$\Delta(1232)$ contribution to real radiative corrections for elastic electron-proton scattering

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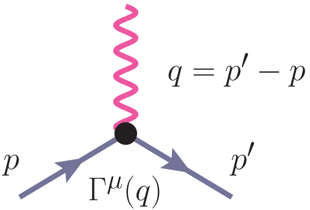
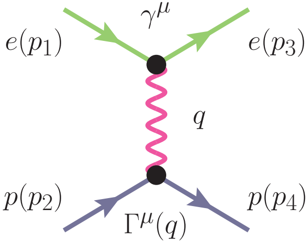
Proton Electromagnetic Form Factors





- Form factors contain information about proton internal structure.

$$\Gamma^\mu(q) = G_M(q^2)\,\gamma^\mu + \left(G_E(q^2) - G_M(q^2)\right) \frac{2M_p\,(p' + p)^\mu}{4M_p^2 - q^2}$$

$$G_E(0) = 1, \quad G_M(0) = \mu_p$$

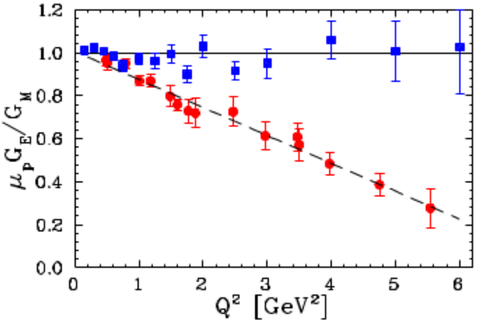
- Form factors can be extracted from unpolarized elastic ep-scattering (since 1950s)

$$\frac{d\sigma_{Rosenbluth}}{d\Omega_3} \propto \bar{\sum} |M_{1\gamma}|^2 \propto \tau G_M^2(q^2) + \epsilon G_E^2(q^2)$$

$$\tau = \frac{-q^2}{4M_p^2}, \quad \epsilon = \left[1 + 2(1+\tau)\tan^2\frac{\theta_3}{2}\right]^{-1}$$

Form Factors Ratio $\mu_p G_E / G_M$: Experimental Results.

- Rosenbluth separation method leads to dipole dependence of form factors w.r.t. squared momentum transfer ($Q^2 = -q^2$), and the ratio $\mu_p G_E / G_M \approx 1$.
- Form factors ratio can be measured in experiments with polarized particles (JLab, 2000;...). The result is significantly different: $\mu_p G_E / G_M$ decreases with the rise of Q^2.



Cross section ratio

- $e^{\pm}p$ scattering cross section ratio

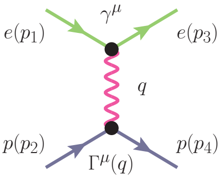
$$R = \frac{d\sigma(e^{+}p)/d\Omega_3}{d\sigma(e^{-}p)/d\Omega_3}$$

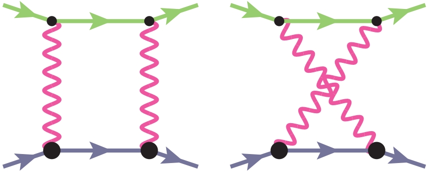
- C-odd radiative corrections make the difference between R and 1

$$R - 1 \approx -2(\delta_{2\gamma} + \delta_{brem,int})$$

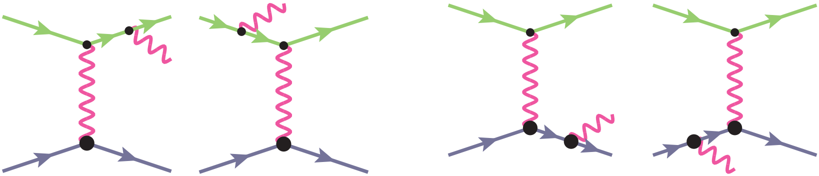
- Virtual radiative corrections $\delta_{2\gamma}$ comes from the interference of born amplitude with the TPE amplitudes. We are interested in the contribution of $M_{2\gamma}^{hard}$

$$\delta_{2\gamma} = \frac{2\,Re\left[\bar{\sum} {M_{1\gamma}}^{\dagger}(M_{2\gamma}^{soft} + M_{2\gamma}^{hard})\right]}{\bar{\sum}|M_{1\gamma}|^2}$$





Real radiative corrections

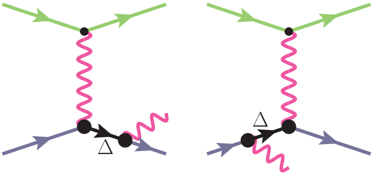


- Real radiative correction $\delta_{brem,int}$ comes from the interference of lepton and proton bremsstrahlung

$$\delta_{brem,int} = \frac{\int 2Re\left[\bar{\sum} M_{brem,e}{}^{\dagger} M_{brem,p}\right]}{\bar{\sum}|M_{1\gamma}|^2}$$

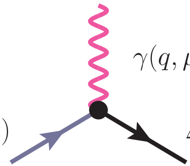
- Integration over final particles phase space is restricted by certain experimental cuts. It can not be done analytically, so ESEPP event generator was used for the experiment at the VEPP-3 storage ring

Real radiative corrections. $\Delta(1232)$



- We can consider $\Delta(1232)$ in the intermediate state. Real radiative corrections are determined by experimental cuts and in principal could alter results of the Rosenbluth separation and the TPE contribution extraction from the $e^{\pm}p$ cross section ratio
- These amplitudes can be calculated for concrete form of $\gamma\Delta p - vertex$ explicitly. It was also done, but the expressions are complex for analyses
- The contribution of Δ appeared to be small, so we will present estimates for the contributions of $|M_{\Delta}|^2$ and C-odd interference term $M_{brem,e}{}^{\dagger}M_{\Delta}$.

Proton-Δ(1232) transition form factors



- 3 form factors for the vertex (TPE-Δ arXiv:1407.2711)

$$\Gamma^{\mu\alpha}_{\gamma p\to\Delta}(p,q) =$$
$$-\sqrt{\frac{2}{3}}\frac{1}{2M_\Delta^2}\gamma^5\left\{G^{(1)}_\Delta(q^2)\left[g^{\mu\alpha}\hat{q}\hat{p}-p^\mu\hat{q}\gamma^\alpha-\gamma^\alpha\gamma^\mu(p\cdot q)+\hat{p}\gamma^\mu q^\alpha\right]\right.$$
$$+\,G^{(2)}_\Delta(q^2)\left[p^\mu q^\alpha-g^{\mu\alpha}(p\cdot q)\right]$$
$$\left.-\frac{G^{(3)}_\Delta(q^2)}{M_\Delta}\left[q^2\left(p^\mu\gamma^\alpha-g^{\mu\alpha}\hat{p}\right)+q^\mu\left(q^\alpha\hat{p}-\gamma^\alpha(p\cdot q)\right)\right]\right\}$$

Helicity amplitudes

- In the following we will express the results in terms of 3 helicity amplitudes for the process $\gamma^* p \to \Delta$.

$$M^{\gamma^* p\to\Delta}_{+1,+\frac{1}{2},+\frac{1}{2}} = C\,\frac{1}{\sqrt{2}}\,A_{1/2}(q^2), \quad M^{\gamma^* p\to\Delta}_{+1,-\frac{1}{2},+\frac{3}{2}} = C\,\sqrt{\frac{3}{2}}\,A_{3/2}(q^2)$$

$$iM^{\gamma^* p\to\Delta}_{0,\frac{1}{2},-\frac{1}{2}} = C\,\frac{\sqrt{-q^2}}{M_\Delta}\,S_{1/2}(q^2)$$

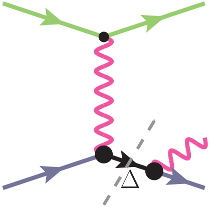
with $C = \frac{\sqrt{(M_\Delta - M_p)^2 - q^2}\sqrt{(M_\Delta + M_p)^2 - q^2}}{2(M_\Delta + M_p)}$, and q^2 is the photon virtuality.

- Decay $\Delta \to \gamma p$

$$\Gamma_{\Delta\to\gamma p} = \frac{1}{8\pi}\,\frac{(M_\Delta^2 - M_p^2)^3}{32 M_\Delta^3 (M_\Delta + M_p)^2}\left[A_{1/2}^2(0) + 3A_{3/2}^2(0)\right]$$

$$\frac{\Gamma_{\Delta\to\gamma p}}{\Gamma_\Delta} = (0.55 - 0.65)\%\ (PDG)$$

Contribution of $|M_\Delta|^2$. Rough estimate



- $ep \to e\Delta$

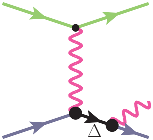
$$\bar{\sum} |M_{ep\to e\Delta}|^2 \propto \left\{ A_{1/2}^2(q^2) + 3A_{3/2}^2(q^2) + 4\epsilon \frac{-q^2}{M_\Delta^2} S_{1/2}^2(q^2) \right\}$$

- A very rough estimate for the contribution of $|M_\Delta|^2$

$$\delta_\Delta \simeq \frac{d\sigma_{ep\to e\Delta}/d\Omega_3}{d\sigma_{ep\to ep}/d\Omega_3} \times \frac{\Gamma_{\Delta\to p\gamma}}{\Gamma_\Delta}.$$

which gives δ_Δ about 0.5%, and it is not small.

Contribution of $|M_\Delta|^2$. More accurate estimate



- $\Delta(1232)$ propagator

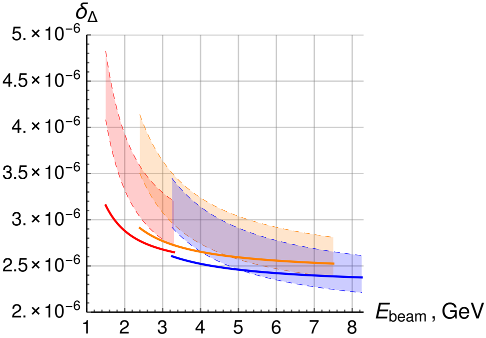
$$\frac{i(\hat{p}_\Delta + M_\Delta)}{p_\Delta^2 - M_\Delta + i\Gamma_\Delta M_\Delta}\left(g^{\alpha\beta} - \frac{\gamma^\alpha\gamma^\beta}{3} - \frac{\hat{p}_\Delta\gamma^\alpha p_\Delta^\beta + p_\Delta^\alpha\gamma^\beta\hat{p}_\Delta}{3p_\Delta^2}\right)$$

- The vertex $\Delta \to \gamma p$ with real photon is proportional to its energy $\omega = (W^2 - M_p^2)/(2W)$ in the special frame $\mathbf{p_4} + \mathbf{k} = 0$, where $W^2 = (p_4 + k)^2$
- A more accurate estimate for the contribution of $|M_\Delta|^2$:

$$\delta_\Delta \approx \frac{\Gamma_{\Delta\to\gamma p}}{\Gamma_\Delta}\frac{\frac{d\sigma_{ep\to e\Delta}}{d\Omega_3}}{\frac{d\sigma_{ep\to ep}}{d\Omega_3}}\frac{1}{\pi}\int_0^{W_{max}^2 - M_p^2}\frac{1}{(M_\Delta^2 - M_p^2)^3}\frac{M_\Delta\Gamma_\Delta x^3 dx}{(x + M_p^2 - M_\Delta^2)^2 + \Gamma_\Delta^2 M_\Delta^2}$$

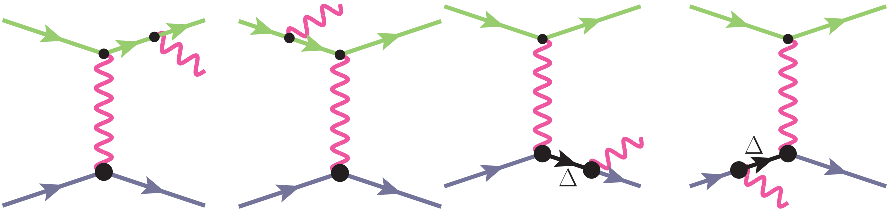
where $x = W^2 - M_p^2$

Contribution of $|M_\Delta|^2$. Numerical



- δ_Δ for the Rosenbluth separation experiment (SLAC, 94). Solid lines — for matrix element $|M_\Delta|^2$ evaluated without approximations, stripes — for the approximate expression: red, orange, blue for $Q^2 = 1, 2, 3$ GeV^2
- The contribution is highly suppressed for $W^2_{max} < (M_p + m_\pi)^2$ below π production threshold.

C-odd interference $M_{brem,e}{}^{\dagger} M_{\Delta}$



- Relatively simple expression for $\delta^{int}_{brem,\Delta}$ can be found using soft photon approximation ($W \to M_p$) and saving leading terms w.r.t. $M_\Delta - M_p$.

$$\delta^{int}_{brem,\Delta} = \frac{\int' \left(1 - \frac{x}{2M_p\epsilon_1}\right) \frac{1}{4} \frac{xdx}{x+M_p^2} \int' \frac{d\Omega_\gamma}{(2\pi)^3 2} \bar{\sum} 2\mathrm{Re}\left[M_{brem,e}{}^{\dagger} M_\Delta\right]}{\bar{\sum} |M_{1\gamma}|^2}$$

C-odd interference $M_{brem,e}{}^{\dagger} M_{\Delta}$

In the following p, ψ — electron momentum and scattering angle in the Breit frame; q_v — virtual photon momentum and $\zeta,\ \phi,\ \epsilon(k)$ — real photon emission angles and polarization vector in the special frame $\mathbf{p}_4 + \mathbf{k}$

$$\bar{\sum} 2\, Re \left[M_{brem,e}^{(soft)}{}^{\dagger} M_{\Delta}^{(1)} \right] \approx -\frac{4}{3}\frac{e^5}{(q^2)^2}\ \ Re\left[\frac{1}{W^2 - M_\Delta^2 + i\Gamma_\Delta M_\Delta} \right] \times$$

$$G_1(0) \left(M_p F_E(q^2)(A_{1/2}(q^2) + 3A_{3/2}(q^2)) + \frac{(-q^2)}{M_\Delta} F_M(q^2)\, S_{1/2}(q^2) \right)$$

$$(p^2 \sin\psi) \left(\frac{\omega}{M_\Delta} \frac{q_v}{M_\Delta} \left[\frac{p_{3,\mu}}{(p_3 k)} - \frac{p_{1,\mu}}{(p_1 k)} \right] \right)$$

$$\left[\frac{\epsilon^{\mu}_{+1}(k) - \epsilon_{-1}(k)}{\sqrt{2}} \cos\phi + \frac{\epsilon^{\mu}_{+1}(k) + \epsilon^{\mu}_{-1}(k)}{\sqrt{2}} i \sin\phi \cos\zeta \right]$$

C-odd interference $M_{brem,e}{}^{\dagger} M_{\Delta}$

- Futher integration within the angular and energy cuts applied in the experiment at the VEPP-3 is not easy.
  It was done numerically
- Using the approximate expression as well as $M_e{}^{\dagger} M_{\Delta}$ without approximations ensures that $\delta_{brem,\Delta} < 0.1\%$

Summary

- We consider a potential contribution of $\Delta(1232)$ resonance to real radiative corrections for unpolarized elastic electron-proton scattering
- The effect is found to be small for the past experiments on Rosenbluth separation as well as for the recent experiment at the VEPP-3 storage ring to investigate the two-photon exchange effects