Quasiclassical approach to high-energy QED processes in the field of heavy atom.

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Outline

Introduction

QED precesses in the field of heavy atom

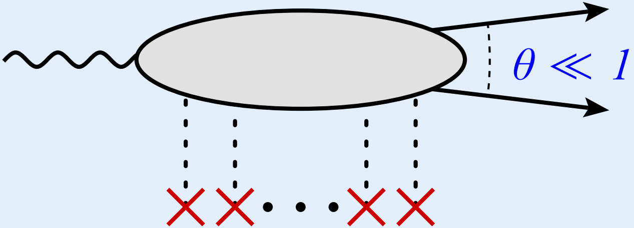
Elastic scattering	$e^{\pm}+Z \to e^{\pm}+Z$
Bremsstrahlung	$e^{\pm}+Z \to e^{\pm}+\gamma+Z$
Pair production	$\gamma+Z \to e^{+}+e^{-}+Z$
Double Bremsstrahlung	$e^{\pm}+Z \to e^{\pm}+\gamma+\gamma'+Z$
PP accompanied by BS	$\gamma+Z \to e^{+}+e^{-}+\gamma'+Z$
PP in peripheral heavy-ion collisions	$Z_1+Z_2 \to Z_1+Z_2+e^{+}+e^{-}$
Delbruck scattering	$\gamma+Z \to (e^{+}+e^{-})+Z \to \gamma+Z$
Photon splitting	$\gamma+Z \to (e^{+}+e^{-})+Z \to \gamma_1+\gamma_2+Z$

- Total cross sections
- Energy spectra
- Differential cross sections
- Polarization effects

Introduction

Typical experimental conditions

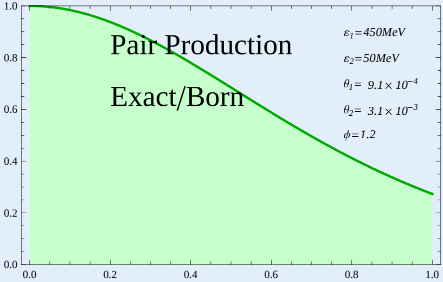
- High energy
- Small scattering angles
- High nuclear charge $\Longrightarrow$ parameter $Z\alpha$ is not small.

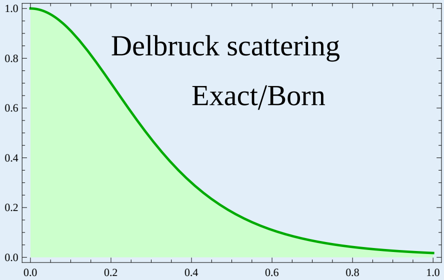


Introduction

Relative magnitude of Coulomb corrections

- Differential cross section: Coulomb corrections (CC) are large.





- Total cross section: Born term contains large logarithm $\Longrightarrow$ $CC \sim 10 \div 20\%$. Exclusion: Delbruck scattering ($CC \sim 100\%$, since there is no one-photon exchange).

Introduction

Perturbation theory in $Z\alpha$

Direct calculation even of the leading Coulomb correction is hardly doable even given a huge progress in multiloop calculations.

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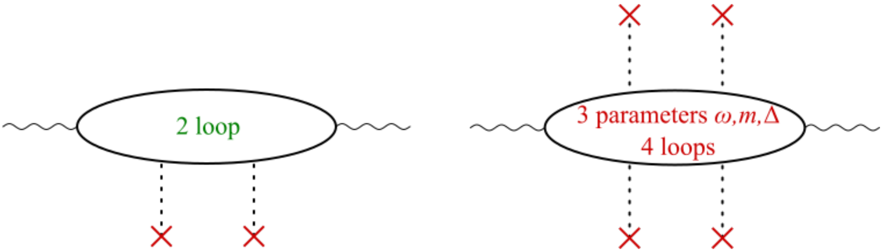
Example: Delbruck scattering



Introduction

Furry representation

Use exact wave functions and propagators (Green's functions) in the external field

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Coulomb Green's function

$$G(\boldsymbol{r}_2,\boldsymbol{r}_1|\varepsilon) = -\frac{i}{4\pi r_2 r_1 \kappa}\int_0^\infty ds \exp[2iZ\alpha s\lambda + i\kappa(r_2+r_1)\coth s]\; T$$

$$T = [1-(\gamma\cdot\boldsymbol{n}_2)(\gamma\cdot\boldsymbol{n}_1)][(\gamma^0\varepsilon+m)\frac{y}{2}\partial_y S_B - iZ\alpha\gamma^0\kappa\coth s\, S_B]$$
$$+[1+(\gamma\cdot\boldsymbol{n}_2)(\gamma\cdot\boldsymbol{n}_1)](\gamma^0\varepsilon+m)S_A + imZ\alpha\gamma^0\gamma\cdot(\boldsymbol{n}_2+\boldsymbol{n}_1)S_B$$
$$+\frac{i\kappa^2(r_2-r_1)}{2\sinh^2 s}\gamma\cdot(\boldsymbol{n}_2+\boldsymbol{n}_1)S_B - \kappa\coth s\;\gamma\cdot(\boldsymbol{n}_2-\boldsymbol{n}_1)S_A\,.$$

Too complicated for applications.

Quasiclassical approach

Small parameters in Coulomb problem

- $\frac{Ze^2}{\hbar v} = \frac{Z\alpha}{\beta}$ — nonrelativistic quantum corrections
- $\frac{Ze^2}{\hbar c} = Z\alpha$ — relativistic quantum corrections
- $\frac{Ze^2}{Lc} = \frac{Z\alpha}{l}$ — relativistic classical corrections
- $\frac{(Z\alpha)^2}{l}$ — relativistic quasiclassical corrections

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High-energy processes in the field of heavy atom

- $\frac{Z\alpha}{\beta} \gtrsim Z\alpha \sim 1$ — should be treated exactly.
- $L_{\text{хар}}\theta_{\text{хар}} \sim \hbar \quad \Longrightarrow \quad \frac{(Z\alpha)^2}{l} \ll 1$ — perturbation in $1/l$ is possible.

Furry-Sommerfeld-Maue wave functions

Furry (1934), Sommerfeld and Maue (1935)

Using partial wave expansion and neglecting terms of order $(Z\alpha)^2/l$ Furry obtained the quasiclasical wave function of Dirac equation in the Coulomb field $V(r) = -Z\alpha/r$:

$$\psi^{(+)} = e^{\pi Z\alpha/2}\Gamma(1-iZ\alpha)\,e^{i\mathbf{kr}}\left(1-i\frac{\alpha\nabla}{2\varepsilon}\right){}_1F_1(iZ\alpha,1,i[kr-\mathbf{kr}])\,u$$

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Useful tool

Pair production, Bremsstrahlung — Bethe and Maximon (1954), Davies et al. (1954)

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Quasiclassical approach:

- How to derive analog of the FSM functions for arbitrary localized potential?
- How to derive quasiclassical Green's functions?
- How to go beyong leading quasiclassical approximation?
- Applications

Eikonal vs Quasiclassical

Klein-Fock-Gordon equation

$$k^2\psi = \left[\mathbf{p}^2 + 2\varepsilon V - V^2\right]\psi,$$

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Substitution

$$\psi = e^{ikz} F$$

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$$\partial_z f_n = \frac{i}{2k}\left(V^2 + F_0{}^{-1}\Delta F_0\right) f_{n-1}$$

Substitution

$$\psi = e^{ikz} F$$

$$F = F_0\left(1 + f_1 + f_2 + \ldots\right)$$

$$F_0 = \exp\left[-\frac{i}{\beta}\int dz V(z,\rho)\right]$$

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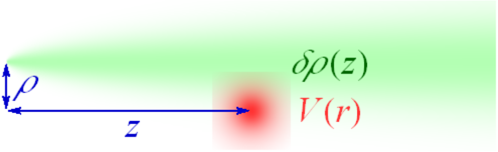
Applicability condition



$$\frac{1}{k}\int dz \left[\int dz \nabla_\perp V(z,\rho)\right]^2 \ll 1$$

$$|z| \ll k\rho^2$$

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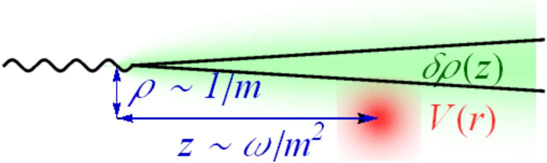
Eikonal approximation is not applicable!



$$|z| \sim k\rho^2$$

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Problem: transverse gradients.

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$$2ik\partial_z F = 2\varepsilon \widetilde{V} F - \widetilde{V^2} F$$

Substitution

$$\psi = e^{ikz} F$$

$$\psi = e^{ikz}\exp\left[iz\Delta_\perp/k\right] F$$

$$\widetilde{V} = e^{-iz\Delta_\perp/k} V e^{iz\Delta_\perp/k}$$

$$\approx V - iz\left[\Delta_\perp, V\right]/k$$

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FSM wave function analog (Lee, Milstein and Strakhovenko 2000)

$$\psi = e^{ikz}\exp\left[iz\Delta_\perp/k\right] F_0(z,\rho) = e^{ikz}\exp\left[iz\Delta_\perp/k\right]\exp\left[-\frac{i}{\beta}\int^z d\zeta V(\zeta,\rho)\right]$$

$$= e^{ikz}\int \frac{d^2\mathbf{q}}{i\pi} e^{iq^2} F_0\left(z, \rho + 2\mathbf{q}\sqrt{z/k}\right)$$

Green's function

Quasiclassical Green's function of KG equation

$$D_{\text{КФГ}}(\mathbf{r}_2,\mathbf{r}_1|\,\varepsilon) = \frac{ie^{ikr}}{4\pi^2 r}\int d^2\mathbf{q}\exp\left[iq^2 - \frac{i}{\beta}r\int_0^1 dxV(\mathbf{R}_x)\right]$$

$$\mathbf{R}_x = \mathbf{r}_1 + x\mathbf{r} + \mathbf{q}\sqrt{2x\bar{x}r/k} \Longleftarrow \text{quantum fluctuations}$$

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$$\times\left\{1 - \frac{1}{2\beta k}\left[2\int_0^1 dx V(\mathbf{R}_x) - V(\mathbf{r}_1) - V(\mathbf{r}_2)\right]\right.$$

$$\left. + \frac{ir^3}{k}\int_0^1 dx\int_0^x dy\left[\sqrt{x\bar{x}y\bar{y}} - \bar{x}y\right](\nabla_\perp V(\mathbf{R}_x))(\nabla_\perp V(\mathbf{R}_y))\right\}$$

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Coulomb potential:

$$D_{\text{K}\Phi\Gamma}(\mathbf{r}_2,\mathbf{r}_1|\varepsilon) = \frac{ik\mathrm{e}^{ikr}}{8\pi^2 r_1 r_2}\int d\mathbf{q}\exp\left[i\frac{krq^2}{2r_1 r_2}\right]\left(\frac{2\sqrt{r_1 r_2}}{|\mathbf{q}-\rho|}\right)^{\frac{2iZ\alpha}{\beta}}\left(1+i\frac{\pi(Z\alpha)^2}{2k|\mathbf{q}-\rho|}\right)$$

Applications

e^+e^- pair photoproduction.

Total cross section

Bethe-Maximon asymptotics for Coulomb corrections

$$\sigma_C^{(0)} = -\frac{28\alpha (Z\alpha)^2}{9m^2} f(Z\alpha)\,,$$

$$f(Z\alpha) = \mathrm{Re}\left[\psi(1 + iZ\alpha) + C\right]\,,$$

is valid formally at $\omega \gg m$.

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Lee, Milstein and Strakhovenko (2004)

Using derived quasiclassical Green's function with the first QC correction we have obtained

$$\sigma_C^{(1)} = \frac{\alpha (Z\alpha)^2 \pi^4}{2m\omega} \mathrm{Im}\, g(Z\alpha).$$

$$g(Z\alpha) = Z\alpha \frac{\Gamma(1-iZ\alpha)\Gamma(1/2+iZ\alpha)}{\Gamma(1+iZ\alpha)\Gamma(1/2-iZ\alpha)}$$

Huge coefficient π^4!

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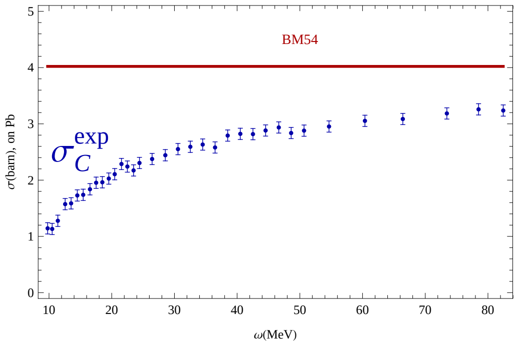

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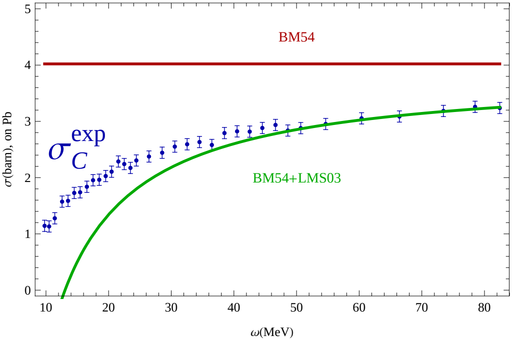

QC correction taken into account

Bremsstrahlung

Differential cross section

Momentum transfer distribution

Thanks to factorization, $d\sigma^{BS} = d\sigma^{el} dW$ at $\Delta_\perp \gg \Delta_{\min} \sim m^2/\varepsilon$

$$\frac{d\sigma_C^{BS}}{d^2\Delta_\perp} \propto \Delta_\perp^2 \left[|A(\Delta_\perp)|^2 - |A_B(\Delta_\perp)|^2 \right],$$

where $A(\Delta_\perp) = \int d\rho e^{-i\Delta_\perp \rho} \left(1 - e^{i\chi(\rho)}\right)$— eikonal amplitude,
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Coulomb potential

$$A(\Delta_\perp) = A_B(\Delta_\perp) \frac{\Gamma(1 - iZ\alpha)}{\Gamma(1 + iZ\alpha)} \left(\frac{4}{\Delta_\perp^2}\right)^{-iZ\alpha} \implies |A(\Delta_\perp)| = |A_B(\Delta_\perp)|$$

Coulomb corrections come from the region $\Delta_\perp \sim \Delta_{\min}$.

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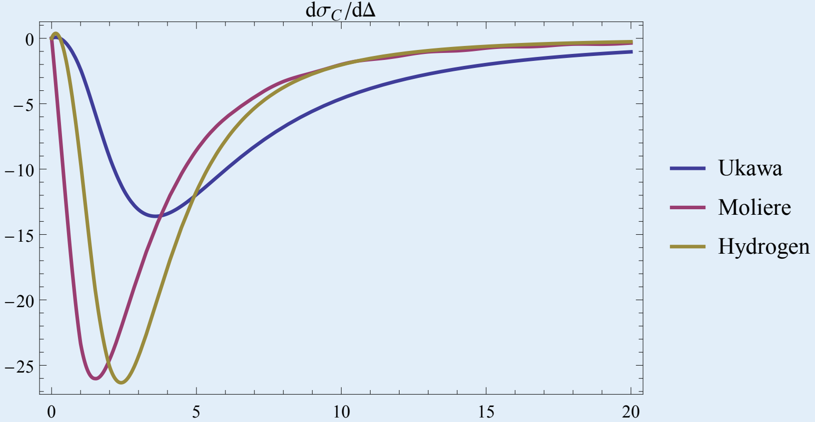
Controversy about influence of screening ($r_{scr}^{-1} \gg \Delta_{min}$)

(BM 1954): $\sigma_C^{BS} = 0$ since reg. $\Delta_\perp \sim \Delta_{\min}$ is suppressed
(Olsen 1955): cross-channel of PP $\Longrightarrow \sigma_C^{BS} = \sigma_C^{PP} \neq 0$
(Olsen 2003): $d\sigma_C^{BS}$ independent of screening

Bremsstrahlung

Differential cross section



$d\sigma_C^{BS}/d^2\Delta_\perp$ essentially depends on screening in the region giving the main contribution to σ_C^{BS}, but the integral $\sigma_C^{BS} \propto (Z\alpha)^2 f(Z\alpha)$ is universal!

Incomplete list of results of QC approach

Results (Exact in $Z\alpha$!)

$e^{\pm}+Z\to e^{\pm}+\gamma+Z$: Differential cross section (next talk), spectrum (Lee, Milstein, Strakhovenko and Schwarz 2005) with the account of the first QC correction and screening.

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Earlier results

$\gamma+Z\to(e^{+}+e^{-})+Z\to\gamma_1+\gamma_2+Z$: Helicity amplitudes, differential cross section (Lee, Milstein and Strakhovenko 1998, 1997)

Summary

- Quasiclassical approach provides effective reliable framework for investigation of the high-energy QED processes in the field of heavy atom.
- Quasiclassical Green's functions and wave functions in arbitrary localized potential are derived with the account of the first QC correction (with typical relative magnitude θ or $1/\gamma$).
- Applications include all basic high-energy QED processes in the field of heavy atom, and more.

Thank you!

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Example of results

Process $\gamma + Z \to e^+ + e^- + \gamma' + Z$, helicity amplitudes

$$M = \frac{32\eta}{\omega_1 \omega_2 Q^2} \int d\boldsymbol{T} \left(\frac{|\boldsymbol{T} + \boldsymbol{Q}_\perp|}{|\boldsymbol{T} - \boldsymbol{Q}_\perp|} \right)^{2iZ\alpha} \boldsymbol{\chi} \cdot \boldsymbol{\nabla}_{\boldsymbol{T}} \left[F(\boldsymbol{p}, \boldsymbol{q}, \boldsymbol{T}) - F(\boldsymbol{q}, \boldsymbol{p}, -\boldsymbol{T}) \right],$$

$$F_{++++} = \sqrt{2} m (\varepsilon_p + \omega_2) \omega_1 (\boldsymbol{e}_- \cdot \boldsymbol{A})$$
$$F_{+++-} = -(\varepsilon_p + \omega_2)^2 \boldsymbol{e}_+ \cdot (\boldsymbol{T} - \boldsymbol{\delta}_q)(\boldsymbol{e}_- \cdot \boldsymbol{A}),$$
$$\vdots$$