Revealing transversity GPDs through the production of a rho meson and a photon

Renaud Boussarie

Laboratoire de Physique Théorique
Orsay

Photon 2015
Budker Institute, Novosibirsk

in collaboration with B. Pire (CPhT, Palaiseau), L. Szymanowski (NCBJ, Warsaw), S. Wallon (LPT Orsay and UPMC)
Transversity of the nucleon using hard processes

**What is transversity?**

- **Transverse spin content of the proton:**
  \[
  |↑⟩(x) \sim |→⟩ + |←⟩ \\
  |↓⟩(x) \sim |→⟩ - |←⟩
  \]
  spin along \(x\) helicity states

- **Observables which are sensitive to helicity flip thus give access to transversity** \(ΔTq(x)\). Poorly known

- **Transversity GPDs are completely unknown**

- **For massless (anti)particles, chirality = (-)helicity**

- **Transversity is thus a chiral-odd quantity**

- **Since QCD and QED are chiral even, the chiral odd quantities which one want to measure should appear in pairs**
Transversity of the nucleon using hard processes: using a two body final state process?

**How to get access to transversity GPDs?**

- the dominant DA of $\rho_T$ is of twist 2 and chiral odd ($[\gamma^\mu, \gamma^\nu]$ coupling)
- unfortunately $\gamma^* N^\uparrow \rightarrow \rho_T N' = 0$
  - This cancellation is true at any order: such a process would require a helicity transfer of 2 from a photon.
- lowest order diagrammatic argument:

\[ \gamma^\alpha [\gamma^\mu, \gamma^\nu] \gamma_\alpha \rightarrow 0 \]
Transversity of the nucleon using hard processes: using a two body final state process?

Can one circumvent this vanishing?

- This vanishing is only occurs at twist 2
- At twist 3 this process does not vanish [Goloskokov, Kroll], [Ahmad, Goldstein, Liuti]
- However processes involving twist 3 DAs may face problems with factorization (end-point singularities)
- One can also consider a 3-body final state process [Ivanov, Pire, Szymanowski, Teryaev], [El Beiyad, Pire, Segond, Szymanowski, Wallon]
Probing transversity using rho meson production

- Processes with 3 body final states can give access to all GPDs
- We consider the process $\gamma N \rightarrow \gamma \rho N'$
- Collinear factorization of the amplitude for $\gamma + N \rightarrow \gamma + \rho + N'$ at large $M_{\gamma\rho}^2$
Master formula based on leading twist 2 factorization

\[ A = \frac{1}{\sqrt{2}} \int_{-1}^{1} dx \int_{0}^{1} dz \left( T^u(x, z) - T^d(x, z) \right) \times \left( H^u_T(x, \xi, t) - H^d_T(x, \xi, t) \right) \Phi_\rho(z) + \cdots \]

- Both the DA and the GPD can be either chiral even or chiral odd.
- At twist 2 the longitudinal rho DA is chiral even and the transverse rho DA is chiral odd.
- Hence we will need both chiral even and chiral odd non-perturbative building blocks and hard parts.
Helicity flip GPD at twist 2:

\[
\int \frac{dz^{-}}{4\pi} e^{ix P^{+} z^{-}} \langle p_2, \lambda_2 | \bar{\psi}_q \left( -\frac{1}{2} z^{-} \right) i \sigma^{+i} \psi \left( \frac{1}{2} z^{-} \right) | p_1, \lambda_1 \rangle \\
= \frac{1}{2P^{+}} \bar{u}(p_2, \lambda_2) \left[ H^q_T(x, \xi, t) i \sigma^{+i} + \bar{H}^q_T(x, \xi, t) \frac{P^{+} \Delta^i - \Delta^{+} P^i}{M_N^2} \right. \\
+ \left. E_T^q(x, \xi, t) \frac{\gamma^{+} \Delta^i - \Delta^{+} \gamma^i}{2M_N} + \bar{E}_T^q(x, \xi, t) \frac{\gamma^{+} P^i - P^{+} \gamma^i}{M_N} \right] u(p_1, \lambda_1)
\]

- We will consider the simplest case when $\Delta_{\perp} = 0$.
- In that case and in the forward limit $\xi \to 0$ only the $H^q_T$ term survives.

Transversity DA at twist 2:

\[
\langle 0 | \bar{u}(0) \sigma^{\mu\nu} u(x) | \rho^0(p, s) \rangle = \frac{i}{\sqrt{2}} \left( \sigma^{\mu}_\rho p^\nu - \sigma^{\nu}_\rho p^\mu \right) f_{\rho}^\perp \int_0^1 du \ e^{-iu p \cdot x} \phi_\perp(u)
\]
Helicity conserving GPDs at twist 2:

\[
\int \frac{dz^-}{4\pi} e^{ixP^+z^-} \langle p_2, \lambda_2 | \bar{\psi} q \left( -\frac{1}{2} z^- \right) \gamma^+ \psi \left( \frac{1}{2} z^- \right) | p_1, \lambda_1 \rangle \\
= \frac{1}{2P^+} \bar{u}(p_2, \lambda_2) \left[ H^q(x, \xi, t) \gamma^+ + E^q(x, \xi, t) \frac{i\sigma^+ + \Delta^\alpha}{2m} \right]
\]

\[
\int \frac{dz^-}{4\pi} e^{ixP^+z^-} \langle p_2, \lambda_2 | \bar{\psi} q \left( -\frac{1}{2} z^- \right) \gamma^+ \gamma^5 \psi \left( \frac{1}{2} z^- \right) | p_1, \lambda_1 \rangle \\
= \frac{1}{2P^+} \bar{u}(p_2, \lambda_2) \left[ \tilde{H}^q(x, \xi, t) \gamma^+ \gamma^5 + \tilde{E}^q(x, \xi, t) \frac{\gamma^5 \Delta^+}{2m} \right]
\]
Helicity conserving DAs at twist 2:

\[ \langle 0 | \bar{u}(0) \gamma^\mu \gamma^5 u(x) | \rho^0(p, s) \rangle = -\frac{1}{4\sqrt{2}} \epsilon_{\mu\nu\sigma\delta} \epsilon_\nu p_\sigma x_\delta f_\rho m_\rho \int_0^1 du e^{-iup\cdot x} g_{(a)}^\perp(u) \]
Kinematics to handle GPD in a 3-body final state process

- use a Sudakov basis:
  light-cone vectors $p$, $n$ with $2p \cdot n = s$

- assume the following kinematics:
  - $\Delta_{\perp} \sim 0$
  - $M^2$, $m_{\pi}^2$, $m_{\rho}^2 \ll M_{\pi\rho}^2$

- initial state particle momenta:
  \[ q^\mu = n^\mu, \quad p_1^\mu = (1 + \xi) p^\mu + \frac{M^2}{s(1 + \xi)} n^\mu \]

- final state particle momenta:
  \[
  p_2^\mu = (1 - \xi) p^\mu + \frac{M^2}{s(1 - \xi)} n^\mu \\
  k^\mu = \alpha n^\mu + \frac{p_t^2}{\alpha s} p^\mu + \rho^\mu \\
  p_\rho^\mu = \alpha_\rho n^\mu + \frac{p_t^2 + m_{\rho}^2}{\alpha_\rho s} p^\mu - p_\perp^\mu
  \]
20 diagrams to compute

The other half can be deduced by $q \leftrightarrow \bar{q}$ (anti)symmetry

Red diagrams cancel in the chiral odd case
The $z$ and $x$ dependence of the amplitude can be factorized

$$A = \mathcal{N}(z, x) T^i$$

$$T^i = (1 - \alpha) \left[ (\epsilon_{q \perp} \cdot p_{\perp}) (\epsilon_{k \perp} \cdot \epsilon_{\rho \perp}) - (\epsilon_{k \perp} \cdot p_{\perp}) (\epsilon_{q \perp} \cdot \epsilon_{\rho \perp}) \right] p_{\perp}^i$$

$$- (1 + \alpha) (\epsilon_{\rho \perp} \cdot p_{\perp}) (\epsilon_{k \perp} \cdot \epsilon_{q \perp}) p_{\perp}^i + \alpha (\alpha^2 - 1) \xi (\epsilon_{q \perp} \cdot \epsilon_{k \perp}) \epsilon_{\rho}^i$$

$$- \alpha (\alpha^2 - 1) \xi \left[ (\epsilon_{q \perp} \cdot \epsilon_{\rho \perp}) \epsilon_{k \perp}^i - (\epsilon_{k \perp} \cdot \epsilon_{\rho \perp}) \epsilon_{q \perp}^i \right]$$

Hence calculating differential cross sections is simple:

$$d\sigma \propto \left| \int_0^1 dz \int_{-1}^1 dx \mathcal{N}(z, x) \phi_{\rho}(z) H_T^q(x) \right|^2 \sum_{\text{helicities},(i,j)} T^i T^j$$
A model based on Double Distribution

Realistic Parametrization of $H_T^q$

- GPDs can be represented in terms of **Double Distribution** (Radyushkin) based on Schwinger representation of a toy model for GPDs which has the structure of a triangle diagram in scalar $\phi^3$ theory

\[
H_T^q(x, \xi, t = 0) = \int_{-1}^{1} d\beta \int_{-1+|\beta|}^{1-|\beta|} d\alpha \, \delta(\beta + \xi \alpha - x) \, f_T^q(\beta, \alpha)
\]

- ansatz for these Double Distribution (Radyushkin):
  - $f_T^q(\beta, \alpha) = \Pi(\beta, \alpha) \, \Delta_T q(\beta)$
  - $\Delta_T q(x) :$ chiral-odd PDF (Anselmino et al.)
  - $\Pi(\beta, \alpha) = \frac{3}{4} \frac{(1-\beta)^2 - \alpha^2}{(1-\beta)^3} :$ profile function ($f_T^q(\beta, 0) = \Delta_T q(\beta)$)

- ansatz for the $t$-dependence:

\[
H_T^q(x, \xi, t) = H_T^q(x, \xi, t = 0) \times F_H(t)
\]

with $F_H(t) = \frac{C^2}{(t-C)^2}$ a standard dipole form factor ($C = 0.71$ GeV)
The chiral even case

- All 20 (10) diagrams must be computed, both with vector and axial coupling.

- The $z$ and $x$ dependence does not factorize.

- No problem with integration over GPDs. Results will be available very soon.
Conclusion

- This is still work in progress but predictions for cross sections and counting rates will be ready very soon.

- Our result will also be applied to electroproduction ($Q^2 \neq 0$) after adding Bethe-Heitler contributions and interferences.

- This mechanism will give us access to transversity GPDs but also to the usual GPDs by analogy with Timelike Compton Scattering, the $\gamma \rho$ pair playing the role of the $\gamma^*$. 

- Possible measurement in JLAB and in COMPASS