Measuring a_μ^{HLO} in the spacelike region

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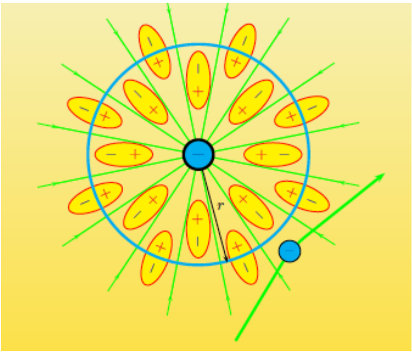
Novosibirsk, 17 June 2015

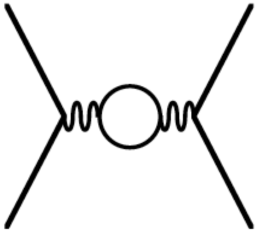
α_{em} running and the Vacuum Polarization



- Due to Vacuum Polarization effects $\alpha_{em}(q^2)$ is a running parameter from its value at vanishing momentum transfer to the effective q^2.

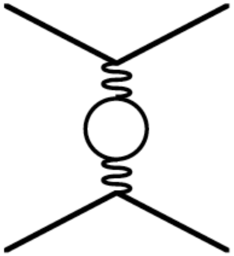
➢ The "Vacuum Polarization" function $\Pi(q^2)$ can be "absorbed" in a redefinition of an effective charge:

$$e^2 \rightarrow e^2(q^2) = \frac{e^2}{1+(\Pi(q^2)-\Pi(0))} \qquad \alpha(q^2) = \frac{\alpha(0)}{1-\Delta\alpha}; \quad \Delta\alpha = -\Re e\left(\Pi(q^2)-\Pi(0)\right)$$

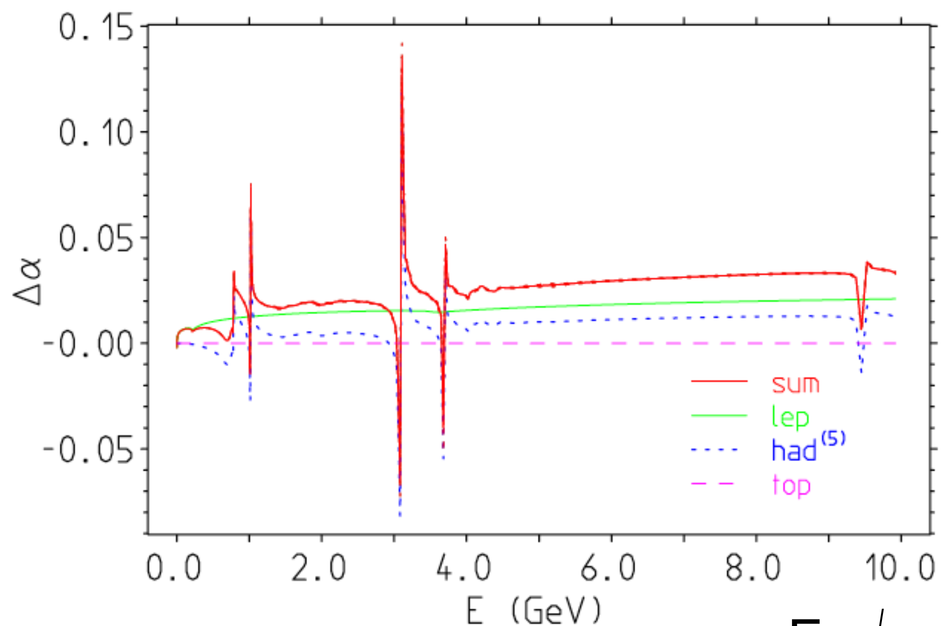


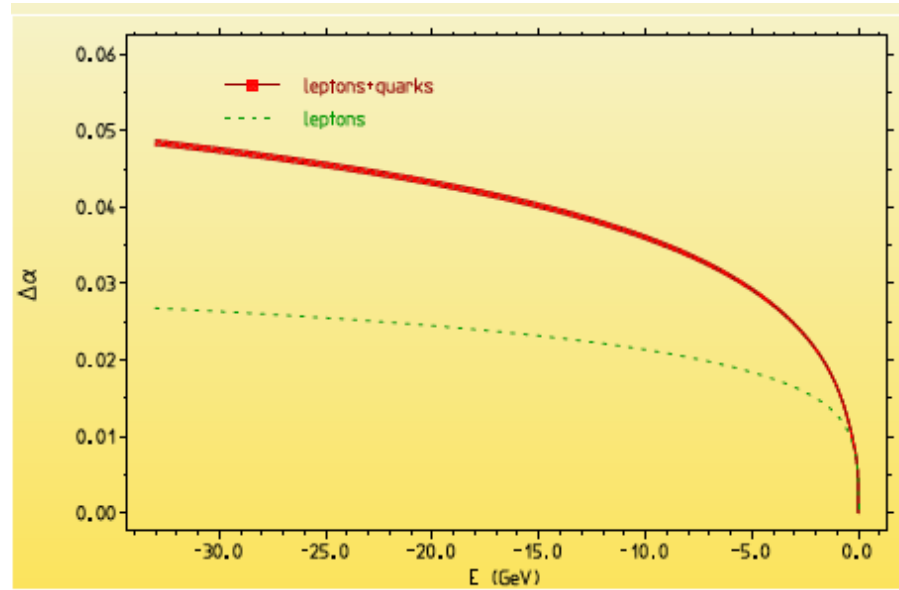
$$\Delta\alpha = \Delta\alpha_l + \Delta\alpha^{(5)}_{had} + \Delta\alpha_{top}$$

➢ $\Delta\alpha$ takes a contribution by non perturbative hadronic effects ($\Delta\alpha^{(5)}_{had}$) which exibits a different behaviour in time-like and spacelike region



Running of α_{em}





$E=\sqrt{s}$

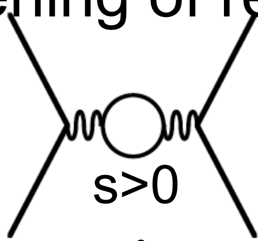
Time-like

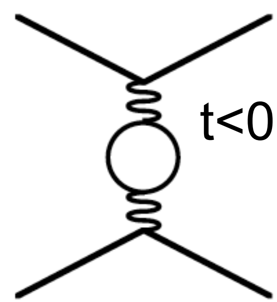
$E=-\sqrt{-t}$

Space-like

Behaviour characterized by the opening of resonances

Very smooth behaviour

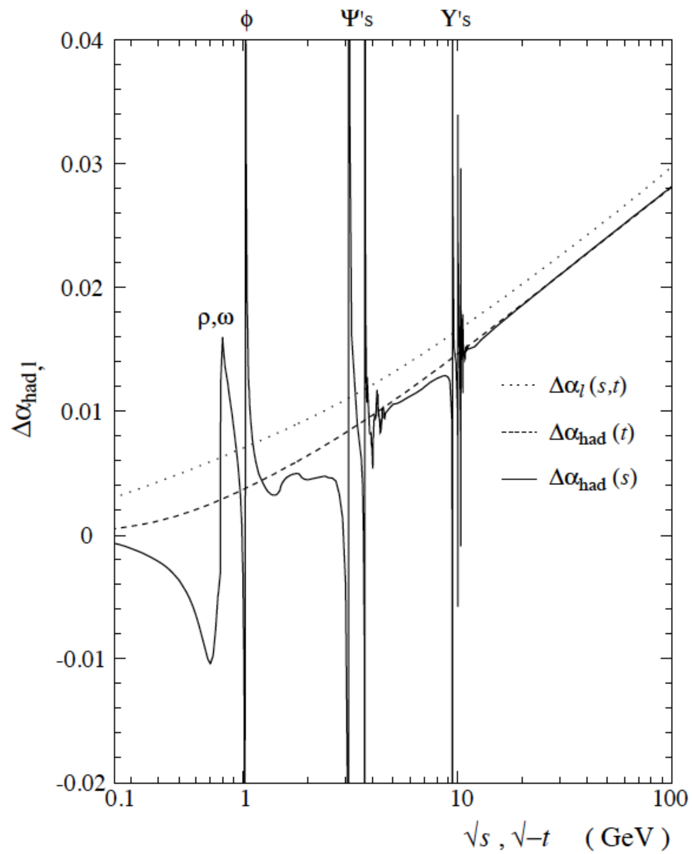




$$\Delta\alpha^{(5)}_{had}(M_Z^2) = -\frac{\alpha M_Z^2}{3\pi}\mathrm{Re}\int_{4m_\pi^2}^{\infty} ds \frac{R(s)}{s(s-M_Z^2-i\varepsilon)}$$

Measurement of α_{em} running

- A direct measurement of $\alpha_{em}(q^2)$ in space/time like region can prove the running of α_{em}
- It can provide a test of "duality" (fare way from resonances)
- It has been done in past by few experiments at e^+e^- colliders by comparing a "well-known" QED process with some reference (obtained from data or MC)



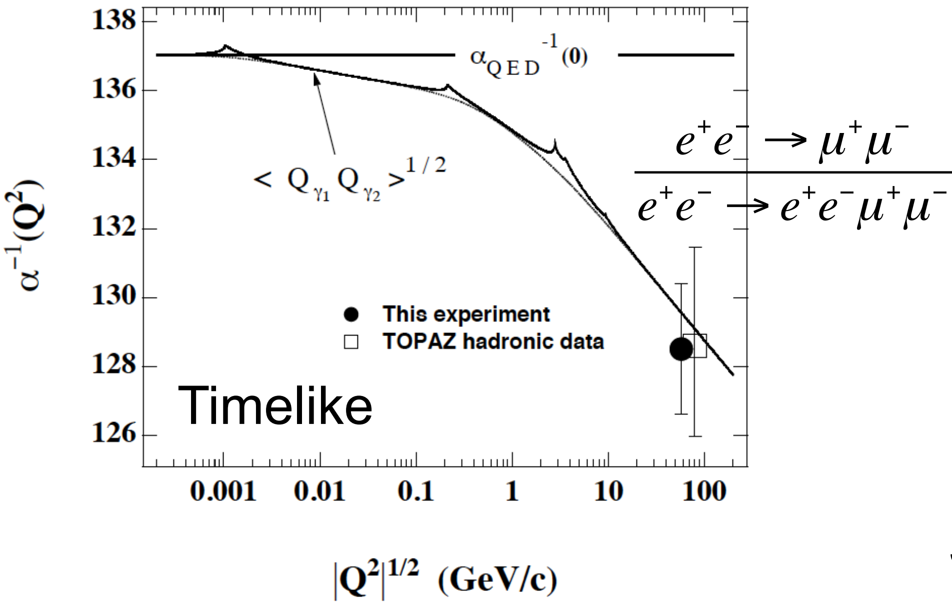
$$\left(\frac{\alpha(q^2)}{\alpha(q_0^2)}\right)^2 \sim \frac{N_{signal}(q^2)}{N_{norm}(q_0^2)}$$

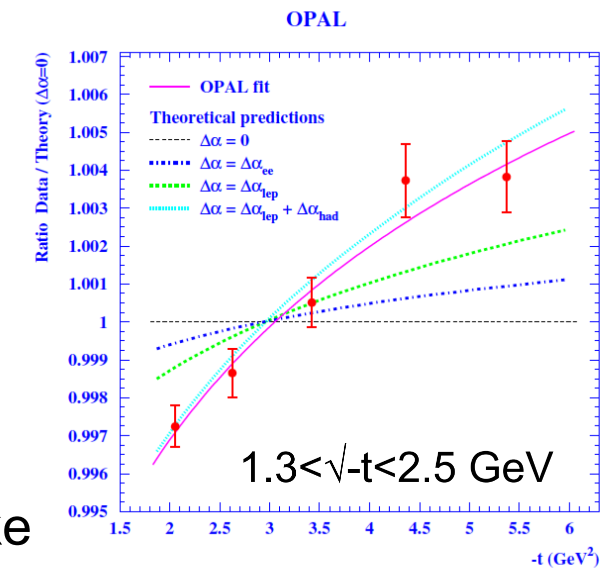
N_{signal} can be Bhabha process, muon pairs, etc…
N_{signal} can be Bhabha process, $\gamma\gamma$ pairs, Theory, etc…

Measurement of α_{em} running

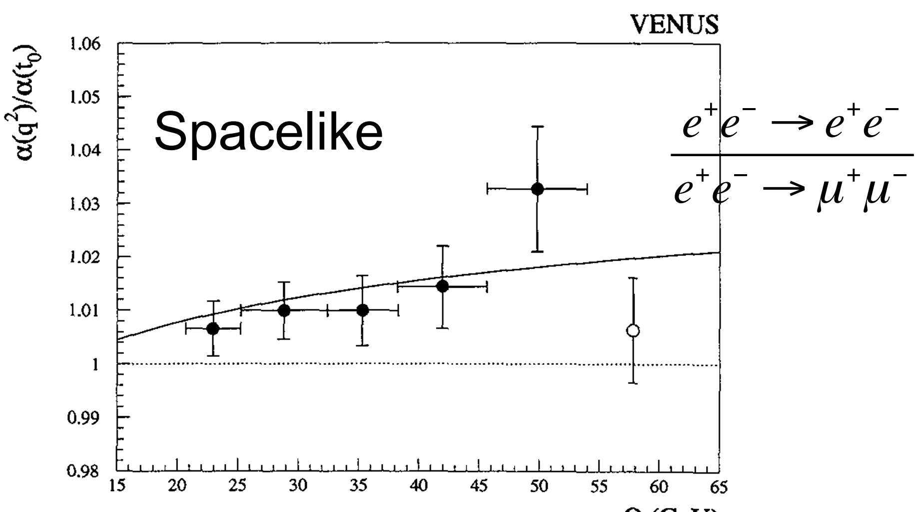
e+e- collider TRISTAN at √s=57.8 GeV,

e+e- collider LEP at √s=189 GeV, using **Bhabha** events

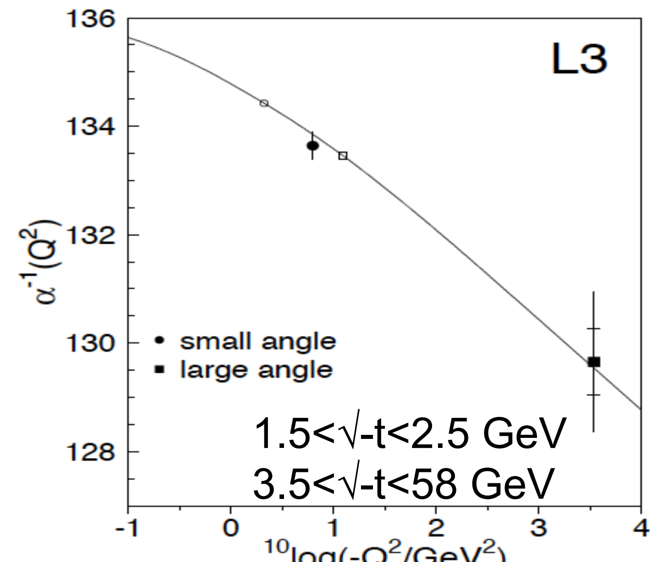




Spacelike



10<√-t<54 GeV

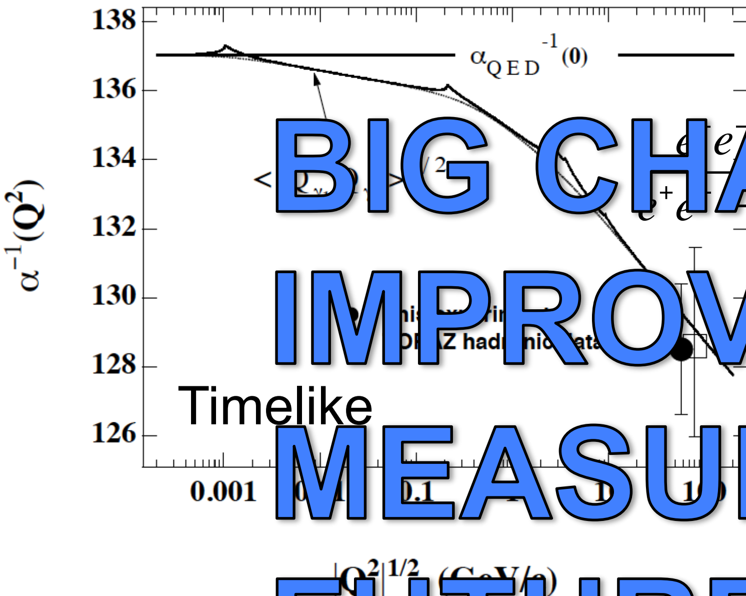


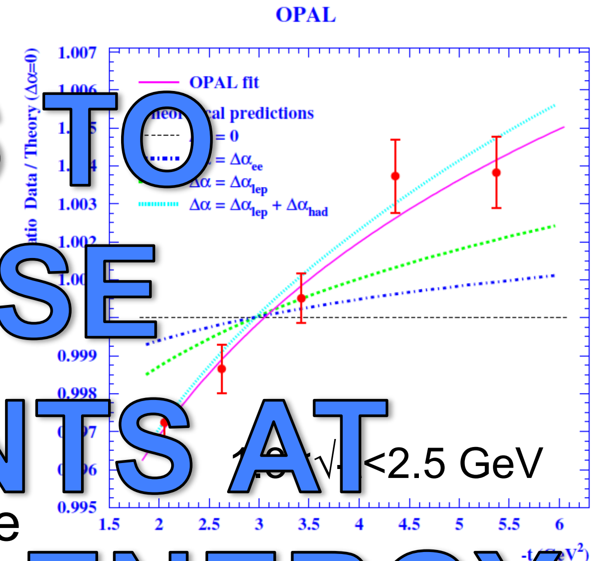
Measurement of α_{em} running

e+e- collider TRISTAN at √s=57.8 GeV,

e+e- collider LEP at √s=189 GeV, using **Bhabha** events

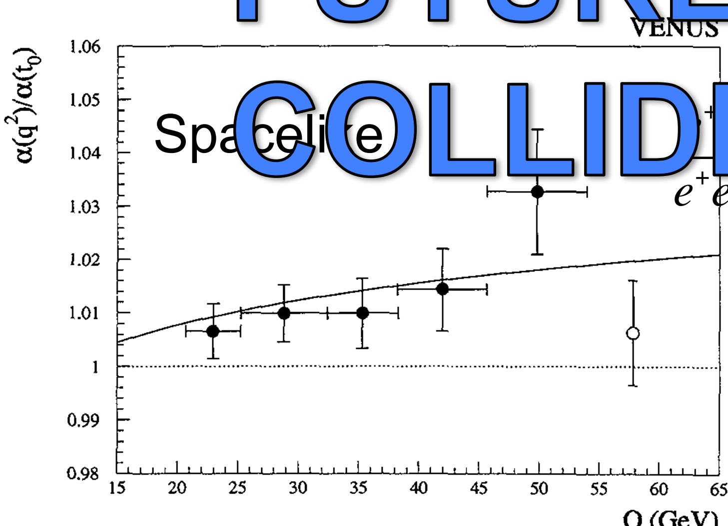
BIG CHANCES TO IMPROVE THESE MEASUREMENTS AT FUTURE HIGH ENERGY COLLIDER (ILC/TLEP)



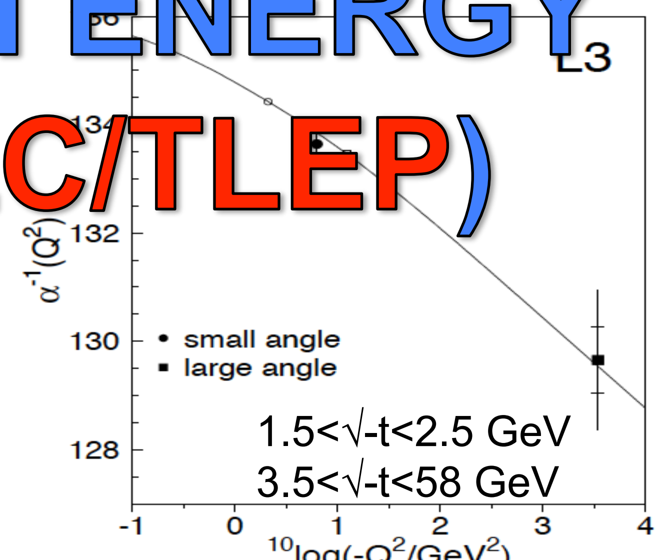


1.8<√-t<2.5 GeV

Spacelike



10<√-t<54 GeV



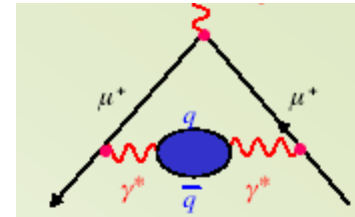
1.5<√-t<2.5 GeV

3.5<√-t<58 GeV

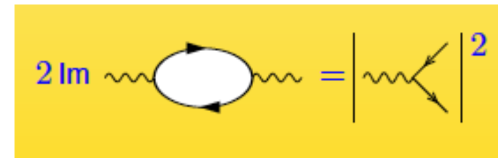
a_μ^{HLO} calculation, traditional way: time-like data

$a_\mu=(g-2)/2$

$$a_\mu^{HLO} = \frac{1}{4\pi^3}\int_{4m_\pi^2}^{\infty}\sigma_{e^+e^-\to hadr}(s)K(s)\,ds$$



$$a_\mu^{HLO.} = \frac{\alpha}{\pi^2}\int_0^\infty \frac{ds}{s}K(s)\,\mathrm{Im}\,\Pi_{had}(s) \qquad \sigma_{e^+e^-\to hadr}(s) = \frac{4\pi}{s}\mathrm{Im}\,\Pi_{had}(s)$$



$$K(s) = \int_0^1 dx\,\frac{x^2(1-x)}{x^2+(1-x)(s/m^2)} \sim \frac{1}{s}$$

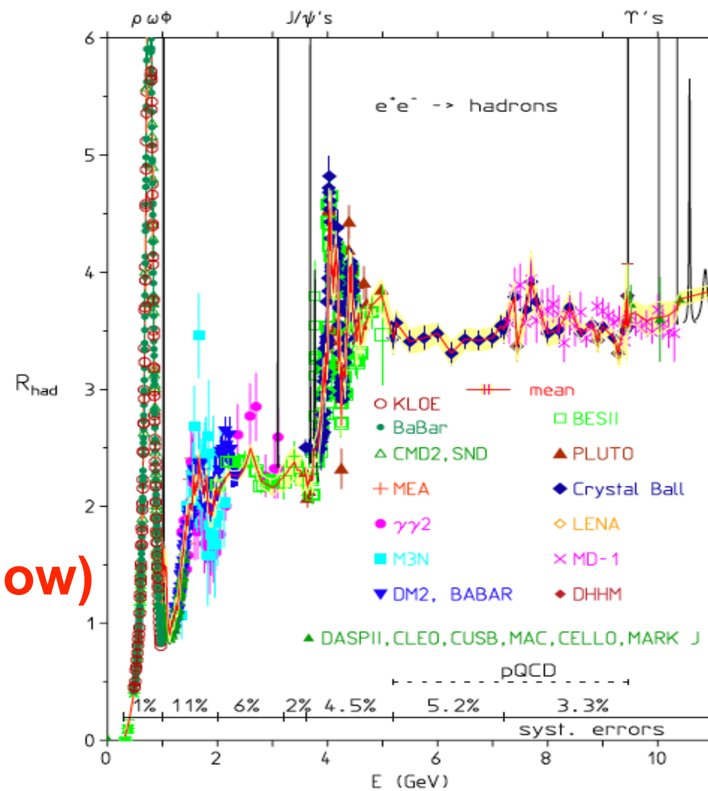
Traditional way: based on precise experimental (time-like) data:

$$a_\mu^{had} = (689.7 \pm 4.4)\cdot 10^{-10}$$

Main contribution in the low energy region

$\delta a_\mu^{exp} \rightarrow$ 1.5 10^{-10} = 0.2% on a_μ^{HLO} (from 0.7% now)

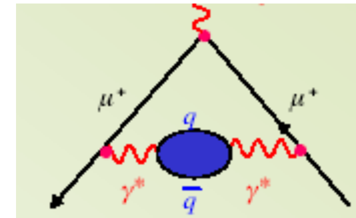
NEW G-2 at FNAL and JPARC



a_μ^{HLO} evaluation in spacelike region: alternative approach

$a_\mu = (g-2)/2$

$$a_\mu^{HLO} = -\frac{\alpha}{\pi}\int_0^1 (1-x)\Pi_{had}\left(-\frac{x^2}{1-x}m_\mu^2\right)dx$$



x =Feynman parameter

See also G.Fedotovich, proceedings of PHIPSI08

$$t = \frac{x^2 m_\mu^2}{x-1} \qquad 0 \le -t < +\infty$$

t<0

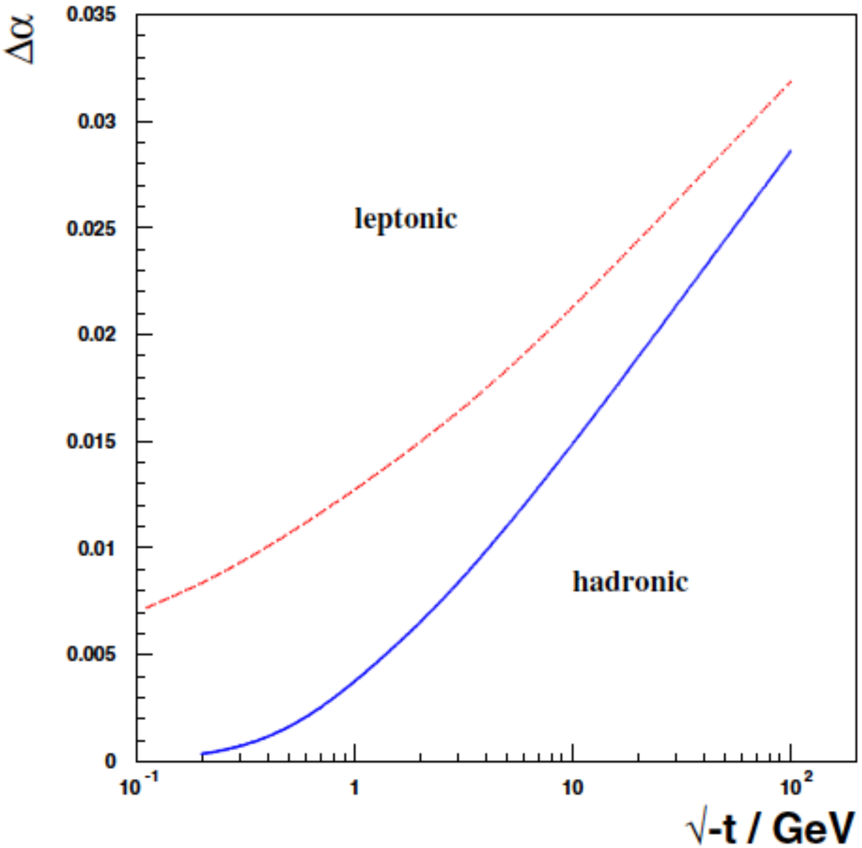
$$x = \frac{t}{2m_\mu^2}\left(1-\sqrt{1-\frac{4m_\mu^2}{t}}\right); \quad 0 \le x < 1; \qquad t = -s\sin^2\left(\frac{\vartheta}{2}\right)$$

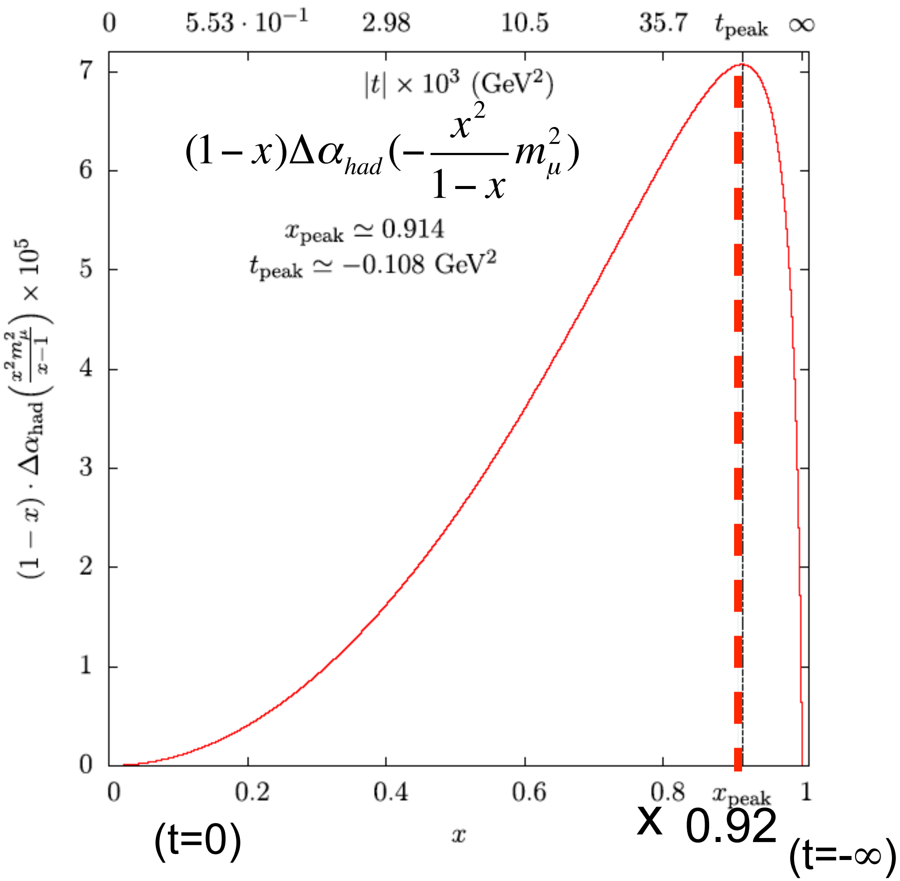
$$\Delta\alpha_{had}(t) = -\Pi_{had}(t) \quad for\ t<0$$

$$a_\mu^{HLO} = -\frac{\alpha}{\pi}\int_0^1 (1-x)\Delta\alpha_{had}\left(-\frac{x^2}{1-x}m_\mu^2\right)dx$$

For t<0

Behaviors





$\Delta\alpha \sim \log(-t)$
Dominated at low |t| by leptonic contribution

A. Arbuzov et al., Eur. Phys. J. C 34 (2004) 267

High |t|-values are depressed by 1-x
(a kind of analogy with time-like region)
The integrand is peaked at ~x=0.92
→t=-0.11 GeV^2 (~330 MeV) for which
$\Delta\alpha_{had}(0.92) \sim 10^{-3}$

Experimental considerations

Using Bhabha at small angle (to emphasize t-channel contribution) to extract $\Delta\alpha$:

$$\left(\frac{\alpha(t)}{\alpha(0)}\right)^2 \sim \frac{d\sigma_{ee\to ee}(t)}{d\sigma^0_{MC}(t)}$$

Where $d\sigma^0_{MC}$ is the MC prediction for Bhabha process with α(t)=α(0), and there are corrections due to RC…

$$\Delta\alpha_{had}(t) = 1 - \left(\frac{\alpha(t)}{\alpha(0)}\right)^{-1} - \Delta\alpha_{lept}(t)$$

$\Delta\alpha_{lep}(t)$ theoretically well known!

Which experimental accuracy we are aiming at?
$\delta\Delta\alpha_{had}$~1/2 fractional accuracy on $d\sigma(t)/d\sigma^0_{MC}(t)$.

If we assume to measure $\delta\Delta\alpha_{had}$ at 5% at the peak of the integrand ($\Delta\alpha_{had}$ ~10^{-3} at x=0.92) → fractional accuracy on $d\sigma(t)/d\sigma^0_{MC}(t)$ ~ 10^{-4} !

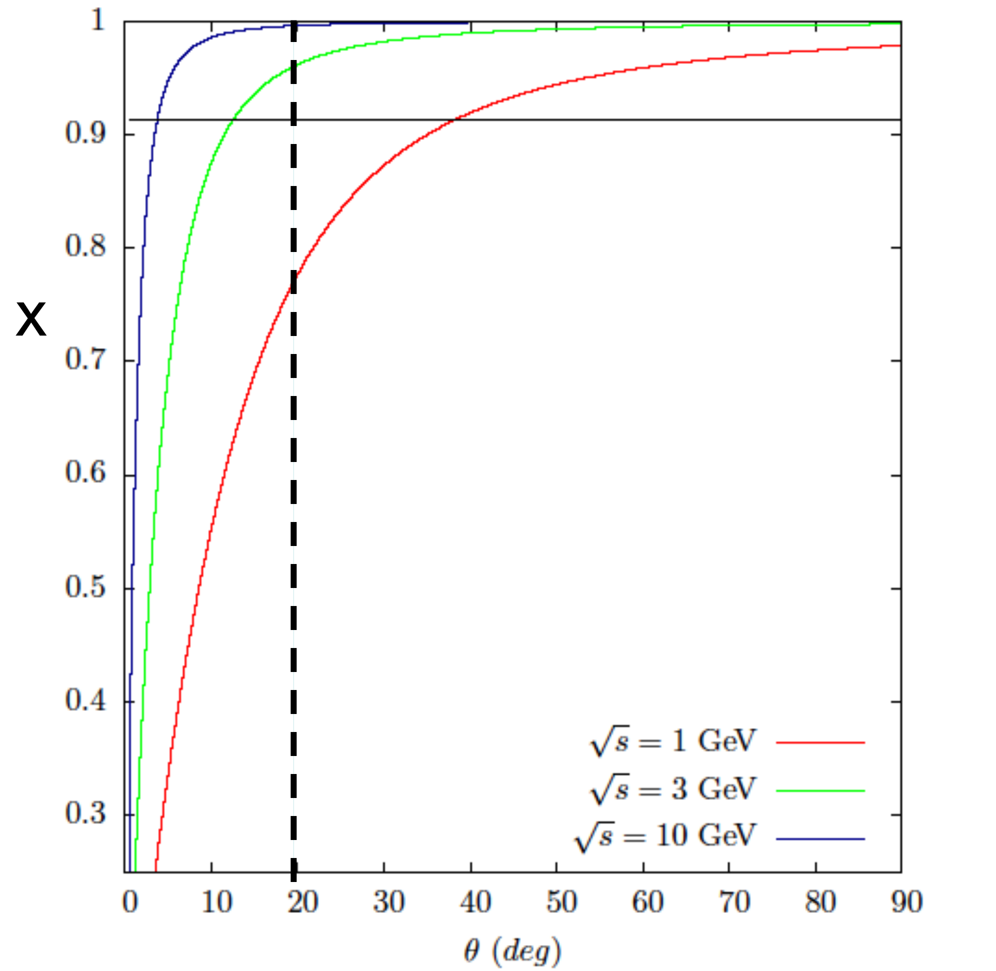
Very challenging measurement (one order of magnitude improvement respect to date) for systematic error

Experimental considerations - II

Most of the region (up to x~0.98) can be covered with a low energy machine (like Dafne/VEPP-2000 or tau/charm-B-factories)

Example:
Covering up to 60^0 at $\sqrt{s}$=1 GeV can arrive at x= 0.95(!)

A different situation can be obtained at tau/charm/ B-factories (and at future ILC/TLEP machines) where smaller angles (below 20^o) are needed



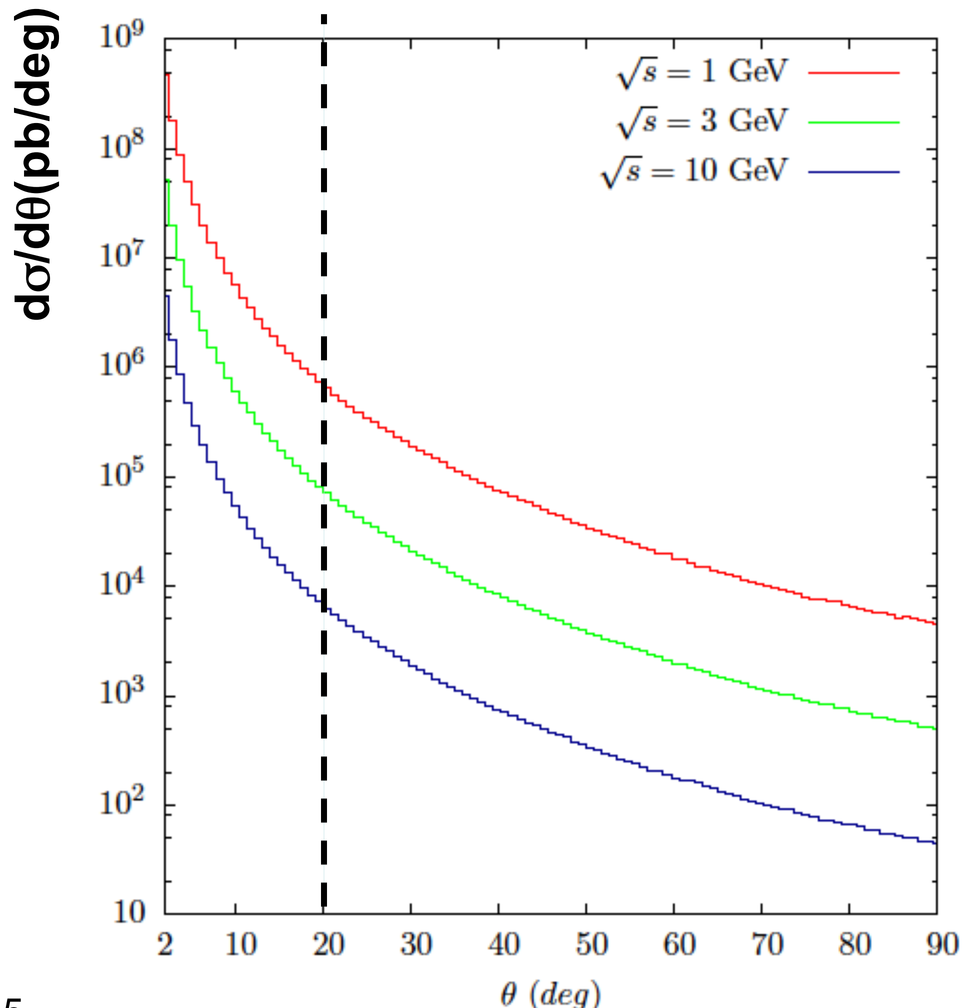
$$t = -s\sin^2(\frac{\vartheta}{2})$$

Statistical consideration

10^{-4} accuracy on Bhabha cross section requires at least 10^8 events which at 20^o mean at least:

- O(1) fb^{-1} @ 1 GeV
- O(10) fb^{-1} @ 3 GeV
- O(100) fb^{-1} @ 10 GeV

These luminosities are within reach at flavour factories!



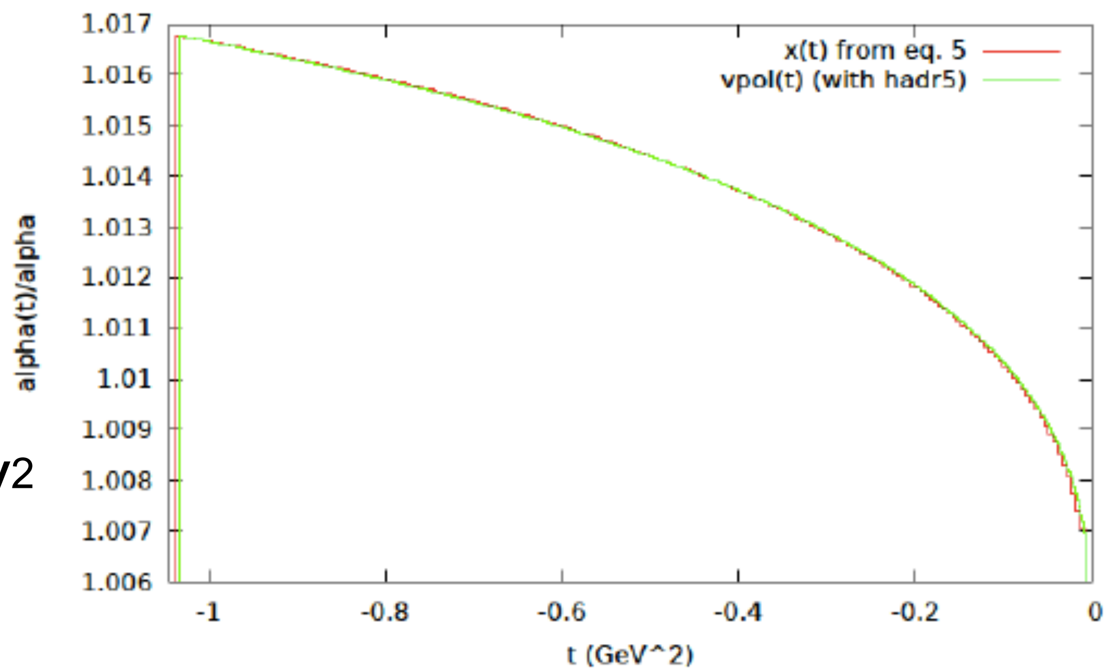
Additional considerations: s-channel

At low energy (<10 GeV) above 10^0 there is still a sizeable contribution from s-channel.
At LO no difficulty to deconvolute the cross section for the s-channel

Test with Babayaga:

s=1 GeV
$10^o<\theta<170^o$

$d\sigma_{born}/dt=1.52\ mb/GeV^2$



However this picture changes with Rad. Corr.

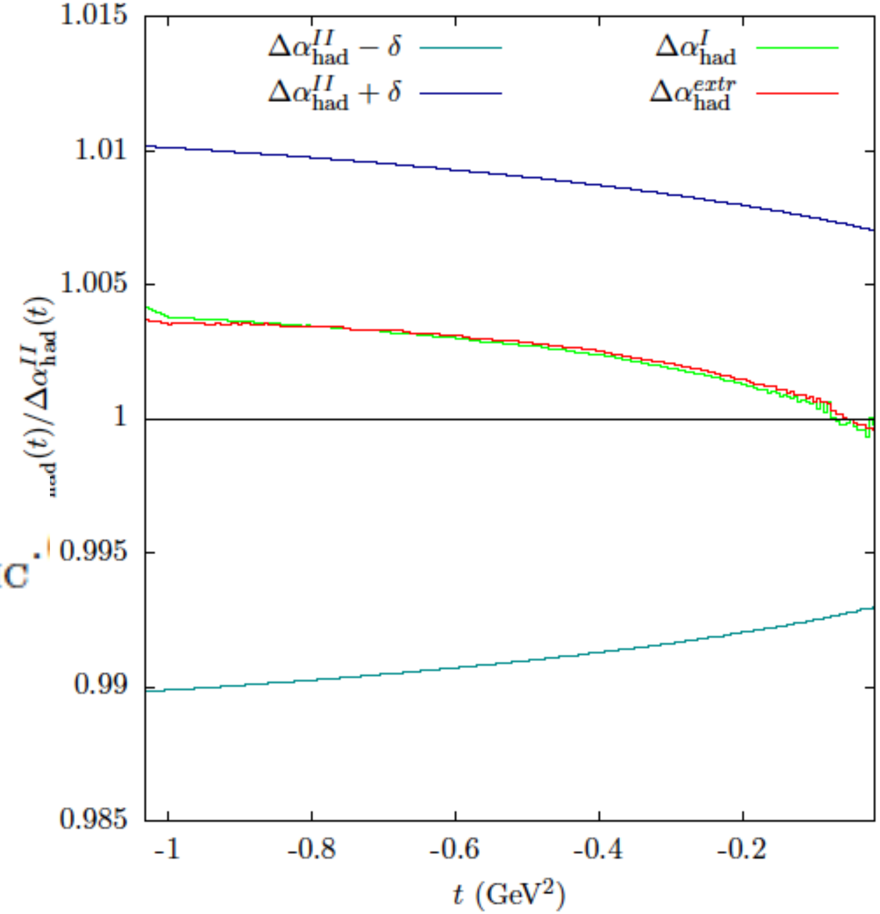
Additional considerations: Rad. Corr.

A Monte Carlo procedure has been developed to check if $\Delta\alpha_{had}(t)$ can be obtained by a minimization procedure with a different $\Delta\alpha_{had}(t)'$ inside

$$\frac{d\sigma}{dt}\Big|_{\mathrm{data}} = \frac{d\sigma}{dt}\Big(\alpha(t), \alpha(s)\Big)\Big|_{\mathrm{MC}},$$

→

$$\frac{d\sigma}{dt}\Big|_{j,\mathrm{data}} = \frac{d\sigma}{dt}\Big(\bar{\alpha}(t) + \frac{i_j}{N}\delta(t), \alpha(s)\Big)\Big|_{j,\mathrm{MC}}.$$



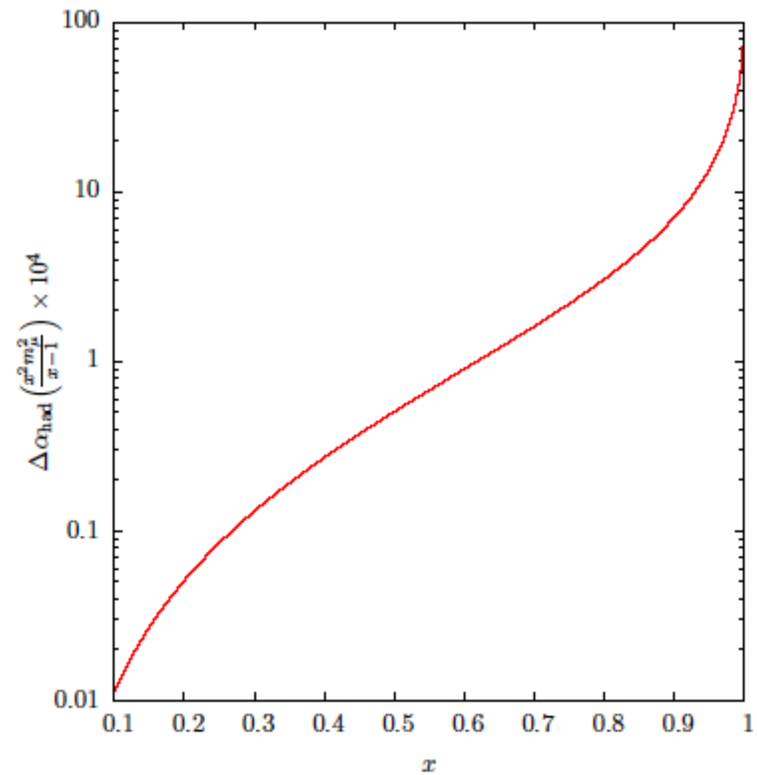
$\Delta\alpha_{had}(t)$ is obtained with$<10^{-4}$ error !

Additional consideration: Normalization

To compare Bhabha absolute cross section from data with MC we need Luminosity of the machine.

Two possibilities:

1) Use Bhabha at very small angle where the uncertainty on $\Delta\alpha_{had}$ can be neglected (for example at E_{beam}=1 GeV and $\theta=5^{o}$, $\Delta\alpha_{had} \sim 10^{-5}$).
2) Use a process with $\Delta\alpha_{had}=0$, like e+e- →$\gamma\gamma$. However very difficult to determine it at 10^{-4} accuracy.



Option 1) looks better to us as some of the common systematics cancel in the measurement !

Measurement of DAFNE Luminosity with KLOE/KLOE-2 at 10^{-4}?

F. Ambrosino et al [KLOE] Eur. Phys. J. C 47, 589–596 (2006)

Table 2. Summary of the corrections and systematic errors in the measurement of the luminosity

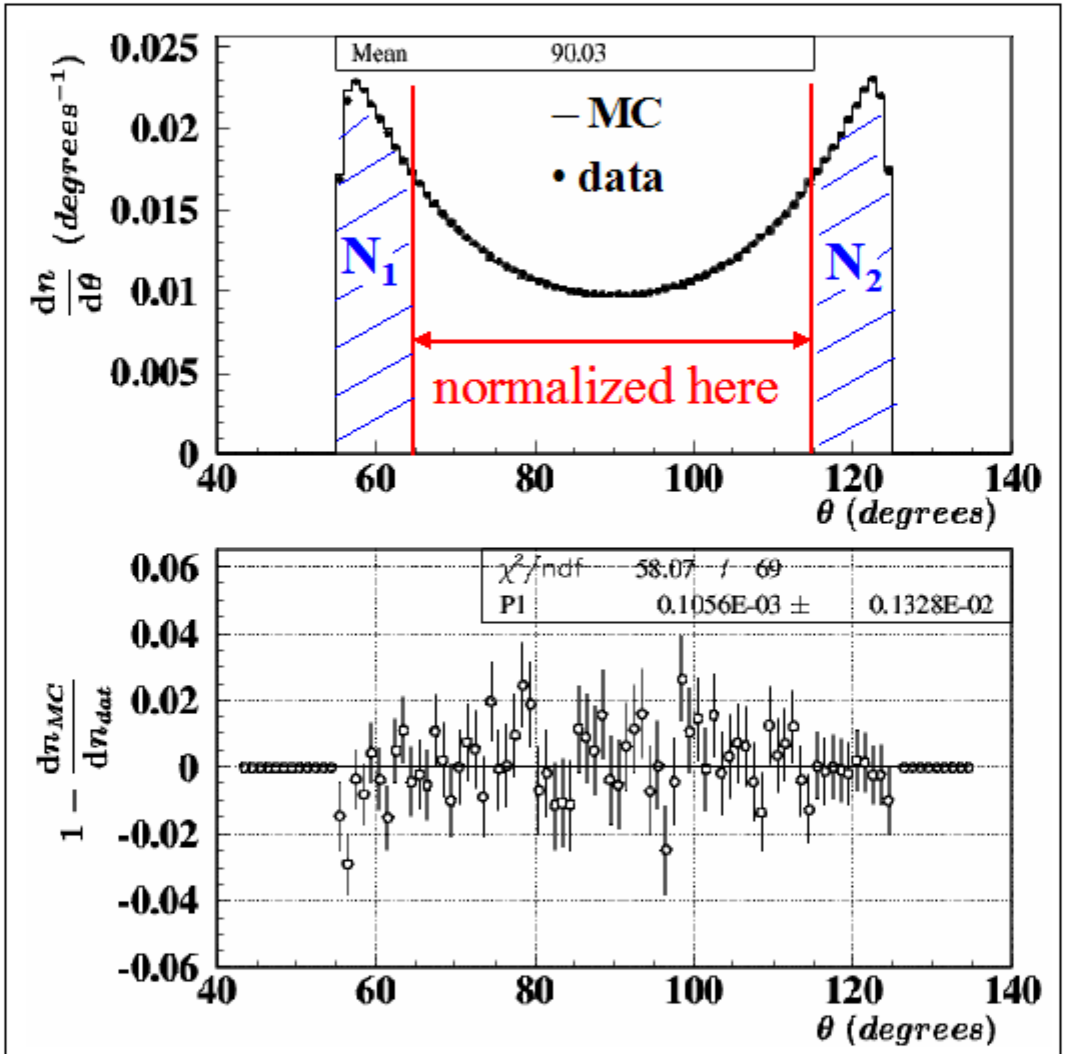
	correction (%)	systematic error (%)
angular acceptance	+0.25	0.25
tracking	–	0.06
clustering	+0.14	0.11
background	−0.62	0.13
cosmic veto	+0.40	–
energy calibration	–	0.10
center of mass energy	+0.10	0.10
	+0.34	0.32

Adding in quadrature: 0.3 %

(can be improved by a factor 10?)

From F. Nguyen 2006

Polar angle systematics



✓ global agreement is very good

but the cut occurs in a steep region of the distributions
⇒ estimate of border mismatches

✓ after normalizing MC to make it coincide with data in the region $65^o < \theta < 115^o$, we estimate as a systematic error:

$$\frac{N^{dat}_{[55:65]+[115:125]} - N^{MC}_{[55:65]+[115:125]}}{N^{dat}_{TOT}} \sim 0.25\%$$

Can be improved at 10^{-4}?

A measurement of the Luminosity at 10^{-4} at LEP

Giovanni Abbiendi

INFN - Bologna

Eur. Phys. J. C 45, 1–21 (2006)
Digital Object Identifier (DOI) 10.1140/epjc/s2005-02389-3

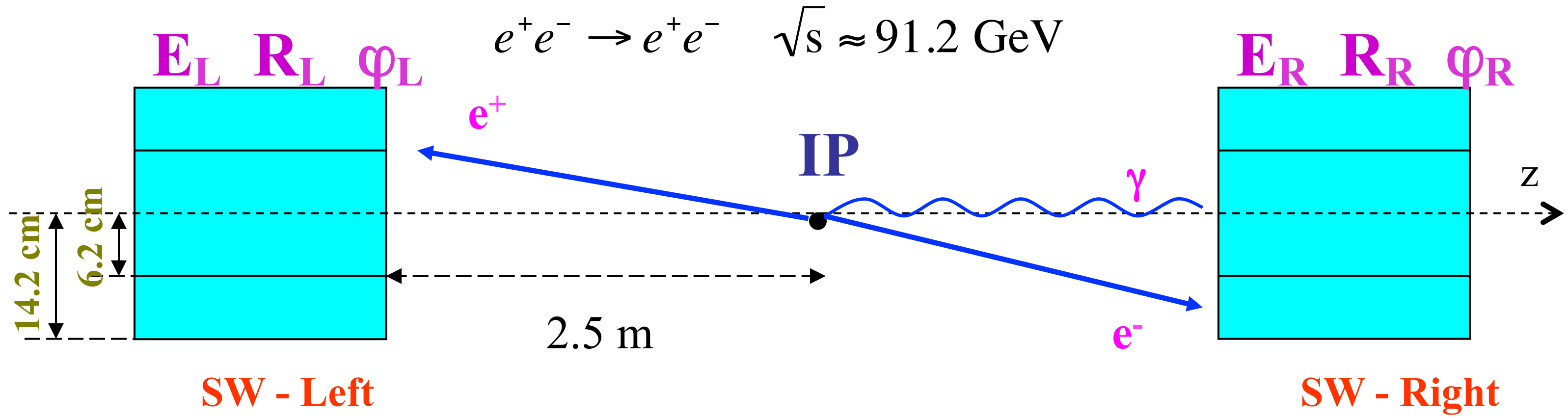
THE EUROPEAN PHYSICAL JOURNAL C

Measurement of the running of the QED coupling in small-angle Bhabha scattering at LEP

The OPAL Collaboration

G. Abbiendi[2], C. Ainsley[5], P.F. Åkesson[3,y], G. Alexander[22], G. Anagnostou[1], K.J. Anderson[9], S. Asai[23], D. Axen[27], I. Bailey[26], E. Barberio[8,p], T. Barillari[32], R.J. Barlow[16], R.J. Batley[5], P. Bechtle[25], T. Behnke[25], K.W. Bell[20], P.J. Bell[1], G. Bella[22], A. Bellerive[6], G. Benelli[4], S. Bethke[32], O. Biebel[31], O. Boeriu[10], P. Bock[11], M. Boutemeur[31], S. Braibant[2], R.M. Brown[20], H.J. Burckhart[8], S. Campana[4], P. Capiluppi[2], R.K. Carnegie[6], A.A. Carter[13], J.R. Carter[5], C.Y. Chang[17], D.G. Charlton[1], C. Ciocca[2], A. Csilling[29], M. Cuffiani[2], S. Dado[21], G.M. Dallavalle[2], A. De Roeck[8], E.A. De Wolf[8,s], K. Desch[25], B. Dienes[30], J. Dubbert[31], E. Duchovni[24], G. Duckeck[31], I.P. Duerdoth[16], E. Etzion[22], F. Fabbri[2], P. Ferrari[8], F. Fiedler[31], I. Fleck[10], M. Ford[16], A. Frey[8], P. Gagnon[12], J.W. Gary[4], C. Geich-Gimbel[3], G. Giacomelli[2], P. Giacomelli[2], R. Giacomelli[2], M. Giunta[4], J. Goldberg[21], E. Gross[24], J. Grunhaus[22], M. Gruwé[8], P.O. Günther[3], A. Gupta[9], C. Hajdu[29], M. Hamann[25], G.G. Hanson[4], A. Harel[21], M. Hauschild[8], C.M. Hawkes[1], R. Hawkings[8], R.J. Hemingway[6], G. Herten[10], R.D. Heuer[25], J.C. Hill[5], D. Horváth[29,c], P. Igo-Kemenes[11], K. Ishii[23], H. Jeremie[18], P. Jovanovic[1], T.R. Junk[6,i], J. Kanzaki[23,u], D. Karlen[26], K. Kawagoe[23], T. Kawamoto[23], R.K. Keeler[26], R.G. Kellogg[17], B.W. Kennedy[20], S. Kluth[32], T. Kobayashi[23], M. Kobel[3], S. Komamiya[23], T. Krämer[25], P. Krieger[6,l], J. von Krogh[11], T. Kuhl[25], M. Kupper[24], G.D. Lafferty[16], H. Landsman[21], D. Lanske[14], D. Lellouch[24], J. Letts[o], L. Levinson[24], J. Lillich[10], S.L. Lloyd[13], F.K. Loebinger[16], J. Lu[27,w], A. Ludwig[3], J. Ludwig[10], W. Mader[3,b], S. Marcellini[2], A.J. Martin[13], T. Mashimo[23], P. Mättig[m], J. McKenna[27], R.A. McPherson[26], F. Meijers[8], W. Menges[25], F.S. Merritt[9], H. Mes[6,a], N. Meyer[25], A. Michelini[2], S. Mihara[23], G. Mikenberg[24], D.J. Miller[15], W. Mohr[10], T. Mori[23], A. Mutter[10], K. Nagai[13], I. Nakamura[23,v], H. Nanjo[23], H.A. Neal[33], R. Nisius[32], S.W. O'Neale[1,*], A. Oh[8], M.J. Oreglia[9], S. Orito[23,*], C. Pahl[32], G. Pásztor[4,g], J.R. Pater[16], J.E. Pilcher[9], J. Pinfold[28], D.E. Plane[8], O. Pooth[14], M. Przybycień[8,n], A. Quadt[3], K. Rabbertz[8,r], C. Rembser[8], P. Renkel[24], J.M. Roney[26], A.M. Rossi[2], Y. Rozen[21], K. Runge[10], K. Sachs[6], T. Saeki[23], E.K.G. Sarkisyan[8,j], A.D. Schaile[31], O. Schaile[31], P. Scharff-Hansen[8], J. Schieck[32], T. Schörner-Sadenius[8,z], M. Schröder[8], M. Schumacher[3], R. Seuster[14,f], T.G. Shears[8,h], B.C. Shen[4], P. Sherwood[15], A. Skuja[17], A.M. Smith[8], R. Sobie[26], S. Söldner-Rembold[16], F. Spano[9], A. Stahl[3,x], D. Strom[19], R. Ströhmer[31], S. Tarem[21], M. Tasevsky[8,s], R. Teuscher[9], M.A. Thomson[5], E. Torrence[19], D. Toya[23], P. Tran[4], I. Trigger[8], Z. Trócsányi[30,e], E. Tsur[22], M.F. Turner-Watson[1], I. Ueda[23], B. Ujvári[30,e], C.F. Vollmer[31], P. Vannerem[10], R. Vértesi[30,e], M. Verzocchi[17], H. Voss[8,q], J. Vossebeld[8,h], C.P. Ward[5], D.R. Ward[5], P.M. Watkins[1], A.T. Watson[1], N.K. Watson[1], P.S. Wells[8], T. Wengler[8], N. Wermes[3], G.W. Wilson[16,k], J.A. Wilson[1], G. Wolf[24], T.R. Wyatt[16], S. Yamashita[23], D. Zer-Zion[4], L. Zivkovic[24]

Small-angle Bhabha scattering in OPAL



2 cylindrical calorimeters encircling the beam pipe at ± 2.5 m from the Interaction Point

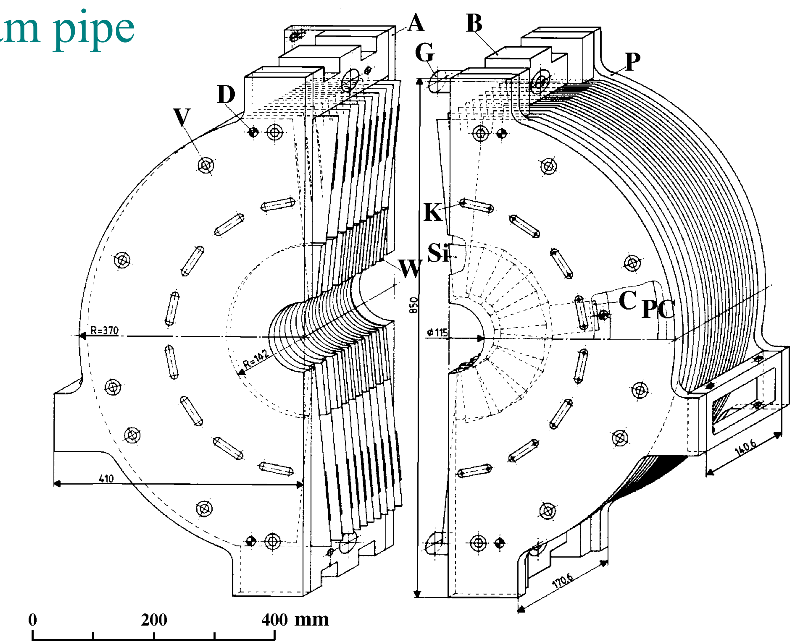
19 Silicon layers

18 Tungsten layers

Total Depth 22 X_0 (14 cm)

Each detector layer divided into 16 overlapping wedges

Sensitive radius: **6.2 – 14.2 cm**, corresponding to **scattering angle** of **25 – 58 mrad** from the beam line



Final Error on Luminosity

After all the effort on Radial reconstruction the dominant systematic error is related to Energy (mostly tail in the E response and nonlinearity) Quantitatively: (OPAL Collaboration, Eur.Phys.J. C14 (2000) 373)

	Systematic Error ($\times 10^{-4}$)
Energy	1.8
Inner Anchor	1.4
Radial Metrology	1.4

Total Experimental Systematic Error : 3.4×10^{-4}

Theoretical Error on Bhabha cross section: 5.4×10^{-4}

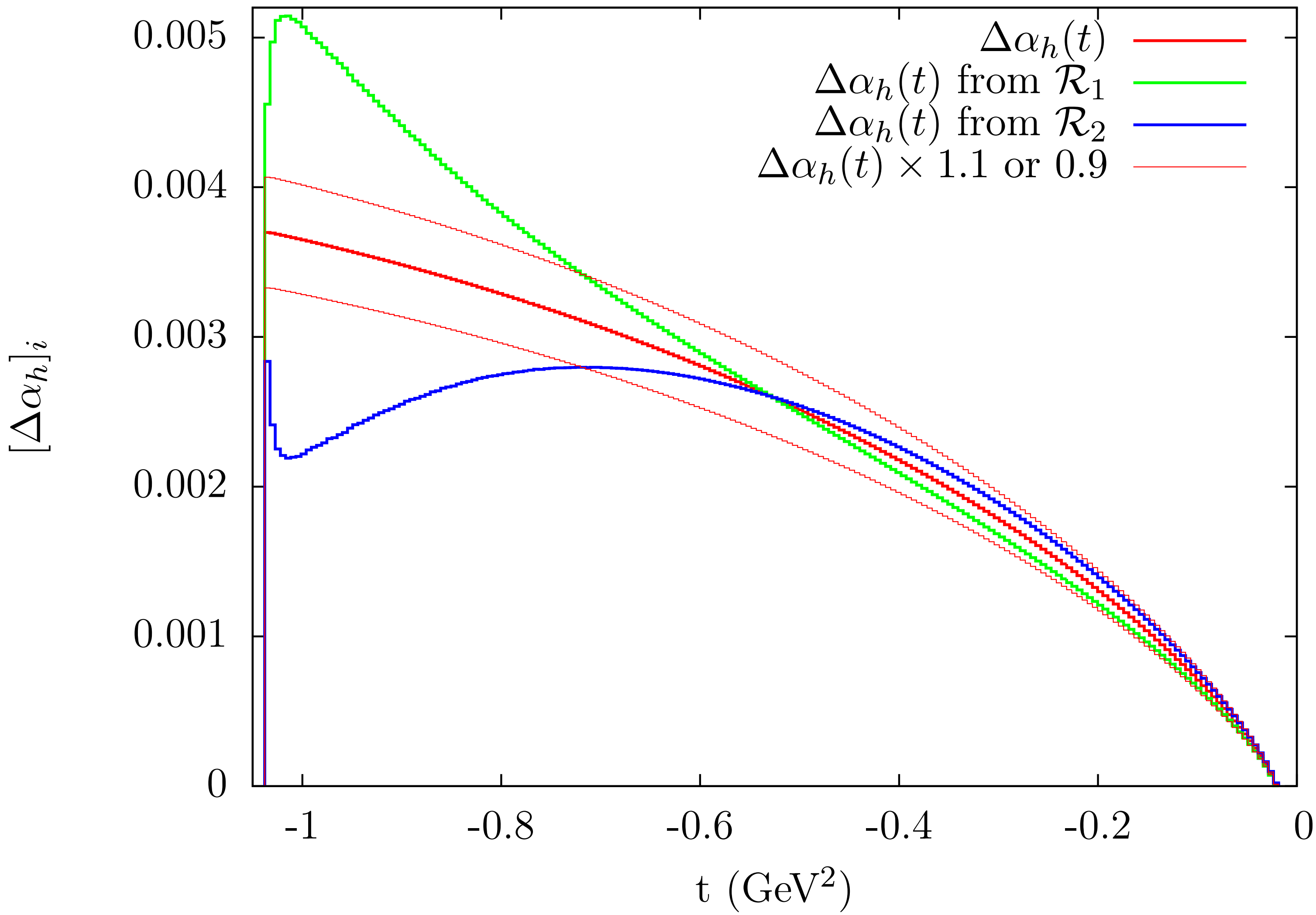
Conclusions

- Measuring α_{em} running in time-like and space like region appears to be very interesting. (Relatively) high q^2-values can be explored at ILC/TLEP
- An alternative formula for a_μ^{HLO} in spacelike region has been studied in details. It emphasizes low values of t (<1 GeV^2) and can be explored at low energy e+e- machines (VEPP2000/ DAFNE, τ/charm, B-factories)
- It requires to measure the Bhabha cross section at relatively small angles at (better than) 10^{-4} accuracy!
- Reaching such an accuracy demands a dedicated experimental and theoretical work for the next few years
- Can this method apply also at other (e^-e^-; fixed target) machines?

Thanks!

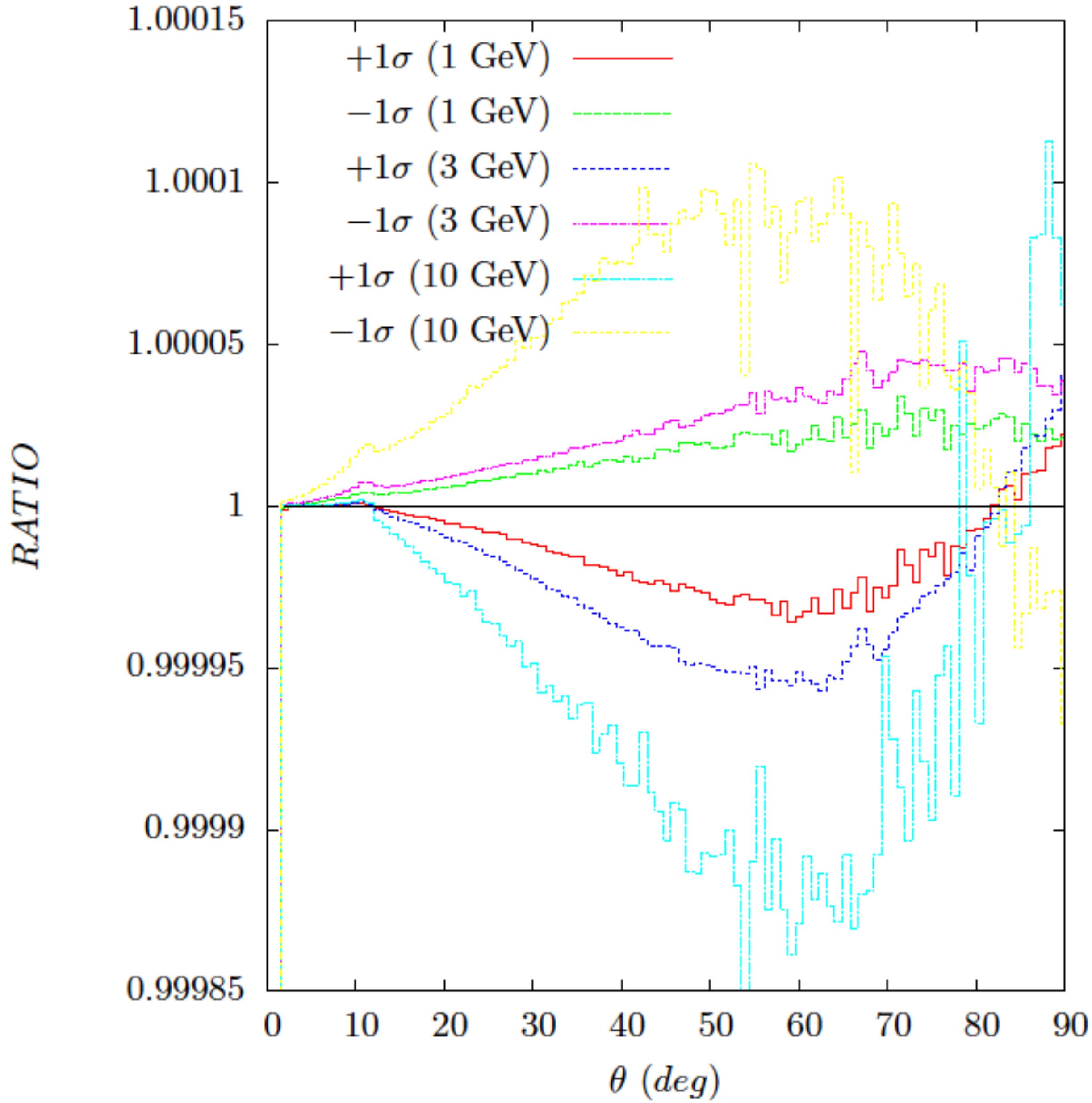
END

test



$\Delta\alpha_h(t)$
$\Delta\alpha_h(t)$ from $\mathcal{R}_1$
$\Delta\alpha_h(t)$ from $\mathcal{R}_2$
$\Delta\alpha_h(t) \times 1.1$ or 0.9
0.005
0.004
0.003
0.002
0.001
0
-1
-0.8
-0.6
-0.4
-0.2
0
$[\Delta\alpha_h]_i$
t (GeV^2)

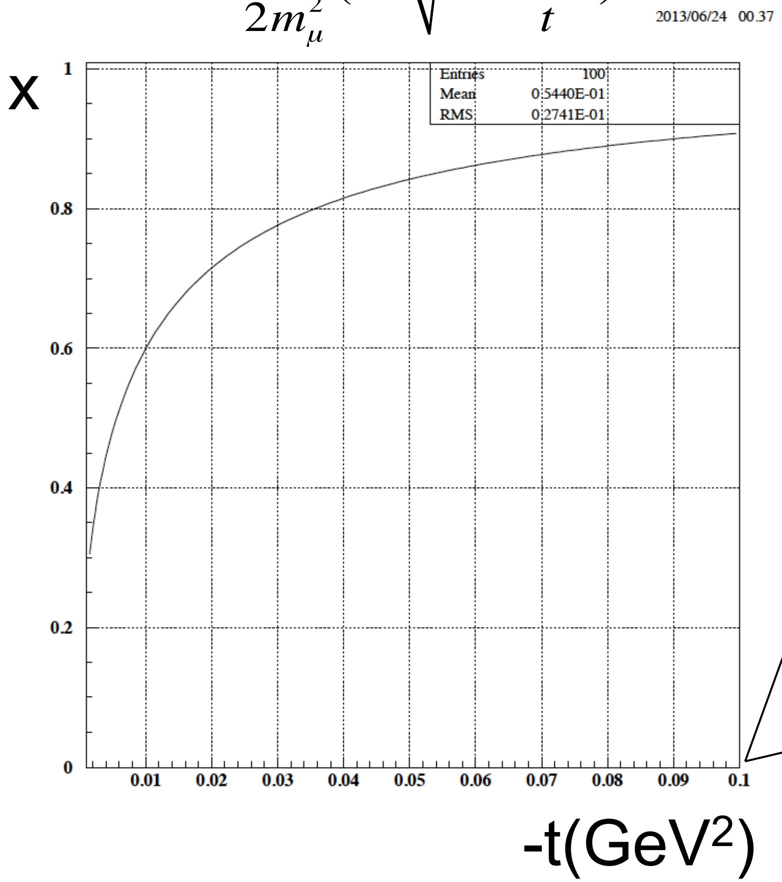
$\Delta\alpha_{em}^{HAD}(s)$ dependence

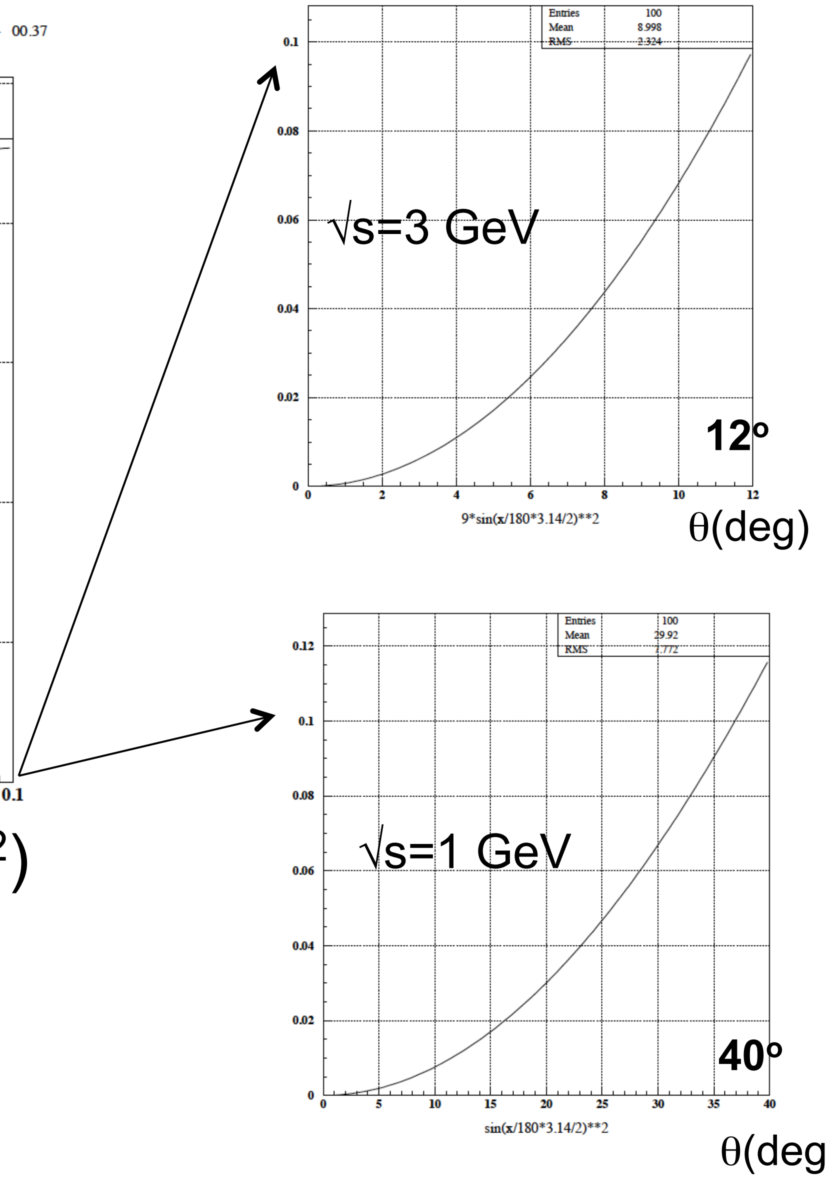


Which is the best energy/angle configuration?

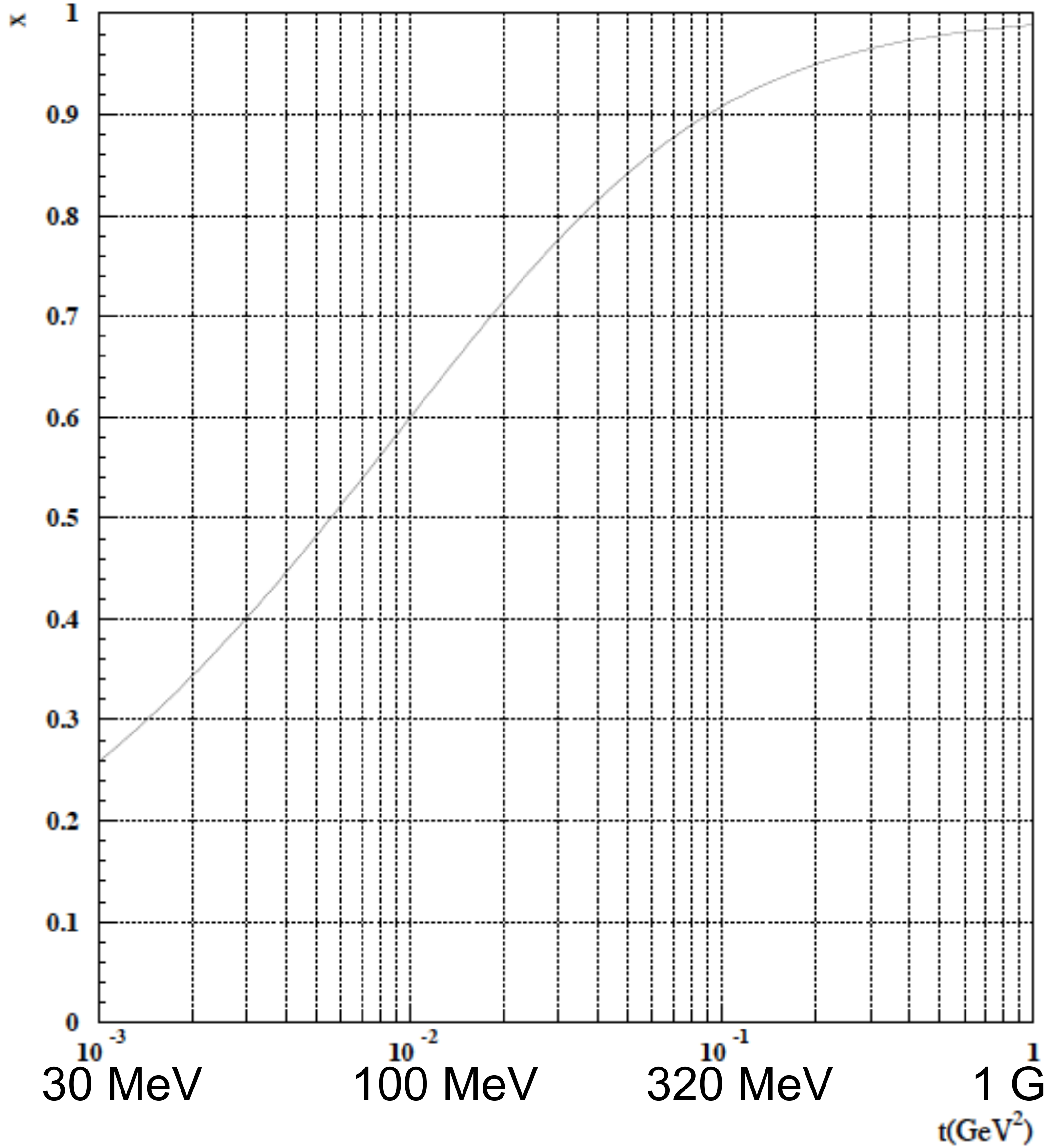
$$-t = 9(1-\cos\theta)/2$$

$$x = \frac{t}{2m_\mu^2}\left(1-\sqrt{1-\frac{4m^2}{t}}\right)$$





x vs t behaviour



x→1
t → ∞

x→0
t→0

30 MeV 100 MeV 320 MeV 1 GeV