Particle-in-Cell Simulation of Field Reversal in Mirror Trap with Neutral Beam Injection

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Abstract. Facility for experimental investigation of field reversal in mirror trap with neutral beam injection is being developed in the Budker INP. Preliminary results of numerical simulation of field reversal in this device are presented. Kinetic instabilities, which cause anomalous losses of fast ions and restrict pressure of ions, are observed in simulations. Methods of suppression of such instabilities are discussed.

INTRODUCTION

One advantage of mirror traps is the possibility of achieving plasma pressure comparable to the magnetic field pressure. It is desirable to increase the plasma pressure since it increases the fusion reaction rate. While the plasma pressure grows, the equilibrium magnetic field inside it decreases. The field reversed configuration (FRC) can be formed if the plasma pressure is high enough. One possible way of FRC formation in a mirror machine is azimuthal current drive by powerful off-axis neutral beam injection (NBI). It forms population of axis-circling fast ions. This method was experimentally tested on 2XIIB device, where the magnetic field inside the plasma decreased by about ten times [1]. Further accumulation of fast ions and field reversal were prevented by NBI-power deficit and ion-cyclotron fluctuations, which caused anomalous ion losses.

Specialized device for investigation of high-pressure plasma in mirror trap with off-axis NBI is under construction in BINP SB RAS [2]. Some effects might prevent the field reversal in it [3]. For this reason a new PIC code for simulating processes that accompany the ion accumulation and field reversal was developed. Brief description of the code and results of preliminary simulations are presented in this article.

BASIC EQUATIONS

The code simulates trapping of neutral beams into warm target plasma and formation of population of fast ions. The ions generate the current, which reverses the magnetic field. The target plasma is confined by a mirror magnetic field generated by a set of coaxial axisymmetric coils; it is surrounded by cylindrical conducting wall and contacts with end-plates along field lines [2]. A high electron thermal conductivity is assumed so that the electron temperature does not depend on time and coordinates. Two different models are used for simulation. In the simplified model the full axial symmetry is assumed and the target plasma currents are neglected. This allows necessary parameters of injection and of the target plasma for field reversal to be determined. The magneto-hydro-dynamic evolution of electrons and the possibility of breaking the axial symmetry of the magnetic field are taken into account in the more sophisticated model.

Fully Axisymmetric Model with Prescribed Distribution of the Target Plasma

The simplified model takes into account injection of neutral beams and motion of fast ions in fully axisymmetric fields only. Evolution of the distribution function of fast ions, f_i, is described by the kinetic equation

$$\frac{\partial f_i}{\partial t} + \frac{\partial f_i}{\partial \vec{r}}\frac{\partial H}{\partial \vec{p}} - \frac{\partial f_i}{\partial \vec{p}}\frac{\partial H}{\partial \vec{r}} = \hat{C} f_i - \hat{L} f_i + S_i, \quad H = \frac{(\vec{p} - e_i\vec{A}/c)^2}{2m_i} + e_i\varphi. \tag{1}$$

Here φ is the scalar potential (in this model only ambipolar potential, $(T_e/e)\ln(n_e)$, is taken into account), $\vec{A}$ is the vector potential, S_i is the ion source due to the NBI, $\hat{C}$ describes ion-ion collisions and the electron drag, $\hat{L}$ describes ion losses caused by charge-exchange, collisions with first wall and losses through mirrors. Only the azimuthal component of vector potential (that does not depend on the azimuthal angle θ) is taken into account.

The fast ions are born on the surface of a sphere segment so that the ballistic focusing of the atomic beam is modeled. The velocities of atoms are directed approximately toward the center of the sphere with a small angular spread, so that the angular divergence due to the injector grid is simulated.

At given parameters of macro-particles the fast-ion density, n_f, and the current, j_f, can be calculated. The ion current allows us to calculate the vector potential by using azimuthal component of the Maxwell equation, $(\nabla \times \nabla \times \vec{A})_\varphi = A_\varphi/r^2 - \Delta A_\varphi = 4\pi j_{f\varphi}/c$. Two different types of conditions at the trap wall can be used: either the magnetic flux, $\Psi \equiv rA_\varphi$, or the z-component of the magnetic field, $B_z = r^{-1}\partial_r\Psi$, do not depend on time. The magnetic field in mirrors is assumed to be equal to the vacuum field.

It is assumed that the target-ion density depends on the magnetic flux only, $n_c = n_0 e^{-\Psi/\Psi_0}$, i.e. the target ions are frozen into the magnetic field. The electrons are treated in this model as a charge-neutralizing background, i.e. the electron density, n_e, is equal to the sum of target- and fast-ion densities.

Three-Dimensional Model

The more sophisticated model takes into account the electron dynamics and deviation of fields from axial symmetry. The kinetic equation (1) is used for description of fast-ion and target-ion motions. The motion of mass-less electrons is described by

$$-\frac{\nabla p_e}{n_e} - e\vec{E} - \frac{e}{c}\vec{v}_e \times \vec{B} - m_i\tau_{ei}^{-1}(\vec{v}_e - \vec{v}_i) = 0, \tag{2}$$

where $\tau_{ei} = 3m_i T_e^{3/2}/4\sqrt{2\pi m_e}\Lambda e^4 n_e$ is the electron drag time, $\vec{E} = -\nabla\varphi - \partial_t\vec{A}/c$, $\vec{B} = \nabla \times \vec{A}$, $\vec{v}_e$ and $\vec{v}_i$ are the electron and ion velocities. Combining (2) with the Maxwell equation $\nabla \times \vec{B} = (4\pi/c)\vec{j}$ (here $\vec{j}$ is the total plasma current) one can derive equation for the vector potential:

$$\frac{\partial \vec{A}}{\partial t} = -c\nabla\Phi + \frac{\vec{j}_i - \vec{j}}{en_e} \times \vec{B} - D_m\frac{4\pi}{c}\vec{j}. \tag{3}$$

Here $\vec{j}_i$ is the fast-ion current, $\Phi \equiv \varphi + (T_e/e_e)\ln(n_i)$, $D_m \equiv m_i c^2/(4\pi n_i e_i^2 \tau_{ie})$ is the magnetic diffusion coefficient. Note that this field dynamics takes into account the ohmic diffusion and the Hall effect: $\partial_t\vec{B} = \nabla \times (\vec{v} \times \vec{B}) + D_m\Delta\vec{B} - \nabla \times (\vec{j} \times \vec{B}/(en_e))$ follows from equation (3). The boundary condition on surfaces of end plates is $\vec{E}_\| = \vec{E}_{\|w} + (T_e/e)(\nabla_\| j_{en})/j_{en}$, where $E_{\|w}$ is the electric field on the plate and subscripts $\|$ and n mean directions along and perpendicular to the surface of plate. The boundary condition follows from the relation $j_{en} = j_{e0}e^{e\Delta\varphi/T_e}$ between the electron saturation current, j_{e0}, the potential drop, $\Delta\varphi$, and the electron current, j_{en}, emitted from the surface into plasma with Maxwellian electrons.

Dependence of fields and plasma parameters on azimuthal coordinate is described through sums of Fourier harmonics, $a(r,\varphi,z) = Re(\sum_{m=-N}^{N} a_m(r,z)e^{im\varphi}$. Equation (3) is multiplied by $e^{im\varphi}$ and is averaged over φ. This results in a set of relations between different azimuthal components of $\vec{A}$, Φ, $\vec{j}$ and $\vec{j}_e$.

To derive equation, describing the scalar potential Φ, we approximate the electric field as $\vec{E} = -\nabla\Phi - \partial_t\Psi\nabla\theta/c + (T_e/e_e)\ln(n_i/n_0)$ (the A_r and A_z are assumed to be small in comparison with A_φ). The potential is searched for in the form $\Phi(r,\theta,z) = \Phi(X(r,z), Y(r,\theta,z))$, where $X \equiv \Psi$ and $Y(r,\theta,z)$ is determined by $\vec{B} = \nabla\Psi \times \nabla Y$. Using the charge conservation condition, $\oint \vec{j}d\vec{S} = 0$, and equations (2) and (3) one can derive

$$\partial_X(D^{XX}\partial_X\Phi + D^{XY}\partial_Y\Phi) + \partial_Y(D^{XY}\partial_X\Phi + D^{YY}\partial_Y\Phi) + \partial_X\Phi\partial_Y h - \partial_Y\Phi\partial_X h = \partial_X S^{(X)} + \partial_Y S^{(Y)} + S^{(Z)}, \tag{4}$$

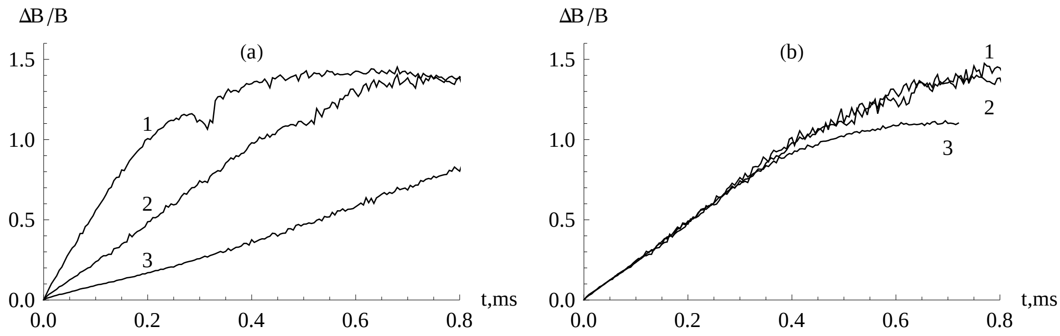

FIGURE 1. The time evolution of field-reversal parameter at different target plasma densities (a) and electron temperatures (b). Target plasma densities: 1 – $10^{14}cm^{-3}$, 2 – $3 \cdot 10^{13}cm^{-3}$, 3 – $10^{13}cm^{-3}$. Electron temperatures: 1 – 100 eV, 2 – 50 eV, 3 – 30 eV.

where

$$h = \oint \frac{dl\tilde{n}}{B},\ D^{(ab)} = \oint \frac{dl\nu\tilde{n}}{B}\nabla a \nabla b,\ S^{(a)} = \oint \frac{dl}{cB}(\tilde{\vec{j}} \cdot \nabla a),\ S^{(Z)} = \left.\frac{j_{\parallel}}{cB}\right|_{z_0}^{z_1},\ \tilde{\vec{j}} = \frac{\vec{j} + \nu \vec{j} \times \vec{b}}{1 + \nu^2} + \tilde{n}\partial_t \Psi \frac{\nabla\theta \times \vec{b} - \nu\nabla\theta}{B}, \quad (5)$$

$\tilde{n}_i \equiv e_i n_i/(1 + \nu^2)$, $\nu \equiv m_i c/(e_i B \tau_{ei})$, $\oint dlf/B$ means integration of variable f along magnetic field line ($X = const$ and $Y = const$), $\vec{b} = \vec{B}/B$, z_0 and z_1 are coordinates of end-plates.

SIMULATION RESULTS

The following basic parameters, which correspond planned CAT experiment [2], are used in simulations: the magnetic field is produced by two co-axial coils of 25 cm in radius, the distance between coils is 60 cm, the magnetic field in the trap center is B_0 =2 kG. The hydrogen neutral beams with energy 15 keV are produced by two injectors with beam current 120 A and angular divergence along and across the mirror field of 30 mrad and 11 mrad [4]. The focal length and diameter of the injector grids are 243 cm and 34 cm, respectively. The injection pitch angle is 90°, the impact parameter is 10 cm. The electron temperature is 50 eV, the initial target plasma density is $3 \cdot 10^{13}cm^{-3}$. In the following, all parameters not given explicitly are the basic parameters.

Field Reversal in the Axisymmetric Case

The figure 1 illustrates the processes of accumulation of fast ions and of field reversal at different densities and temperatures of the target plasma in the absence of non-axisymmetric perturbations. The field reversal parameter is defined as $\Delta B/B = 1 - B_z(z = 0, r = 0, t)/B_0$. The higher the target plasma density is, the faster are the ion accumulation and the field reversal. The ions losses caused by the electron drag are too large at low electron temperature, so that the maximal field-reversal parameter is close to unity. At higher temperature the losses of fast ions are caused mainly by the charge-exchange, so that the field reversal parameter depends on the target plasma temperature weakly. It follows from figure 1 that the field reversal is attainable at planned parameters if no non-axisymmetric perturbations arise.

It should be noted that the field reversal parameter increases slower in comparison with results of the simplest model, which does not take into account the target-plasma currents. It can be concluded that a significant azimuthal target-plasma current appears, which cancels the ion current partially, but does not prevent the field reversal. The plasma current can be driven by the ion-electron collisions as well as by the azimuthal electric field $E_\varphi \sim \partial_t B$. The rise of the target-plasma current is probably limited by the high-frequency (time scale is much smaller than the inverse ion-cyclotron frequency), small-scale, non-regular fluctuations observed in simulations.

Kinetic Instabilities

The average transverse kinetic energy of fast ions is large in comparison with the longitudinal one, so that the kinetic instabilities can arise due to the high anisotropy. To investigate possibility of kinetic instabilities developing, simulations taking into account non-axisymmetric perturbations (with azimuthal mode number $|m| \leq 2$) were performed.

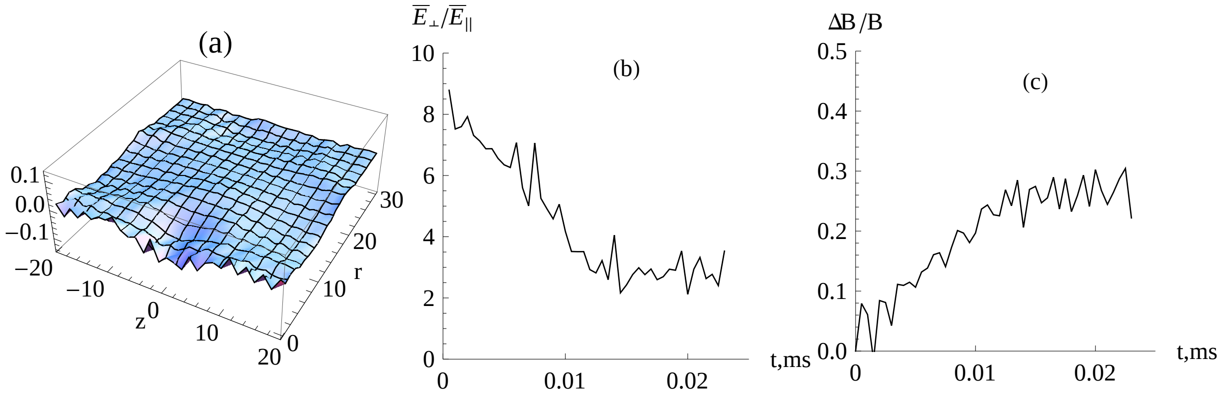

FIGURE 2. Spatial structure of the azimuthal component of the magnetic field (a), fast ions anisotropy (b), and field-reversal parameter (c) at 8.9*μsec* after start of injection. The injection current is increased four times in comparison with basic parameters.

Coherent monochromatic ion-cyclotron perturbations were observed that transform into non-regular turbulence when the ion pressure increases. Sample spatial structure of the first azimuthal component of the azimuthal field is shown in figure 2, a. The spatial structure of the perturbation corresponds to standing wave with longitudinal wave-vector of the order of the length scale of fast-ion population. The magnetic field rotates in the direction of the ion-cyclotron gyration. The perturbation frequency is $0.71\Omega_{ci}$ is this example, where Ω_{ci} is the ion cyclotron frequency at trap center.

The ion-cyclotron fluctuations cause pitch-angle scattering and isotropization of fast ions. An example of how the ion anisotropy depends on time is shown in figure 2, b. Synchronous with reduction of anisotropy the ion population widens in longitudinal direction, so that the ion-current density decreases and the longitudinal losses rise. Such anomalous ion losses limit the field reversal to $\Delta B/B < 0.3$.

Some ways to decrease anisotropy were studied, such as reducing the injection angle and increasing the injector-grid radius. The most effective method to suppress the instability seems to be the magnetic shear, driven by longitudinal electron current. The azimuthal field of order of $0.01B_0$ reduces the fluctuation amplitude essentially, and allows the field reversal parameter to exceed unity. Further study of stabilization methods is needed.

CONCLUSION

Results of numerical simulation of the field reversal in a mirror trap at parameters of the projected CAT device are presented. The injection power is high enough for field reversal if the particle losses remain classical. The main effect preventing the field reversal is the excitation of kinetic instabilities, which cause anomalous losses of fast ions. Improvement of methods of instability suppression, such as introducing the magnetic shear, is needed to prevent anomalous losses. It seems that such methods can allow accumulation of sufficient amount of fast ions for field reversal.

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