Measuring $a_{\mu}^{\text{HLO}}$ in the spacelike region

C.M.C. Calame$^1$, M. Passera$^2$, L. Trentadue$^3$, G. Venanzoni$^4$

$^1$Universita' di Pavia, Pavia, Italy
$^2$INFN, Sezione di Padova, Padova, Italy
$^3$Universita' di Parma, Parma, Italy and Sezione INFN Milano Bicocca, Milano, Italy
$^4$INFN, Laboratori Nazionali di Frascati, Frascati, Italy

Novosibirsk, 17 June 2015
\( \alpha_{\text{em}} \) running and the Vacuum Polarization

• Due to Vacuum Polarization effects \( \alpha_{\text{em}}(q^2) \) is a running parameter from its value at vanishing momentum transfer to the effective \( q^2 \).

➢ The “Vacuum Polarization” function \( \Pi(q^2) \) can be “absorbed” in a redefinition of an effective charge:

\[
e^2 \rightarrow e^2(q^2) = \frac{e^2}{1 + (\Pi(q^2) - \Pi(0))} \quad \quad \alpha(q^2) = \frac{\alpha(0)}{1 - \Delta \alpha}; \quad \Delta \alpha = -\Re\text{e} \left( \Pi(q^2) - \Pi(0) \right)
\]

\[
\Delta \alpha = \Delta \alpha_1 + \Delta \alpha_{\text{(5)had}} + \Delta \alpha_{\text{top}}
\]

➢ \( \Delta \alpha \) takes a contribution by non perturbative hadronic effects (\( \Delta \alpha_{\text{(5)had}} \)) which exhibits a different behaviour in time-like and spacelike region
Running of $\alpha_{\text{em}}$

**Time-like**

$E = \sqrt{s}$

Behaviour characterized by the opening of resonances

$s > 0$

$$\Delta \alpha^{(5)}(M_Z^2) = -\frac{\alpha M_Z^2}{3\pi} \text{Re} \int_{-\frac{2M_Z^2}{1-m_c^2}}^\infty ds \frac{R(s)}{s(s - M_Z^2 - i\epsilon)}$$

**Space-like**

$E = -\sqrt{-t}$

Very smooth behaviour

$t < 0$
Measurement of $\alpha_{\text{em}}$ running

- A direct measurement of $\alpha_{\text{em}}(q^2)$ in space/time like region can prove the running of $\alpha_{\text{em}}$

- It can provide a test of “duality” (far way from resonances)

- It has been done in past by few experiments at $e^+e^-$ colliders by comparing a “well-known” QED process with some reference (obtained from data or MC)

$$\left(\frac{\alpha(q^2)}{\alpha(q_0^2)}\right)^2 \sim \frac{N_{\text{signal}}(q^2)}{N_{\text{norm}}(q_0^2)}$$

$N_{\text{signal}}$ can be Bhabha process, muon pairs, etc…

$N_{\text{signal}}$ can be Bhabha process, $\gamma\gamma$ pairs, Theory, etc…
Measurement of $\alpha_{\text{em}}$ running

**e+e- collider TRISTAN at $\sqrt{s}=57.8$ GeV,**

- $e^+e^- \rightarrow \mu^+\mu^-$
- $e^+e^- \rightarrow e^+e^-\mu^+\mu^-$

**Timelike**

\[ |Q^2|^{1/2} \text{ (GeV/c)} \]

**Spacelike**

\[ \alpha^{-1}(Q^2) \]

---

**e+e- collider LEP at $\sqrt{s}=189$ GeV, using Bhabha events**

**OPAL**

- $1.3<\sqrt{-t}<2.5$ GeV
- $1.5<\sqrt{-t}<2.5$ GeV
- $3.5<\sqrt{-t}<58$ GeV

**VENUS**

- $10<\sqrt{-t}<54$ GeV

**L3**

- small angle
- large angle
Measurement of $\alpha_{em}$ running

e+e- collider TRISTAN at $\sqrt{s}=57.8$ GeV,

$e^+e^- \rightarrow \mu^+\mu^-$

$e^+e^- \rightarrow e^+e^-$

Spacelike $1.3<\sqrt{-t}<2.5$ GeV

Timelike $1.5<\sqrt{-t}<2.5$ GeV

$3.5<\sqrt{-t}<58$ GeV

10<\sqrt{-t}<54$ GeV

**BIG CHANCES TO IMPROVE THESE MEASUREMENTS AT FUTURE HIGH ENERGY COLLIDER (ILC/TLEP)**
\[ a_\mu^{HLO} \] calculation, traditional way: time-like data

\[ a_\mu^{HLO} = \frac{1}{4\pi^3} \int_{4m^2_\pi}^{\infty} \sigma_{e^+e^-\rightarrow hadr} (s) K(s) ds \]

Traditional way: based on precise experimental (time-like) data:

\[ a_\mu^{HLO} = \frac{\alpha}{\pi^2} \int_0^{\infty} ds K(s) \text{Im} \Pi_{had}(s) \sigma_{e^+e^-\rightarrow hadr} (s) = \frac{4\pi}{s} \text{Im} \Pi_{had}(s) \]

\[ K(s) = \int dx \frac{x^2 (1-x)}{x^2+(1-x)(s/m^2)} \sim \frac{1}{s} \]

Main contribution in the low energy region

\[ \delta a_\mu^{\text{exp}} \rightarrow 1.5 \times 10^{-10} = 0.2\% \text{ on } a_\mu^{HLO} \text{ (from 0.7% now)} \]

NEW G-2 at FNAL and JPARC
$a_{\mu}^{HLO}$ evaluation in spacelike region: alternative approach

$$a_{\mu}^{HLO} = -\frac{\alpha}{\pi} \int_{0}^{1} (1 - x)\Pi_{had}(-\frac{x^2}{1 - x} m_{\mu}^2) dx$$

$x =$Feynman parameter

$$t = \frac{x^2 m_{\mu}^2}{x - 1} \quad 0 \leq -t < +\infty$$

$$x = \frac{t}{2 m_{\mu}^2} (1 - \sqrt{1 - \frac{4 m_{\mu}^2}{t}}) \quad 0 \leq x < 1$$

$$\Delta \alpha_{had}(t) = -\Pi_{had}(t) \quad \text{for } t < 0$$

$$a_{\mu}^{HLO} = -\frac{\alpha}{\pi} \int_{0}^{1} (1 - x)\Delta \alpha_{had}(-\frac{x^2}{1 - x} m_{\mu}^2) dx$$

For $t < 0$

$a_{\mu} = (g-2)/2$

See also G. Fedotovich, proceedings of PHIPSI08
\[ \Delta \alpha \sim \log(-t) \]

Dominated at low \(|t|\) by leptonic contribution

\[ \frac{(1-x) \Delta \alpha_{\text{had}}}{1-x} \left( \frac{x^2 m_{\mu}^2}{1-x} \right) \]

\[ |t| \times 10^3 \text{ (GeV}^2) \]

\[ x_{\text{peak}} \approx 0.914 \]

\[ t_{\text{peak}} \approx -0.108 \text{ GeV}^2 \]


High \(|t|\)-values are depressed by \(1-x\) (a kind of analogy with time-like region)

The integrand is peaked at \(~x=0.92\)

\[ t=-0.11 \text{ GeV}^2 (~330 \text{ MeV}) \] for which

\[ \Delta \alpha_{\text{had}}(0.92) \sim 10^{-3} \]
Experimental considerations

Using Bhabha at small angle (to emphasize t-channel contribution) to extract $\Delta \alpha$:

$$\left( \frac{\alpha(t)}{\alpha(0)} \right)^2 \sim \frac{d\sigma_{ee\rightarrow ee}(t)}{d\sigma^0_{MC}(t)}$$

Where $d\sigma^0_{MC}$ is the MC prediction for Bhabha process with $\alpha(t)=\alpha(0)$, and there are corrections due to RC...

$$\Delta \alpha_{had}(t) = 1 - \left( \frac{\alpha(t)}{\alpha(0)} \right)^{-1} - \Delta \alpha_{lep}(t)$$

$\Delta \alpha_{lep}(t)$ theoretically well known!

Which experimental accuracy we are aiming at?

$\delta \Delta \alpha_{had} \sim 1/2$ fractional accuracy on $d\sigma(t)/d\sigma^0_{MC}(t)$.

If we assume to measure $\delta \Delta \alpha_{had}$ at 5% at the peak of the integrand ($\Delta \alpha_{had} \sim 10^{-3}$ at $x=0.92$) $\rightarrow$ fractional accuracy on $d\sigma(t)/d\sigma^0_{MC}(t) \sim 10^{-4}$!

Very challenging measurement (one order of magnitude improvement respect to date) for systematic error
Experimental considerations - II

Most of the region (up to $x \sim 0.98$) can be covered with a low energy machine (like Dafne/VEPP-2000 or tau/charm-B-factories)

Example:
Covering up to $60^\circ$ at $\sqrt{s}=1$ GeV can arrive at $x=0.95(!)$

A different situation can be obtained at tau/charm/B-factories (and at future ILC/TLEP machines) where smaller angles (below $20^\circ$) are needed
10^{-4} accuracy on Bhabha cross section requires at least $10^8$ events which at 20\(^\circ\) mean at least:

- $O(1) \text{ fb}^{-1} @ 1 \text{ GeV}$
- $O(10) \text{ fb}^{-1} @ 3 \text{ GeV}$
- $O(100) \text{ fb}^{-1} @ 10 \text{ GeV}$

These luminosities are within reach at flavour factories!
Additional considerations: s-channel

At low energy (<10 GeV) above $10^0$ there is still a sizeable contribution from s-channel. At LO no difficulty to deconvolute the cross section for the s-channel.

Test with Babayaga:

$s=1$ GeV
$10^o<\theta<170^o$

$d\sigma_{\text{born}}/dt=1.52$ mb/GeV$^2$

However this picture changes with Rad. Corr.
A Monte Carlo procedure has been developed to check if $\Delta\alpha_{\text{had}}(t)$ can be obtained by a minimization procedure with a different $\Delta\alpha_{\text{had}}(t)'$ inside

\[ \frac{d\sigma}{dt} \bigg|_{\text{data}} = \frac{d\sigma}{dt} \left( \alpha(t), \alpha(s) \right) \bigg|_{\text{MC}}', \]

$\rightarrow$

\[ \frac{d\sigma}{dt} \bigg|_{j,\text{data}} = \frac{d\sigma}{dt} \left( \bar{\alpha}(t) + \frac{i_j}{N} \delta(t), \alpha(s) \right) \bigg|_{j,\text{MC}}', \]

$\Delta\alpha_{\text{had}}(t)$ is obtained with $<10^{-4}$ error!
Additional consideration: Normalization

To compare Bhabha absolute cross section from data with MC we need Luminosity of the machine. Two possibilities:

1) Use Bhabha at very small angle where the uncertainty on $\Delta \alpha_{\text{had}}$ can be neglected (for example at $E_{\text{beam}} = 1$ GeV and $\theta = 5^\circ$, $\Delta \alpha_{\text{had}} \sim 10^{-5}$).

2) Use a process with $\Delta \alpha_{\text{had}} = 0$, like $e^+e^- \rightarrow \gamma\gamma$. However very difficult to determine it at $10^{-4}$ accuracy.

Option 1) looks better to us as some of the common systematics cancel in the measurement!
Measurement of DAFNE Luminosity with KLOE/KLOE-2 at $10^{-4}$?

F. Ambrosino et al [KLOE]  


Table 2. Summary of the corrections and systematic errors in the measurement of the luminosity

<table>
<thead>
<tr>
<th></th>
<th>correction (%)</th>
<th>systematic error (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>angular acceptance</td>
<td>+0.25</td>
<td>0.25</td>
</tr>
<tr>
<td>tracking</td>
<td>–</td>
<td>0.06</td>
</tr>
<tr>
<td>clustering</td>
<td>+0.14</td>
<td>0.11</td>
</tr>
<tr>
<td>background</td>
<td>−0.62</td>
<td>0.13</td>
</tr>
<tr>
<td>cosmic veto</td>
<td>+0.40</td>
<td>−</td>
</tr>
<tr>
<td>energy calibration</td>
<td>–</td>
<td>0.10</td>
</tr>
<tr>
<td>center of mass energy</td>
<td>+0.10</td>
<td>0.10</td>
</tr>
<tr>
<td></td>
<td>+0.34</td>
<td>0.32</td>
</tr>
</tbody>
</table>

Adding in quadrature: 0.3 %

(can be improved by a factor 10?)

G. Venanzoni, Seminar at LNF, Frascati, 20 May 2015
Polar angle systematics

- global agreement is very good

but the cut occurs in a steep region of the distributions ⇒ estimate of border mismatches

- after normalizing MC to make it coincide with data in the region $65^\circ < \theta < 115^\circ$, we estimate as a systematic error:

$$\frac{N_{\text{dat}}^{[55:65]} + [115:125]}{N_{\text{TOT}}^{\text{dat}}} - \frac{N_{MC}^{[55:65]} + [115:125]}{N_{\text{TOT}}^{MC}} \approx 0.25\%$$

Can be improved at $10^{-4}$?

G. Venanzoni, Seminar at LNF, Frascati, 20 May 2015

From F. Nguyen 2006
Measurement of the running of the QED coupling in small-angle Bhabha scattering at LEP

The OPAL Collaboration

Small-angle Bhabha scattering in OPAL

\[ e^+ e^- \rightarrow e^+ e^- \quad \sqrt{s} \approx 91.2 \text{ GeV} \]

2 cylindrical calorimeters encircling the beam pipe at ± 2.5 m from the Interaction Point

19 Silicon layers \quad \text{Total Depth 22 } X_0
18 Tungsten layers \quad \text{(14 cm)}

Each detector layer divided into 16 overlapping wedges

Sensitive radius: 6.2 – 14.2 cm, corresponding to scattering angle of 25 – 58 mrad from the beam line
Final Error on Luminosity

After all the effort on Radial reconstruction the dominant systematic error is related to Energy (mostly tail in the E response and nonlinearity).


<table>
<thead>
<tr>
<th></th>
<th>Systematic Error ((\times 10^{-4}))</th>
</tr>
</thead>
<tbody>
<tr>
<td>Energy</td>
<td>1.8</td>
</tr>
<tr>
<td>Inner Anchor</td>
<td>1.4</td>
</tr>
<tr>
<td>Radial Metrology</td>
<td>1.4</td>
</tr>
</tbody>
</table>

Total Experimental Systematic Error : \(3.4 \times 10^{-4}\)

Theoretical Error on Bhabha cross section: \(5.4 \times 10^{-4}\)
Conclusions

• Measuring $\alpha_{\text{em}}$ running in time-like and space like region appears to be very interesting. (Relatively) high $q^2$-values can be explored at ILC/TLEP
• An alternative formula for $a_{\mu}^{\text{HLO}}$ in spacelike region has been studied in details. It emphasizes low values of $t$ (<1 GeV$^2$) and can be explored at low energy e+e- machines (VEPP2000/DAFNE, $\tau$/charm, B-factories)
• It requires to measure the Bhabha cross section at relatively small angles at (better than) $10^{-4}$ accuracy!
• Reaching such an accuracy demands a dedicated experimental and theoretical work for the next few years
• Can this method apply also at other (e-e; fixed target) machines?

Thanks!
END
\( \Delta \alpha_{em}^{HAD}(s) \) dependence
Which is the best energy/angle configuration?

\[ x = \frac{t}{2m^2} \left(1 - \sqrt{1 - \frac{4m^2}{t}}\right) \]

\[-t = 9(1 - \cos \theta)/2\]

\[ \sqrt{s} = 3 \text{ GeV} \]

\[ \theta = 12^\circ \]

\[ \sqrt{s} = 1 \text{ GeV} \]

\[ \theta = 40^\circ \]
$x$ vs $t$ behaviour

$x \to 1$
$t \to \infty$

$x \to 0$
$t \to 0$

30 MeV  100 MeV  320 MeV  1 GeV