Effect of Alfvén Ion-Cyclotron Instability on Ion Dynamic in an Axisymmetric Mirror Trap

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Abstract. The ions dynamic in an mirror trap in presence of ion-cyclotron oscillations is studied. The oscillations cause the stochastic diffusion of ions with low transversal energy so providing anomalous losses. The diffusion coefficient is estimated analytically and way of calculating energy spectrum of lost ions is proposed.

INTRODUCTION

The Alfvén ion cyclotron (AIC) instability is an electromagnetic instability which can develops in magnetized plasmas with non-isotropic ions distribution. The instability developing leads to generation of elliptically polarized waves with frequency of the order of the ion-cyclotron frequency propagating along magnet field. The instability-driven anomalous ion transport and losses was observed in TMX [1], GAMMA-10 [2] and GDT facilities [3].

This paper is aimed to study the anomalous ion transport in an mirror trap with high mirror ratio and skew neutral beam injection. The transport is caused by the AIC instability driven by neutral injection. At given parameters of injection and target plasma the fast ions distribution function can be calculated and parameters of unstable perturbations can be found [4] in the eikonal approximation [5]. The unstable fluctuation structure is discussed briefly in second section. The dynamic of fast ions in presence of the fluctuation is studied in next two sections with using methods developed in [6, 7]. It is shown that ions with low transversal energy move stochastically so they can arrive loss cone. The anomalous longitudinal losses caused are discussed in fifth section. The main results are briefly concluded in Conclusion.

SPATIAL STRUCTURE OF FLUCTUATIONS

The Hamiltonian $H = (\vec{p} - e\vec{A}_0/c - e\vec{A}_w/c)^2/(2m_i)$ is used, here e and m_i are ion charge and mass, vector potential $\vec{A}_0 = A_0(r,z)\vec{e}_\varphi$ describes unperturbed mirror magnet field, $\vec{A}_w$ is wave vector potential. The wave potential being found from the linear theory in the WKB (eikonal) approximation is used which is the sum of monochromatic waves propagating in the opposite sides $\vec{A}_w(r,\theta,z,t) = Re(\vec{a}(r,\theta,z,t) + \vec{a}(r,\theta,-z,t)e^{i\varphi_0})$, $\vec{a} = (A_+e^{-i\theta} + A_-e^{i\theta})\vec{e}_r - i(A_+e^{-i\theta} - A_-e^{i\theta})\vec{e}_\varphi$. A_+ and A_- represent fields rotating in the direction of ion and electron gyration:

$$A_+(r,z) = A_{+0}(\partial_{k_a}D)^{-1/2}e^{-k_{\perp a}^2(z)r^2}e^{i\int_{z_1}^{z}k_a dz - i\omega t}H(z-z_{t1}) + A_{+0}(\partial_{k_r}D)^{-1/2}e^{-k_{\perp r}^2(z)r^2}e^{i\int_{z_2}^{z}k_r dz - i\omega t}H(z_{t2}-z),$$

$$A_- = -\frac{\partial^2_{k_\perp k_\perp}D^{(rl)}}{2D^{(rr)}}\left(\partial_x - i\partial_y\right)^2 A_+ \qquad (1)$$

here $D^{(ll)}$, $D^{(lr)}$, $D^{(rl)}$ and $D^{(rr)}$ are components of the dispersion function [5], $D(k_\parallel, z, \omega) \equiv D^{(ll)} + sign(Re(k_0^2))k_0^2\partial^2_{k_\perp,k_\perp}D^{(ll)}$ is dispersion function taking transversal non-uniformity into account in the Pearlstein-Berk approximation [4], $k_0^2 \equiv \sqrt{\partial^2_{rr}D^{(ll)}/\partial^2_{k_\perp k_\perp}D^{(ll)}}$ (derivatives evaluate at $r = 0$ and $k_\perp = 0$), ω is perturbation frequency, $k_a(z)$ and $k_r(z)$ are roots of the equation $D(k_\parallel, z, \omega) = 0$, z_1 and z_2 are points where $k_a = k_r$ (turning points), z_{t1} and z_{t2}

are intercepts of real z axis of the Stoke's lines emanating from z_1 and z_2, $k_\perp^2(z) = (\partial_r^2 D/\partial_{k_\perp}^2 D)^{1/2}$ (derivatives evaluate at $r = 0$, $k_\perp = 0$ and longitudinal wave vector equal k_a or k_r), $H(x)$ is the Heaviside function. To avoid unphysical discontinuities of wave field in numerical simulations the Heaviside functions $H(z - z_t)$ are replaced by smoothed functions $1/2 + \tanh(2(z - z_t)/z_t)/2$.

The expression (1) is appropriate for numerical simulation of ion motion but cannot be used for analytical investigation. It should be noted that particle energy and magnet momentum varies near trap center. So the following approximation of wave field found in the Pearlstein-Berk approximation [5, 4] can be used

$$A_+(r,z) = A_w e^{-k_{\perp 0}^2 r^2/2} e^{-\varkappa^2 z^2/2} \sin(k_{\|0} z + \varphi_0) e^{-i\omega t}, \tag{2}$$

which approximates field (1) near mirror midplane. Here ω and $k_{\|0} = (k_a(0) + k_r(0))/2$ are roots of equations $D(k_\|, 0, \omega) = 0$ and $\partial_{k_\|} D(k_\|, 0, \omega) = 0$, $\varkappa^2 = (k_a(0) - k_r(0))^2/4 = (\partial_z^2 D/\partial_{k_\|}^2 D)^{1/2}$ and $k_{\perp 0}^2 = (\partial_r^2 D/\partial_{k_\perp}^2 D)^{1/2}$, all derivatives are evaluated at $k_\| = k_{\|0}$, $k_\perp = 0$, $z = 0$ and $r = 0$.

HAMILTONIAN

In the paraxial approximation mirror magnet field is described by vector potential $\vec{A}_0 = (r/2)B(z)\vec{e}_\varphi$. The ion Larmour radius ρ is assumed to be small in comparison with magnet field scale length $B(z)/B'(z)$. Neglecting by terms of order of $(B'\rho/B)^2$ and $(k_{\|0}A_w/B)^2$ and using vector potential (2) one can transform the ion Hamiltonian to

$$H(\mu, P_\theta, P_z; \Phi, \theta, Z) = (\Omega_{ci}(Z) - \omega)\mu + \frac{P_z^2}{2m_i} + \omega P_\theta - w\frac{m_i\Omega_0}{k_{\|0}}\sqrt{\frac{2\Omega_{ci}(Z)\mu}{m_i}} Re\left(e^{i\Phi} e^{-\varkappa^2 Z^2/2} e^{-k_{\perp 0}^2 R^2/2} \sin(k_{\|0} Z + \varphi_0),\right) \tag{3}$$

here $w \equiv k_{\|0}A_w/B(0)$ is the ratio of maximal wave magnet field to mirror field, $\Omega_{ci}(z) \equiv e_i B(z)/(m_i c)$, $\Omega_0 \equiv \Omega_{ci}(0)$,

$$\mu = \frac{p_r^2 + (p_\theta/r - m_i\Omega_{ci} r/2)^2}{2m_i\Omega_{ci}} + O(\frac{\rho B'}{B}), \quad \Phi = \arcsin\left(\left(r - \frac{2p_\theta}{m_i\Omega_{ci} r}\right)\sqrt{\frac{m_i\Omega_{ci}}{8\mu}}\right) - \theta - \omega t + O(\frac{\rho B'}{B}) \tag{4}$$

is magnet momentum and conjugated phase,

$$P_\theta = p_\theta + \mu,\ \Theta = \arcsin\left(\left(r + \frac{2p_\theta}{m_i\Omega_{ci} r}\right)\sqrt{\frac{m_i\Omega_{ci}}{8\mu + 8p_\theta}}\right) + \theta + \omega t + O(\frac{\rho B'}{B}),\ P_z = p_z + O(\frac{\rho B'}{B}),\ Z = z + O(\frac{\rho B'}{B}), \tag{5}$$

and $R^2 = 2(P_\theta + \mu - 2\sqrt{\mu P_\theta}\cos(\Phi + \Theta))/(m_i\Omega_{ci})$ is the squared distance between ion and trap axis. The ion Larmour radius and distance between center of Larmor circle and axis are $\sqrt{2\mu/(m_i\Omega_{ci})}$ and $\sqrt{2P_\theta/(m_i\Omega_{ci})}$. The small additions in (4) and (5) are needed to vanish the terms of the order of $(\rho B(z)/B'(z))^2$ in the hamiltonian.

VELOCITY DIFFUSION

The case $k_{\perp 0} = 0$ is studied now, so P_θ is the integral of the motion and particle dynamic is two-dimensional. In the case of small wave amplitude $w \ll 1$ evolution of particle magnet momentum can be described by twist mapping [6]:

$$\mu_{n+1} = \mu_n + Re\left(ie^{i\Phi_n}(F_+(\mu_{n+1}) - F_(\mu_{n+1}))\right),\ \Phi_{n+1} = \Phi_n + 2\pi\alpha(\mu_{n+1}) - Re\left(e^{i\Phi_n}\frac{\partial}{\partial\mu}(F_+(\mu_{n+1}) - F_-(\mu_{n+1}))\right),$$
$$F_\pm = e^{i\pi\alpha}(e^{\pm i\varphi_0} I(-z_t, z_t) - e^{\mp i\varphi_0 + i\pi\alpha} I_\pm(-z_t, 0) - e^{\mp i\varphi_0 - i\pi\alpha} I_\pm(0, z_t)),$$
$$I_\pm(z_1, z_2) = w\frac{m_i\Omega_0}{2k_{\|0}}\int_{z_1}^{z_2}\sqrt{\frac{2\Omega_{ci}(z)\mu}{m_s}} e^{-\kappa^2 z^2/2} e^{i\phi \mp ik_{\|0} z}\frac{dz}{V(z)}, \tag{6}$$

here μ_n and Φ_n are particle momentum and phase at $z = 0$, n is number of bounce oscillation, $\alpha \equiv (\langle\Omega_{ci}\rangle - \omega)/\Omega_b$ is rotation number, $\Omega_b \equiv \pi/(\int_{-z_t}^{z_t} dz/V(z))$ is the bounce frequency, $\langle\Omega_{ci}\rangle \equiv 2\Omega_b\int_{-z_t}^{z_t} dz\Omega_{ci}(z)/V(z)$ is the ion cyclotron frequency averaged over bounce oscillation, $V(z) \equiv \sqrt{2\varepsilon/m_i - 2(\Omega_{ci}(z) - \omega)\mu/m_i}$ is unperturbed longitudinal velocity,

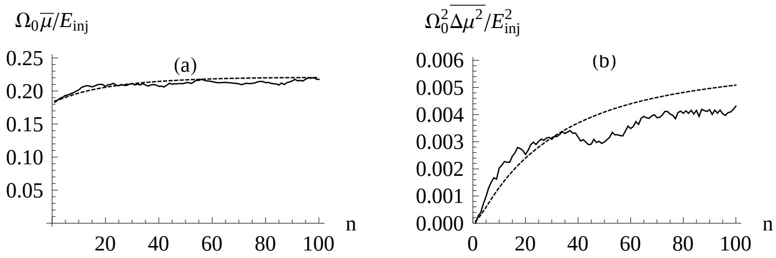

FIGURE 1. The mean magnet momentum (a) and standard deviation (b) obtained from numerical simulation (solid line) and solving diffusion equation (dashed line). Parameters: $w = 10^{-3}$, $k_{\|0}v_{inj}/\Omega_0 = 1.3 + 0.15i$, $\kappa^2 v_{inj}^2/\Omega_0^2 = 0.017 + 0.065i$, $\omega/\Omega_0 = 0.35$, $\varphi_0 = 0$, other parameters given in the text.

ε is the Hamiltonian value, z_t is the solution of equation $V(z) = 0$, $\phi(z) \equiv \int_0^z(\Omega_{ci}(z') - \omega)dz'/V(z')$. The $I_\pm(z_1, z_2)$ is magnet momentum variation during particle motion from z_1 to z_2, sign indicates interaction with wave propagating in same or opposite direction with the particle. Dynamic described by the mapping (6) is typical for weakly-perturbed two-dimensional Hamilton system. Set of resonant trajectories surrounded by separatrices exists. Positions of resonant trajectories satisfy condition $\alpha = n$, here n an positive integer. The resonant condition can be written in the form $\langle\Omega_{ci}\rangle - \omega = n\Omega_b$ and states that resonant particle makes n cyclotron rotations during bounce period. Separatrixes width $\Delta\mu \approx 2\sqrt{|F_+ - F_-|/(2\pi\partial_\mu\alpha)}$ decreases slower that distance between neighboring resonances when magnet moment decreases. So neighboring separatrixes overlap in the area of small transversal energy and ions motion becomes stochastic.

The main contribution into integrals (6) is given by intervals where phase varies slowly, $\partial_{v_\|}(\phi + \int k_{\|0}dz) = (\Omega_{ci}(z) - \omega + k_{\|0}v_\| + 2i\kappa^2 z v_\|)/v_\| = 0$. It is assumed that resonant interaction between wave and fast ion takes place near trap midplane so all quantities can be expanded near $z = 0$: $\Omega_{ci}(z) \approx \Omega_0(1 + z^2/L^2)$, $\phi(z) \approx n_\|\{1 - \omega/\Omega_0 + \gamma^2 z^2/(3L^2)\}z/L$, here $n_\| \equiv \Omega_0 L/v_{\|0}$, $\gamma^2 \equiv 1 + (1 - \omega/\Omega_0)v_{\perp 0}^2/(2v_{\|0}^2)$, $v_{\|0} = V(z = 0)$ and $v_{\perp 0} = \sqrt{2\Omega_0\mu/m_i}$ are the longitudinal and transversal velocities at trap center. The phase derivative equal to zero at $z = -iLZ_\pm = -iL(-\kappa^2 L^2 \pm \sqrt{\gamma^2 n_\|^2\Delta\omega + \kappa^4 L^4})/(\gamma^2 n_\|)$, where $\Delta\omega \equiv \Omega_0 - \omega - k_{\|0}v_{\|0}$. Note that phase changes relatively fast because typically $n_\| \gg 1$. Applying stationary phase method one can evaluate $I_+(-z_t, z_t)$ to

$$I_+(-z_t, z_t) \approx w\frac{m_i\Omega_0}{2k_{\|0}}\frac{v_{\perp 0}}{v_{\|0}}Le^{-\gamma^2 n_\|(Z_+ - 3Z_-)Z_+^2/6}\frac{\sqrt{\pi}}{\gamma\sqrt{n_\|(Z_+ - Z_-)/2}} \tag{7}$$

Integrals $I_\pm(-z_t, 0)$ and $I_\pm(0, z_t)$ are evaluated analogically. The estimation (7) is in good agreement with precise integrals (6) while transversal energy is small enough $\Omega_0\mu \ll \kappa^2 L^2\varepsilon$.

The diffusion coefficient can be estimated [6] as squared magnet momentum variation during particle bounce-oscillation multiplied on bounce frequency,

$$D_\mu = \Omega_b\left|e^{i\varphi_0}I_+(-z_t, z_t) - e^{-i\varphi_0}I_-(-z_t, z_t)\right|^2. \tag{8}$$

Applicability of the diffusion coefficient was verified by numerical simulation of motion of 156 ions in the mirror field with perturbation (1). The wave spatial distribution and frequency corresponds following parameters of injection and target plasma: the injection pitch angle 45°, the ratio of the electron temperature to the injection energy is $T_e/E_{inj} = 3.6 \cdot 10^{-3}$, the time of ion charge-exchange with beam atoms exceeds four times the drag time of fast atoms in collisions with electrons [4]. All particles has same velocities (longitudinal velocity is $0.67v_{inj}$, azimuthal is $v_{\varphi 0} = -0.6v_{inj}$, radial velocity is zero, $v_{inj}^2 = E_{inj}/m_i$), same azimuthal momentum equal $-m_i v_{\varphi 0}^2/(2\Omega_0)$ and is placed at trap midplane at the initial moment. The wave causes ions diffusion on magnet momentum. The average magnet momentum $\overline{\mu}$ and standard deviation $\overline{\Delta\mu^2}$ after n bounce oscillations are shown on figure 1. Also results of solving diffusion equation [7] $\partial_\mu D_\mu \partial_\mu f = \partial_t f$ with diffusion coefficient (8) and initial condition $f(\mu, t = 0) = \delta(\mu - m_i v_{\varphi 0}^2/2)$ are shown. There is agreement between numerical simulation and analytical estimations.

Transversal non-uniformity ($k_{\perp 0} \neq 0$) leads to variation of P_θ, in particular the radial diffusion arises for particles with small transversal energy. Typically $k_{\perp 0} \sim 1/R_p$, here R_p is the plasma radius. The radial diffusion as well as

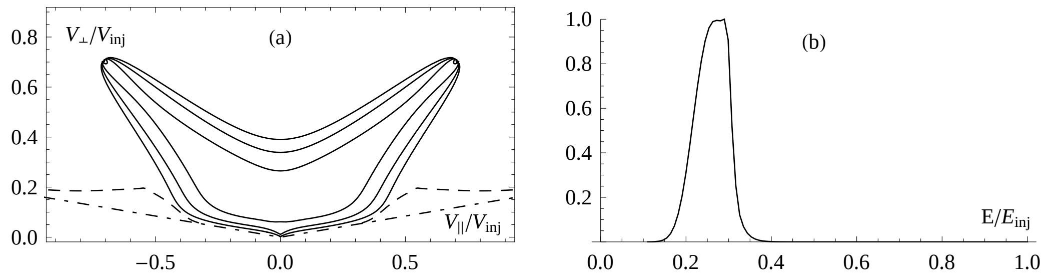

FIGURE 2. An example of distribution function isolines (solid lines), chaos margin (dashed lines) and loss cone margin (dot-dashed lines) (a) and energy spectrum of lost ions (b). Parameters are the same as in figure 1 except $w = 10^{-4}$.

velocity diffusion occurs for ions with low transversal energy. The condition $k_{\perp 0}\rho_\perp$ is satisfies for such particles, so influence of transversal motion on magnet momentum evolution can be neglected. As well as in the case of magnet momentum diffusion the radial diffusion coefficient can be estimated as squared variation of P_θ during bounce oscillation: $D_P \approx \Omega_b \Delta P_\theta^2 \approx 2k_{\perp 0}^4(4\mu P_\theta/(m_s\Omega_0))e^{-k_{\perp 0}^2 P_\theta/(m_i\Omega_0)}D_\mu$.

LONGITUDINAL LOSSES

It is shown in the previous section that motion of ions with low transversal energy can be described as diffusion on magnet momentum with diffusion coefficient (8). The diffusion enlarges rate of particles arriving the loss cone and provides anomalous longitudinal losses.

An example of the chaos margin and energy spectrum of ions being lost during time period $10^3\Omega_0$ is shown on figure 2. The ions distribution function is the same as in example in previous section. The mirror field is $B(z)/B_0 = 1 + z^2/L^2 + r(1 + (z - z_0)^2/l^2)^{-3/2} + r(1 + (z + z_0)^2/l^2)^{-3/2}$, here $L = 30\rho_{inj}$, $z_0 = 39\rho_{inj}$, $l = 1.3\rho_{inj}$, $r = 31.7$, $\rho_{inj} = v_{inj}/(\sqrt{2}\Omega_0)$, $v_{inj}^2 = 2E_{inj}/m_i$. The chaos margin is found from the condition of resonances overlapping [6] $K \equiv (2\pi\partial_\mu\alpha)|F_+ - F_-| > 1$. The spectrum is calculated by numerical solving the diffusion equation with initial condition $f(t = 0, \varepsilon, \mu) = F_i(\sqrt{2\varepsilon/m_i - 2(\Omega_0 - \omega)\mu/m_i}, \sqrt{2\Omega_0\mu/m_i})$ and boundary conditions $f_i = 0$ at loss cone margin and $\partial_\mu f_i = 0$ at chaos margin.

Note that ions with relatively small energy are lost only so longitudinal losses and energy spectrum of lost particles depends strongly on details of distribution function at low energies.

CONCLUSION

The AIC instability developing in mirror trap with skew injection leads to stochastic magnet momentum diffusion of particles with low transversal energy and longitudinal velocity similar to the longitudinal injection velocity. So the instability provides anomalous longitudinal losses. The diffusion coefficient for most particles is insignificantly small if injection angle differs from 90° essentially and energy spectrum of anomalously lost ions depends strongly on details of distribution function at low energies.

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