finite, so that the axial losses appear. In a unit of time the "bubble" looses particles from the layer of width λ and radius a by axial outflow, hence $\tau_n \sim a\tau_\parallel/\lambda$.

CONCLUSION

A new scheme for confining high-β fusion plasmas in a linear trap is described. It promises huge improvement of confinement quality as compared to the gas-dynamic scheme. A stable confinement of the $\beta \approx 1$ plasma cannot be easy, but there seems to be a straightforward way to use the conducting-shell stabilization method that is shown to work for FRCs [9]. Although there is still no detailed theory of stability and transport, it is probably worthwhile to attempt an experimental check of the predicted "bubble" formation and of the related improvement in confinement time. Such initial concept-exploration experiments are now in the planning stage in the Budker Institute of Nuclear Physics in Novosibirsk. Some additional details about the concept of diamagnetic confinement in linear traps can be found in Ref.[7].

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REFERENCES

[1] A. A. Ivanov, V. V. Prikhodko, Plasma Phys. and Contr. Fusion **55**, 006301 (2013).
[2] A. V. Anikeev, P. A. Bagryansky, A. D. Beklemishev, et al., Materials **8**, 8452 (2015).
[3] A. Beklemishev, A. Anikeev, V. Astrelin, et al., Fusion Sci. Technol. **63** (1T), 46 (2013).
[4] V. V. Postupaev, A. V. Sudnikov, A. D. Beklemishev, I. A. Ivanov, Fusion Eng. and Design **106**, 29 (2016).
[5] T. B. Kaiser and L. D. Pearlstein, Phys. Fluids **28**, 1003 (1985).
[6] I. A. Kotelnikov, Fusion Science & Technology **59** (1T), 47 (2011).
[7] A. D. Beklemishev, Physics of Plasmas **23**, (2016).
[8] D. D. Ryutov, Fusion Science & Technology **47**, 148 (2005).
[9] M. W. Binderbauer, T. Tajima, L. C. Steinhauer, et al., Phys. Plasmas **22**, 056110 (2015).