Low-energy hadronic cross sections measurements at BaBar, and implication for the g-2 of the muon

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**PEP-II e+e- collider, Baar detector**

- $E_+ = 3.1 \text{ GeV}$, $E_- = 9 \text{ GeV}$

- $E_{\text{CM}} = M(\Upsilon(4S)) = 10.6 \text{ GeV}$
- 2000 - 2008 yrs
- $\Delta L = 500 \text{ fb}^{-1}$
- $N(B) = 10^9$

**Equation**

$$
\frac{d\sigma(s,x)}{dx d(\cos \theta)} = W(s,x,\theta) \cdot \sigma_0(s(1-x)),$$

$$W(s,x,\theta) = \frac{\alpha}{\pi x} \left( \frac{2 - 2x + x^2}{\sin^2 \theta} - \frac{x^2}{2} \right), \quad x = \frac{2E_+}{\sqrt{s}}$$

$\theta$ - photon polar angle in c.m.
ISR study at BaBar

- ISR at BaBar gives competitive (even dominates!) statistics
- BaBar has excellent capability for ISR study
- Many major hadronic processes have been studied

Some reactions are being updated to the full BaBar data with ~500 fb⁻¹
To calculate $R$ in the energy range $1-2$ GeV the processes $\pi^+\pi^-3\pi^0$, $\pi^+\pi^-4\pi^0$, $K_S K_L$, $K_S K_L\pi^+\pi^-$, $K_S K^+ \pi^-\pi^0$ are under study. $\pi^+\pi^-2\pi^0$ will come soon. Work is in progress for $K_S K_L\pi^0\pi^0$, $K_S K_L\pi^0$.
BaBar measured: $e^+e^- \rightarrow \pi^+\pi^-$

Motivation: dominance of the E<1GeV region, accessed through $\pi^+\pi^-$

Features of the cross section distribution
- Includes possible FSR
- Dominated by $\rho(770)$ resonance
- $\rho-\omega$ interference
- Dip at 1.6GeV: interference between $\rho'$ and $\rho''$
- Dip at 2.2GeV: higher mass $\rho$ state

LO Hadr., 0.28-1.8 GeV
Babar
$a_\mu=(514.1\pm3.8) 10^{-10}$
all $e^+e^-$
$a_\mu=(505.8\pm3.0) 10^{-10}$

Systematic uncertainties at the $\rho$ region
BABAR: 0.5%
CMD2: 0.8%
SND: 1.5%
KLOE: 0.8%
BaBar measured: $e^+e^- \rightarrow K^+K^-$

Published Phys. Rev. D 88, 032013 (2013)

Our result is more precise than the current world average
BaBar updated: $e^+e^- \rightarrow \pi^+\pi^-\pi^+\pi^-$

Published PRD 85 112009 (2012)
Based on 454 fb$^{-1}$ dataset (statistical uncertainties are shown)
Our result is more precise than the current world average (<3% systematic error)
Contribution of missing channels to $a_\mu$

<table>
<thead>
<tr>
<th>SM-to-experiment comparison [in units $10^{-10}$]</th>
</tr>
</thead>
<tbody>
<tr>
<td>QED</td>
</tr>
<tr>
<td>Leading hadronic vacuum polarization (VP)</td>
</tr>
<tr>
<td>Sub-leading hadronic vacuum polarization</td>
</tr>
<tr>
<td>Hadronic light-by-light</td>
</tr>
<tr>
<td>Weak (incl. 2-loops)</td>
</tr>
<tr>
<td>Theory</td>
</tr>
<tr>
<td>Experiment</td>
</tr>
<tr>
<td>Exp − theory</td>
</tr>
</tbody>
</table>

$|a_\mu (\sqrt{s} < 1.8 \text{ GeV})| K^+K^- 2(\pi^+ \pi^-) 3(\pi^+ \pi^-) 2(\pi^+ \pi^- \pi^0)$

<table>
<thead>
<tr>
<th></th>
<th>without BABAR</th>
<th>with BABAR</th>
</tr>
</thead>
<tbody>
<tr>
<td>$a_\mu (\sqrt{s} &lt; 1.8 \text{ GeV})$</td>
<td>21.63 ± 0.70</td>
<td>22.95 ± 0.26</td>
</tr>
<tr>
<td>$K^+K^-$</td>
<td>13.35 ± 0.90</td>
<td>13.64 ± 0.36</td>
</tr>
<tr>
<td>$2(\pi^+ \pi^-)$</td>
<td>0.10 ± 0.10</td>
<td>0.11 ± 0.02</td>
</tr>
<tr>
<td>$3(\pi^+ \pi^-)$</td>
<td></td>
<td>0.89 ± 0.09</td>
</tr>
<tr>
<td>$2(\pi^+ \pi^- \pi^0)$</td>
<td></td>
<td>1.42 ± 0.30</td>
</tr>
</tbody>
</table>

Missing channels contribute 5.98 ± 0.42 or 12.46 ± 0.76 if $\sqrt{s} < 2.0 \text{ GeV}$

Contribution from $KK\pi$, $KK2\pi$, $2\pi3\pi^0$, $2\pi4\pi^0$, $(7\pi, 8\pi)$... added using iso-spin relations (in particular, using measured $K^+K^-\pi^+\pi^- (\pi^0\pi^0)$ channels)
The cross section comparison – BaBar data

Naively expect: $N(K^+K^-\pi^+\pi^-) = 2 \times N(K^+K^-\pi^0\pi^0)$

Many intermediate states break the relation.

Study of intermediate states is important!
Decomposition of $K^+K^-\pi^+\pi^-$ mass spectrum

$K^+K^-\pi^+\pi^-$
$K^{*0}(892)K\pi$
$K^+K^-\rho(770)$
$\phi\pi^+\pi^-$
$K_2^{*0}(1430)K\pi$

Tables with cross sections (corrected for BF) are provided

We present (with more details) results on the study of the processes:

\[ e^+ e^- \rightarrow K_S K_L, \ K_S K_L \pi^+ \pi^-, \ K_S K_S \pi^+ \pi^-(K^+ K^-) \]

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And new (preliminary) results on the process

\[ e^+ e^- \rightarrow K_S K^+ K^- \]

ready for publication

Based on 469 fb\(^{-1}\) integrated luminosity.
**K_S selection (in \( \pi^+\pi^- \) decay)**

Loop over all K_S candidates with ISR photon with \( E_\gamma > 3 \) GeV, and select events with:

- Good quality K_S coming from IP
- No electron ID for either charged track

Simulation of \( \phi \gamma \rightarrow K_S K_L \gamma \) compare to data

Dominated by \( \phi \gamma \rightarrow K_S K_L \gamma \) process if require NO additional tracks from IP
\[ e^+e^- \rightarrow \phi \gamma \rightarrow K_SK_L \gamma \text{ (without } K_L \text{ detection)} \]

\[ E_0 = E^+ + E^- \]
\[ p_0 = p^+ + p^- \]
\[ p_\gamma = nE_\gamma \]

Assuming \( e^+e^- \rightarrow \phi\gamma \) reaction
Use \( \phi \) mass to get \( E_{\gamma\text{ISR}} \)

\[ E^c_\gamma = \frac{E_0^2 - p_0^2 - m_\phi^2}{2\left(E_0 - p_0 \cdot n_\gamma\right)} \]

Using energy-momentum conservation and detected \( K_S \)
we determine \( K_L \) mass and direction:

\[ m^2(K_L) = \left(E^+ + E^- - E^c_\gamma - E_{K_S}\right)^2 - \left(p^+ + p^- - p^c_\gamma - p_{K_S}\right)^2 \]

Using this events we can study \( K_L \) detection.
**K_L mass using φ mass constraint**

Very low background!

MC normalized to two bins at peak

After background subtraction (5.6%) we have $81012\pm285$ events (447434±669 MC).

We estimate ~0.5% systematic error for background subtraction uncertainty.
How $K_L$ cluster in Calorimeter looks like?

1. **Search for EMC cluster closest to $K_L$ direction:**

   - **Energy cut:** $E(K_L) > 0.2$ GeV cut is set
   - **Angular cut:** Apply loose cut $d\Psi < 0.5$

   Data/MC = $0.9394 \pm 0.0052$ (0.6%)
Use events with $\chi^2<15$ and reconstructed parameters of $K_S$ and $K_L$ to calculate $m(K_{S}K_{L})$.
**Fit to \( \phi \) parameters**

**CMD-2**
\[
\begin{align*}
\sigma_0 &= 1376 \pm 6 \pm 23 \text{ nb} \\
m &= 1019.483 \pm 0.011 \pm 0.025 \text{ MeV/c}^2 \\
\Gamma_0 &= 4.280 \pm 0.033 \pm 0.025 \text{ MeV} \\
\Gamma_{ee} &= 1.235 \pm 0.006 \pm 0.022 \text{ keV}
\end{align*}
\]

**PDG2010-2012**
\[
\begin{align*}
m &= 1019.455 \pm 0.020 \text{ MeV/c}^2 \\
\Gamma_0 &= 4.26 \pm 0.04 \text{ MeV} \\
\Gamma_{ee} &= 1.27 \pm 0.04 \text{ keV} \\
B_{ee} \cdot B_{KS KL} &= 0.342 \pm 0.004 \\
B_{ee} \cdot B_{KS KL} &= 1.006 \pm 0.016
\end{align*}
\]

**BaBar**
\[
\begin{align*}
\Gamma_{ee} \cdot B_{KS KL} &= 0.4200 \pm 0.0033 \pm 0.0122 \pm 0.0019 \text{ keV} \\
\Gamma_{ee} &= 1.228 \pm 0.037 \pm 0.014 \text{ (PDG BKSKL) keV} \\
B_{ee} \cdot B_{KS KL} &= 0.986 \pm 0.030 \pm 0.009 \text{ (PDG } \Gamma KS KL) 
\end{align*}
\]

**Systematic uncertainty:**
- \( K_L \) efficiency (+ \( \chi^2 \)) 0.6%
- BGFilter efficiency 2.3%
- \( K_S \) efficiency 1.0%
- ISR photon 0.5%
- Luminosity 0.5%
- Rad.corr. 1.0%
- Track overlap 0.6%
- Background sub. 2.9%
$e^+e^- \rightarrow K_S K_L$ cross section for $m(K_S K_L) > 1.06$ GeV

After background subtraction, the systematic error is ~10% (~30% for $\sigma < 0.3$ nb), dominated by the background subtraction procedure.
Is it $\phi(1680)$ ? Fit with single BW

$$\alpha(s) = \frac{P(s)}{s^{5/2}} \left| \frac{A_{\phi(1020)}}{\sqrt{P(m_\phi)}} + \frac{A_X}{\sqrt{P(m_X)}} \cdot e^{i\varphi} + A_{bkg} \right|^2$$

$$P(s) = \left(\frac{s}{2}\right)^2 - m_{K^0}^2 \right)^{3/2}$$

$$A(s) = \frac{\Gamma(m^2) \cdot m^3 \sqrt{\sigma_0 \cdot m}}{s - m^2 + i\sqrt{s}\Gamma(s)}$$

$$\Gamma(s) = \Gamma \cdot \sum_f B_f \cdot \frac{P_f(s)}{P_f(m_f^2)}$$

$$A_{\phi(1020)} = A_{\phi} + A_{\omega} - A_{\rho}, \quad f = K^* K, \phi\eta, \phi\pi\pi, K_SK_L$$

$\sigma_0 = 0.46 \pm 0.10 \pm 0.04 \text{ nb}$

$m = 1674 \pm 12 \pm 6 \text{ MeV}/c^2$

$\Gamma_0 = 165 \pm 38 \pm 70 \text{ MeV}$

$\varphi = 3.01 \pm 0.38 - \text{fixed to } \pi$

$\sigma_{bkg} = 0.36 \pm 0.18 \text{ nb}$

$\Gamma_{ee} \cdot B_{KSKL} = 14.3 \pm 2.4 \pm 1.5 \pm 6.0 \text{ eV}$

Simultaneous $K_SK_L$ and $K^+K^-$ (and $\pi\pi$) fit is needed to separate $l=0,1$ states and $\omega(1420, 1650)$, $\rho(1450, 1700)$ contribution
$K_S K_L \pi^+ \pi^- \gamma$ event selection

- Select (best) $K_S$
- Select ISR photon with $E > 3$ GeV
- Two additional tracks from IP (no kaon ID)
- Cycle over remaining clusters with $E > 0.2$ GeV – $K_L$ candidates
- Best $\chi^2$ for 3C fit ($K_L$ momentum float)
- $\chi^2 > 100$ and $\text{Im} \gamma \gamma_L - 0.135 > 0.03$ for the $K_S K \pi \pi^0 \gamma$ hypothesis

1580 events after background subtraction

No other measurements exist
Some mass distributions in $K_S K_L \pi^+ \pi^- \gamma$

![Graphs showing mass distributions for $m(K_S \pi^+)$ and $m(K_L \pi^+)$](image)

Very clear $K^*(892)^\pm$ signals with
1322 ± 70 for $K^{*\pm}(K_S \pi)$ and 1362 ± 78 for $K^{*\pm}(K_L \pi)$
Plus 183 ± 48 events for $K_2(1430)^\pm$

How large is $K^*(892)^+K^*(892)^-$?
Fit slice in $m(K_L \pi^\mp)$ for number of $K_S \pi^{\mp}$

Very clear signal with 913 ± 37 events (70%) of $K^*(892)^+K^*(892)^-$ correlated production!
And 90 ± 16 for $K^*(892)^+K_2^*(1430)^-$.

We have negligible contribution from $K^*(892)^0K^*(892)^0$
from our $K^+K^-\pi^+\pi^-$ analysis!
\( \phi(1020) \pi^+ \pi^- \) contribution in \( K_S K_L \pi^+ \pi^- \gamma \)

\[ 230 \pm 20 \text{ ev.} \]

\[ 0.85 < m(\pi\pi) < 1.1 \]

\( \phi \pi^+ \pi^- (\phi f_0(980)) \)

seen as expected in agreement with our \( K^+ K^- \pi^+ \pi^- \) study
$K_S K_S \pi^+ \pi^- (K^+ K^-) \gamma$ event selection

- Select 2 (best) $K_S$
- Select ISR photon with $E > 3$ GeV
- Two additional tracks from IP with pion or kaon ID
- Best $\chi^2$ for 4C fit assuming $K_S K_S \pi^+ \pi^- (K^+ K^-) \gamma$ hypotheses

About 3000 ISR events with 2 good $K_S$

Simulation of $\phi \gamma \rightarrow K_S K_L \gamma$

compare to data

Six tracks with ISR photon – very low background!
$e^+e^- \rightarrow K_S K_S \pi^+ \pi^- (K^+K^-)$ cross sections

No other measurements exist

$J/\psi$ is excluded
Some mass distributions in $K_S K_S \pi^+ \pi^-$

Very clear $K^*(892)^\pm$ signals with 829 ± 49 for $K^{*+} (K_S \pi^+)$ and 856 ± 50 for $K^{*-} (K_S \pi^-)$

Plus 116 ± 40 (70±34) events for $K_2(1430)^\pm$

How large is $K^*(892)^+K^*(892)^-$?

Fit slice in $m(K_S \pi^-)$ for number of $K_S \pi^+$

Very clear signal with 742 ± 30 ± 100 events (50%) of $K^*(892)^+K^*(892)^-$ correlated production!

No $K^*(892)^+K_2^*(1430)^+$ seen.
Some mass distributions in $K_S K_S \pi^+ \pi^-$

If we exclude $K^*(892)^+ K^*(892)^-$ by $|m(K_S \pi) - m(K^*)| < 0.15 \text{ GeV/c}^2$ in both combinations:

Plus some number of $K^*(892) K_S \pi$ events
Some mass distributions in $K_S K_S K^+ K^-$

$N (K^+ K^- f_{2}') = 29 \pm 7$ events

$m(K_S K_S) = 1.526 \pm 0.007 \text{ GeV/c}^2$

$\Gamma = 0.037 \pm 0.013 \text{ GeV}$

PDG:

$m(f_{2}') = 1.525 \pm 0.005 \text{ GeV/c}^2$

$\Gamma = 0.073 \pm 0.006 \text{ GeV}$
$K_S K_L \pi^+ \pi^-$, $K_S K_S \pi^+ \pi^-$ signal decomposition

![Graphs showing event distributions for $m(K_S K_L \pi^+ \pi^-)$ and $m(K_S K_S \pi^+ \pi^-)$ with different signal components graphed.](image-url)
The cross section comparison – BaBar data

Naively expect:
\[ N(K^+K^-\pi^+\pi^-) = 2 \times N(K^+K^-\pi^0\pi^0) \]
Iso-spin relations for $K^+K^-\pi^+\pi^+$ vs. $K^+K^-\pi^0\pi^0$ vs. $K_SK_L\pi^+\pi^-$ vs. $K_SK_S\pi^+\pi^-$

Only $K^*(892)^+K^*(892)^-$ contribution can be compared using iso-spin relations, and we expect:

\[ N(K^+K^-\pi^0\pi^0) = \frac{1}{4} N(K^0K^0 \pi^+\pi^-) \]
\[ N(K_SK_L\pi^+\pi^-) = \frac{1}{2} N(K^0K^0 \pi^+\pi^-) \]
\[ N(K_SK_S\pi^+\pi^-) = N(K_LK_L\pi^+\pi^-) = \frac{1}{4} N(K^0K^0 \pi^+\pi^-) \]

We detect correlated pairs:

\[ N(K^+K^-\pi^0\pi^0) = 1750 \pm 60 \quad \text{eff}= 8\% \]
\[ N(K_SK_L\pi^+\pi^-) = 2098 \pm 209 \quad \text{eff}= 5\% \]
\[ N(K_SK_S\pi^+\pi^-) = 742 \pm 104 \quad \text{eff}= 4.5\% \]

Should be equal numbers after efficiency normalized to 5% and iso-spin correction:

\[ 2188 \pm 76 \sim 2098 \pm 209 \sim 1648 \pm 232 \]

Some tension (~2 sigma)

30% 63% 50% of all events – how the rest are added to the g-2 calculation?
$K_S K^{-+} \pi^{+-} \pi^0(\eta) \gamma$ event selection

- Select 1 (best) $K_S$
- Select ISR photon with $E > 3$ GeV
- Two additional tracks from IP with pion or kaon ID
- Loop over remaining photons in $\pi^0$ or $\eta$ mass windows
- Best $\chi^2$ for 5C fit assuming $K_S K^{-+} \pi^{+-} \pi^0(\eta) \gamma$ hypotheses

No other measurements exist
We observe a $J/\psi$ signal in all studied channels.

$N = 24.6 \pm 7.5$

$N = 154 \pm 19$

$N = 248 \pm 27$

$N = 28.5 \pm 5.1$

$N = 393 \pm 23$

$N = 44 \pm 7$
# J/ψ decay results

<table>
<thead>
<tr>
<th>Measured Quantity</th>
<th>Measured value (eV)</th>
<th>This work Br $(10^{-3})$ $\Gamma_{ee} = 5.55 \pm 0.14$ keV</th>
<th>PDG 2014 Br $(10^{-3})$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\Gamma_{ee} \cdot \text{Br}(J/\psi \to K_S K_L)$</td>
<td>1.13±0.34±0.11</td>
<td>0.20 ± 0.06 ± 0.02</td>
<td>0.146 ± 0.026 $S=2.7$</td>
</tr>
<tr>
<td>$\Gamma_{ee} \cdot \text{Br}(J/\psi \to K_S K_L \pi^+ \pi^-)$</td>
<td>20.9±2.7±2.1</td>
<td>3.7 ± 0.6 ± 0.4</td>
<td>no entry</td>
</tr>
<tr>
<td>$\Gamma_{ee} \cdot \text{Br}(J/\psi \to K_S K_{S} \pi^+ \pi^-)$</td>
<td>9.3±0.9±0.5</td>
<td>1.68 ± 0.16 ± 0.08</td>
<td>no entry</td>
</tr>
<tr>
<td>$\Gamma_{ee} \cdot \text{Br}(J/\psi \to K_S K_S K^+ K^-)$</td>
<td>2.3±0.4±0.1</td>
<td>0.42 ± 0.08 ± 0.02</td>
<td>no entry</td>
</tr>
<tr>
<td>$\Gamma_{ee} \cdot \text{Br}(J/\psi \to K_S K_S \phi) \cdot \text{Br}(\phi \to K^+ K^-)$</td>
<td>1.6±0.4±0.1</td>
<td>0.58 ± 0.14 ± 0.03</td>
<td>no entry</td>
</tr>
<tr>
<td>$\Gamma_{ee} \cdot \text{Br}(J/\psi \to f_2' \phi) \cdot \text{Br}(\phi \to K^+ K^-)$ \cdot $B(f_2' \to K_S K_S)$</td>
<td>0.88±0.34±0.04</td>
<td>0.45±0.17 ± 0.02</td>
<td>0.8 ± 0.4 $S=2.7$</td>
</tr>
<tr>
<td>$\Gamma_{ee} \cdot \text{Br}(J/\psi \to K_S K^+ \pi^- \pi^0)$</td>
<td>31.7±1.9±1.8</td>
<td>5.7 ± 0.3 ± 0.4</td>
<td>no entry</td>
</tr>
<tr>
<td>$\Gamma_{ee} \cdot \text{Br}(J/\psi \to K_S K^+ \pi^- \eta)$</td>
<td>7.3±1.4±0.4</td>
<td>1.30 ± 0.25 ± 0.07</td>
<td>2.2 ± 0.4</td>
</tr>
</tbody>
</table>

We measure:

$B(J/\psi \to f_2') = (0.48 \pm 0.18) \cdot 10^{-3}$ (MarkII)
$B(J/\psi \to f_2') = (1.23 \pm 0.026 \pm 0.20) \cdot 10^{-3}$ (DM2)

\[ B_{J/\psi \to f} \cdot \Gamma_{ee}^{J/\psi} = \frac{N_{J/\psi \to f} \cdot m_{J/\psi}^2}{6\pi^2 \cdot dL/dE \cdot \epsilon_f (m_{J/\psi}) \cdot C} \]
Summary

• BaBar continues analysis of collected data and ISR studies in particular

• Most published results for $e^+e^- \rightarrow \text{hadrons}$ reactions have the best to date accuracy.

• Recently obtained $e^+e^- \rightarrow K_S K_L \pi^+ \pi^-$, $K_S K_S \pi^+ \pi^-$, $K_S K_S K^+ K^-$, $K_S K^- \pi^+ \pi^0(\eta)$ cross sections were never studied before. Intermediate states study is performed.

• Using these cross sections we can reduce uncertainty in the muon g-2 calculation.

• $J/\psi$ decays to above modes have been measured for the first time.

• Results for $K_S K_L \pi^0(\pi^0)$ final state should come out soon – it will completely close iso-spin relations problem in the g-2 calculation for the $KK\pi\pi$ modes.
SM prediction for muon $g-2$

$$a_{\mu}^{\text{experimental}} = \frac{(g-2)^{\mu}}{2}$$

11659 ± 208.9

$\pm 6.3 \times 10^{-10}$ world average

$\delta a_{\mu}$ contribution to precision at $\sqrt{s} < 1.8$ GeV

- $\pi^+\pi^-$: 505.65 ± 3.09
- $\pi^+\pi^- 2\pi 0$: 18.62 ± 1.15
- $\pi^+\pi^- \pi 0$: 47.38 ± 0.99 (mostly from omega region)
- $2\pi + 2\pi^-$: 13.64 ± 0.36 (buckler)
- K+K-: 22.95 ± 0.26 (buckler)

From isospin relations $5.98 \pm 0.42$ for not measured KKpi,KK2pi,2pi4pi0,2pi3pi0

(or $12.46 \pm 0.76$ for $\sqrt{s} < 2$ GeV) (1.5-3$\sigma$ of total error - crucial in case of isospin violation)

$$R_{QCD}(2.109 GeV) = 41.19 \pm 0.82$$

Light-by-light

Theory TOTAL $\pm 2.6$

need more theory, probably with help of experimental Transition FormFactors

$\Delta \text{Exp - Theory} \sim 3.3 - 3.6 \sigma$

<table>
<thead>
<tr>
<th>Experiment</th>
<th>Theory</th>
</tr>
</thead>
<tbody>
<tr>
<td>HMNT 07 (e-e' based)</td>
<td>-285 ± 51</td>
</tr>
<tr>
<td>JN 09 (e-e')</td>
<td>-299 ± 65</td>
</tr>
<tr>
<td>Davier et al. 09/1 (t-based)</td>
<td>-167 ± 52</td>
</tr>
<tr>
<td>Davier et al. 09/1 (e'e)</td>
<td>-312 ± 51</td>
</tr>
<tr>
<td>Davier et al. 09/2 (e'e' w/ BABAR)</td>
<td>-255 ± 49</td>
</tr>
<tr>
<td>HLMNT 10 (e'e' w/ BABAR)</td>
<td>-259 ± 48</td>
</tr>
<tr>
<td>DHMZ 10 (t newest)</td>
<td>-196 ± 54</td>
</tr>
<tr>
<td>DHMZ 10 (e'e' newest)</td>
<td>-267 ± 49</td>
</tr>
</tbody>
</table>

New $g-2$ experiments at FNAL and J-PARC have plans to reduce precision to $1.5 \times 10^{-10}$
charmonium branching ratios

\[ \mathcal{B}_{J/\psi \rightarrow 2(\pi^+\pi^-)} \cdot \sigma_{J/\psi}^{int} = \frac{N(J/\psi \rightarrow 2(\pi^+\pi^-))}{d\mathcal{L}/dE \cdot \epsilon_{MC}} = (48.9 \pm 2.1_{\text{stat}} \pm 1.0_{\text{syst}}) \text{MeV}/c^2 \text{nb} \]

\[ \mathcal{B}_{J/\psi \rightarrow 2(\pi^+\pi^-)} = (3.67 \pm 0.16_{\text{stat}} \pm 0.08_{\text{syst}} \pm 0.09_{\text{ext}}) \cdot 10^{-3} \]

\[ \mathcal{B}_{J/\psi \rightarrow 2(\pi^+\pi^-)}^{PDG} = (3.55 \pm 0.23) \cdot 10^{-3} \]

→ agrees with PDG, higher in precision

\[ \mathcal{B}_{\psi(2S) \rightarrow J/\psi \pi^+\pi^-} \cdot \mathcal{B}_{J/\psi \rightarrow \mu^+\mu^-} \cdot \sigma_{J/\psi}^{(2S)} = \frac{N(\psi(2S) \rightarrow \pi^+\pi^- - \mu^+\mu^-)}{d\mathcal{L}/dE \cdot \epsilon_{MC}} \]

\[ = (84.7 \pm 2.2_{\text{stat}} \pm 1.8_{\text{syst}}) \text{MeV}/c^2 \text{nb} \]

\[ \mathcal{B}_{\psi(2S) \rightarrow J/\psi \pi^+\pi^-} = 0.354 \pm 0.009_{\text{stat}} \pm 0.007_{\text{syst}} \pm 0.007_{\text{ext}} \]

\[ \mathcal{B}_{\psi(2S) \rightarrow J/\psi \pi^+\pi^-}^{PDG} = 0.336 \pm 0.005 \]

\[ \mathcal{B}_{\psi(2S) \rightarrow J/\psi \pi^+\pi^-}^{CLEO} = 0.3504 \pm 0.009_{\text{stat}} \pm 0.0007_{\text{syst}} \pm 0.0077_{\text{ext}} \]

→ agrees with recent CLEO result (PRD 78, 011102 (2008))
Small systematic errors allow BaBar to improve BF for major decay modes.
In MC we know all inputs and can create a “test” \( m(K_L) \) distribution and compare with data. And the only free parameter is \( \phi(1020) \) mass. By varying f mass we calculate \( \chi^2 \) value by fitting data-MC difference with “ARGUS” function. We obtain:

\[
    m_\phi = 1019.483 \pm 0.040 \pm 0.036 \text{ MeV/c}^2 : 24 \text{ keV} - K^0 \text{ mass uncertainty, 20 keV} - K_S \text{ momentum, 18 keV} - \text{DCH-EMC mis-alignment.}
\]
How other distributions look like

Clean events with small systematic errors - 1% from KS, 0.5% ISR photon, 0.5% background, 0.6% from overlap effect.

Additional +1.5±0.6% correction due to not fully compensated overlap effect
**What we know about \( \phi(1680) \)**

<table>
<thead>
<tr>
<th>Energy dependence significantly increase width.</th>
<th>BaBar has measured ( \phi(1680) ) parameters in major decay modes:</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \phi(1680) \rightarrow K_S K \pi, , KK \pi^0 , (K^* K), , \phi \eta, , \phi \pi \pi, , K_S K_L ) - still no info in PDG</td>
<td></td>
</tr>
</tbody>
</table>