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# Light-quark mesons in two-photon processes at Belle

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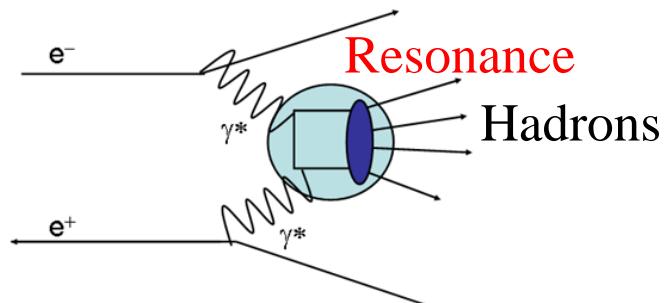
Sadaharu Uehara (KEK)  
Belle Collaboration



*PHOTON2015*  
*Budker INP, Novosibirsk,*  
*June 15-19, 2015*

# Resonance production and quantum numbers

## Resonance formation from two photon collisions



$$Q = 0, \mathbf{C} = +,$$

for real-photon collisions

$$J^P = 0^+, 0^-, 2^+, 2^-, 3^+, 4^+, 4^-, 5^+ \dots (\text{even})^\pm, (\text{odd} \neq 1)^+$$

This talk:

Pseudoscalar-meson pair production:  $J^P = (\text{even})^+$  only

Vector-meson pair production

$\eta' \pi^+ \pi^-$  production

Strict constraints for quantum numbers → Determination of  $J^P$  by PWA

Quasi-real photon case, “Zero-tag”:  $\gamma^*$  can be interpreted as a real  $\gamma$

$\Gamma_{\gamma\gamma}$ : The cross section is proportional to the two-photon partial decay width of the resonance, useful information to explore meson's internal structure

Decay properties of the resonance

Production of isospin  $I=0$  ( $f$ , mesons) and  $I=1$  ( $a$ , mesons)

Searches/Discoveries of new resonances

Form factors / Test of QCD



# KEKB Accelerator and Belle Detector

- Asymmetric  $e^- e^+$  collider

8 GeV  $e^-$  (HER) x 3.5 GeV  $e^+$  (LER)

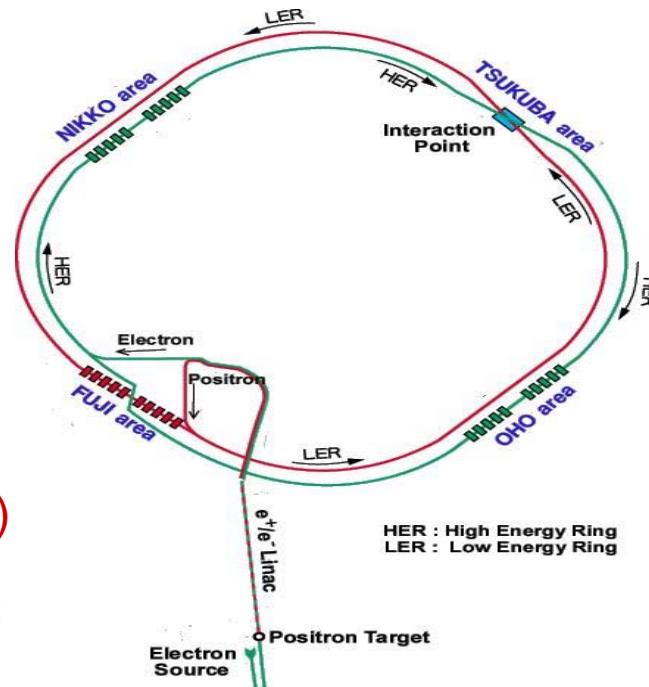
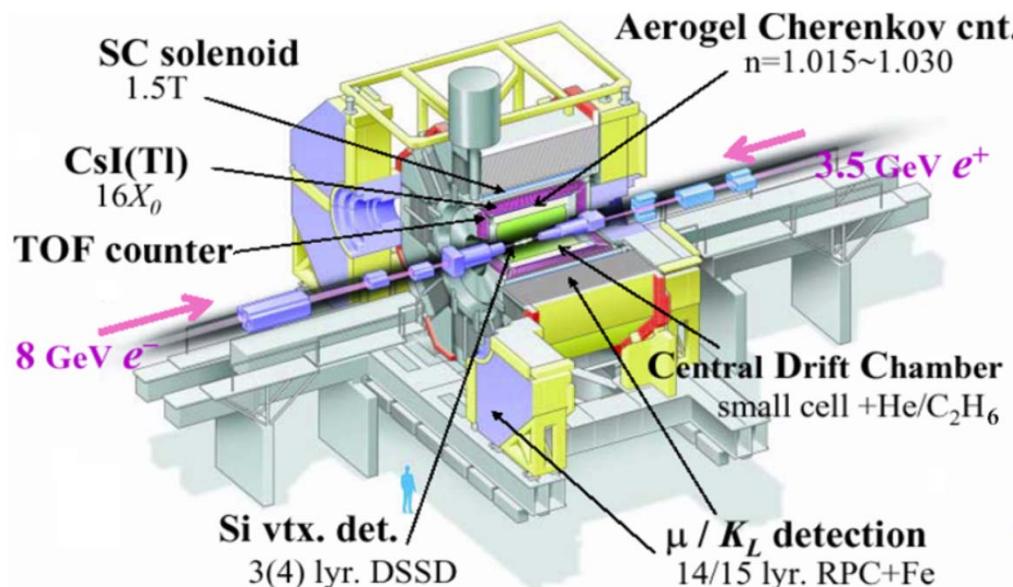
$\sqrt{s} =$  around 10.58 GeV  $\Leftrightarrow \Upsilon(4S)$

Beam crossing angle: 22mrad

- World-highest Luminosity

$L_{\max} = 2.1 \times 10^{34} \text{ cm}^{-2}\text{s}^{-1}$

∫  $Ldt \sim 1040 \text{ fb}^{-1}$  (Completed in Jun.2010)



High momentum/energy resolutions

CDC+Solenoid, CsI

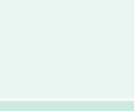
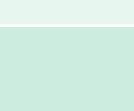
Vertex measurement – Si strips

Particle identification

TOF, Aerogel, CDC-dE/dx,  
RPC for  $K_L/\mu$ on

# “ $\gamma\gamma \rightarrow$ Pseudoscalar-meson pair” from Belle

10 papers for 6 processes

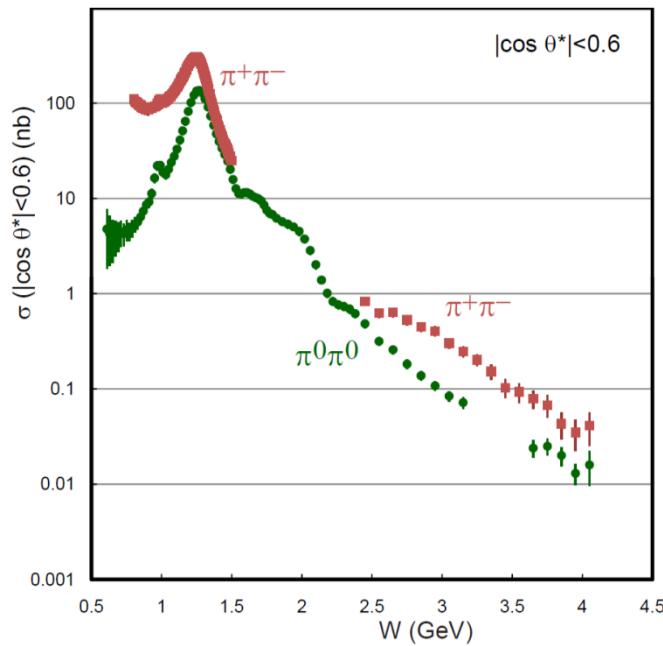
Process	Reference	BELLE	Int.Lum. (fb <sup>-1</sup> )	$\gamma\gamma$ c.m. Energy (GeV)	Light Mesons	QCD	Char- monia
$\pi^+\pi^-$	PLB 615, 39 (2005)		87.7	2.4 - 4.1	✓ ✓	✓	✓
	PRD 75, 051101(R) (2007)		85.9	0.8 - 1.5			
	J. Phys. Soc. Jpn. 76, 074102 (2007)		85.9	0.8 – 1.5			
$K^+K^-$	EPJC 32, 323 (2003)		67	1.4 – 2.4	✓	✓	✓
	PLB 615, 39 (2005)		87.7	2.4 – 4.1			
$\pi^0\pi^0$	PRD 78, 052004 (2008)		95	0.6 – 4.0	✓ ✓	✓	✓
	PRD 79, 052009 (2009)		223	0.6 – 4.0			
$K^0_S K^0_S$	PLB 651, 15 (2007)		397.1	2.4 – 4.0	✓	✓	✓
	PTEP 2013, 123C01 (2013)		972	1.05 – 4.0			
$\eta\pi^0$	PRD 80, 032001 (2009)		223	0.84 – 4.0	✓	✓	
$\eta\eta$	PRD 82, 114031 (2010)		393	1.1 – 3.8	✓	✓	✓

Differential cross section  $d\sigma/d|\cos\theta^*|$  for these processes are measured.



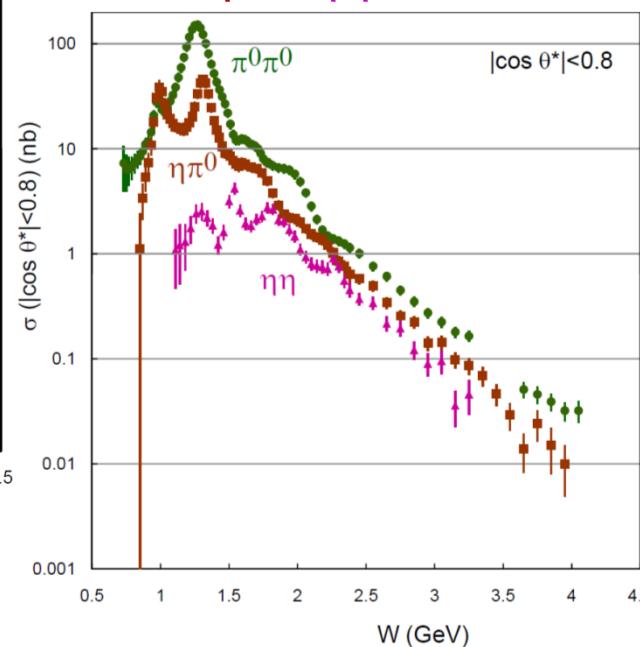
# The six processes; in total $\sim$ 20 peaks

## Charged vs Neutral $\pi\pi$

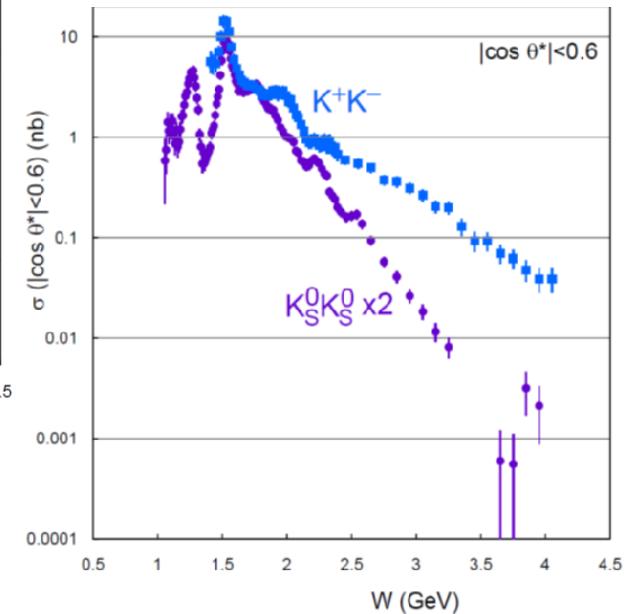


## Three neutral-pair processes

$\pi^0\pi^0, \eta\pi^0, \eta\eta$



## Charged vs Neutral $K\bar{K}$



- PWA Formalism
- $f_0(980)$  and  $a_0(980)$
- Tensor mesons  $f_2(1270)$ ,  $a_2(1320)$ ,  $f_2'(1525)$
- Scalars in 1.2 – 1.6 GeV
- 1.6 – 1.8 GeV region
- 1.8 – 2.2 GeV region
- 2.2 – 2.6 GeV region

Horizontal axis:  
W --  $\gamma\gamma$  c.m. energy = invariant mass of the two-meson system



# Formalism of PWA for P-meson pair final-state processes

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- We consider up to  $J=4$  (for  $W < 3$  GeV).

$$\frac{d\sigma}{d\Omega} = |SY_0^0 + D_0Y_2^0 + G_0Y_4^0|^2 + |D_2Y_2^2 + G_2Y_4^2|^2$$

- $S, D_0, G_0, D_2, G_2$  Partial-wave amplitudes for each wave  $J_\lambda$   
 $J = L = 0, 2, 4$  (even only) with the helicity  $\lambda = 0$  or  $2$  (to the  $\gamma\gamma$  axis)

- $Y_J^\lambda$ : spherical harmonics
  - $|Y_J^\lambda|$  are NOT mutually independent, as we have no information for the azimuthal-angle direction.

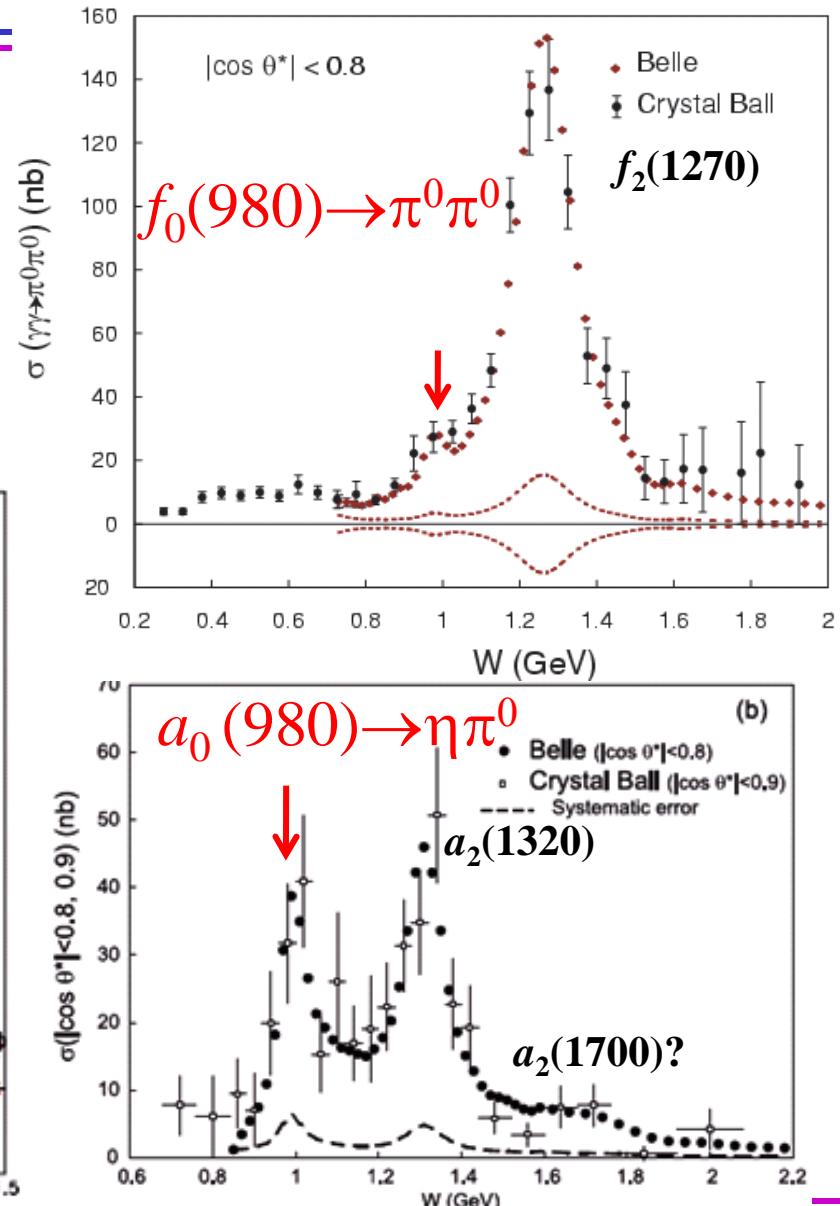
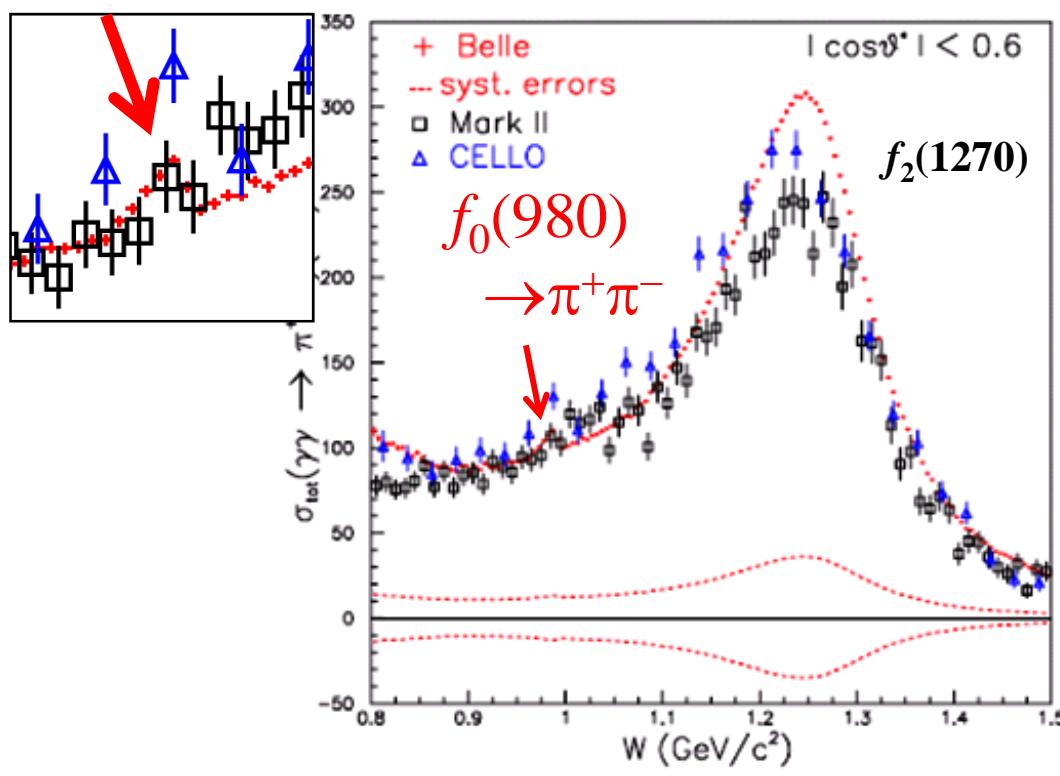
- We cannot determine the partial waves model independently;  
We need **parameterization based on a model** including the  $W$  dependence of resonances and continuum components.
- Ancillary model-independent way: **Hat amplitudes**;  $|Y_J^m|^2$  mutually independent

$$\frac{d\sigma}{d\Omega} = \hat{S}^2 |Y_0^0|^2 + \hat{D}_0^2 |Y_2^0|^2 + \hat{G}_0^2 |Y_4^0|^2 + \hat{D}_2^2 |Y_2^2|^2 + \hat{G}_2^2 |Y_4^2|^2$$

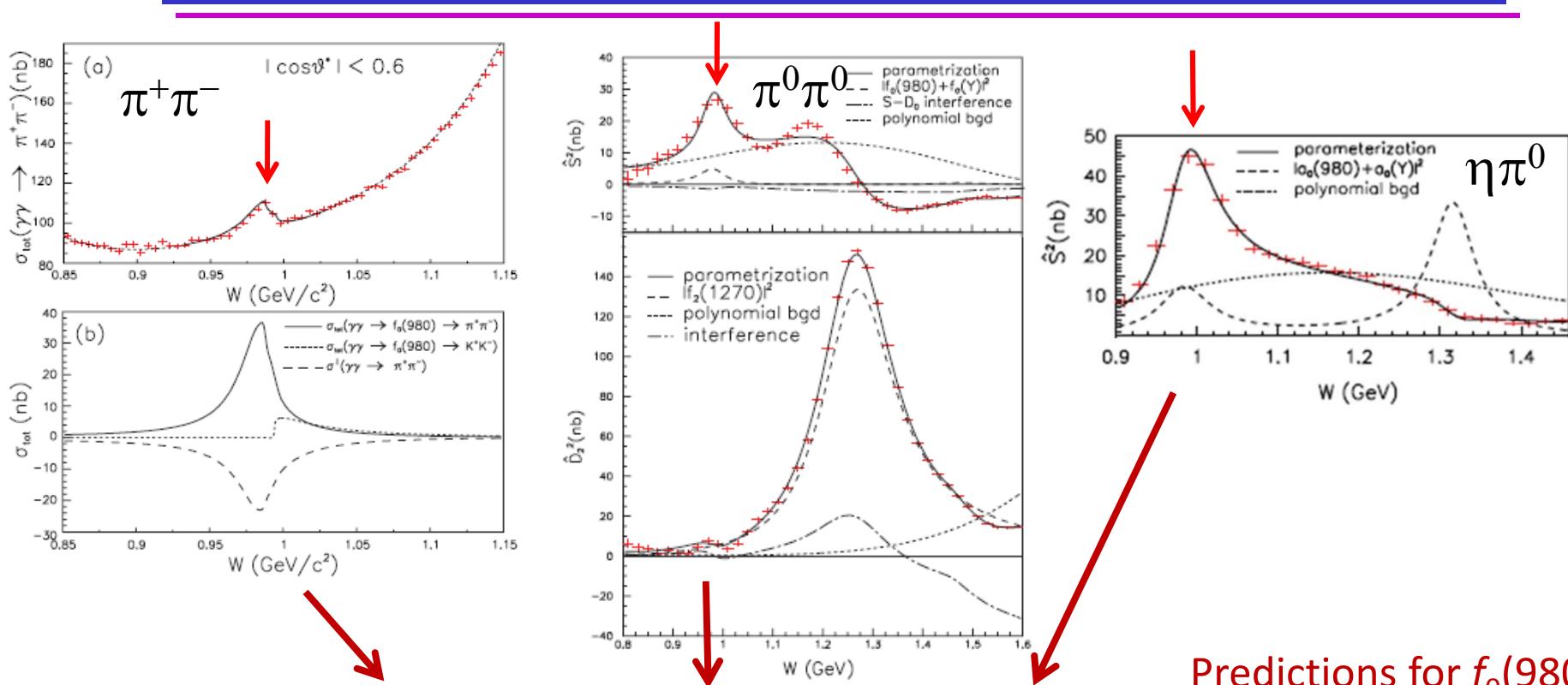


# Confirmations of $f_0(980)$ and $a_0(980)$ formations

$f_0(980)$  and  $a_0(980)$ :  
Observed as a peak very clearly in two-photon production, for the first time.



# Two-photon decay width of $f_0(980)$ and $a_0(980)$



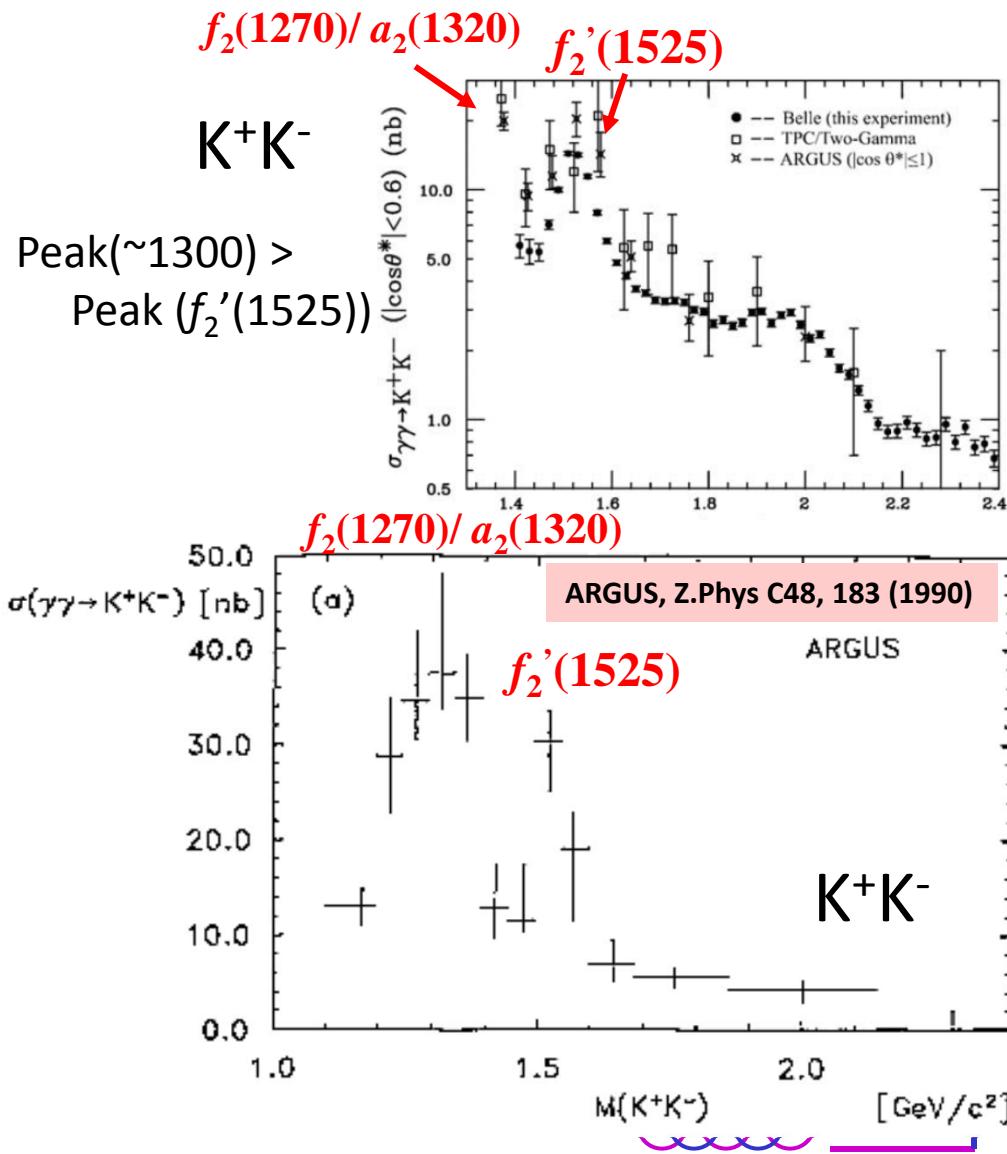
Meson	$f_0(980)$	$f_0(980)$	$a_0(980)$
$M [\text{MeV}/c^2]$	$985.6^{+1.2+1.1}_{-1.5-1.6}$	$982.2 \pm 1.0^{+8.1}_{-8.0}$	$982.3^{+0.6+3.1}_{-0.7-4.7}$ $(\Gamma_{\text{tot}})$
$\Gamma_{\pi\pi/\text{tot}} [\text{MeV}]$	$51.3^{+20.9+13.2}_{-17.7-3.8}$	$66.9^{+13.9+8.8}_{-11.8-2.5}$	$75.6 \pm 1.6^{+17.4}_{-10.0}$
$\Gamma_{\gamma\gamma} [\text{eV}]$	$205^{+95+147}_{-83-117}$	$286 \pm 17^{+211}_{-70}$	$128^{+3+502}_{-2-43} / \mathcal{B}_{\pi^0\eta}$

## Predictions for $f_0(980)$

Model	$\Gamma_{\gamma\gamma} [\text{eV}]$
$uubar, dbar$	$1300 - 1800$
$ssbar$	$300 - 500$
<b><math>KKbar</math> molecule</b>	$200 - 600$
<b>Four-quark</b>	270



# $f_2(1270)$ - $a_2(1320)$ interference in $\bar{K}K$



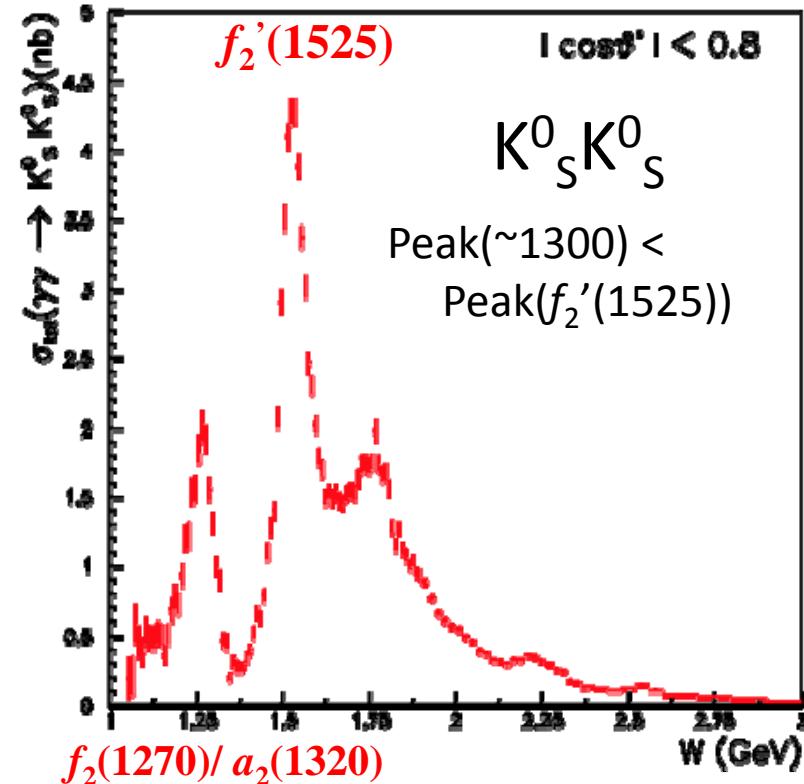
Constructive interference

$f_2(1270)+a_2(1320)$  in  $K^+K^-$

Destructive interference

$f_2(1270)-a_2(1320)$  in  $K^0_S K^0_S$

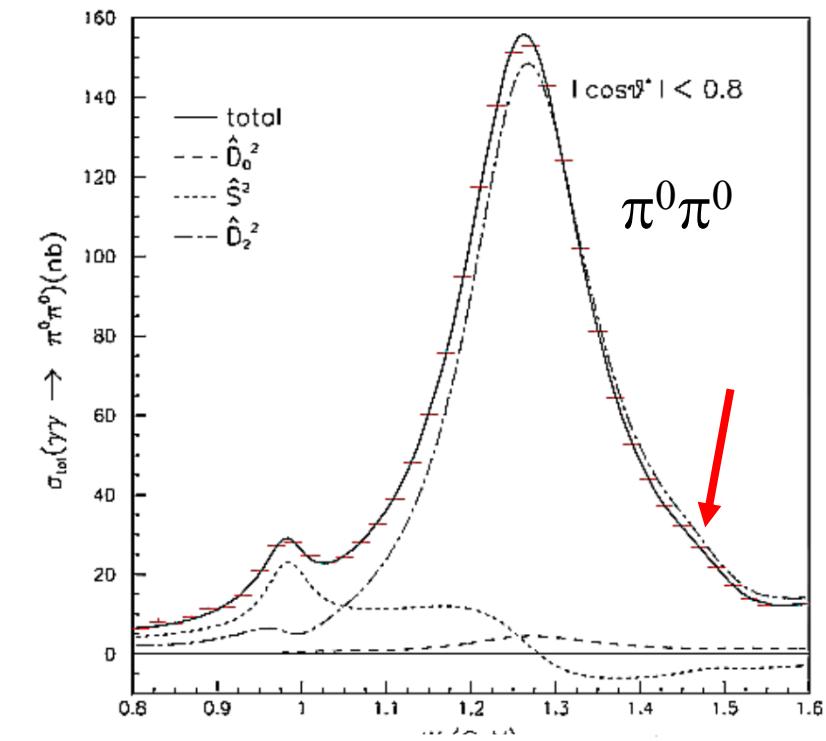
Explained by a phase relation in isospin composition



# Scalars in the 1.2 – 1.6 GeV region

- Hadron experiments report a wide  $f_0(1370)$  and a narrow  $f_0(1500)$ .
- Some of previous two-photon measurements provide a hint of  $f_0(1100\text{--}1400) \rightarrow \pi\pi$  under the huge peak of  $f_2(1270)$
- Belle's  $\pi^0\pi^0$  measurement reports  $f_0(1470)$ .  
May be visible in the line shape.  
→ favorable to the narrow  $f_0(1500)$ ,  
but also consistent with  $f_0(1370)$ .

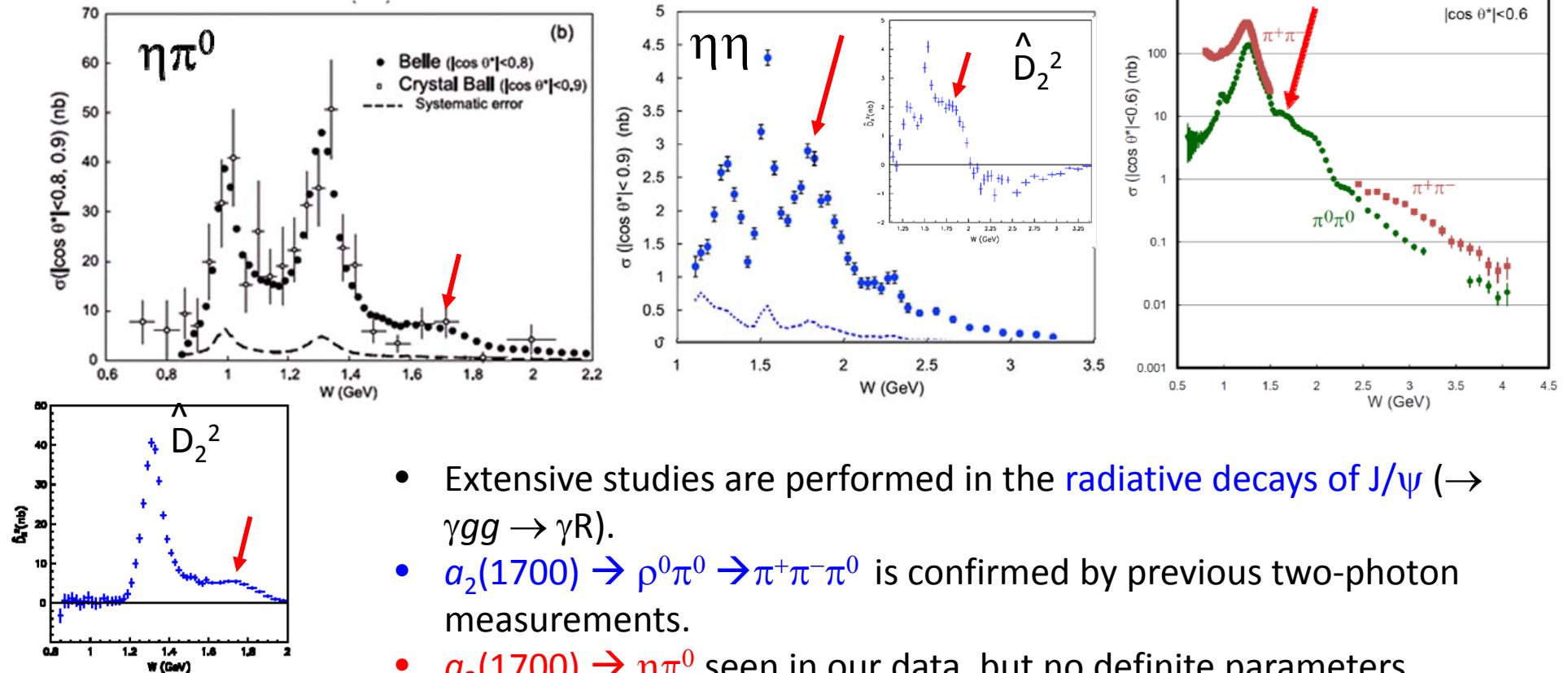
$f_0(1370)$ [v]	$J^G(J^{PC}) = 0^+(0^{++})$
Mass $m = 1200$ to $1500$ MeV	
Full width $\Gamma = 200$ to $500$ MeV	
$f_0(1370)$ DECAY MODES	Fraction ( $\Gamma_i/\Gamma$ )
$\pi\pi$	seen
	672
$f_0(1500)$ [n]	$J^G(J^{PC}) = 0^+(0^{++})$
Mass $m = 1505 \pm 6$ MeV (S = 1.3)	
Full width $\Gamma = 109 \pm 7$ MeV	
$f_0(1500)$ DECAY MODES	Fraction ( $\Gamma_i/\Gamma$ )
$\pi\pi$	( $34.9 \pm 2.3$ ) %
	Scale factor $p$ (MeV/c)
	1.2 741



Parameter	Belle ( $\pi^0\pi^0$ )	Crystal Ball	Unit
Mass	$1470^{+6+72}_{-7-255}$	1250	$\text{MeV}/c^2$
$\Gamma_{\text{tot}}$	$90^{+2+50}_{-1-22}$	$268 \pm 70$	MeV
$\Gamma_{\gamma\gamma} \mathcal{B}(\pi^0\pi^0)$	$11^{+4+603}_{-2-7}$	$430 \pm 80$	eV

10

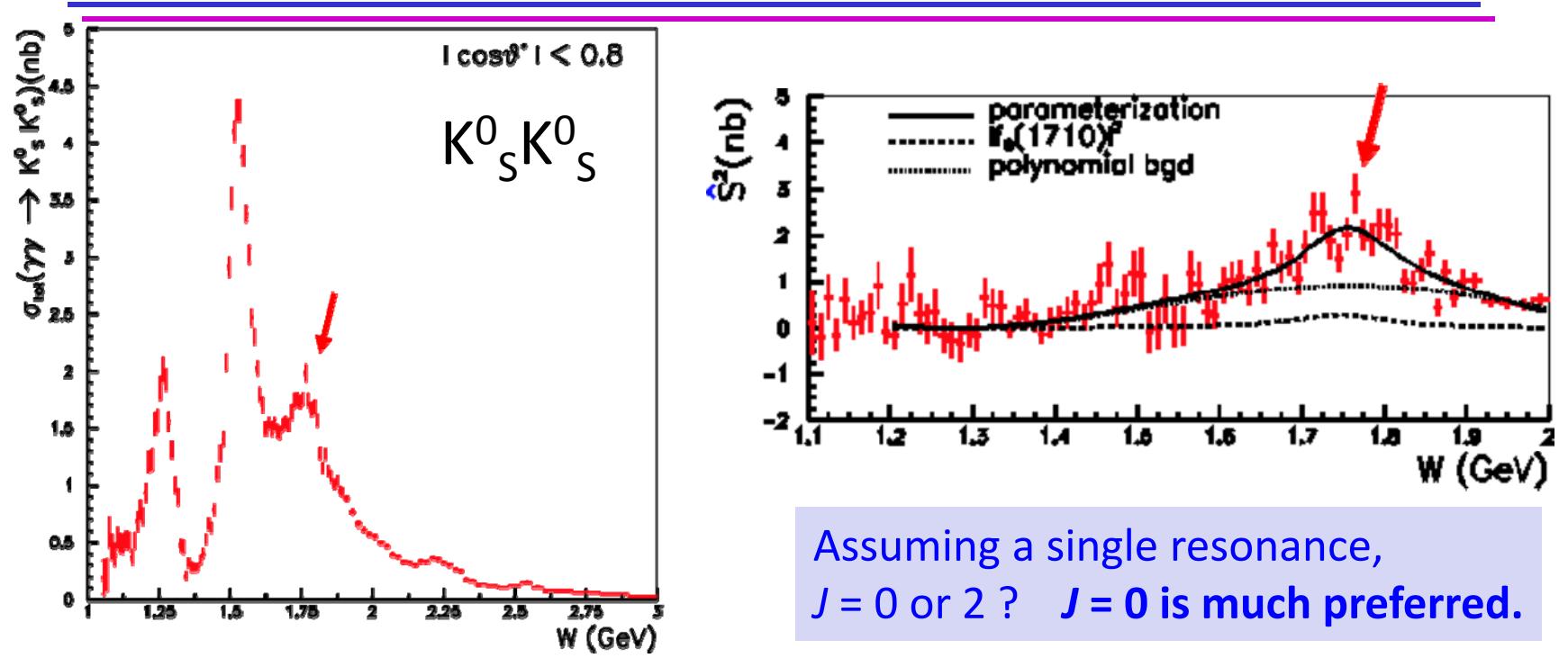
# 1.6 – 1.8 GeV: Mass region of the greatest difficulty



- Extensive studies are performed in the radiative decays of  $J/\psi$  ( $\rightarrow \gamma gg \rightarrow \gamma R$ ).
- $a_2(1700) \rightarrow \rho^0\pi^0 \rightarrow \pi^+\pi^-\pi^0$  is confirmed by previous two-photon measurements.
- $a_2(1700) \rightarrow \eta\pi^0$  seen in our data, but no definite parameters obtained.
- $f_2(1810) \rightarrow \eta\eta$  is confirmed in two-photon process.
- An unidentified structure around  $\sim 1.6$  GeV is seen in  $\pi^0\pi^0$ . But, its correspondence to a single resonance of the mass is not sure.



# $f_0(1710)$ formation in $K^0_S K^0_S$

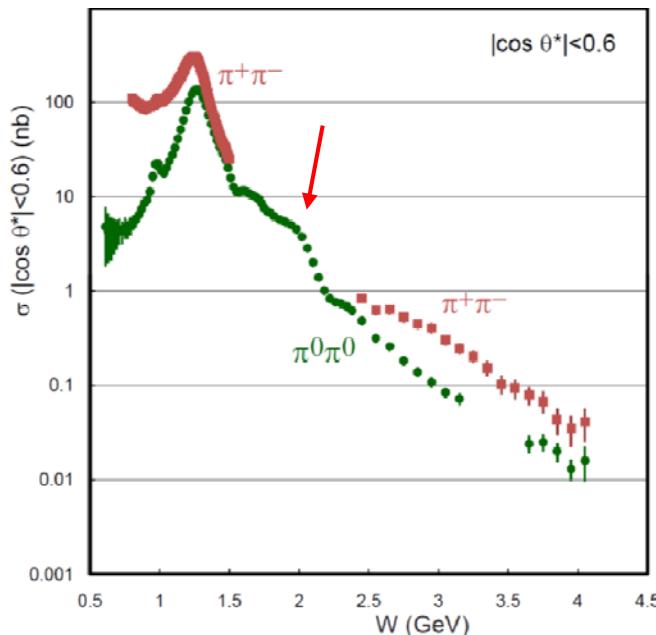


Parameter $f_J(1710)$	$f_0(1710)$ fit				$f_2(1710)$ fit	
	fit-H	fit-L	H,L combined	PDG	fit-H	fit-L
$\chi^2/ndf$	694.2/585	701.6/585		Two solutions of interference	796.3/585	831.5/585
Mass( $f_J$ ) (MeV/ $c^2$ )	$1750^{+5+29}_{-6-18}$	$1749^{+5+31}_{-6-42}$	$1750^{+6+29}_{-7-18}$	$1720 \pm 6$	$1750^{+6}_{-7}$	$1729^{+6}_{-7}$
$\Gamma_{\text{tot}}(f_J)$ (MeV)	$138^{+12+96}_{-11-50}$	$145^{+11+31}_{-10-54}$	$139^{+11+96}_{-12-50}$	$135 \pm 6$	$132^{+12}_{-11}$	$150 \pm 10$
$\Gamma_{\gamma\gamma}\mathcal{B}(K\bar{K})_{f_J}$ (eV)	$12^{+3+227}_{-2-8}$	$21^{+6+38}_{-4-26}$	$12^{+3+227}_{-2-8}$	unknown	$2.1^{+0.5}_{-0.3}$	$1.6 \pm 0.2$

$f_0(1710) \rightarrow K^0_S K^0_S$  is confirmed in two-photon process.

# The 1.8 – 2.2 GeV region

- $f_2(1950) \rightarrow \pi^0\pi^0$  shows a broad structure
- Similar structure exists in  $K^+K^-$  (but, they can be different states)
- No peak in  $\eta\pi^0$ ,  $\eta\eta$  and  $K^0_S K^0_S$  in this mass region



Parameter	$f_4(2050)$	" $f_2(1950)$ "	Unit
Mass	$1885^{+14}_{-13} {}^{+218}_{-25}$	$2038^{+13}_{-11} {}^{+12}_{-73}$	$\text{MeV}/c^2$
$\Gamma_{\text{tot}}$	$453 \pm 20 {}^{+31}_{-129}$	$441^{+27}_{-25} {}^{+28}_{-192}$	MeV
$\Gamma_{\gamma\gamma} \mathcal{B}(\pi^0\pi^0)$	$7.7^{+1.2}_{-1.1} {}^{+23.5}_{-5.2}$	$54^{+23}_{-14} {}^{+379}_{-68}$	eV
$\chi^2(ndf)$		323.2 (311)	

$|l=0 (f)$  and  $|l=1 (a)$  interference in  $KK^-$

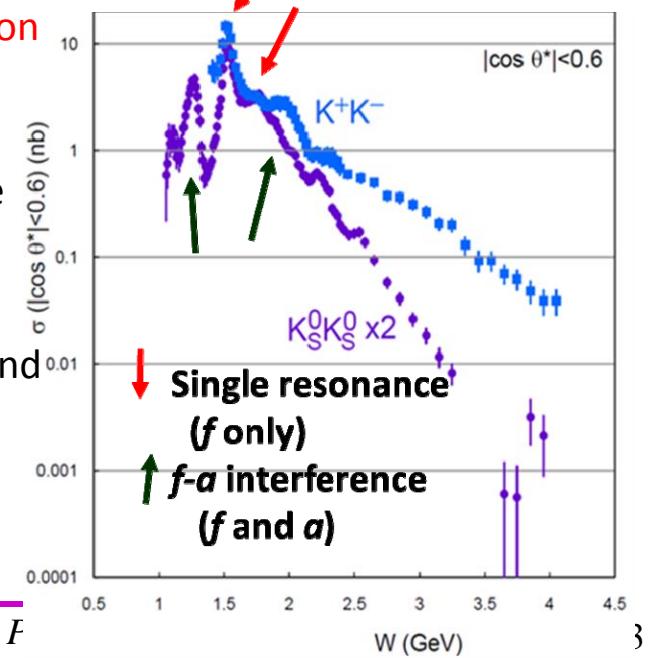
Size of the cross sections for  $K^+K^-$  and  $K^0_K^0$

A single resonance production of  $f$  or  $a$  decaying with the strong interaction

→ The cross sections are similar size.

If they are very different

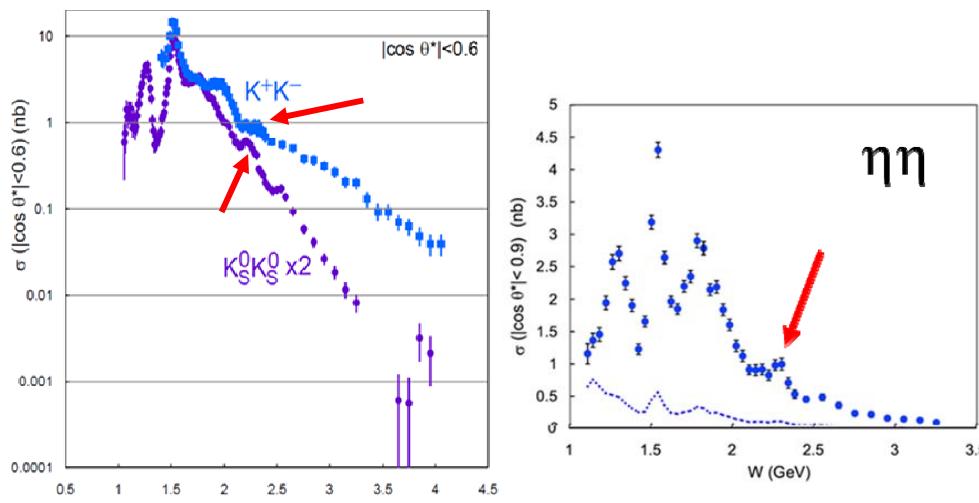
Interference between  $|l=0$  and  $|l=1$  resonances



# The 2.2 – 2.6 GeV region

- The very narrow  $f_J(2220)$  (was  $\xi(2220)$ ) and a wide  $f_2(2300)$  are suggested.
- Do the both exist? Really narrow?
- Our  $\pi^0\pi^0$  result does not need  $f(\sim 2300)$ ; the high mass  $f_2(1950)$  can explain the observed line shape.
- Surely something narrow(?) peaks are found in  $K^+K^-$ ,  $K_S^0 K_S^0$  and  $\eta\eta$ .

An  $s\bar{s}$  state or a glueball flavor insensible?



$f_J(2220)$		$I^G(J^{PC}) = 0^+(2^{++} \text{ or } 4^{++})$			
OMITTED FROM SUMMARY TABLE		Needs confirmation. See our mini-review in the 2004 edition of this Review, PDG 04.			
$f_J(2220)$ MASS					
VALUE (MeV)	EVTS	DOCUMENT ID	TECN	COMMENT	
$2231.1 \pm 3.5$ OUR AVERAGE					
$f_J(2220)$ WIDTH					
VALUE (MeV)	CL%	EVTS	DOCUMENT ID	TECN	COMMENT
$23^+ \pm 8$ OUR AVERAGE					
$f_J(2220)$ DECAY MODES					
Mode	Fraction ( $\Gamma_i/\Gamma$ )				
$\Gamma_1 \pi\pi$	seen				
$\Gamma_2 \pi^+\pi^-$	seen				
$\Gamma_3 K\bar{K}$	seen				
$\Gamma_4 p\bar{p}$					
$\Gamma_5 \gamma\gamma$	not seen				
$\Gamma_6 \eta\eta'(958)$	seen				
$\Gamma_7 \phi\phi$	not seen				
$\Gamma_8 \eta\eta$	not seen				

$f_2(2300)$		$I^G(J^{PC}) = 0^+(2^{++})$			
Mass $m = 2297 \pm 28$ MeV					
Full width $\Gamma = 149 \pm 40$ MeV					
$f_2(2300)$ DECAY MODES		Fraction ( $\Gamma_i/\Gamma$ )	$p$ (MeV/c)		
$\phi\phi$		seen	529		
$K\bar{K}$		seen	1037		
$\gamma\gamma$		seen	1149		

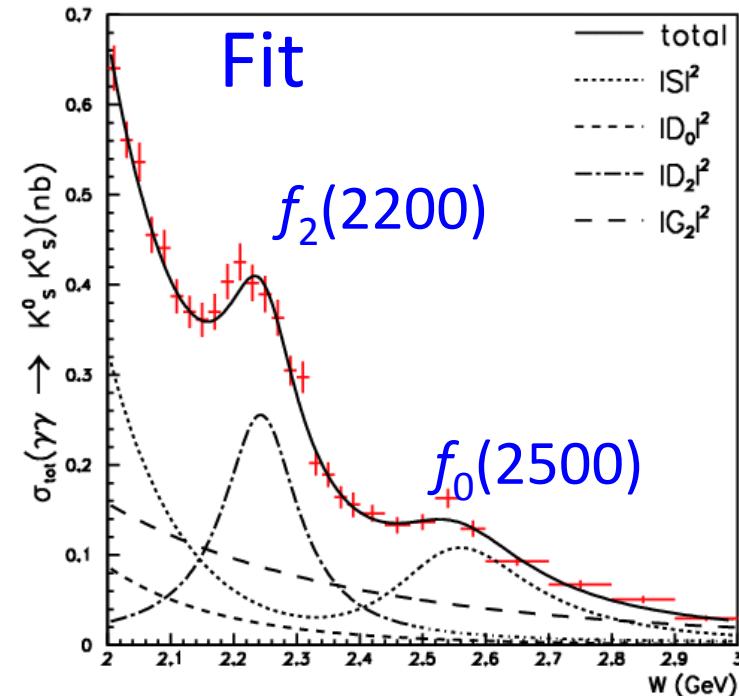
# Fit Results for resonances in $K_s^0 K_s^0$

$f_2(2200)$ - $f_0(2500)$  is the best solution (in all the  $J=0, 2, 4$  combinations)

Parameter	$f_2(2200)$	$f_0(2500)$
Mass (MeV/ $c^2$ )	$2243^{+7+3}_{-6-29}$	$2539 \pm 14^{+38}_{-14}$
$\Gamma_{\text{tot}}$ (MeV)	$145 \pm 12^{+27}_{-34}$	$274^{+77+126}_{-61-163}$
$\Gamma_{\gamma\gamma} \mathcal{B}(K\bar{K})$ (eV)	$3.2^{+0.5+1.3}_{-0.4-2.2}$	$40^{+9+17}_{-7-40}$

## Significances

- $3.4\sigma$  for  $f_2(2200)$  over  $f_0(2200)$
- $4.3\sigma$  for  $f_0(2500)$  over  $f_2(2500)$

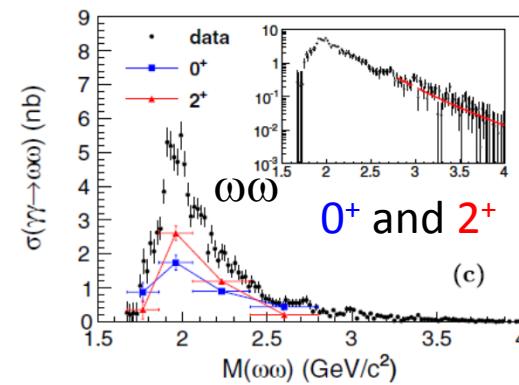
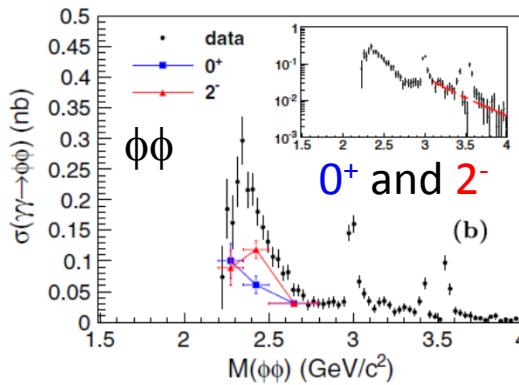
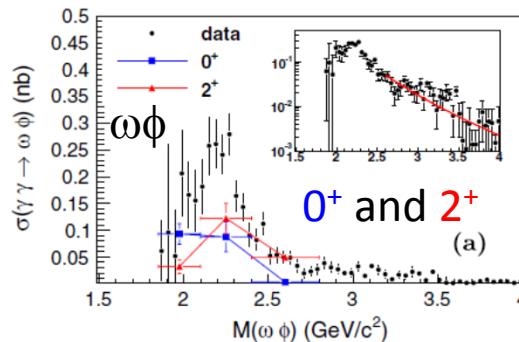
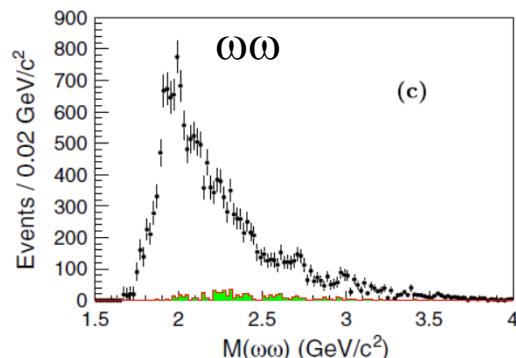
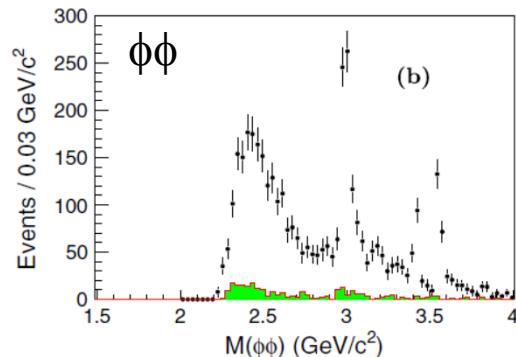
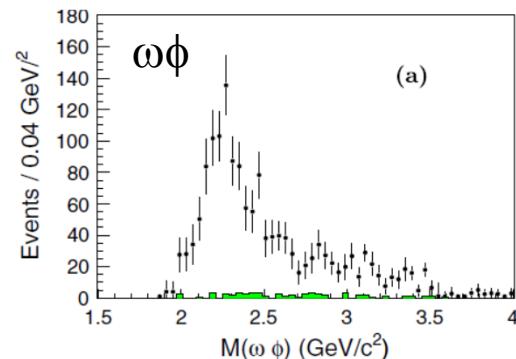


- There can be **an only wide state** around 2240 MeV.
- Narrow appearances in previous measurements may be due to an interference effect and/or statistical fluctuation.
- A high-mass state at 2.5 GeV may be **the heaviest light-quark scalar meson** so far found.



# “ $\gamma\gamma \rightarrow$ Vector-meson pair” from Belle

## Observation of New Resonant Structures in $\gamma\gamma \rightarrow \omega\phi, \phi\phi$ , and $\omega\omega$



Belle, PRL 108, 232001 (2012)

There is a resonance-like structures at  $2.0 - 2.5 \text{ GeV}$  in each of the final states.

Preferred  $J^P$  combinations are determined by the angular analysis of production and decay of  $\omega$  and  $\phi$ .

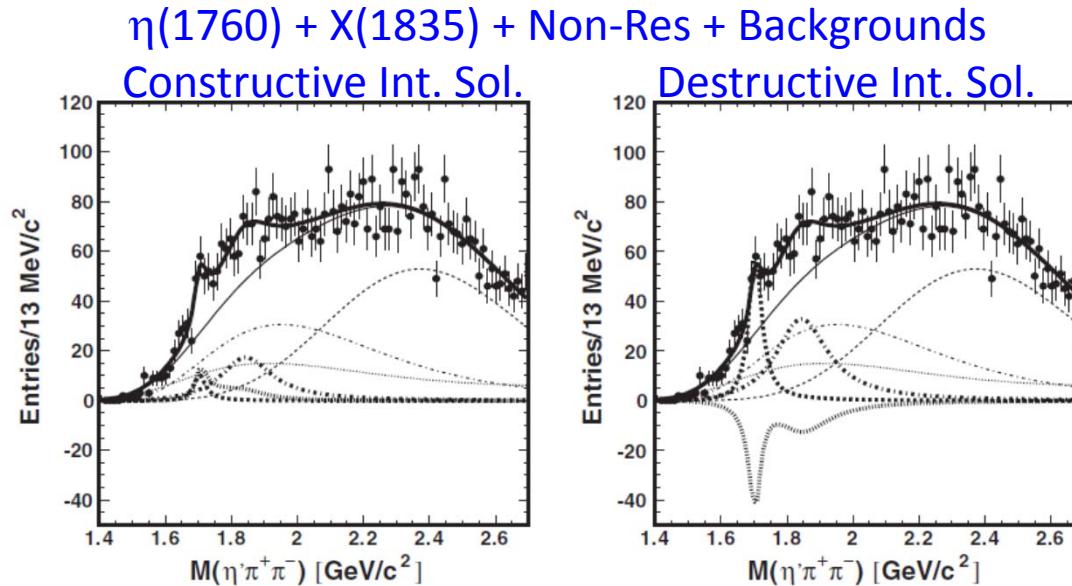
$$\gamma\gamma \rightarrow \eta'\pi^+\pi^-$$

Production of light-quark mesons decaying to the **three pseudoscalar meson final state**. (The  $\eta_c$  production is also presented.)

Belle, PRD 86, 052002 (2012)

**X(1835)** is an exotic resonance candidate found in the radiative decay of J/ $\psi$  by BES. Is it gluon-rich, or q $\bar{q}$ -rich?

A hint of X(1835) –  $2.8\sigma$ ,  
but it is not very significant.



Parameter	Two interfering resonances		Reference
	Solution I	Solution II	
<b><math>X(1835)</math></b>			
$M$ , MeV/c <sup>2</sup>	1836.5 (fixed)		$1836.5 \pm 3.0^{+5.6}_{-2.1}$
$\Gamma$ , MeV/c <sup>2</sup>	190 (fixed)		$190 \pm 9^{+38}_{-36}$
$\Gamma_{\gamma\gamma}\mathcal{B}$ , eV/c <sup>2</sup>	$18.2^{+7.7}_{-6.7} \pm 4.0$	$35^{+12}_{-13} \pm 8$	
$(\Gamma_{\gamma\gamma}\mathcal{B})_{90}$ eV	<35.6	<83	
$S, \sigma$		2.8	
<b><math>\eta(1760)</math></b>			
$M$ , MeV/c <sup>2</sup>	$1703^{+12}_{-11} \pm 1.8$		$1756 \pm 9$
$\Gamma$ , MeV/c <sup>2</sup>	$42^{+36}_{-22} \pm 15$		$96 \pm 70$
$\Gamma_{\gamma\gamma}\mathcal{B}$ , eV/c <sup>2</sup>	$3.0^{+2.0}_{-1.2} \pm 0.8$	$18^{+13}_{-10} \pm 5$	
$S, \sigma$		4.1	
$\phi$	$(287^{+42}_{-51})^\circ$	$(139^{+19}_{-9})^\circ$	



# Summary

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- $\gamma\gamma \rightarrow$  pseudoscalar-meson pair have been measured in six different final states.
- The  $\gamma\gamma$ -invariant-mass region, 0.6 – 2.6 GeV, is studied for light-meson spectroscopy.
- We measure  $\Gamma_{\gamma\gamma}(\times BF)$  for various  $J^{PC}=(even)^{++}$  meson states.
- We have confirmed:
  - $\gamma\gamma$  coupling of the scalar mesons  $f_0(980)$ ,  $a_0(980)$ ,  $f_0(1710)$
  - $f - a$  interference in the  $K\bar{K}$  final states
  - Many clear resonant structures found in 1.6 – 2.6 GeV regions.
- Resonant signals in the 1.6 – 2.6 GeV region are also found in the  $VV$  and  $\eta'\pi\pi$  final states.



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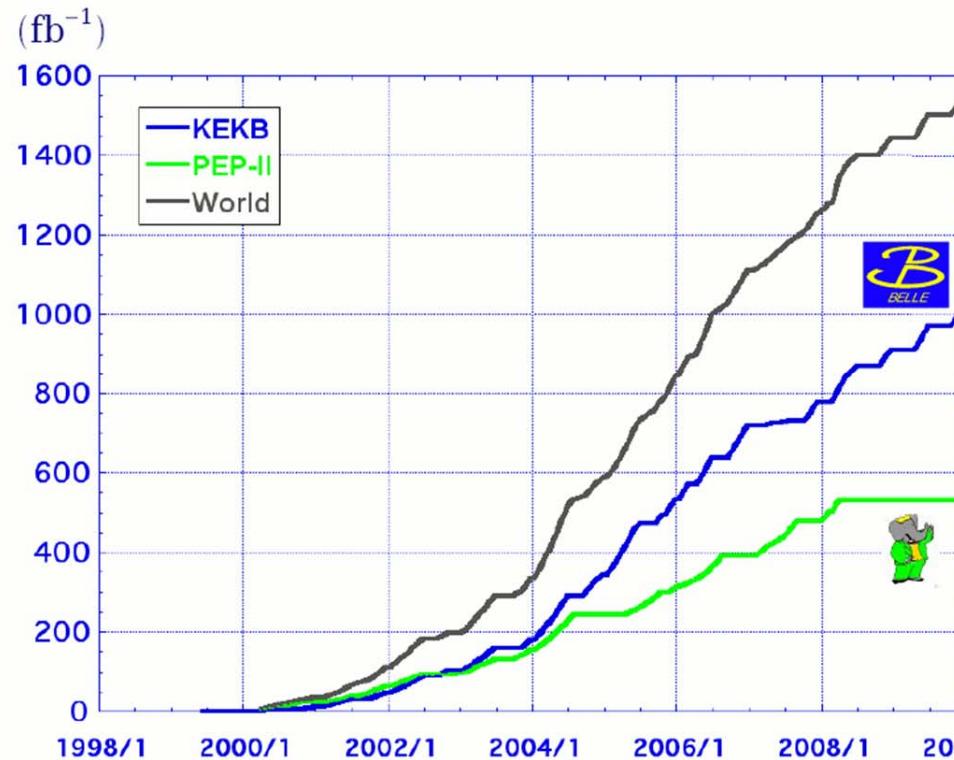
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# Backup



# History of integrated luminosity at Belle

## Luminosity at B factories



**> 1 ab<sup>-1</sup>**  
**On resonance:**  
 $Y(5S): 121 \text{ fb}^{-1}$   
 $Y(4S): 711 \text{ fb}^{-1}$   
 $Y(3S): 3 \text{ fb}^{-1}$   
 $Y(2S): 24 \text{ fb}^{-1}$   
 $Y(1S): 6 \text{ fb}^{-1}$   
**Off reson./scan:**  
 $\sim 100 \text{ fb}^{-1}$

**~ 550 fb<sup>-1</sup>**  
**On resonance:**  
 $Y(4S): 433 \text{ fb}^{-1}$   
 $Y(3S): 30 \text{ fb}^{-1}$   
 $Y(2S): 14 \text{ fb}^{-1}$   
**Off resonance:**  
 $\sim 54 \text{ fb}^{-1}$



1999	The Belle experiment started
2001	CP violation in B mesons was verified and the KEKB accelerator achieved the world's highest luminosity
2002	Anomalous CP violation in $b \rightarrow s$ was measured
2003	The $B \rightarrow K\bar{l}l$ decay was discovered
2004	The New particle X (3872) was discovered
2005	Direct violation of CP in $B \rightarrow K\pi$ was found. The $B \rightarrow \rho\gamma$ decay was discovered
2006	$B \rightarrow \tau\nu$ was observed
2007	D meson mixing was discovered. A new particle composed of 4 quarks Z (4430) + was discovered
2008	Dr. Makoto Kobayashi and Dr. Toshihide Maskawa were awarded the Nobel Prize in Physics
2010	The Belle experiment was completed

# Formalism of PWA for P-meson pair final-state processes

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- We consider up to  $J=4$  (for  $W < 3$  GeV).

$$\frac{d\sigma}{d\Omega} = |SY_0^0 + D_0Y_2^0 + G_0Y_4^0|^2 + |D_2Y_2^2 + G_2Y_4^2|^2$$

- $S, D_0, G_0, D_2, G_2$  Partial-wave amplitudes for each wave  $J_\lambda$   
 $J = L = 0, 2, 4$  (even only) with the helicity  $\lambda = 0$  or  $2$  (to the  $\gamma\gamma$  axis)
  - $Y_J^\lambda$ : spherical harmonics
  - $|Y_J^\lambda|$  are NOT mutually independent, as we have no information for the azimuthal-angle direction.
- We cannot determine the partial waves model independently;  
We need **parameterization** based on a model including the  $W$  dependence of resonances and continuum components.



# Nature of $I=0$ and $I=1$ interference in $K\bar{K}$

- Consider both isospin  $I=0$  and  $I=1$ , e.g.,  $f_J$  and  $a_J$
- Their Constructive and Destructive interference based on OZI (Okubo-Zweig-Iizuka) rule and isospin  $I_z$  inversion.

D. Faiman, H.J. Lipkin and H.R. Rubinstein, PL 59B,269 (1975)

Two Feynman diagrams illustrating the production of  $K^+$ ,  $K^-$ ,  $K^0$ , and  $\bar{K}^0$  mesons through quark loop processes:

- $K^+$ : A quark  $u$  and an antiquark  $\bar{s}$  form a loop, with an incoming quark  $u$  and outgoing antiquark  $\bar{s}$ .
- $K^-$ : An antiquark  $\bar{u}$  and a quark  $s$  form a loop, with an incoming antiquark  $\bar{u}$  and outgoing quark  $s$ .
- $K^0$ : A quark  $d$  and an antiquark  $\bar{s}$  form a loop, with an incoming quark  $d$  and outgoing antiquark  $\bar{s}$ .
- $\bar{K}^0$ : An antiquark  $\bar{d}$  and a quark  $s$  form a loop, with an incoming antiquark  $\bar{d}$  and outgoing quark  $s$ .

$$|u\bar{u}\rangle = \frac{1}{\sqrt{2}}(f_J + a_J)$$

$$|d\bar{d}\rangle = \frac{1}{\sqrt{2}}(f_J - a_J)$$

## Size of the cross sections for $K^+K^-$ and $K^0\bar{K}^0$

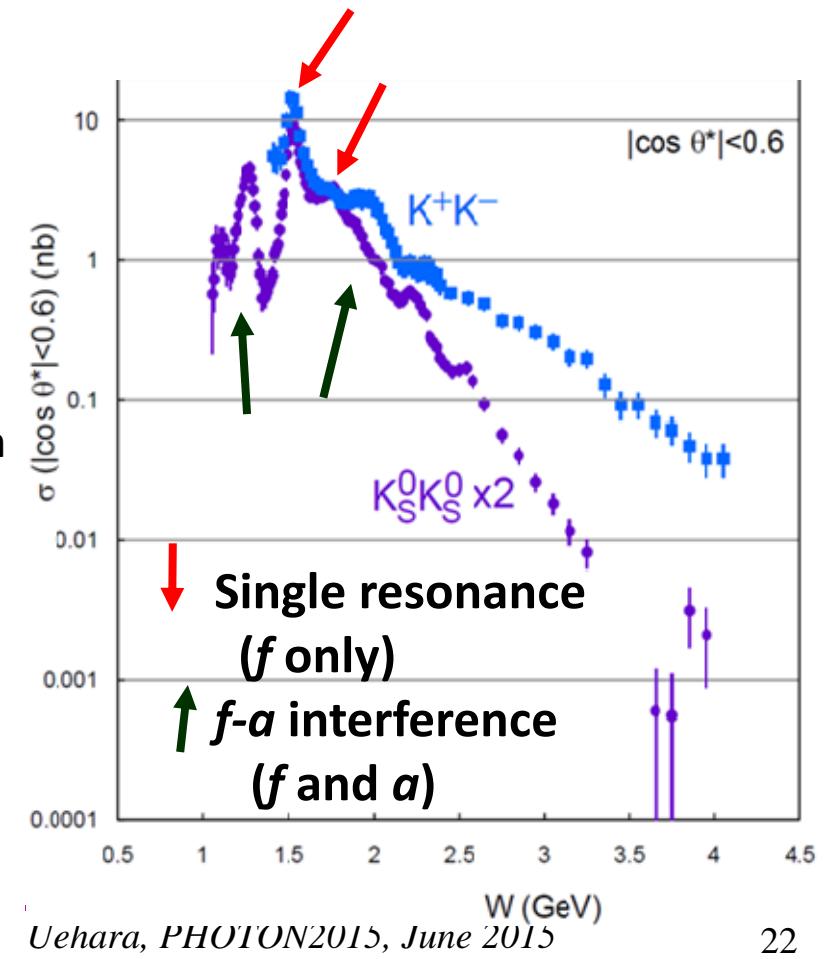
A single resonance production of  $f$  or  $a$  decaying with the strong interaction

→ The cross sections are **similar size**.

If they are **very different** →

Interference between  $I=0$  and  $I=1$  resonances, or effective (electromagnetic) continuum production

The difference above  $>\sim 2.4\text{GeV}$  is explained by electric-charge difference of the quarks.

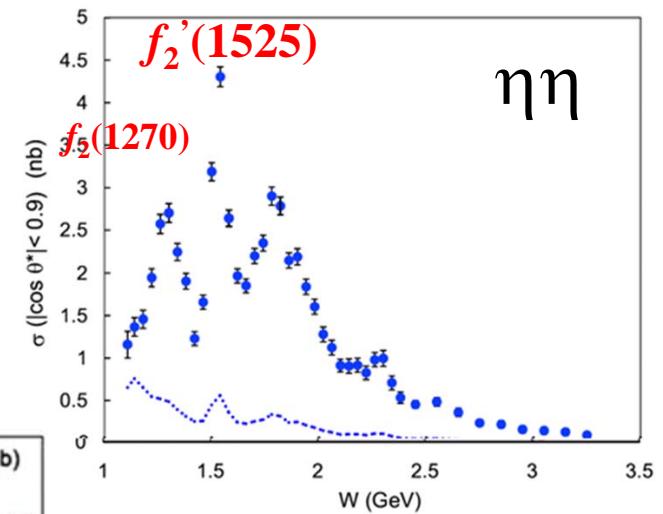
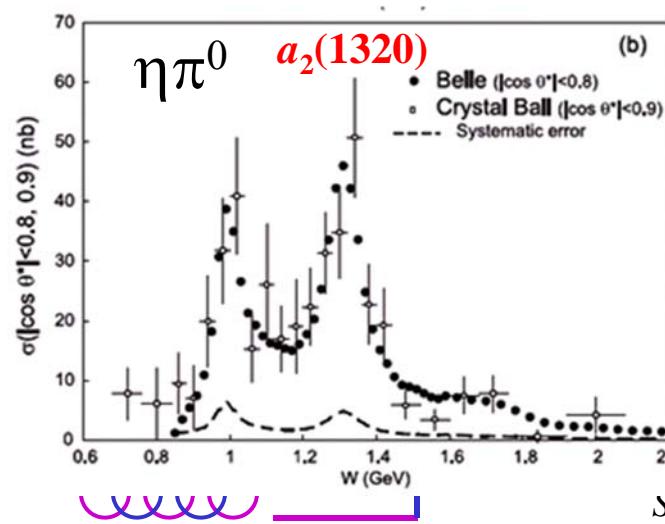
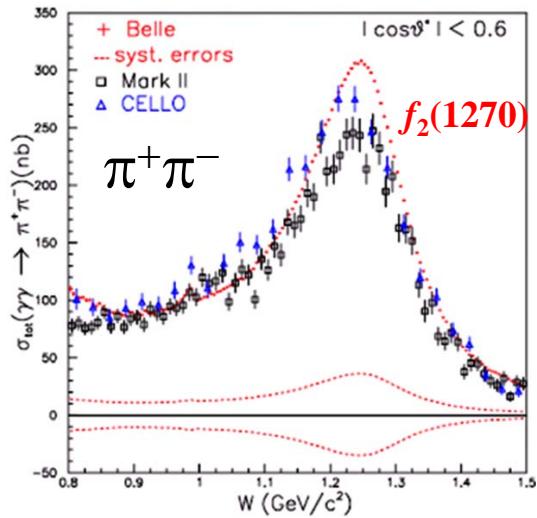


# The tensor-meson triplet, $f_2(1270)$ , $a_2(1320)$ , $f_2'(1525)$

$f_2(1270)$  : The largest peak in  $\pi^+\pi^-$  and  $\pi^0\pi^0$ . Also seen in  $\eta\eta$

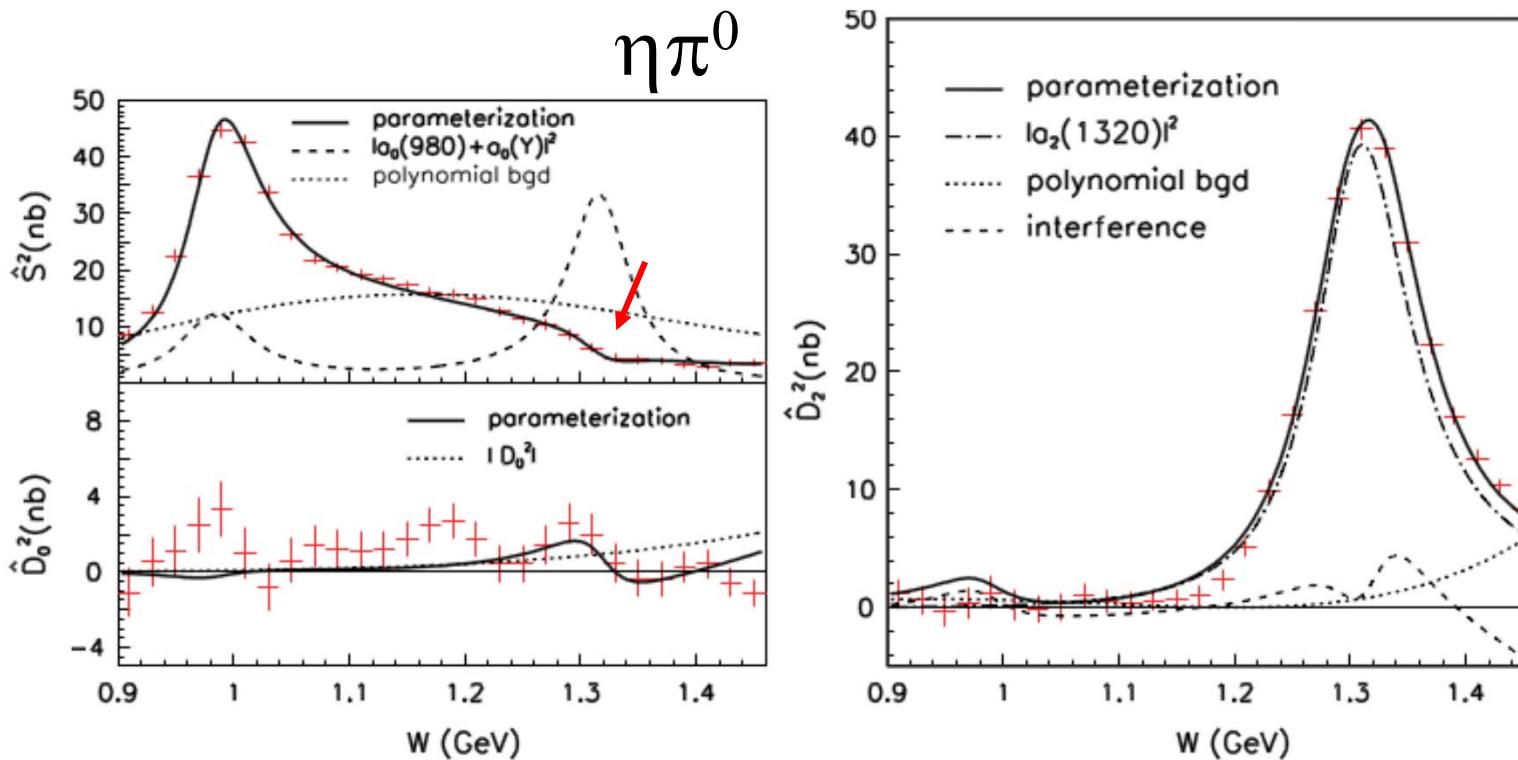
$a_2(1320)$ : Large peak in  $\eta\pi^0$

$f_2'(1525)$ : Large peak in  $\eta\eta$ ,  $K^+K^-$ , and  $K_S^0 K_S^0$



# The $J=1$ sector

- We find  $a_0(1320) \rightarrow \eta\pi^0$  just under  $a_2(1320)$ .
- The mass is not compatible with  $a_0(1450)$ ?

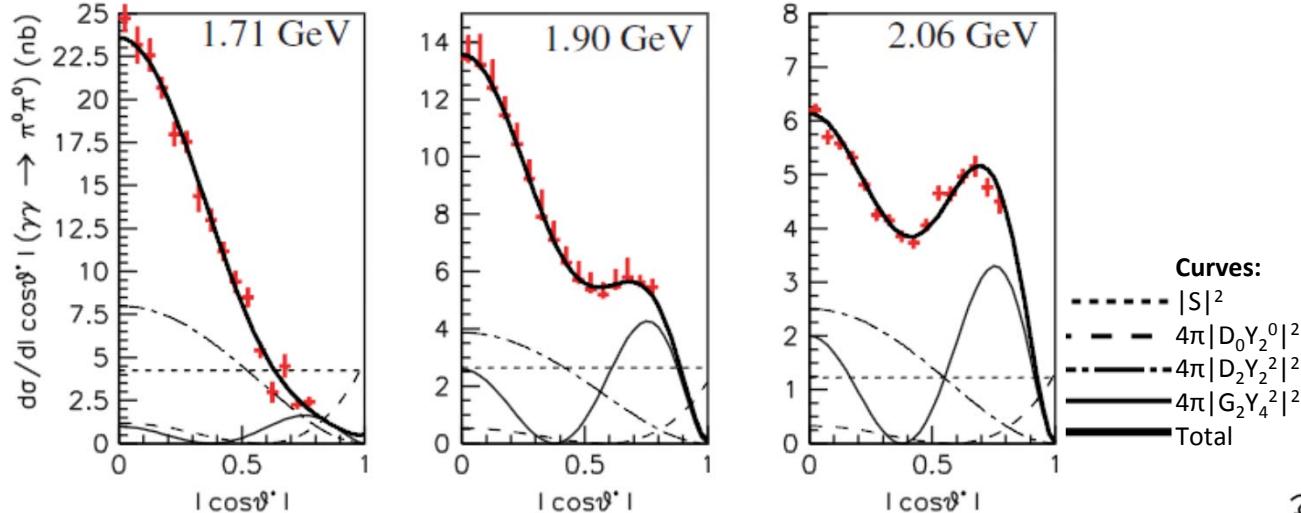


Parameter	This work	$a_0(1450)$ (PDG)	Unit
Mass	$1316.8^{+0.7+24.7}_{-1.0-4.6}$	$1474 \pm 19$	$\text{MeV}/c^2$
$\Gamma_{\text{tot}}$	$65.0^{+21+99.1}_{-5.4-32.6}$	$265 \pm 13$	MeV
$\Gamma_{\gamma\gamma}\mathcal{B}(\eta\pi^0)$	$432 \pm 6^{+1073}_{-256}$	unknown	eV

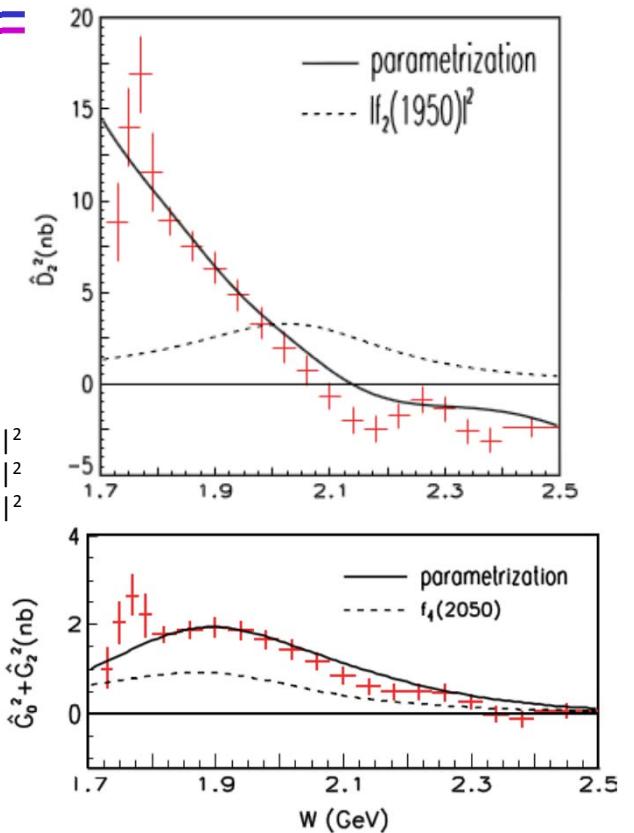
See Discussion in PDG-RPP2014,  
NOTE ON SCALAR MESONS  
BELOW 2GeV (pp 784 – 791)

# $J=2$ and $J=4$ components in $\pi^0\pi^0$

Angular ( $|\cos \theta^*|$ ) dependence of the differential cross section



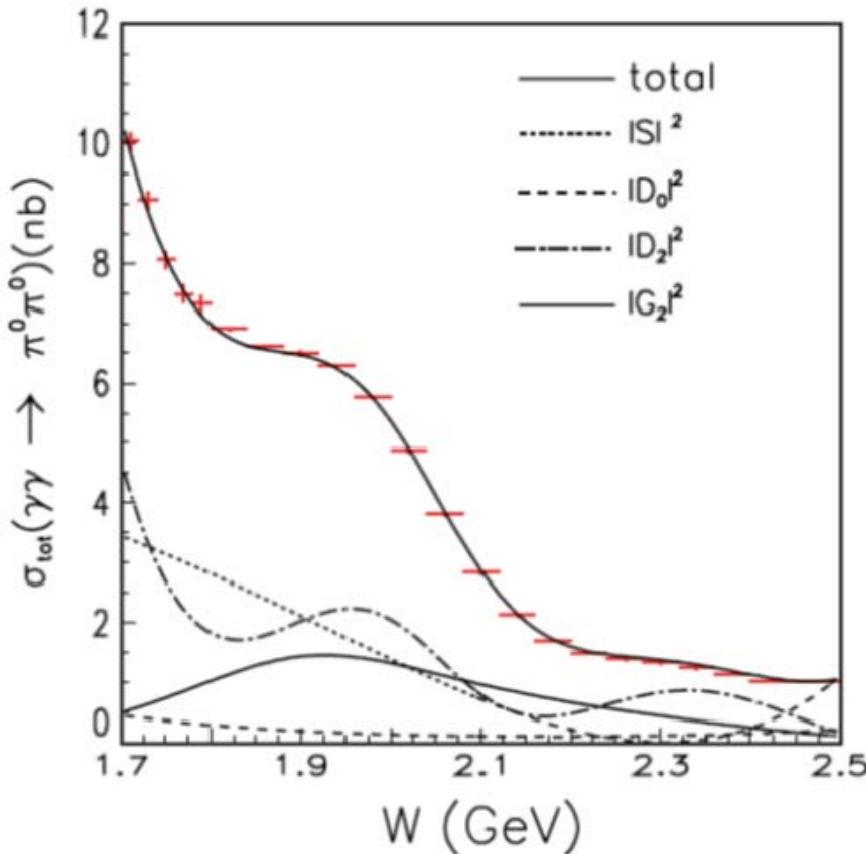
Parameter	$f_4(2050)$	" $f_2(1950)$ "	Unit
Mass	$1885^{+14}_{-13} {}^{+218}_{-25}$	$2038^{+13}_{-11} {}^{+12}_{-73}$	$\text{MeV}/c^2$
$\Gamma_{\text{tot}}$	$453 \pm 20 {}^{+31}_{-129}$	$441^{+27}_{-25} {}^{+28}_{-192}$	MeV
$\Gamma_{\gamma\gamma}\mathcal{B}(\pi^0\pi^0)$	$7.7^{+1.2}_{-1.1} {}^{+23.5}_{-5.2}$	$54^{+23}_{-14} {}^{+379}_{-68}$	eV
$\chi^2(\text{ndf})$	323.2 (311)		



The mass-magnitude relation to the spin between  $f_2$  and  $f_4$  is opposite between our measurement and PDG.  
 (That is possible between the  $J=2(2P)$  and  $J=4(1F)$  states.)



# Fit to $\pi^0\pi^0$ ( $W = 1.7 - 2.5$ GeV)

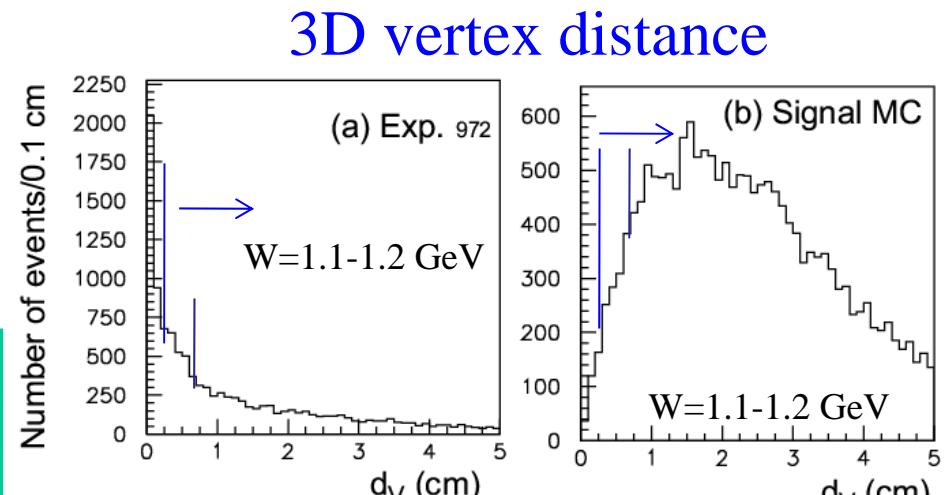
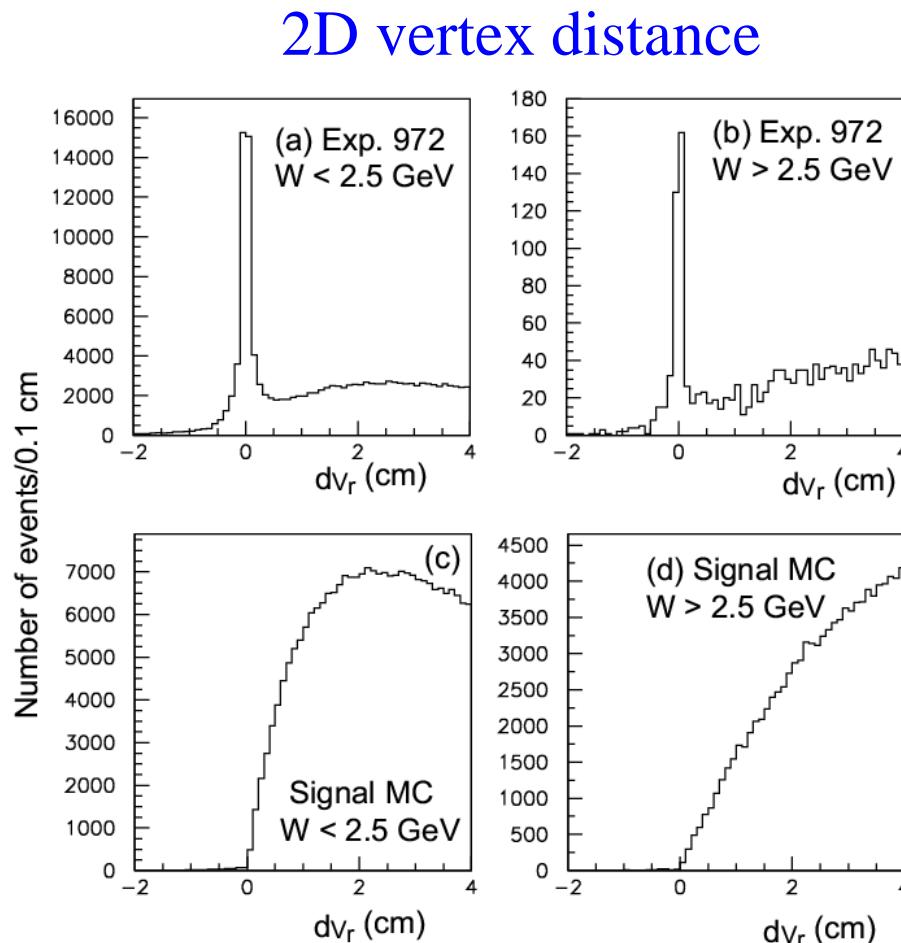


Parameter	$f_4(2050)$	" $f_2(1950)$ "	Unit
Mass	$1885^{+14}_{-13} {}^{+218}_{-25}$	$2038^{+13}_{-11} {}^{+12}_{-73}$	$\text{MeV}/c^2$
$\Gamma_{\text{tot}}$	$453 \pm 20 {}^{+31}_{-129}$	$441 {}^{+27}_{-25} {}^{+28}_{-192}$	MeV
$\Gamma_{\gamma\gamma}\mathcal{B}(\pi^0\pi^0)$	$7.7^{+1.2}_{-1.1} {}^{+23.5}_{-5.2}$	$54^{+23}_{-14} {}^{+379}_{-68}$	eV

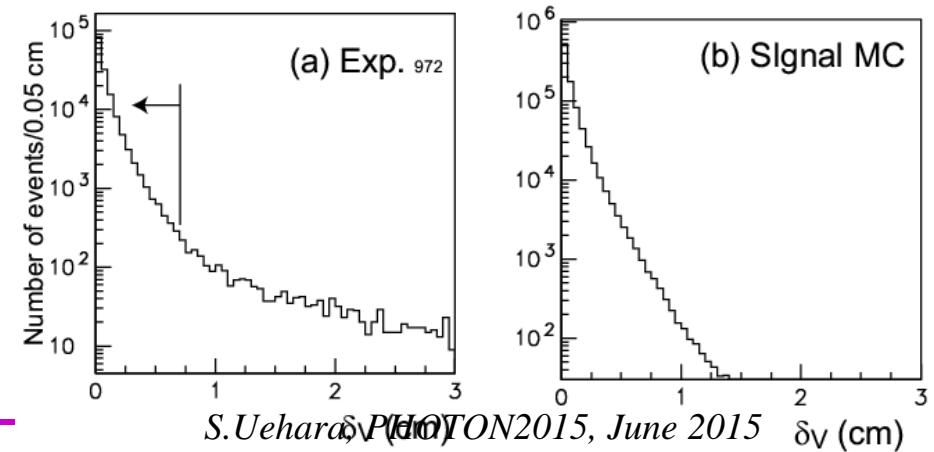
$\chi^2(ndf)$       323.2 (311)



# Ks Ks vertex distances



Tr. mometum diff. and vertex position diff.  
must be in parallel



Sharp peaks near 0cm seen only in Exp. are from  
Direct  $4\pi (\pi^+\pi^-\pi^+\pi^-)$  production backgrounds.



# Systematic errors

Source	Uncertainty(%)	
Tracking efficiency (for 4 tracks)	2	
Beam background effect	1	From correlation study of different Exp# settings in data and signal MC
Pion identification (for 4 tracks)	2	
Non-exclusive and four-pion backgrounds	2 – 19	A Half of the subtraction + 2% from pt-fit (quad.sum)
Geometrical coverage and fit uncertainty	4	
$K_S^0 K\pi$ background subtraction	1 – 2	
$K_S^0$ -pair reconstruction	5 – 3	Loose-cut sample
Trigger efficiency	5 – 7	Correlation of the two triggers
$E_{ECL}$ cut	1	
Integrated luminosity and luminosity function	5 – 4	
L4 efficiency	1 – 10	About 10% of the inefficiency
Total	9 – 25, typically 10	



# Fitting the region $W > 2 \text{ GeV}$

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- Parameterization

$$i\text{-wave} = B.W.e^{i\phi_i} + B_i$$

$$B_i = b_i \left( \frac{W}{W_0} \right)^{-c_i} e^{i\phi_i}$$

(assume power behavior  
for non-resonant background:

$$i = S, D_0, D_2 \text{ and } G_2; (\text{we assume } G_0=0)$$

B.W.= $f_J(2200)$  and/or  $f_J(2500)$  with  $J=0, 2$  and  $4$

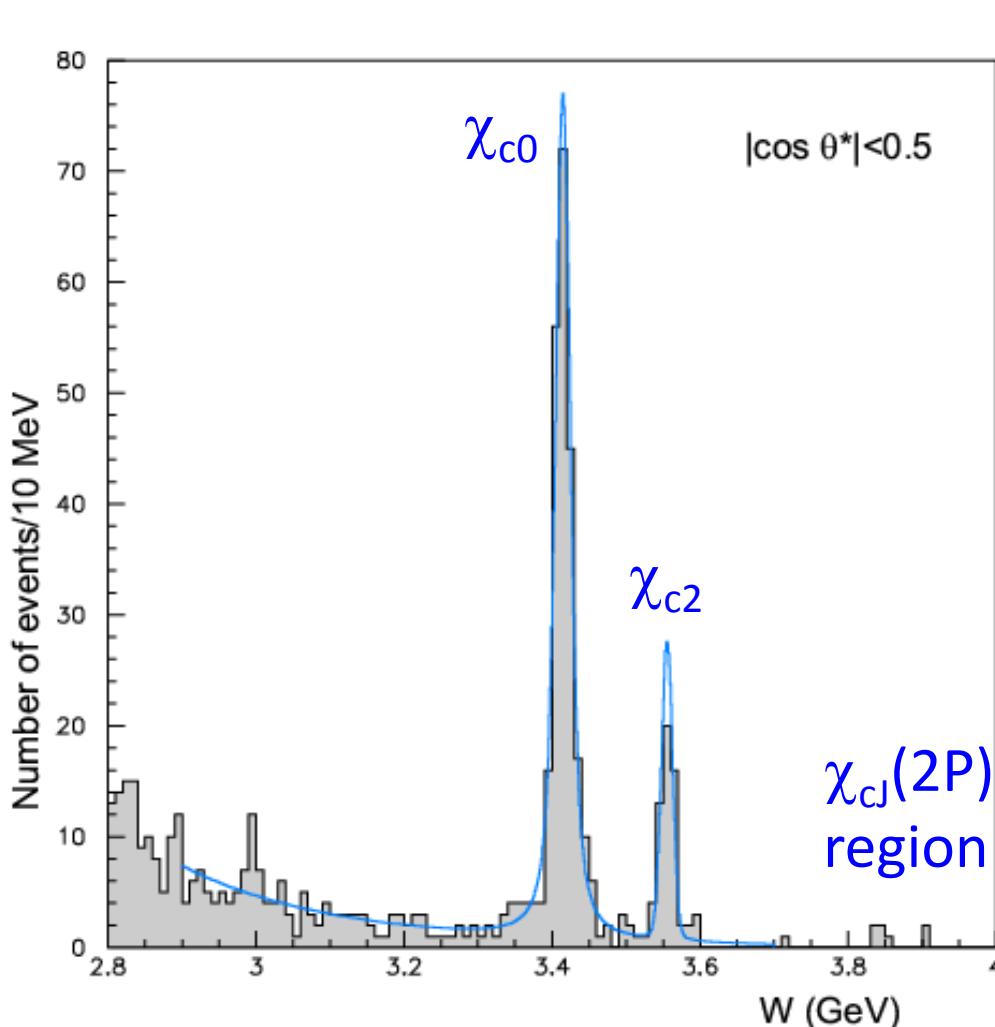
- Then fit  $d\sigma/d\Omega$  (typically 16 free parameters)



# Fit results for 13 assumptions

Assumption	No. of sol.	$\chi^2$	<i>ndf</i>
$f_0-f_0$	2	293.3, 293.9	214
$f_0-f_2$	4	320.9, 321.9, 324.5, 327.6	214
$f_0-f_4$	1	291.4	214
$f_2-f_0$	1	228.3	214
$f_2-f_2$	1	260.4	214
$f_2-f_4$	1	323.6, 306.7	214
$f_4-f_0$	1	411.6	214
$f_4-f_2$	2	468.6, 472.1	214
$f_4-f_4$	4	459.6, 464.1, 466.4, 467.5	214
Only- $f_0$	1	390.0	218
Only- $f_2$	1	323.6	218
Only- $f_4$	1	518.7	218
No resonances	1	659.32	222

# Charmonia $\chi_{c0}$ and $\chi_{c2}$



## Yield

	$N_{\chi_{c0}}$	$N_{\chi_{c2}}$	$-2 \ln \mathcal{L}/ndf$
Interference not included	$248.3^{+17.9}_{-17.2}$	$53.0^{+8.1}_{-7.4}$	57.34/73
Interference included	$266 \pm 53$	$53^{+14}_{-12}$	57.22/71

Interference between  $\chi_{c0}$  and continuum

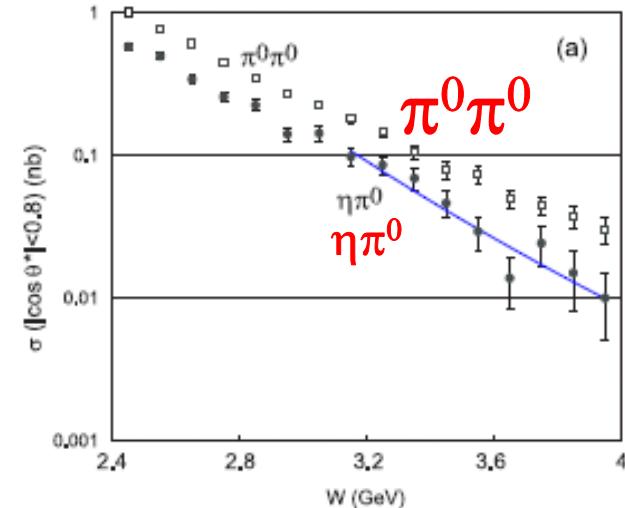
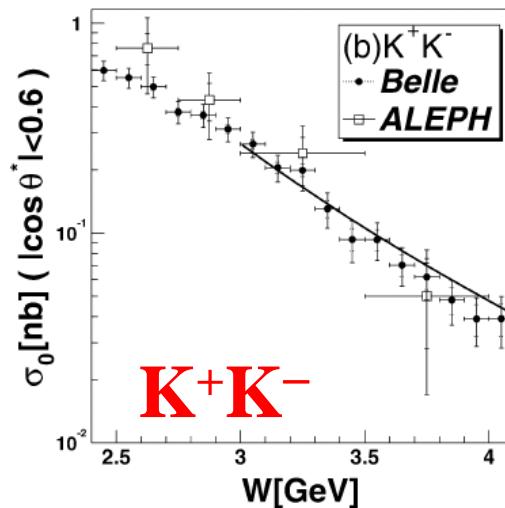
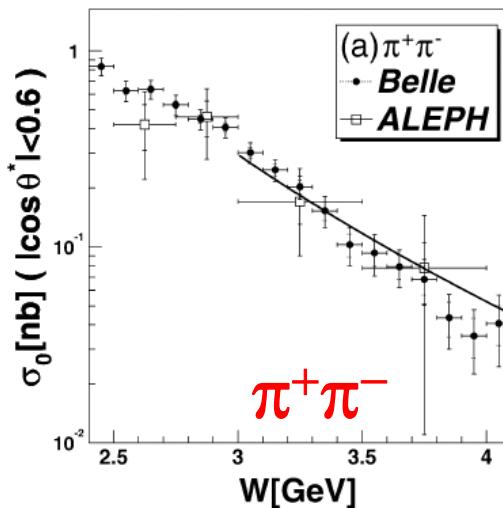
Product of  
two-photon decay width  
and  $B(K_S^0 K_S^0)$

Interference	$\Gamma_{\gamma\gamma} B(\chi_{c0})$ (eV)	$\Gamma_{\gamma\gamma} B(\chi_{c2})$ (eV)
not included	$8.09 \pm 0.58 \pm 0.83$	$0.268^{+0.041}_{-0.037} \pm 0.028$
included	$8.7 \pm 1.7 \pm 0.9$	$0.27^{+0.07}_{-0.06} \pm 0.03$
Belle 2007	$7.00 \pm 0.65 \pm 0.71$	$0.31 \pm 0.05 \pm 0.03$
PDG 2012	$7.3 \pm 0.5$	$0.297 \pm 0.026$



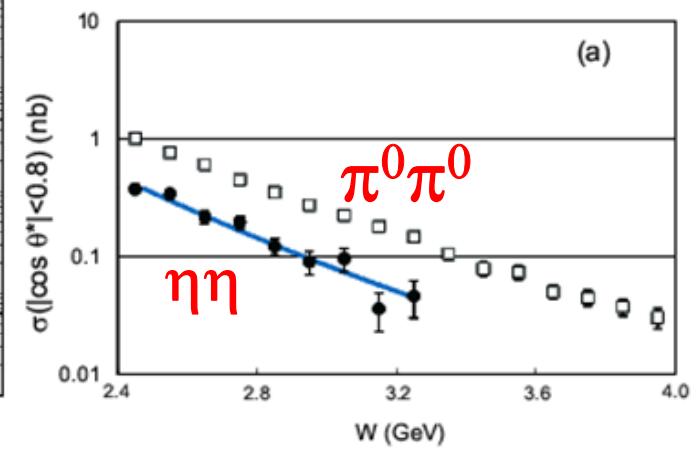
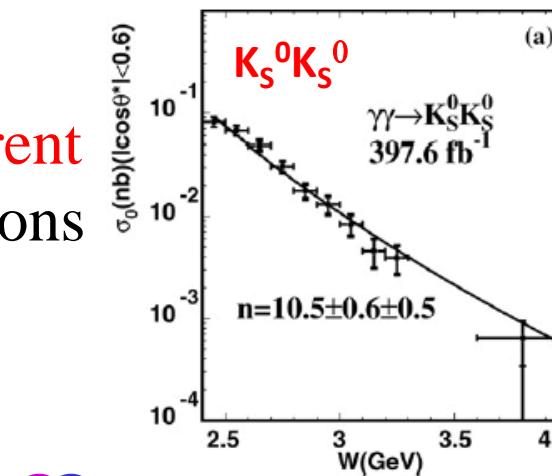
# W-dependences at high energies

Assume or expect  $\sigma(W) \sim W^{-n}$



Fitted and reproduced  
Slope parameter **n** different  
among the reactions

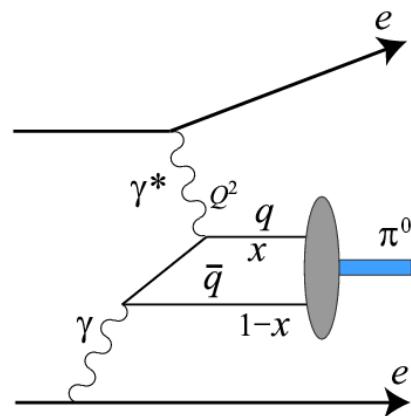
Charmonium contributions  
not included/removed



# $\pi^0$ Transition Form Factor

$$\gamma\gamma^*\rightarrow\pi^0$$

Coupling of neutral pion with two photons  
Good test for QCD at high  $Q^2$



Single-tag  $\pi^0$  production in two-photon process with a large- $Q^2$  and a small- $Q^2$  photon

Theoretically calculated from pion distribution amplitude and decay constant

$$F(Q^2) = \frac{\sqrt{2}f_\pi}{3} \int T_H(x, Q^2, \mu) \phi_\pi(x, \mu) dx$$

Measurement:

$$|F(Q^2)|^2 = |F(Q^2, 0)|^2 = (\text{d}\sigma/\text{d}Q^2)/(2A(Q^2)) \quad A(Q^2) \text{ is calculated by QED}$$

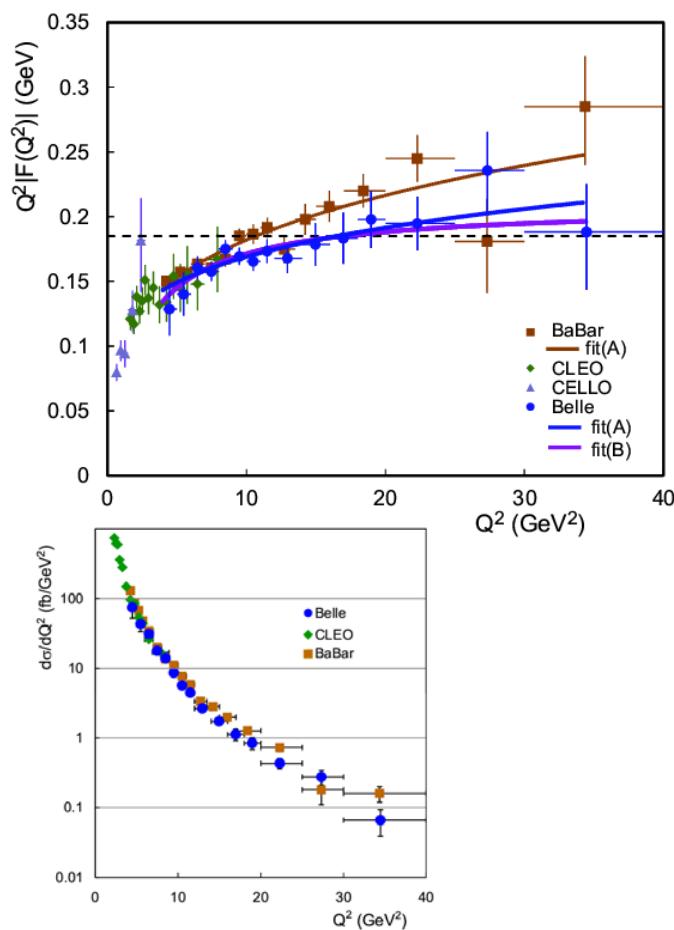
$$|F(0, 0)|^2 = 64\pi\Gamma_{\gamma\gamma}/\{(4\pi\alpha)^2 m_R^3\}$$

Detects  $e$  (tag side) and  $\pi^0$

$Q^2 = 2EE'(1 - \cos \theta)$  from energy and polar angle of the tagged electron



# Comparisons with Previous Measurements and Fits



No rapid growth above  $Q^2 > 9 \text{ GeV}^2$  is seen in Belle result.

~  $2.3\sigma$  difference between Belle and BaBar in  $9 - 20 \text{ GeV}^2$

## Fit A (suggested by BaBar)

$$Q^2 |F(Q^2)| = A (Q^2/10\text{GeV}^2)^\beta$$

BaBar: —

$$A = 0.182 \pm 0.002 (\pm 0.004) \text{ GeV}$$

$$\beta = 0.25 \pm 0.02$$

Belle: —

$$A = 0.169 \pm 0.006 \text{ GeV}$$

$$\beta = 0.18 \pm 0.05$$

$\chi^2/\text{ndf} = 6.90/13$  ~ $1.5\sigma$  difference from BaBar

## Fit B (with an asymptotic parameter)

$$Q^2 |F(Q^2)| = B Q^2 / (Q^2 + C)$$

Belle: —

$$B = 0.209 \pm 0.016 \text{ GeV}$$

$$C = 2.2 \pm 0.8 \text{ GeV}^2$$

$$\chi^2/\text{ndf} = 7.07/13$$

B is consistent with the QCD value (0.185 GeV)

