Radial and Axial Transport in Trap Sections with Helical Corrugation

Alexei D. Beklemishev[1,2,a)]

[1]Budker Institute of Nuclear Physics, Novosibirsk, Russia.
[2]Novosibirsk State University, Novosibirsk, Russia.

[a)]Corresponding author: bekl@bk.ru

Abstract. Plasma outflow through helically corrugated mirrors is influenced by its $E \times B$ rotation due to friction between the trapped and passing particles. If the relative velocity of these populations exceeds the thermal velocity, the onset of the two-stream instability will make the friction effective even in weakly-collisional plasmas. This regime can be achieved by plasma biasing. The momentum transfer between the plasma and the magnetic corrugation is accompanied by radial drifts and currents. The radial pinch effect is shown to be proportional to the axial pressure gradient that is created by the helical mirror. However, it can be directed either to or from the axis, depending on the symmetry of the helix and the sign of biasing. Equations of the transport model are presented and analyzed.

INTRODUCTION

Sections with helical corrugation of the magnetic field are currently considered for supplemental improvement of axial confinement in gas-dynamic mirror traps [1] as a possible modification of the GDMT project [2]. The corresponding concept exploration experiment is under development [3, 4].

The $E \times B$ plasma rotation in a helically corrugated field leads to the effective axial motion of magnetic mirrors. If it is coupled to enhanced plasma scattering (to facilitate ion exchanges between trapped and passing populations) such motion is capable of transferring axial momentum from the magnetic field coils to the plasma flow. Theory also predicts radial pinch effect coupled to this momentum transfer that is capable of contracting or expanding the discharge in radius. The fact is that different components of plasma transport in helically corrugated sections are inherently coupled and should be considered together. The use of helical mirrors is illustrated by Fig.1.

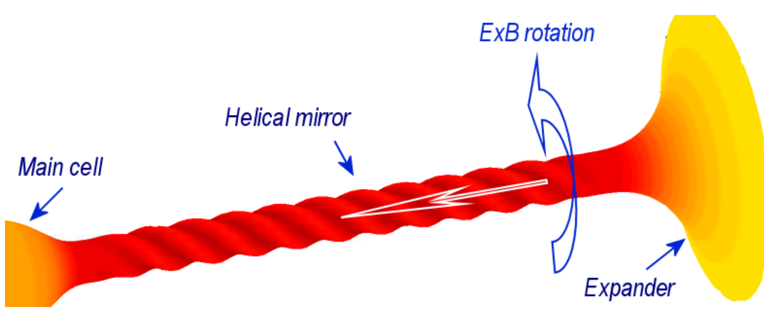

FIGURE 1. Scheme of the helical mirror section and its pumping effect.

This paper will attempt theoretical description of the radial and axial transport components in a helical-mirror section that is attached to a plasma reservoir (main trap cell). Different models of ion scattering (binary and turbulent) can be considered. Since the classical binary scattering is too weak to work efficiently in stationary reactors, the GDMT scheme relies on self-consistent or driven turbulence. Here we suggest that the easily-reached condition of supersonic relative velocity of trapped and passing ion populations in a helical mirror would lead to excitation of two-stream instabilities and thus naturally result in the necessary momentum transfer.

MODEL OF TRANSPORT

Let's consider force components on a plasma within the flux band $d\psi, dz$, while defining the nested flux surfaces as magnetic surfaces with constant values of electrostatic potential. The azimuthal Ampere's force is due to radial current

$$dF_\phi = -\frac{1}{c}\int j_\psi B dV, \tag{1}$$

where dV is the volume of the flux band. In a stationary state this force is compensated by the axial diamagnetic force on trapped particles that in turn experience friction with the axial plasma flow. Situation is similar to a worm gear where dF_ϕ is the driving force, while there is a friction-type load. The energy conservation law in this case looks like

$$u_\phi dF_\phi + u_z dF_z = 0, \tag{2}$$

where u_z is the effective axial velocity of magnetic wells, u_ϕ is the velocity of azimuthal plasma rotation, and dF_z is the axial friction force. The velocity components relate as $u_\phi/u_z = \alpha(\psi, z)$, where α is the average inclination of the magnetic field lines to the magnetic axis on the given flux surface, then $dF_z = -\alpha dF_\phi$, which is just the gear ratio of the worm. Thus, the axial force that is generated within a flux belt is

$$dF_z = \frac{\alpha}{c}\bar{B} j_\psi \ell dz dr, \tag{3}$$

where j_ψ is the radial current density through the surface of the flux belt $dS = \ell dz$, $\bar{B}$ is the average magnetic field in it, ℓ is the length along the belt, and r is its effective radius, defined via $dV = \ell dz dr$. Subsequent equations will be written in the same simplified representation, when the plasma parameters are averaged over the flux surface belt (but the averaging bars will be omitted for shortness).

The friction force on trapped particles can be related to the plasma outflow velocity relative to the motion of trapped ions

$$dF_z = \nu\varkappa\rho\left(U_z + u_z\right)\ell dz dr, \tag{4}$$

where ρ is the mass density, ν is the momentum transfer rate, U_z is the plasma outflow velocity, $\varkappa$ is the fraction of trapped particles, and, as before, the axial velocity of magnetic wells is proportional to the ExB rotation velocity,

$$u_z = u_\phi/\alpha = \frac{c}{\alpha B}\frac{\partial\varphi}{\partial r}, \tag{5}$$

where $\varphi(\psi)$ is the plasma potential. The friction force causes reduction in the axial momentum flux

$$\nu\varkappa\rho\left(U_z + u_z\right) = -\frac{\partial P}{\partial z} - \rho U_z\frac{\partial U_z}{\partial z}, \tag{6}$$

where P and ρ are the plasma pressure and its mass density.

In presence of a radial particle flux the continuity equation looks like

$$\frac{\partial}{\partial z}\left(\rho\left(U_z - \varkappa u_z\right)\ell\right) + \frac{\partial}{\partial r}\left(\rho U_r \ell\right) = 0, \tag{7}$$

while the radial particle flux is due to two factors: the ohmic diffusion and the pinch, caused by the radial current j_ψ in ions:

$$\rho U_r = -D\frac{\partial\rho}{\partial r} + \frac{M}{q}j_\psi. \tag{8}$$

Here D is the radial diffusion coefficient, and M/q is the mass-to-charge ratio of ions.

Finally, we need the current closure condition that would relate the radial currents, the radial electric filed within the plasma, and the electrode potentials that are our handles to the worm gear of the helical mirror. The current closure looks like

$$\frac{\partial}{\partial r}\int j_\psi \ell dz = -\ell I_z, \tag{9}$$

where I_z is the current density to the end wall (or biasing electrodes) from the helical section, and the integral is over the whole flux surface, rather than over some belt of it. The current I_z passes through the expander sheath [5] and causes deviation of the plasma potential φ from its stationary "ambipolar" value $\varphi_A(r)$:

$$I_z = J_i\left(1 - \exp\left[(\varphi - \varphi_A)\, e/T_e\right]\right). \tag{10}$$

Here J_i is the equilibrium ion current density to the end wall (loss current), T_e is the electron temperature. For small deviations $I_z \approx -J_i(\varphi - \varphi_A)\, e/T_e$, so that the current closure condition becomes

$$\frac{\partial}{\partial r}\int j_\psi \ell dz = \frac{e\ell J_i}{T_e}(\varphi - \varphi_A). \tag{11}$$

Summarizing the model, we express the rotation and translation velocities, u_ϕ and u_z, in terms of the electrostatic potential. We can also exclude the radial particle flux, U_r, and the axial force, dF_z. Then, besides Eq.(11), the system contains three more equations:

$$j_\psi = \nu\varkappa\rho\frac{c}{\alpha B}\left(U_z + \frac{c}{\alpha B}\frac{\partial\varphi}{\partial r}\right), \tag{12}$$

$$\frac{\partial}{\partial z}\left(\rho\left(U_z - \frac{\varkappa c}{\alpha B}\frac{\partial\varphi}{\partial r}\right)\ell\right) + \frac{\partial}{\partial r}\left(\frac{M}{q}\ell j_\psi\right) = \frac{\partial}{\partial r}\left(\ell D\frac{\partial\rho}{\partial r}\right), \tag{13}$$

$$\nu\varkappa\rho\left(U_z + \frac{c}{\alpha B}\frac{\partial\varphi}{\partial r}\right) = -\frac{\partial P}{\partial z} - \frac{\rho}{2}\frac{\partial U_z^2}{\partial z}. \tag{14}$$

Five unknown functions, j_ψ, φ, ρ, P and U_z should satisfy four equations, that makes it a closed system if the pressure and density are additionally related throgh some equation of state, $P = P(\rho, r)$. However, in a more realistic setting one would want to describe the variability of the fraction of trapped particles, $\varkappa$, as well. This would require some model for the changes of the trapping state along field lines, and is outside of the scope of the present paper. Boundary conditions can be set on radial distributions of pressure and density in the main cell (at $z = 0$).

Let's normalize radius r by a, z by L, φ by T_e/e, U_z by $c_s = \sqrt{T_e/M}$, replace $\rho = Mn_0 n$, $c_s/\nu = \lambda$, and $P = Mn_0 c_s^2 p$. Then

$$\frac{1}{n}\frac{\partial p}{\partial z} + \frac{1}{2}\frac{\partial U^2}{\partial z} + \varkappa\Lambda\left(U + \zeta\frac{\partial\varphi}{\partial r}\right) = 0, \tag{15}$$

where $\zeta = cT_e/(eB\alpha a c_s)$ and $\Lambda = L/\lambda$ are dimensionless parameters of the problem. The continuity equation reads

$$\frac{\partial}{\partial z}\left(n\left(U - \varkappa\zeta\frac{\partial\varphi}{\partial r}\right)\ell\right) + \frac{\partial}{\partial r}\left(Z^{-1}n\varkappa\Lambda\zeta\left(U + \zeta\frac{\partial\varphi}{\partial r}\right)\ell\right) = \frac{\partial}{\partial r}\left(\ell D\frac{\partial n}{\partial r}\right), \tag{16}$$

where now $D = DL/c_s a^2$ and is just a small correction. Finally, the current closure condition becomes

$$\frac{\partial}{\partial r}\int_0^1 \varkappa n\Lambda\zeta\left(U + \zeta\frac{\partial\varphi}{\partial r}\right)\ell dz = \ell J(\varphi - \varphi_A), \tag{17}$$

where $J = J_i/en_0c_s$. The curve length ℓ describes the geometry of the flux surfaces and can be set to r for quasi-cylindrical cases.

ANALYSIS

When $\varkappa = 0$, i.e., without trapping, we regain the standard MHD outflow:

$$\frac{1}{n}\frac{\partial p}{\partial z} + \frac{1}{2}\frac{\partial U^2}{\partial z} = 0, \qquad \frac{\partial}{\partial z}(nU) = \frac{1}{\ell}\frac{\partial}{\partial r}\left(\ell D\frac{\partial n}{\partial r}\right), \tag{18}$$

and $\varphi = \varphi_A$. When $\Lambda = 0$ but $\varkappa \neq 0$, the continuity equation is modified to include the (counter)flux of trapped particles. The most interesting limit is when $\varkappa\Lambda \gg 1$, i.e., the helical mirror is in principle capable of absorbing the plasma pressure. In this case the plasma outflow does not reach the sound speed, so that from Eq.(15) we get

$$U + \zeta\frac{\partial\varphi}{\partial r} \approx -\frac{1}{\varkappa\Lambda n}\frac{\partial p}{\partial z}. \tag{19}$$

As a result, the potential can be expressed explicitly

$$\varphi = \varphi_A + \frac{1}{\ell J}\int_0^1 \zeta \frac{\partial p}{\partial z}\ell dz, \tag{20}$$

and the system simplifies to the single equation (of continuity):

$$\frac{\partial}{\partial z}\ell\left(\frac{1}{\varkappa\Lambda}\frac{\partial p}{\partial z} + (1+\varkappa)n\zeta\frac{\partial}{\partial r}\left[\varphi_A + \frac{1}{\ell J}\int_0^1 \zeta\ell\frac{\partial p}{\partial z}dz\right]\right) + \frac{\partial}{\partial r}\ell\left(Z^{-1}\zeta\frac{\partial p}{\partial z} + D\frac{\partial n}{\partial r}\right) = 0. \tag{21}$$

The second term here represents the divergence of the radial particle flux. For $\zeta < 0, \partial p/\partial z < 0$ (which is possible since $\zeta \propto 1/\alpha$) the net flux may be directed toward the axis, i.e., there seems to be a radial pinch that is independent of the electrode biasing, φ_A. However, in the absence of biasing it is impossible to get both ζ and the pressure gradient negative simultaneously: one needs an energy source for plasma pumping, and it can be either the external biasing or the radial plasma expansion. The natural plasma rotation in its ambipolar field can be used for suppression of the axial outflow, but only at the cost of increased radial losses. This price tag can be waived in case of negative biasing of the plasma axis. However, one should note, that the directions of plasma rotation in the ambipolar and biasing cases are opposite, so that they cannot be used in a single discharge, since the pumping effects of the fixed helical structure would have opposite directions.

The momentum transfer rate ν and its dimensionless analog Λ represent complex nonlinear processes and cannot be considered as just constants. In particular, when the relative velocity of the trapped and passing populations $U_z + u_z$ exceeds the sound speed c_s one should expect this rate to increase enormously due to the onset of the two-stream instability. Since the binary scattering is too weak, this regime looks like the desired mode of operation of helical mirrors in fusion regimes. If the instability is triggered, the first term in Eq.(21) becomes highly nonlinear, as Λ becomes a complex function of the axial pressure gradient.

It is necessary to note that all helical mirrors have a hole on the magnetic axis, since the field corrugation there is zero. On the other hand, the inclination of field lines to the axis in its vicinity is very small, so that the relative flow speed will be supersonic. Thus it is likely that the vicinity of the magnetic axis will be a turbulent area where the simplified treatment via Eq.(21) will fail. One should note the role of coefficient $J(r)$. Since this is essentially the ion flux density to the end wall, it cannot go entirely to zero, since otherwise the plasma biasing schemes would be difficult to realize.

A new method of plasma refueling by means of the radial pinch of cold ions from the plasma periphery can be suggested. Design of the ionization zone in the low-field phase of the corrugation as in the helical plasma thruster [6] allows to inject a stream of cold ions into the trapped state of the helical mirror without significant cold-ion outflow into the expander. This cold inflow would facilitate the effective momentum transfer by two ways: 1) the fraction of trapped ions, $\varkappa$, will go up, especially in low-density areas, and 2) the excitation of the two-stream turbulence in presence of a cold beam is much more effective. Refueling by ions rather than by neutrals would also lower its energy cost by eliminating the charge-exchange losses.

ACKNOWLEDGMENT

This work has been supported by Russian Science Foundation (project N 14-50-00080).

REFERENCES

[1] A. D. Beklemishev, Fusion Science & Technology **63** (1T), 355 (2013).
[2] A. Beklemishev, A. Anikeev, V. Astrelin, et al., Fusion Science & Technology, **63** (1T), 46 (2013).
[3] V. V. Postupaev, A. V. Sudnikov, A. D. Beklemishev, I. A. Ivanov, Fusion Eng. and Design, **106**, 29 (2016).
[4] A.V. Sudnikov, A.D. Beklemishev, V.V. Postupaev, A.V. Burdakov, I.A. Ivanov, N.G. Vasilyeva, K.N. Kuklin, A.G. Makarov, E.N Sidorov, Helical Mirror Concept Exploration: Design and Status, AIP Conf. Proc. (these proceedings).
[5] D. D. Ryutov, Fusion Science & Technology **47** , 148 (2005).
[6] A. D. Beklemishev, Physics of Plasmas **22**, 103506 (2015).