Theory of Electromagnetic Wave Generation via a Beam-Plasma Antenna

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Abstract. In this work, we propose a theory describing generation of electromagnetic waves in a thin beam-plasma system with a characteristic transverse size comparable with the radiation wavelength. Such a theory can be useful for the description of the processes responsible for EM emission in beam-plasma experiments at the GOL-3 multi mirror trap.

INTRODUCTION

The processes of electromagnetic (EM) emission from a beam-plasma system have long been of great interest with regard to different astrophysical phenomena. In the terahertz frequency range (0.1—0.5 THz), similar radiation processes are actively studied in laboratory beam-plasma experiments at the GOL-3T open trap. The main peculiarities of these experiments are the small plasma size and relatively strong guiding magnetic field. It has been experimentally found that the beam power converts to radiation power much more efficiently when the typical transverse size of the beam-plasma system becomes comparable to the radiation wavelength [1]. These experiments have motivated theoretical studies on the mechanism of EM emission in the thin plasma case [2]. The feature of this regime is that a thin plasma column actually acts as an antenna which can efficiently radiate EM waves if a superluminal wave of electric current is excited inside the plasma. The wave growing inside a homogeneous plasma column under the two-stream instability cannot be the source for this radiation, since being in the Cherenkov resonance with the beam such a wave has the sublight phase velocity. In a plasma with a longitudinal density modulation, however, the beam-driven wave generates a long-wavelength sattelite in which the phase can move faster than light. Only such a scattered wave enables to exchange the energy with vacuum electromagnetic waves. It is clear that a necessary condition for efficient generation of EM radiation in such a scheme is the existence of a longitudinal inhomogeneity of the plasma density. In turbulent plasmas typical to the experiments mentioned, such inhomogeneities are characterized by wide spectra and radiate EM waves in all possible directions. But in the plasma with a fixed periodic density perturbation EM waves generated via the antenna mechanism can form a narrow beam which is radiated in a single direction and can be useful for various applications. Also the numerical simulations [3] of the beam injection into a thin plasma column have shown that such a quasi-regular periodic density perturbation can be created self-consistently even in an initially uniform plasma.

BEAM-PLASMA ANTENNA

Let us study how efficiently EM radiation is generated by a beam-plasma system with a finite transverse size. Firstly, we consider the two-dimensional problem in which beam and plasma particles occupy the infinite plane layer with the thickness $2l$, immersed in the uniform magnetic field $\mathbf{B_0}$ (Fig. 1). If the plasma density is modulated along the layer $n = n_0 + \delta n \cos qz$, the electron beam with the density n_b and velocity v_b is able to transfer its energy to EM radiation via the plasma antenna mechanism. Indeed, if the beam propagating inside the plasma excites the wave of longitudinal electric current with the superluminal phase velocity ($\omega/\mathcal{K}_\parallel > c$), the plasma layer acts as an antenna radiating EM waves with the frequency ω and wave vector $\mathbf{K} = (\mathcal{K}_\parallel, 0, \mathcal{K}_\perp)$, where the transverse to this layer component of wave

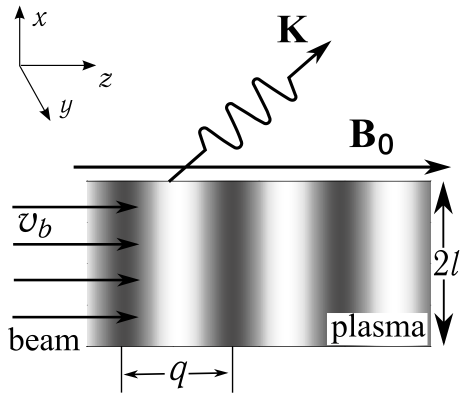

FIGURE 1. Geometry of the problem.

vector takes form $\mathcal{K}_\perp^2 = 1 - \mathcal{K}_\parallel^2$, in units of ω/c. Inside the plasma, EM fields $\mathbf{E(x)}\exp(i(\mathcal{K}_\parallel \omega/c)z - i\omega t)$ driven by this current of nonuniformly distributed electrons can be found from the following system of differential equations

$$E_z'' + a_1 E_z - a_2 E_y' = -\frac{i\mathcal{J}}{\omega}\left(1 - \mathcal{K}_\parallel^2/\varepsilon\right), \tag{1}$$

$$E_y'' + a_3 E_y + a_4 E_z' = 0, \tag{2}$$

where the prime denotes the derivative with respect to the transverse coordinate x, $\mathcal{J} = j_0/(en_0c)$ is the dimensionless amplitude of radiating current density,

$$a_1 = \eta\left(1 - \mathcal{K}_\parallel^2/\varepsilon\right), \quad a_2 = \mathcal{K}_\parallel g/\varepsilon, \quad a_3 = \varepsilon - \mathcal{K}_\parallel^2 - \frac{g^2}{\varepsilon - \mathcal{K}_\parallel^2}, \quad a_4 = \frac{\mathcal{K}_\parallel g}{\varepsilon - \mathcal{K}_\parallel^2}, \tag{3}$$

and ε, g and η are the components of the dielectric tensor for the cold magnetized plasma. Hereinafter, all quantities are presented in the dimensionless form: the wave frequency $\hat{\omega}$ and electron cyclotron frequency $\hat{\Omega}$ are measured in units of the plasma frequency $\omega_p = \sqrt{4\pi n_0 e^2/m_e}$, EM fields — in units of $m_e c\omega_p/e$, lengths — in units of c/ω. The solution of the system can be represented as the sum of two different plasma eigenmodes and the forced term originating from the current:

$$E_z^{in} = b_1\left(C_1 e^{i\varkappa_1 x} + C_2 e^{-i\varkappa_1 x}\right) + b_2\left(C_3 e^{i\varkappa_2 x} - C_4 e^{-i\varkappa_2 x}\right) - \frac{i\mathcal{J}}{\eta\hat{\omega}}, \tag{4}$$

$$E_y^{in} = b_3\left(C_1 e^{i\varkappa_1 x} - C_2 e^{-i\varkappa_1 x}\right) + b_4\left(C_3 e^{i\varkappa_2 x} + C_4 e^{-i\varkappa_2 x}\right), \tag{5}$$

where

$$\varkappa_{1,2}^2 = \tfrac{1}{2}\left(a_1 + a_3 + a_2 a_4 \mp \sqrt{(a_1 + a_3 + a_2 a_4)^2 - 4a_1 a_3}\right), \tag{6}$$

$$b_1 = a_3 - \varkappa_1^2, \quad b_2 = i\varkappa_2 a_2, \quad b_3 = -i\varkappa_1 a_4, \quad b_4 = a_1 - \varkappa_2^2. \tag{7}$$

At the plasma boundaries, this solution should be matched with the vacuum EM fields:

$$E_z^{out} = C_5 e^{i\mathcal{K}_\perp(x-l)}, \qquad E_y^{out} = C_6 e^{i\mathcal{K}_\perp(x-l)}, \qquad (x > l); \tag{8}$$

$$E_z^{out} = C_7 e^{-i\mathcal{K}_\perp(x+l)}, \qquad E_y^{out} = C_8 e^{-i\mathcal{K}_\perp(x+l)}, \qquad (x < -l). \tag{9}$$

Finding the energy flux density and taking into account the slow dependence of the current amplitude on longitudinal coordinate z, we can write the full radiation power P_{rad} in units of beam power P_b:

$$\mathcal{P} = \frac{2\mathcal{K}_\perp \hat{\omega}^3 (F_1 + F_2)}{(\gamma_b - 1)\hat{n}_b \hat{v}_b l(\hat{\omega}^2 - 1)^2} \int_0^{L_z} |\mathcal{J}|^2 dz. \tag{10}$$

Here, L_z is the plasma length, γ_b — the beam relativistic factor, $\hat{n}_b$ is the beam density in units of n_0 and $\hat{v}_b$ is the beam velocity in units of c. In this expression we separate contributions from different plasma eigenmodes:

$$F_1 = \left| \frac{b_5 \sin(\varkappa_1 l) + G b_6 \sin(\varkappa_2 l)}{Z} \right|^2, \qquad F_2 = \left| \frac{b_3 \sin(\varkappa_1 l) + G b_4 \sin(\varkappa_2 l)}{Z} \right|^2, \tag{11}$$

where

$$Z = b_1 \cos(\varkappa_1 l) + i\mathcal{K}_\perp b_5 \sin(\varkappa_1 l) + G\left(b_2 \cos(\varkappa_2 l) + i\mathcal{K}_\perp b_6 \sin(\varkappa_2 l)\right), \tag{12}$$

$$G = -\tfrac{b_3}{b_4}\left(\varkappa_1 \cos(\varkappa_1 l) - i\mathcal{K}_\perp \sin(\varkappa_1 l)\right) / \left(\varkappa_2 \cos(\varkappa_2 l) - i\mathcal{K}_\perp \sin(\varkappa_2 l)\right), \tag{13}$$

$$b_5 = -\varkappa_1 b_1 - i\mathcal{K}_\parallel b_3(\varepsilon - \mathcal{K}_\parallel^2 - \varkappa_1^2)/g, \tag{14}$$

$$b_6 = -\varkappa_2 b_2 - i\mathcal{K}_\parallel b_4(\varepsilon - \mathcal{K}_\parallel^2 - \varkappa_2^2)/g. \tag{15}$$

Let's now specify the process responsible for the excitation of the superluminal current wave in the plasma. The unstable spectrum of the cold beam-plasma system is dominated by the longitudinally propagating wave ($k_\parallel = \omega/v_b$, $\omega = \omega_p(1 - n_b^{1/3}/(2^{4/3}\gamma_b))$). In a uniform plasma, such a wave cannot radiate. However, in the periodically perturbed plasma with the wave number q, it excites the wave of longitudinal electric current with the frequency ω and the wave number $k_\parallel - q$. EM emission is possible, when the following inequality is true: $1 - \hat{v}_b < Q < 1 + \hat{v}_b$, where $Q = q/k_\parallel$ is the dimensionless period of density modulation. The angle of EM radiation: $\theta = \arctan(\sqrt{\hat{v}_b^2 - (1-Q)^2}/(1-Q))$. The current amplitude in (10) and the wave number of this current wave are determined by the expressions: $\mathcal{J} = i\hat{\delta n} E_0(z)/(4\hat{\omega})$ and $\mathcal{K}_\parallel = (1-Q)/\hat{v}_b$, where $\hat{\delta n}$ is the modulation depth in units of n_0 and $E_0(z)$ — the dimensionless wave amplitude of the dominant beam mode. Substituting this current amplitude, we can rewrite the relative radiation power in the form

$$\mathcal{P} = \frac{\hat{\delta n}^2 \mathcal{F}_1(l)}{8(\gamma_b - 1)\hat{n}_b \hat{v}_b \sqrt{1-\hat{\omega}^2}} \int_0^{L_z} E_0^2 dz, \qquad \mathcal{F}_1(l) = \frac{\mathcal{K}_\perp \hat{\omega}(F_1 + F_2)}{l(1-\hat{\omega}^2)^{3/2}}. \tag{16}$$

In the particular case, when the modulation period equals to the wavelength of the beam-driven mode ($Q = 1$), EM

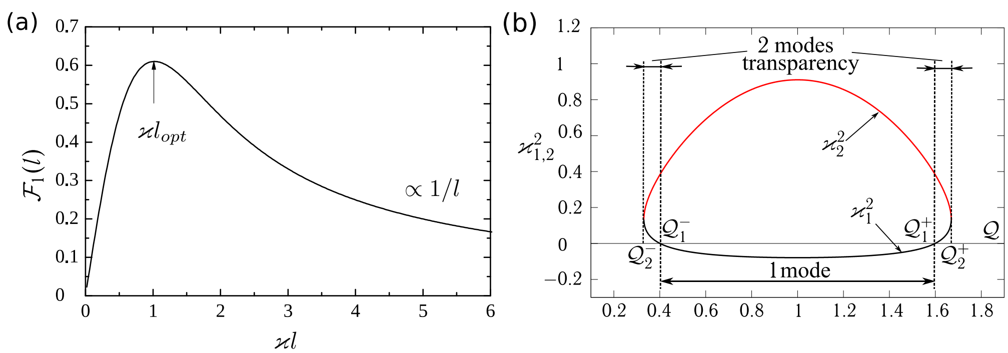

FIGURE 2. (a) The factor $\mathcal{F}_1(l)$ as a function of plasma thickness for $\hat{\omega} = 0.94$ and $Q = 1$. (b) Window of plasma transparency to the generated EM waves ($\hat{n}_b = 0.01$, $\hat{v}_b = 0.9$, $\hat{\Omega} = 0.9$, $\hat{\delta n} = 0.1$).

radiation is directed across the plasma layer. In this case, the first plasma eigenmode corresponds to the O-mode polarized along the magnetic field with the dispersion relation $\varkappa_1 = i\sqrt{\eta} = i\varkappa$, and the second one corresponds to the X-mode polarized perpendicularly to the magnetic field. The latter mode cannot interact with the longitudinal current. Hence, only the first mode makes a contribution in the radiation power. Since the transverse wave number of O-mode becomes purely imaginary, this mode can penetrate in the plasma only to the skin-depth. The factor $\mathcal{F}_1(l)$ takes the simple form

$$\mathcal{F}_1(l) = \frac{\sinh^2(\varkappa l)}{\varkappa l\left[\hat{\omega}^2 + \sinh^2(\varkappa l)\right]} \tag{17}$$

and describes the decrease of radiation efficiency in thick plasmas according to the law $1/l$ (Fig. 2(a)). In this case, the radiation efficiency can reach $5-10\%$. This result coincides with the results obtained in Ref. [4]. The conclusion about the dependence of the radiation efficiency on the plasma thickness has been confirmed by numerical simulations. But this is not the most optimal case. More efficient generation of EM waves should be achieved in the regime of oblique emission, when the plasma is transparent for the emitted waves.

Let's find out how the plasma transparency changes with variations of the parameter Q. For this purpose, we study the dependences $\varkappa^2_{1,2}(Q)$. On Fig. 2(b) one can see that for a sufficiently strong magnetic field there are the regions of plasma transparency for both modes. These regions are bounded by the equations $\varkappa_1^2 = 0$ and $\varkappa_1^2 = \varkappa_2^2$. Corresponding solutions for Q:

$$Q_1^{\pm} = 1 \pm \hat{v}_b \sqrt{\varepsilon + g}, \quad Q_2^{\pm} = 1 \pm \hat{v}_b \sqrt{\varepsilon + g\xi}, \tag{18}$$

where $\xi = -(g(\eta+\varepsilon) + 2\sqrt{\varepsilon\eta(g^2 - (\eta-\varepsilon)^2)})/(\eta-\varepsilon)^2$. In this way, the region of complete plasma transparency is bounded by $Q_1^+ < Q < Q_2^+$ and $Q_2^- < Q < Q_1^-$. Let us plot the relative power of EM emission for this regime. The amplitude of the beam-driven wave in (16) is assumed to reach the level of saturation: $E_0 = \gamma_b^3 \hat{\Gamma}^2 \hat{v}_b$, where $\hat{\Gamma} = \Gamma/\omega_p = \sqrt{3}\,\hat{n}_b^{1/3}/(2^{4/3}\gamma_b)$. The radiating plasma region is characterized by the length $L_z \approx 3\hat{v}_b/\hat{\Gamma}$, required for beam trapping.

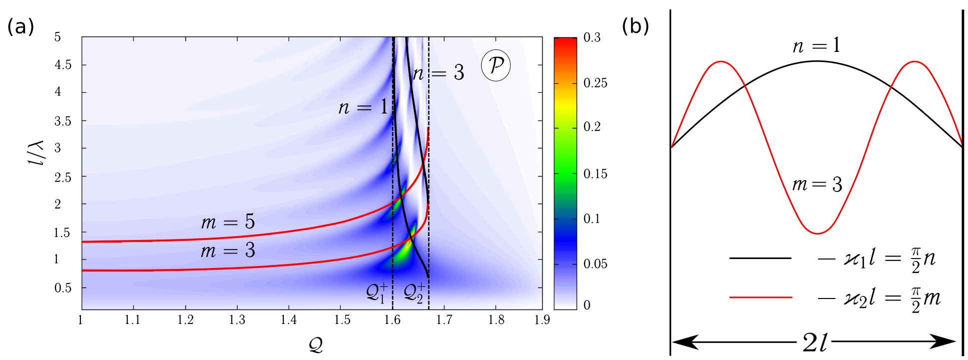

FIGURE 3. (a) Radiation efficiency $\mathcal{P}$ as a function of the modulation period and plasma thickness for the plane antenna with the parameters $\hat{n}_b = 0.01$, $\hat{v}_b = 0.9$, $\hat{\Omega} = 0.9$, $\hat{\delta n} = 0.1$. (b) Transverse structure of electric fields for plasma eigenmodes at the point of the global maximum of $\mathcal{P}$.

From Fig.3(a) we can see that in the region of complete plasma transparency the radiation efficiency significantly increases. Moreover, in this region we observe a pronounced periodic structure. This can be explained by the fact that the plasma thickness reaches an even or odd number of transverse half-wavelengths of plasma eigenmodes. The curves corresponding to conditions $\varkappa_1 l = \pi n/2$ (black) and $\varkappa_2 l = \pi m/2$ (red) for odd n and m are shown in Fig.3(a). The largest maximum is reached at the position where the plasma thickness equals to one half-wavelength of the first plasma mode and three half-wavelengths of the second mode. This situation is schematically shown in Fig.3(b). This is explained by the fact that the work of the uniform radiation current under the field of plasma wave becomes the maximum when we integrate over the half-wavelength plasma and the minimum when the plasma width is raised to the wavelength.

To apply these results to laboratory experiments, we can rewrite the obtained formula in a more realistic cylindrical geometry. As a result, we get a picture similar to the previous case (Fig.4).

SUMMARY

The proposed analytical theory has predicted that, at certain emission angles, plasma becomes transparent to radiation and the whole plasma volume may be involved in generation of EM waves. In this regime conversion of the beam power to radiation power becomes very efficient and, moreover, remains high enough (1—10%) even in a relatively thick plasma, the transverse size of which is almost an order of magnitude greater than the radiation wavelength. In

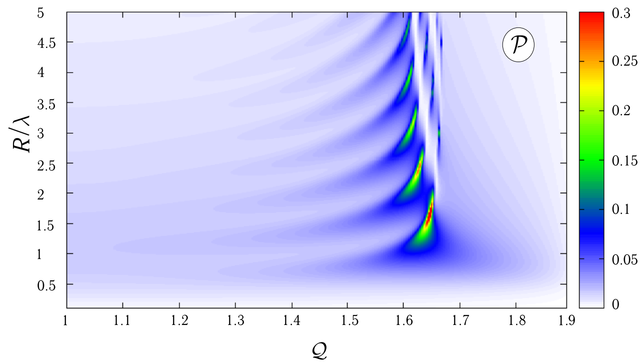

FIGURE 4. Radiation efficiency $\mathcal{P}$ as a function of the modulation period and plasma radius.

particular, it means that, in the laboratory experiments, the terahertz radiation with the frequency 0.5 THz can be generated by the beam with the diameter up to 5 mm.

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