

Effects of strong electromagnetic field
in ultra-relativistic
heavy-ion collisions

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Outline:

1. Introduction
 2. Strong-field effects in the e^+e^- pair production
 3. Strong-field effects in the $\mu^+\mu^-$ pair production
 4. Large contribution of the virtual Delbrück scattering to nuclear bremsstrahlung
 5. Production of bound-free e^+e^- pairs at LHC
- Concluding remarks

Problems with strong-field (SF) effects were considered in many old books and recent reviews, for example:

Heitler “The quantum theory of radiation,” 1954;

Greiner, Müller, Rafelsky “QED of strong fields,” 1985

Baur, Hencken, Trautmann. Phys. Rep. 453, 1 (2007)

Baltz et al. Phys. Rep. 458, 1 (2008)

This report is based mainly on papers written in collaboration with colleagues from the **Basel, Dresden, Jena, Heidelberg, Leipzig and Novosibirsk Universities**:

Ginzburg, Jenschura, Karshenboim, Krauss, Serbo, Soff. *Phys. Rev. C* **58**, 3565 (1998);

Ivanov, Schiller, Serbo. *Phys. Lett. B* **454**, 155 (1999);

Lee, Milstein, Serbo. *Phys. Rev. A* **65**, 022102 (2002);

Hencken, Kuraev, Serbo. *Phys. Rev. C* **75**, 034903 (2007)

Jenschura, Hencken, Serbo. *Eur. Phys. J. C* **58**, 281 (2008);

Jenschura, Serbo. *Eur. Phys. J. C* **64**, 309 (2009);

Artemyev, Jenschura, Serbo, Surzhykov. *Eur. Phys. J. C* **72**, 1935 (2012);

Artemyev, Serbo, Surzhykov. *Eur. Phys. J. C* **74**, 2829 (2014)

1. Introduction

For the RHIC and LHC colliders, the charge numbers of nuclei $Z_1 = Z_2 \equiv Z$ and their Lorentz factors $\gamma_1 = \gamma_2 \equiv \gamma$ are given as follows:

Collider	Z	γ
RHIC, Au-Au	79	108
LHC, Pb-Pb	82	3000

Only a few EM processes are related to Fundamental Physics, but some of EM processes are of great importance mainly for two reasons: they are **dangerous** or they are **useful**.

Two examples:

1) The e^+e^- pair production. The number of the produced electrons is so huge that some of them can be captured by nuclei, that immediately leads to loss of these nuclei from the beam. Thus, this very process is determined mainly **the life time of the beam and a possible luminosity of a machine.**

2) Coherent bremsstrahlung, not ordinary bremsstrahlung

$$Z_1 Z_2 \rightarrow Z_1 Z_2 \gamma$$

but coherent one! The number of the produced photons at the RHIC is so huge in the region of the infrared light, that this process can be used for **monitoring beam collisions**:

*R. Engel, A. Schiller, V.G. Serbo. A new possibility to monitor collisions of relativistic heavy ions at LHC and RHIC, Particle Accelerators **56**, 1 (1996)*

D. Trbojevic, D. Gasner, W. MacKay, G. McIntyre, S. Peggs, V. Serbo, G. Kotkin. Experimental set-up to measure coherent bremsstrahlung and beam profiles in RHIC. 8th European Particle Accelerator Conference (EPAC 2002, 3–7 June, 2002, Paris) p. 1986

It means that various EM processes

have to be estimated

(their cross sections and distributions)

not to miss

something interesting or dangerous.

How **strong** is nuclear field?

The typical electric field of nucleus is of the order of

$$\mathcal{E} \sim \frac{Ze}{\rho^2} \gamma = \gamma Z\alpha \mathcal{E}_{\text{Schwinger}} \text{ at } \rho = \frac{\hbar}{m_e c},$$

$$\mathcal{E}_{\text{Schwinger}} = \frac{m_e^2 c^3}{e\hbar} = 1.3 \cdot 10^{16} \frac{\text{V}}{\text{cm}},$$

therefore,

$$\frac{\mathcal{E}}{\mathcal{E}_{\text{Schwinger}}} \sim \mathbf{60} \text{ for RHIC and } \sim \mathbf{1800} \text{ for LHC,}$$

but interaction time is **very short**.

As a result, one can use **Perturbation Theory**, but the perturbation parameter $Z\alpha \approx \mathbf{0.6}$ for Au-Au and Pb-Pb collisions.

2. Strong-field effects in the e^+e^- pair production

The cross section of **one pair production in the Born approximation** (described by Feynman diagram of Fig. 1)

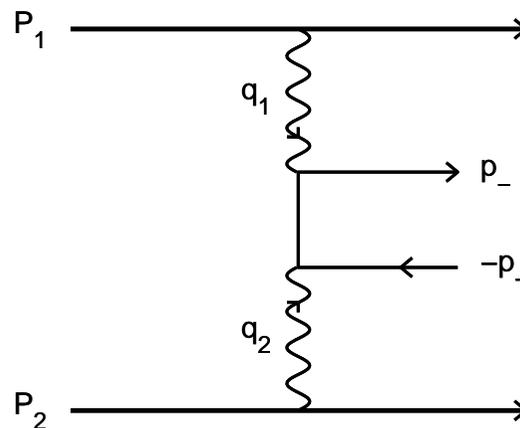


Fig. 1

with **two photon production** was obtained many years ago by

Landau, Lifshitz (1934) and *Racah (1937)*:

$$\sigma_{\text{Born}} = \sigma_0 \left[L^3 - 2.198 L^2 + 3.821 L - 1.632 \right],$$

where

$$\sigma_0 = \frac{28}{27\pi} \frac{(Z_1\alpha Z_2\alpha)^2}{m_e^2}, \quad \alpha = \frac{1}{137}, \quad L = \ln(\gamma_1\gamma_2) \gtrsim 10,$$

m_e is the electron mass and $c = 1, \hbar = 1$.

Since **the parameter Z_α is not small**

the **whole** series in Z_α **has to be summed**

to obtain the cross section with sufficient accuracy.

Fortunately, there is an **important small parameter**

$$\frac{1}{L} < 0.11, \quad L = \ln(\gamma^2),$$

and therefore, in some (**but not in all!**) cases it is sufficient to calculate the corrections

in **the leading logarithmic approximation (LLA) only.**

Note!

In the literature, there were **a lot of controversial and incorrect** statements in papers devoted to this subject.

For example, three groups had published papers with **the wrong statement that $Z\alpha$ corrections are absent in this process:**

B. Segev, J.C. Wells, Phys. Rev. A 57 (1998) 1849; physicsr9805013;
A.J. Baltz, L. McLerran, Phys. Rev. C 58 (1998) 1679;
U. Eichmann, J. Reinhardt, W. Greiner, nucl-thr9806031.

This mistake was criticize by

D.Yu. Ivanov, A. Schiller, V.G. Serbo. Phys. Lett. B 454 (1999) 155;
R.N. Lee, A.I. Milstein. Phys. Rev. A 61 (2000) 032103;
Phys. Rev. A 64 (2001) 032106 (2001).

Further **critical remarks and references** can be found in

Lee, Milstein, Serbo. Phys. Rev. A **65**, 022102 (2002);

Aste, Baur, Hencken, Trautmann, Scharf. Eur. Phys. J. C23 (2002)
545;

Jentschura, Hencken, Serbo. Eur. Phys. J. C58 (2008) 281;

Jentschura, Serbo. Eur. Phys. J. C 64 (2009) 309;

M. Klusek-Gawenda, A. Szczurek. Phys.Rev.C82 (2010) 014904.

The exact cross section for **one pair production** σ_1 can be written in the form

$$\sigma_1 = \sigma_{\text{Born}} + \sigma_{\text{Coul}} + \sigma_{\text{unit}},$$

where **two different types of SF-corrections** have been distinguished.

2.1. Results for the SF-corrections

The **Coulomb** corrections σ_{Coul} correspond to **multi-photon exchanges** of the produced e^\pm with the nuclei:

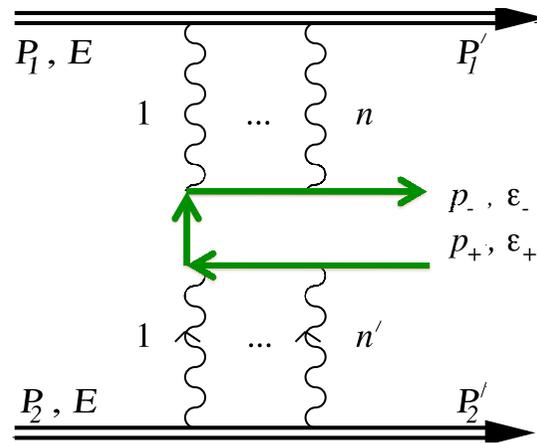


Fig. 2

$$\sigma_{\text{Coul}} = -A(Z\alpha) [L^2 - B(Z\alpha) L] \sigma_0,$$

where the leading coefficient

$$A(Z\alpha) = 6f(Z\alpha) = 6(Z\alpha)^2 \sum_{n=1}^{\infty} \frac{1}{n(n^2 + (Z\alpha)^2)} \approx 1.9$$

was calculated by

D.Yu. Ivanov, A. Schiller, V.G. Serbo. Phys. Lett. B 454 (1999) 155

and next-to-leading coefficient $B(Z\alpha) \approx 5.5$ was calculated by

R.N. Lee, A.I. Milstein. ЖЭТФ 136 (2009) 1121.

It was also shown by ISS that **the Coulomb corrections disappear** for **large transverse momenta of the produced leptons**,

at $p_{\pm\perp} \gg m_e$.

The **unitarity** corrections σ_{unit} correspond to the exchange of the virtual light-by-light blocks between the nuclei (Fig. 3)

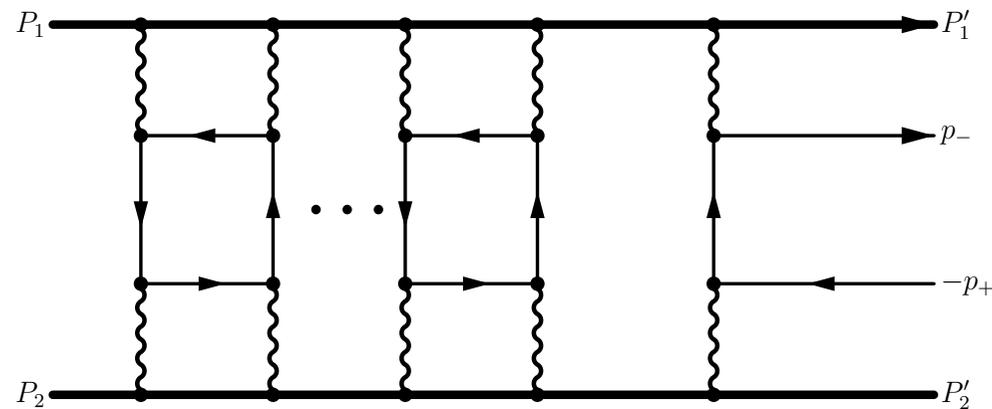


Fig. 3

They were calculated by [R.N. Lee, A.I. Milstein, V.G. Serbo. Phys. Rev. A 65 \(2002\) 022102](#)

and updated by

[U.D. Jentschura, K. Hencken, V.G. Serbo. EPJ C58 \(2008\) 281.](#)

It was found that **the Coulomb corrections are about 10 %** while **the unitarity corrections are about two times smaller**:

Coulomb and unitarity corrections to the e^+e^- pair production

Collider	$\frac{\sigma_{\text{Coul}}}{\sigma_{\text{Born}}}$	$\frac{\sigma_{\text{unit}}}{\sigma_{\text{Born}}}$	$\frac{\sigma_{\text{Coul}}}{\sigma_{\text{Born}}}$ [Baltz]
RHIC, Au-Au	-10%	-5.0%	-17%
LHC, Pb-Pb	-9.4%	-4.0%	-11%

In the last column is shown the result of [A. Baltz. Phys.Rev. C71 \(2005\) 024901; Erratum-ibid. C71 \(2005\) 039901](#) obtained by numerical calculations using formula for the cross section resulting from “exact solution of the semiclassical Dirac equations”. In fact, this formula allows to calculate the Coulomb correction in **the LLA only**, which is insufficient in this case.

3. Strong-field effects in the $\mu^+\mu^-$ pair production

Motivation: **muon pair production may be easier for an experimental observation.**

Moreover, this process can be used for the luminosity monitoring.

This process was considered in detail by

Hencken, Kuraev, Serbo. Phys. Rev. C 75 (2007) 034903;

Jetschura, Hencken, Serbo. EPJ C58 (2008) 281;

Jentschura, Serbo. Eur. Phys. J. C 64, 309 (2009);

M. Klusek-Gawenda, A. Szczurek Phys.Rev.C82 (2010) 014904.

It was found out that:

1. **The Coulomb** corrections are **small**. This result **justifies using the Born approximation for numerical simulations** of the discussed process at RHIC and LHC.
2. **Unitarity** corrections are **large**. **The exclusive** cross section **differs** considerable from its Born value, but an experimental observation is difficult.
3. **The inclusive** cross section **coincides** with the Born cross section.

Born cross section for one $\mu^+\mu^-$ pair production

Let us consider the production of **one $\mu^+\mu^-$ pair**

$$Z_1 + Z_2 \rightarrow Z_1 + Z_2 + \mu^+\mu^-,$$

using EPA or Weizsäcker-Williams approximation,



Carl Fridrich von Weizsäcker

but **taking into account** nuclear electromagnetic **form factors** (Fig. 5):

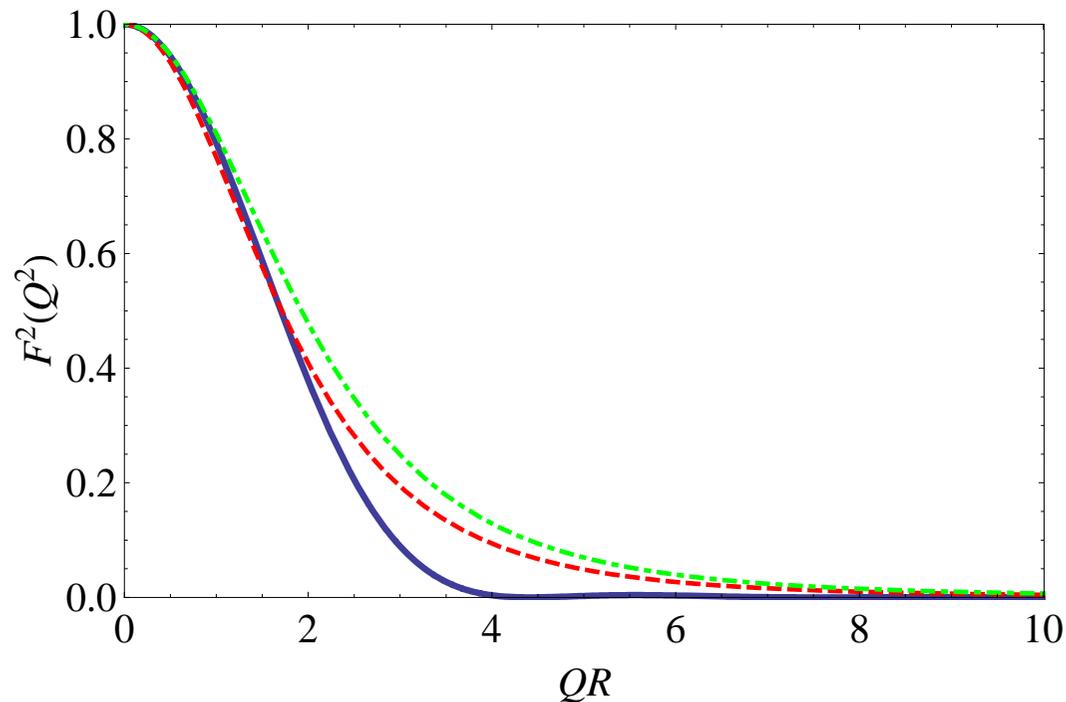


Fig. 5. Realistic (solid line) and simplified (dashed and dot-dashed lines) form factors vs. QR for Au; here R is the radius of nucleus

The Born differential cross section $d\sigma_B$ for the considered process is related to the cross section $\sigma_{\gamma\gamma}$ for **the real $\gamma\gamma \rightarrow \mu^+\mu^-$ process** by the equation

$$d\sigma_B = dn_1 dn_2 d\sigma_{\gamma\gamma},$$

where dn_i is the number of **equivalent photons**.

As a result, the cross section for the case of **the realistic nuclear form factor** reads:

$$\sigma_B = 0.21 \text{ barn for RHIC and } 2.5 \text{ barn for LHC.}$$

The accuracy of this calculation is of the order of few percents.

The Coulomb correction corresponds to the Feynman diagram of Fig. 6 with **a multi-photon exchange**.

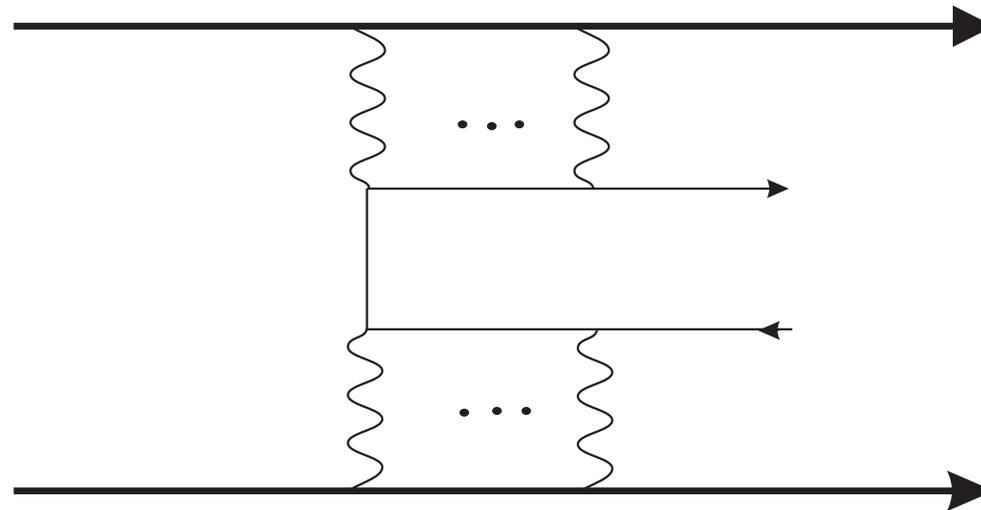


Fig. 6

Estimation: Due to the **restriction of transverse momenta** of additional exchange photons on the level of $1/R$ (**nuclear form factor!**), the effective parameter of the perturbation series is not $(Z\alpha)^2$, **the real suppression parameter** is of the order of

$$\eta_2 = \frac{(Z\alpha)^2}{(R\mu)^2 L}, \quad L = \ln(\gamma^2), \quad \frac{1}{R} \approx 30 \text{ MeV},$$

which corresponds to **a Coulomb correction of the order of a percent.**

Our calculation shows that **Coulomb corrections** to the $\mu^+\mu^-$ pair production is **small**:

Coulomb corrections to the $\mu^+\mu^-$ pair production

Collider	$\frac{\sigma_{\text{Coul}}}{\sigma_{\text{Born}}}$	$\frac{\sigma_{\text{Coul}}}{\sigma_{\text{Born}}}$ [Baltz]
RHIC, Au-Au	-3.7%	-22%
LHC, Pb-Pb	-1.3%	-14%

In the last column is shown the recent result of [A. Baltz. Phys. Rev. C80 \(2009\) 034901](#). In fact, this calculations **do not** take into account the nuclear form factors **properly**. Their trend contradicts the physical requirement that Coulomb corrections should vanish for an infinite mass of the produced lepton pair, not grow with the lepton mass.

The unitarity correction σ_{unit} to **one muon pair production** is described by the exchange of blocks, corresponding to light-by-light scattering via a **virtual electron loop**, between the nuclei (Fig. 7).

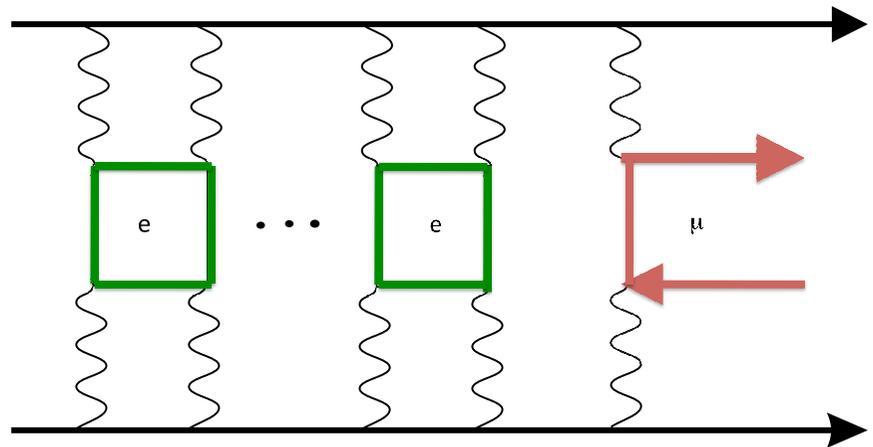


Fig. 7

The unitarity corrections are large because there is a **logarithmic enhancement from the region of small impact parameters**

$$2R < \rho < 1/m_e.$$

However, the experimental study of the **exclusive** muon pair production seems to be **a very difficult task**.

Indeed, this process requires that the muon pair should be registered **without any electron–positron pair production**, including e^\pm emitted at **very small angles**.

Otherwise, the corresponding inclusive cross section will be **close to the Born cross section** (for detail see

Hencken, Kuraev, Serbo. Phys. Rev. C 75 (2007) 034903).

**4. Large contribution
of the virtual Delbrück scattering
into nuclear bremsstrahlung**

Ordinary nuclear bremsstrahlung

The ordinary nuclear bremsstrahlung without excitation of the final nuclei is given by Feynman diagrams of Fig. 8

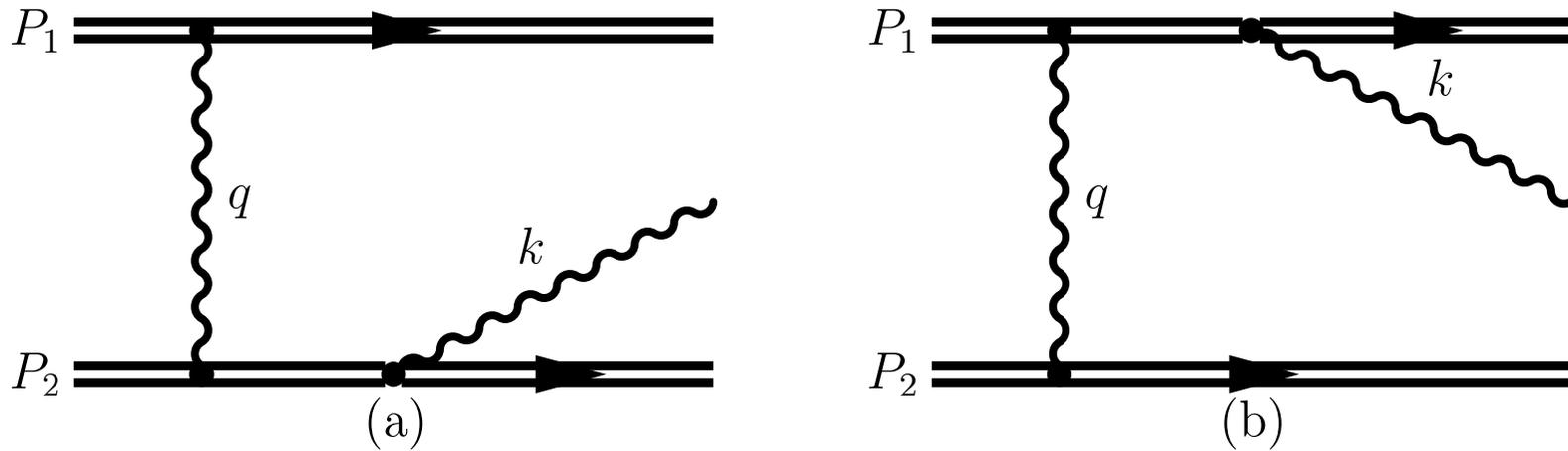


Fig. 8

and was known in detail many years ago
Bertulany, Baur Phys. Rep. 163, 299 (1988)

It can be described as **the Compton scattering** of the equivalent photon off opposite nucleus:

$$d\sigma_{\text{br}} = d\sigma_{\text{br}}^a + d\sigma_{\text{br}}^b,$$

and

$$d\sigma_{\text{br}}^a = dn_1 d\sigma_{\text{C}}(\omega, E_\gamma, E_2, Z_2).$$

Here, dn_1 is the number of equivalent photons emitted by nucleus 1 and $d\sigma_{\text{C}}(\omega, E_\gamma, E_2, Z_2)$ is the differential cross section for the Compton scattering off nucleus Z_2 .

Let us consider emission of photons not via the virtual Compton subprocess, but via another one –

the virtual Delbrück scattering subprocess (Fig. 9)

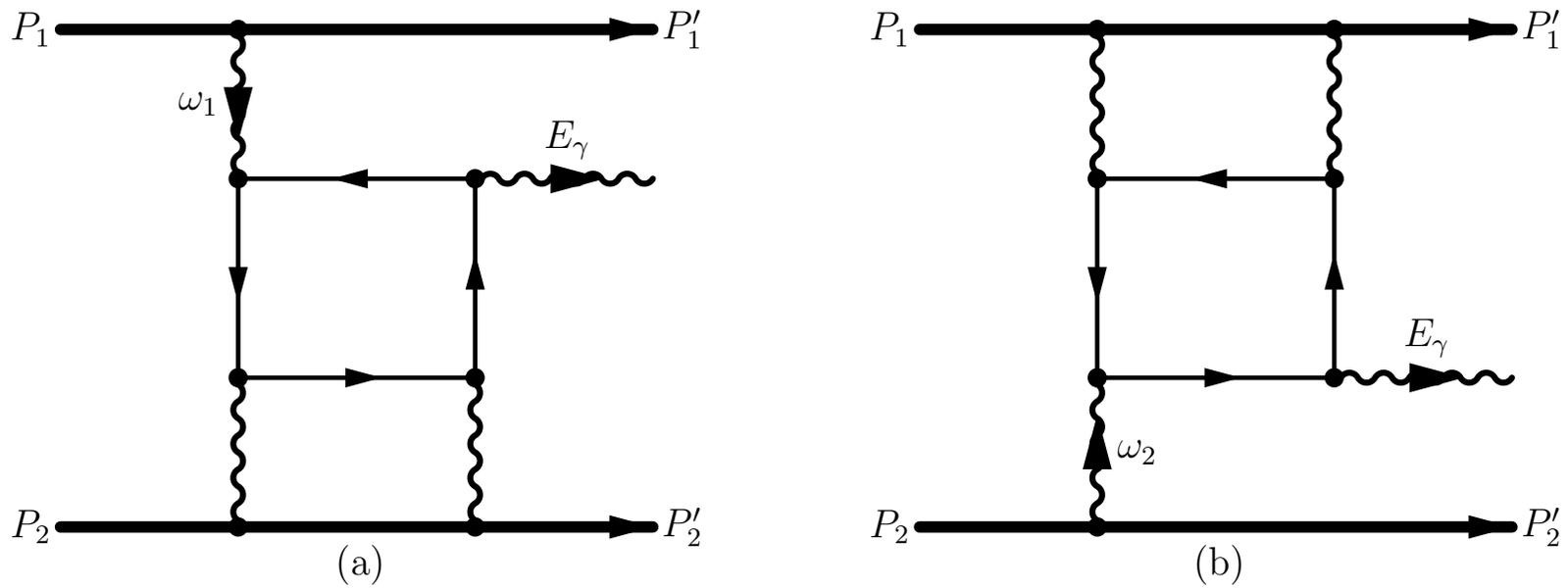


Fig. 9

First note: *Baur, Bertulany* *Z. f. Phys.* **A 330**, 77 (1988)

At first sight, this is a process of **a very small cross section** since

$$\sigma \propto \alpha^7.$$

But at second sight, we should **add a very large factor**

$$Z^6 \sim 10^{11}$$

and take into account that the cross section **scale** is

$$1/m_e^2.$$

And the last, but not the least, we found that this cross section has **an additional logarithmic enhancement** of the order of

$$L^2 \gtrsim 100, \quad L = \ln(\gamma^2).$$

Thus, the estimate is

$$\sigma \sim \frac{(Z\alpha)^6 \alpha}{m_e^2} L^2.$$

Our analytical result

Ginzburg, Jentschura, Serbo, *Phys. Lett. B* **658**, 125 (2008);

Ginzburg, Jentschura, Serbo, *Eur. Phys. J. C* **54**, 267 (2008)

$$\sigma = C \frac{(Z\alpha)^6 \alpha}{m_e^2} L^2$$

with

$$C \approx 0.4.$$

This cross sections is **considerably larger** than that for ordinary nuclear bremsstrahlung in the photon energy range:

$$m_e \ll E_\gamma \ll m_e \gamma.$$

Thus, the discussed cross section for Au-Au collisions at the RHIC collider is

$$\sigma = 14 \text{ barn}$$

and for Pb-Pb collisions at the LHC collider is

$$\sigma = 50 \text{ barn}.$$

That is quite a serious number!

Note for comparison, that the last cross section is **6 times larger than for the total hadronic/nuclear cross section in Pb–Pb collisions**, which is roughly **8 barn**.

5. Production of bound-free e^+e^- pair at LHC

This part of the report based on the recent papers:

A. N. Artemyev, U. D. Jentschura, V. G. Serbo, A. Surzhykov
“Bound-free pair production in ultra-relativistic ion collisions at the LHC collider: Analytic approach to the total and differential cross sections”

European Physical Journal C 72 (2012) 1935

A. N. Artemyev, V. G. Serbo, A. Surzhykov
”Double lepton pair production with electron capture in relativistic heavy-ion collisions”

European Physical Journal C 74 (2014) 2829

5.1. Introduction

January 2010 — TRENTO (Italy) and ALICE (CERN)

Reiner Schicker

Summer 2011 — Physikalisches Institut der Universität Heidelberg

Group of **Andrey Surzhykov**,

Questions from **Reiner Schicker**

The Landau-Lifshitz process

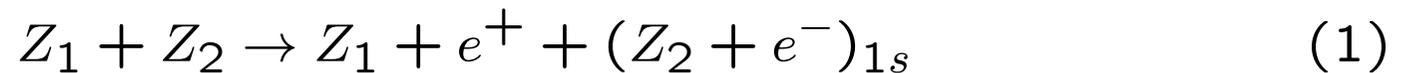
difficult for observation

has no clear trigger



Rainer Schicker

A process with **an electron capture** (on the K -shell, for definiteness)

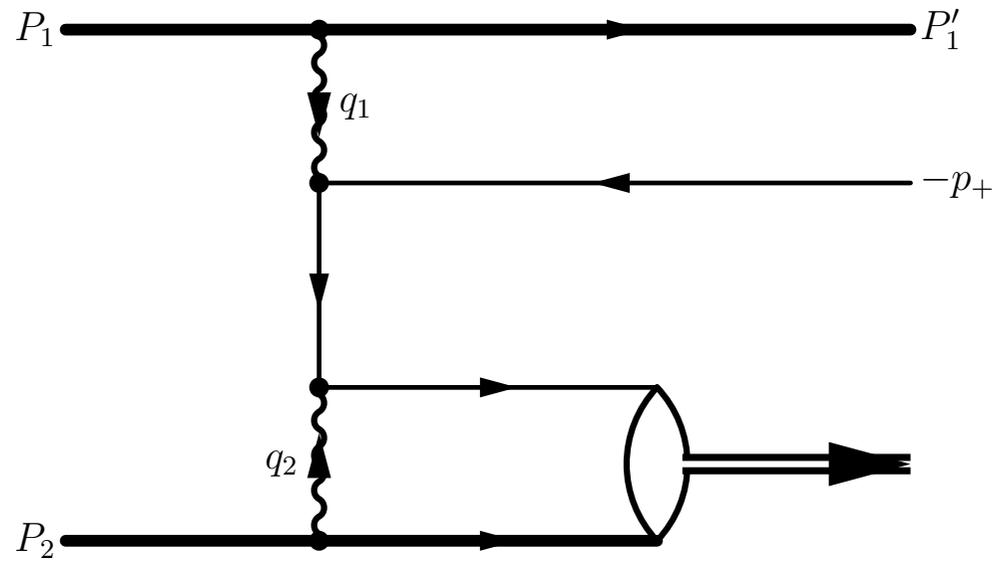


has considerable smaller cross section ~ 100 barn, but it is very important — see reviews and discussions in:

G. Baur et al., Phys. Rep. 364, 359 (2002);

J. M. Jowett, R. Bruce, S. Gilardoni, Proc. of the Particle Accelerator Conf. 2005, Knoxville p. 1306 (2005);

R. Bruce, D. Bocian, S. Gilardoni, J. M. Jowett, Phys. Rev. ST Accel. Beams 12, 071002 (2009).



WHY?

1. The hydrogen-like ion Pb^{81+} is bent out from the beam.->

limitation of the luminosity $L_{\text{Pb-Pb}} \sim L_{\text{pp}}/10^7$.

2. The secondary beam of down-charged ions hit beam-pipe and deposit a considerable portion of energy at a small spot, which may in turn lead to ->

the quenching of superconducting magnets

SPS experiments ($\gamma_L = 168$) ultra-relativistic collisions of highly-charged Pb ions with solid-state and gas targets (there was a qualitative agreement with theory):

H. F. Krause et al., Phys. Rev. Lett. 80, 1190 (1998);

H. F. Krause et al., Phys. Rev. A 63, 032711 (2001).

The first observation of the beam losses at RHIC with nuclei of Cu^{29+} (energy 100 GeV/nucleon):

R. Bruce, et al. Phys. Rev. Lett. 99 (2007) 144801.

But all these experiments are related to **the total cross section, i.e. to** $p_{+\perp} \lesssim m_e$.

This region was studied in the theoretical papers:

R. H. Pratt, Phys. Rev. 117 (1960) 1017;

A. I. Milstein and V. M. Strakhovenko, Zh. Eksp. Teor. Fiz. 103 (1993) 1584;

C.K. Agger, A.H. Sørensen. Phys. Rev. A 55 (1997) 402;

H. Meier, Z. Halabuka, K. Hencken, D. Trautmann, G. Baur, Eur. Phys. Jour. C 5 (1998) 287; Phys. Rev. A 63 (2001) 032713;

A. Aste. EPL 81 (2007) 61001;

G. Baur et al. Phys. Rep. 364 (2002) 359

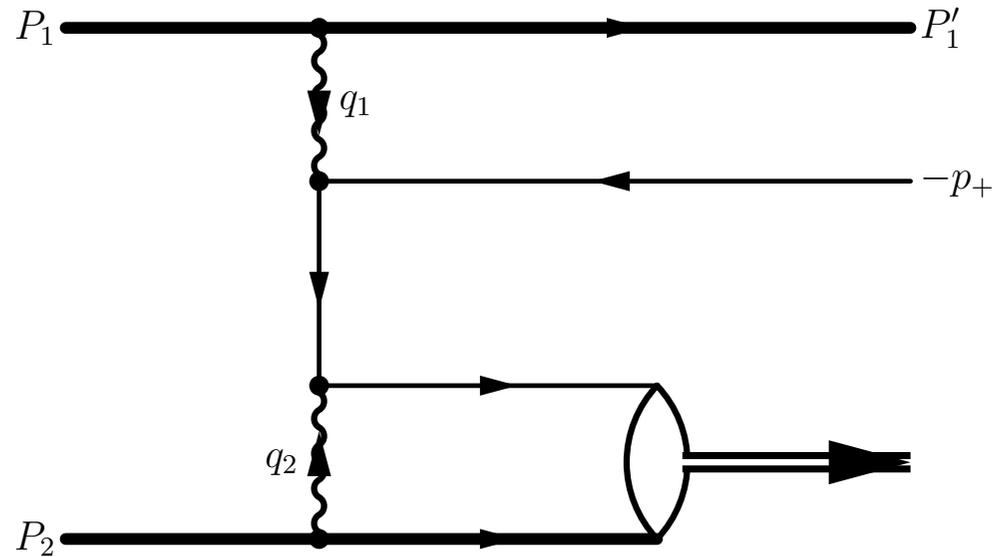
In the LHC collider the bound-free pair production could be measured, in principle, in the following set-up: **a positron is registered in the center detector** with $p_{+\perp} \gg m_e$ **in coincidence with the bent hydrogen-like ion** Pb^{81+} in the very forward detector.

It demands new calculations!

The exact calculations in this region is very difficult.

We present here **the approximate calculations for the ALICE group.**

Besides, we present the simple approximate analytical formulae **for the total cross section also.**



Positrons are observed in **the central detector** with limitations on **a transverse momentum and rapidity** ($m \equiv m_e$):

$$p_{+\perp} \geq p_{\min} \gg m \quad (2)$$

$$y_+ = \frac{1}{2} \ln \frac{\varepsilon_+ + p_{+z}}{\varepsilon_+ - p_{+z}} \approx -\ln \left[\tan \left(\frac{1}{2} \theta_+ \right) \right], \quad |y_+| \leq y_{\max} \quad (3)$$

The first scenario:

$$p_{\min} = 1 \text{ GeV}, \quad y_{\max} = 1 \quad (4)$$

then

$$\theta_{\min} = 40^\circ, W \geq 0.75 \text{ GeV}, \quad (5)$$

The second scenario:

$$p_{\min} = 0.05 \text{ GeV}, \quad y_{\max} = 1.5, \quad (6)$$

then

$$\theta_{\min} = 25^\circ, W \geq 0.13 \text{ GeV}. \quad (7)$$

5.2. Method of calculation

EQUIVALENT PHOTON APPROXIMATION (EPA)

In this approximation, the cross section of the discussed process can be presented in the form

$$d\sigma_{ZZ}^{\text{EPA}} = dn_{\gamma}(\omega_L) d\sigma_{\gamma Z}(\omega_L, p_{+\perp}), \quad (8)$$

where the number of equivalent photons is

[Jentschura, *Serbo. Eur. Phys. J. C* **64**, 309 (2009)]

$$dn_{\gamma}(\omega_L) = \frac{Z_1^2 \alpha}{\pi} \frac{d\omega_L}{\omega_L} \left[2 \ln \frac{\gamma_L}{\omega_L R} - 0.163 \right] \quad (9)$$

($R = 1/(28 \text{ MeV})$ is the radius of nucleus) and

$d\sigma_{\gamma^*Z}(\omega_L, p_{+\perp})$ — **cross sections of the photoprocess:**

$$\gamma + Z_2 \rightarrow e^+ + (Z_2 + e^-)_{1s}, \quad (10)$$

which depends on the energy ω_L of the equivalent photon in the rest frame of the second nucleus. This cross section has been considered in a number of papers

Meier, Halabuka, Hencken, Trautmann, Baur, Eur. Phys. J. C 5, 287 (1998)

Agger, Sörensen, Phys. Rev. A 55, 402 (1997)

but in the region of **small transverse momenta** of positrons only.
Problems...

We found out the **simple approximation** for this cross section. Moreover, just at that time we were informed about **new calculation** for the photo-production cross section in the needed region by
Di Piazza and Milstein, Phys. Rev. A85 (2012) 042107

5.3. Results

To estimate the number of events for the possible LHC experiment we integrate the differential cross section taking into account the experimental limitations. It gives:

$$\Delta\sigma_{ZZ} \approx \frac{32}{3} f(Z) \frac{(Z\alpha)^7}{m^2} \frac{e^{y_{\max}}}{\gamma} \left(\frac{m}{p_{\min}} \right)^3 L, \quad (11)$$

where

$$L = \left[2 \ln \left(\frac{\gamma}{R p_{\min}} \right) + 2 y_{\max} - 1.44 \right] \left(1 - e^{-2y_{\max}} \right) + 4 y_{\max} e^{-2y_{\max}}.$$

Assuming luminosity $L = 10^{27} \text{ cm}^{-2} \text{ s}^{-1}$ we have

For the first scenario: one event per 67 days

For the second scenario: by about 16 events per hour

WHAT NEXT?

We were asked to make estimates for two following processes at LHC:

1. **Production of two bound-free e^+e^- pairs;**
2. **Production of bound-free e^+e^- pair and free $\mu^+\mu^-$ pair.**

Results of the paper

A. N. Artemyev, V. G. Serbo, A. Surzhykov, 2014

1. **$\sim 40\,000$ events per hour**
2. **$\sim 15\,000$ events per hour**

CONCLUDING REMARKS:

Coulomb and unitarity corrections, and loop effects (virtual Delbrück scattering) are essential for an accurate quantitative understanding of photon and lepton production in ultrarelativistic heavy-ion collisions.

The extremely strong fields encountered in these processes lead to a physical situation **not encountered anywhere else in nature**, and thus, surprising effects (like loop-dominance over the tree-level graphs for photon production) represent testimonies of **the extreme state of matter**.

THANK YOU FOR ATTENTION!

