Oscillating Mirror Instability in Plasma with Sloshing Ions

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Abstract. Nonstationary kinetic approach has been used to study the influence of the high beta on the dispersion of slow magnetic sound wave. It is shown that in plasma with sloshing ions the oscillating unstable mode could exist even in limit of cold electrons if beta is high enough. The threshold of the instability depends strongly on the distribution of ions near injection energy. The unstable wave is maintained by the perturbation of magnetic field and become aperiodic mirror mode far beyond the instability threshold.

INTRODUCTION

In experiment on the GDT the longitudinal oscillations of plasma were observed [1, 2]. It has typically zero angular wave-number, seems to be radially correlated and has the frequency about 100 kHz. Plasma in the GDT consists of warm Maxwellian component and fast ions produced by inclined neutral injection. The frequency of the observed wave is close to bounce frequency of fast component. While the oscillation looks like standing sound wave it is well known that in plasma with hot ions the longitudinal sound should be damped strongly because of Landau damping [3]. If the electron temperature is small enough the electric field can not synchronize motion of ions and the mode disappears completely. Namely, longitudinal ion sound mode does not exist at all if the condition $T_e > 3,5T_i$ is not satisfied [4]. In case of GDT plasma fast ions have mean energy about 10 keV while temperature of electrons is about 200 - 300 eV (up to 1 keV with ECR heating) [5]. However in case of GDT experiment the distribution function of ions is far from equilibrium in contradiction to usual assumptions of ion sound theory. Besides, the GDT plasma has high beta and sound wave could not treated as electrostatic one. It begs the question: could the perturbation of magnetic field maintain the wave instead of electric field? For instance, there is at least one collective mode in high-β plasma with anisotropic ions – the mirror instability. It is well known that in MHD approximation this instability is aperiodic one unlike the considered low-frequency wave. Slow magnetic sound wave turns to the mirror instability mode when the frequency $\omega = 0$. That is why kinetic criterion of mirror mode was derived using quasi-stationary approximation [6]. This instability is usually treated for the case of bi-Maxwellian distribution function of ions. At the same time in GDT fast ions have anisotropic beam-like distribution function. In present work nonstationary kinetic approach is used to study the influence of the high beta on the dispersion of slow magnetic sound wave. It is aimed to discovering of oscillating solution of dispersion relation in case of plasma with sloshing ions.

BASIC EQUATIONS OF MODEL

In this paper we study plasma oscillations in the next conditions:

- The frequency of the mode is much less than cyclotron frequencies of particles. In this case drift kinetic equation can be used.
- Plasma has axial symmetry and the oscillations has zero angular mode. The oscillations is radially correlated.
- The wave is much slower than fast magnetic sound. So a transverse paraxial quasi-equilibrium could be used to calculate the perturbation of magnetic field.

- The plasma is thin and the wavelength is much bigger than plasma radius. Because of last three assumptions we consider par-axial equilibrium further to determine the magnetic field variation. The same results can be obtained for the wave which propagate nearly orthogonal to magnetic field in homogeneous plasma (it is usual assumption in mirror mode theory [6]).
- The wavelength is high enough to use quasineutrality condition.

Further consideration is based on the drift kinetic equation:

$$\frac{\partial f_a}{\partial t} + v_{\|}\frac{\partial f_a}{\partial s} + \frac{1}{m_a}\left(-q_a\frac{d\varphi}{ds} - \mu\frac{dB}{ds}\right)\frac{\partial f_a}{\partial v_{\|}} = 0, \tag{1}$$

where subscript "a" enumerate electrons and ions.

The quasineutrality condition could be used to determine the perturbation of electric potential:

$$\sum_a q_a n_a = 0, \tag{2}$$

and the par-axial equilibrium condition for the variation of magnetic field:

$$\sum_a p_{\perp a} + \frac{B^2}{8\pi} = const. \tag{3}$$

Linearizing Equations 1, 2 and 3 we obtain:

$$\frac{\partial \tilde{f}_a}{\partial t} + v_{\|}\frac{\partial \tilde{f}_a}{\partial s} = \frac{1}{m_a}\left(q_a\frac{d\tilde{\varphi}}{ds} + \mu\frac{d\tilde{B}}{ds}\right)\frac{\partial f_a}{\partial v_{\|}}, \tag{4}$$

$$\sum_a q_a \int \tilde{f}_a d^3v = 0, \tag{5}$$

$$-\frac{4\pi}{\tilde{B}}\sum_a \int \mu \tilde{f}_a d^3v = 1 + \beta_{\perp}. \tag{6}$$

where $\beta_{\perp}$ is the unperturbed ratio of traversal pressure to magnetic pressure and tilde denotes perturbations.

DISPERSION RELATION

The Equation 4 can be solved in usual way using Fourier transform :

$$\tilde{f}_a = \frac{-k}{\omega - kv_{\|}}\frac{\partial f_a}{\partial v_{\|}}\frac{\left(q_a\tilde{\varphi} + \mu\tilde{B}\right)}{m_a} \tag{7}$$

Substituting solution 7 to Equations 5 and 6 we obtain dispersion relation:

$$\left(Z^4\frac{\beta_{\|e}}{\beta_{\|i}}G_i^0 + G_e^0\right)\left(\frac{\beta_{\perp i}^2}{\beta_{\|i}}G_i^2 + \frac{\beta_{\perp e}^2}{\beta_{\|e}}G_e^2 + 2(1+\beta_{\perp})\right) = \beta_{\|e}\left(Z^2\frac{\beta_{\perp i}}{\beta_{\|i}}G_i^1 - \frac{\beta_{\perp e}}{\beta_{\|e}}G_e^1\right)^2, \tag{8}$$

where G_a^l are dimensionless dispersion functions:

$$G_a^l = \frac{n_a^2 m_a}{p_{\|a}}\left(\frac{p_{\perp a}}{n_a B}\right)^l \int \frac{-k}{\omega - kv_{\|}}\frac{\partial f_a}{\partial v_{\|}}\mu^l d^3v, \tag{9}$$

where so-called Landau's rule of complex integral calculation is assumed.

First term on the left-hand side of Equation 8 represents ion sound wave and second one describes mirror mode. In the case $\beta_e \ll \beta_i$ the dispersion relation can be simplified:

$$\frac{\beta_{\perp i}^2}{\beta_{\|i}}G_i^2 + 2(1+\beta_{\perp i}) = 0 \tag{10}$$

Further the influence of electrons on the dispersion of oscillations will be neglected thus we will omit subscript "i". It should be noted that Equation 10 match with mirror mode criterion derived by Vedenov and Sagdeev [6] in the case of zero frequency.

OSCILLATING MODE

In the GDT device fast ions is produced by inclined neutral injection. After the trapping fast ions is mainly slowed down by electron drag and slightly scattered by warm ions. Neglecting scattering of ions in pith angle and energy at all the well known model of ions distribution function can be derived:

$$f = \frac{p_{\perp}}{4\pi W^{*} sin^{3}\theta^{*}} \frac{\delta(\theta - \theta^{*}) + \delta(\theta - \pi/2 + \theta^{*})}{v^{3}}, v_{\|} < v_{\|}^{*} \tag{11}$$

where W^{*}, θ^{*} and $v_{\|}^{*}$ are energy, pitch angle and parallel velocity of injected fast ions respectively. It is assumed that $f(v_{\|} > v_{\|}^{*}) = 0$. The distribution function 11 diverges as v approaches zero. However the integral G_i^2 in Equation 10 still converges because of the factor μ^2. Substituting distribution function 11 in the Equation 10 we obtain:

$$\int \frac{1}{y-u} \cdot \frac{dF}{du} du = \frac{8(1+\beta_{\perp})}{tg^{2}\theta^{*}\beta\perp} = P \tag{12}$$

where $y = \frac{1}{v_{\|}^{*}}\frac{\omega}{k}$ and $u = \frac{v_{\|}}{v_{\|}^{*}}$ ($v_{\|}^{*}$ are dimensionless variables and function $F(u)$ is defined as follows:

$$F = \begin{cases} |u|^3 & \text{if } |u| <= 1 \\ 0 & \text{otherwise.} \end{cases} \tag{13}$$

The integral in Equation 12 is similar with standard dispersion function for ion sound wave except that function F is integrated instead of distribution f. If $|u| <= 1$, the function F is increasing, so 12 could have unstable solutions. As the function F is piecewise smooth the left-hand side of Equation 12 $I(y) = \int \frac{1}{y-u} \cdot \frac{dF}{du} du$ is analytic only if $Im(y) > 0$ and has singular points at real line. Substituting the function F in Equation 12 we obtain in upper half-plane:

$$I(y) = \int \frac{1}{y-u} \cdot \frac{dF}{du} du = y^2 \left(3 - \frac{2y^2}{y^2-1} - \frac{1+y^2}{y^2} + 3\, ln\frac{y^2}{y^2-1}\right) \tag{14}$$

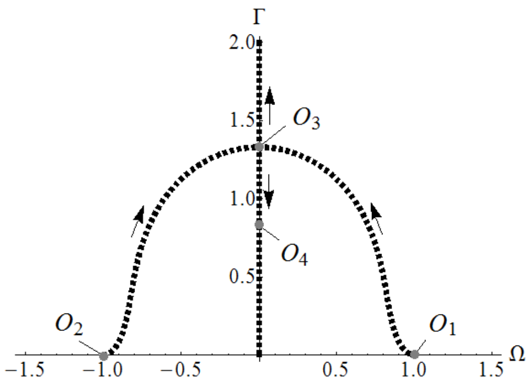

FIGURE 1. The lines $Im\,(I(y)) = 0$. O_1 and O_2 are singular points, O_3 is crossing point of dispersion curves, O_4 is zero point of the function $I(y)$.

The right-hand side of Equation 12 consist of real positive constant P. Because of that the solutions of Equation 12 have to be plasced on lines $Im\,(I(y)) = 0$ ($Re\,(I(y))$ should be positive). Let us introduce real variables Ω, Γ so that $y = \Omega + i\Gamma$. The lines $Im\,(I(y)) = 0$ is shown on the Fig. 1. It consist of two symmetric curves (with positive and negative Ω) and the imaginary axis corresponds aperiodic instability. The limit of I as y approaches the points O_1 and O_2 is $+\infty$ so they are singular points. In the case $\beta_{\perp} \to 0$ the constant $P \to +\infty$. Thus, in case of low beta there are solutions of dispersion relation 12 close to the points O_1 and O_2. As $\beta_{\perp}$ or θ^{*} increase the value of P decreases and solutions of Equation 12 are shifted in the direction of point O_3 along the dispersion curves. In this point the roots of Equation 12 became imaginary so the mode became aperiodic one. Further, one of them increases up to $\Gamma \to +i\infty$ while another one tend to the point O_4 which is zero point of I. The solutions of Equation 12 can not approach the

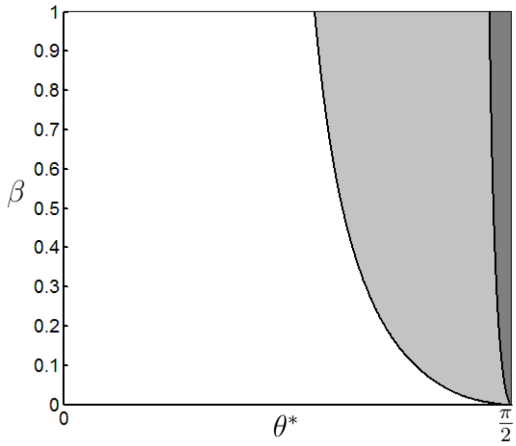

FIGURE 2. Instability threshold in the plane $\beta_\perp$-θ^* in case of $\Delta u = 0.1$. Light gray area corresponds oscillating instability and dark gray ares corresponds aperiodic one.

point $y = 0$ corresponding the criterion of mirror mode stability derived by Vedenov and Sagdeev [6]. Thus, in case of considered distribution function this criterion is satisfied while the unstable waves still can exist. At the beginning the instability is oscillating one and become aperiodic only far beyond the boundary of stability. At the same time the Vedenov-Sagdeev criterion is derived using quasi-stationary approximation. Thereby, this criterion is insufficient one in general and the complete dispersion relation 12 should be used to determine the stability of beam-like distribution functions. The appearance of singular point O_1 and O_2 on the real line caused by piecewise smoothness of the function F. It results from neglecting of energy scattering of ions and energy spread of injected neutral beams (in the GDT latest is about 10%). Thus, let us examine smoothed distribution $F \sim \cdot|u|^3 \left(th\left(\frac{(u+1)}{\Delta u}\right) + th\left(-\frac{(u-1)}{\Delta u}\right)\right)$. In this case the singular points are shifted down from the real line and case of large P the wave becomes damped one. Instability threshold in the plane $\beta_\perp$-θ^* is shown on the Fig. 2. In case of high enough injection pitch-angle the threshold value of β can be moderate one.

CONCLUSIONS

- In high β plasma with beam-like distribution function of ions oscillating wave with sound frequency could exist even in case of cold electrons.
- The unstable wave is maintained by the perturbation of magnetic field and become aperiodic mirror mode far beyond the instability threshold.
- The Vedenov-Sagdeev criterion of mirror mode stability is insufficient one. There are unstable distribution functions even if the criterion is satisfied.

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