To study isospin breaking decay $\eta(1405) \to f_0(980)\pi^0 \to 3\pi$

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1. Introduction: $a_0^0(980)-f_0(980)$ mixing as driving phenomenon
2. The BESIII data on $J/\psi\to\gamma\eta(1405)\to\gamma\,\pi^+\pi^-\pi^0$
3. Mechanism of the $\eta(1405)\to K\bar{K}\pi\to f_0(980)\pi^0\to\pi^+\pi^-\pi^0$ decay via the $K^*\bar{K}+\bar{K}^*K$ intermediate states
4. About description of the $\eta(1405)\to K^*\bar{K}+\bar{K}^*K\to K\bar{K}\pi$ decay
5. Decay $\eta(1405)\to a_0^0(980)\pi^0\to f_0(980)\pi^0\to\pi^+\pi^-\pi^0$
6. Conclusion

Introduction: $a_0^0(980) - f_0(980)$ mixing

The violating isospin $a_0^0(980) - f_0(980)$ mixing was first suggested by

N. N. Achasov, S. A. Devyanin, G. N. Shestakov, "$S^* - \delta^0$ mixing as a threshold phenomenon", Phys. Lett. B 88, 367 (1979); Sov. J. Nucl. Phys. 33, 715 (1981)

Our following contributions are

N. N. Achasov and G. N. Shestakov, Phys. Rev. D 56, 212 (1997)

N. N. Achasov and A. V. Kiselev, Phys. Lett. B 534, 83 (2002)

N. N. Achasov and G. N. Shestakov, Phys. Rev. Lett. 92 182001 (2004)

N. N. Achasov and G. N. Shestakov, Phys. Rev. D 70, 074015 (2004)

N. N. Achasov, A. A. Kozhevnikov, and G. N. Shestakov, arXiv:1504.02844

This phenomenon was discussed approximately in hundred publications. Recently, the interest in the $a_0^0(980) - f_0(980)$ mixing has been renewed. New proposals for searching it have appeared, and the results of the first experiments reporting its discovery with the help of detectors VES (IHEP, Protvino, 2008, 2011) and BESIII (Beijing, 2011, 2012, 2015) have been presented.

Introduction: $a_0^0(980) - f_0(980)$ mixing

We keep in mind the experiments performed by

VES on the decay $f_1(1285) \to f_0(980)\pi^0 \to \pi^+\pi^-\pi^0$ V. Dorofeev et al., Eur. Phys. J. A 38, 149 (2008); Eur. Phys. J. A 47, 68 (2011),

BESIII on the decays $J/\psi \to \phi f_0(980) \to \phi a_0(980) \to \phi\eta\pi^0$ and $\chi_{c1} \to a_0(980)\pi^0 \to f_0(980)\pi^0 \to \pi^+\pi^-\pi^0$ (from $\psi' \to \gamma\chi_{c1}$)
M. Ablikim et al., Phys. Rev. D 83, 032003 (2011),

BESIII on the decays $J/\psi \to \gamma\eta(1405) \to \gamma f_0(980)\pi^0 \to \gamma\pi^+\pi^-\pi^0$, $\gamma\pi^0\pi^0\pi^0$ M. Ablikim et al., Phys. Rev. Lett. 108, 182001 (2012), and

BESIII on the decays $J/\psi \to \phi\pi^0 f_0(980)$ with $f_0(980) \to \pi\pi$, including $J/\psi \to \phi f_1(1285) \to \phi\pi^0 f_0(980) \to \phi\pi^+\pi^-\pi^0, \phi\pi^0\pi^0\pi^0$
M. Ablikim et al., arXiv:1505.06283.

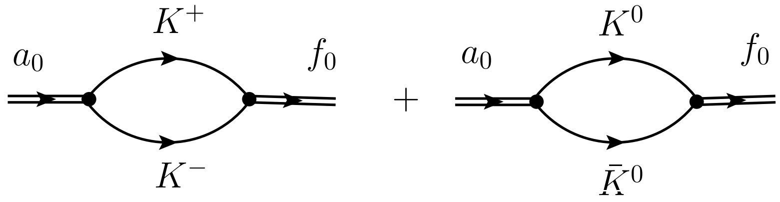
A large isospin violation is observed in all these reactions.

Introduction: $a_0^0(980)-f_0(980)$ mixing

The aim of our talk is the elucidation of the possible mechanism of the decay $\eta(1405)\to f_0(980)\pi^0\to\pi^+\pi^-\pi^0$, discovered by BESIII (2012) in the radiative $J/\psi\to\gamma\pi^+\pi^-\pi^0$ decay.

First of all we recall how the $a_0^0(980)-f_0(980)$ mixing violates isospin. It arises owing to the K^+ and K^0 meson mass difference, that leads to the incomplete compensation between the K^+K^- and $K^0\bar{K}^0$ intermediate state contributions in the transition amplitude $a_0^0(980)\to K^+K^-+K^0\bar{K}^0\to f_0(980)$ which is shown below

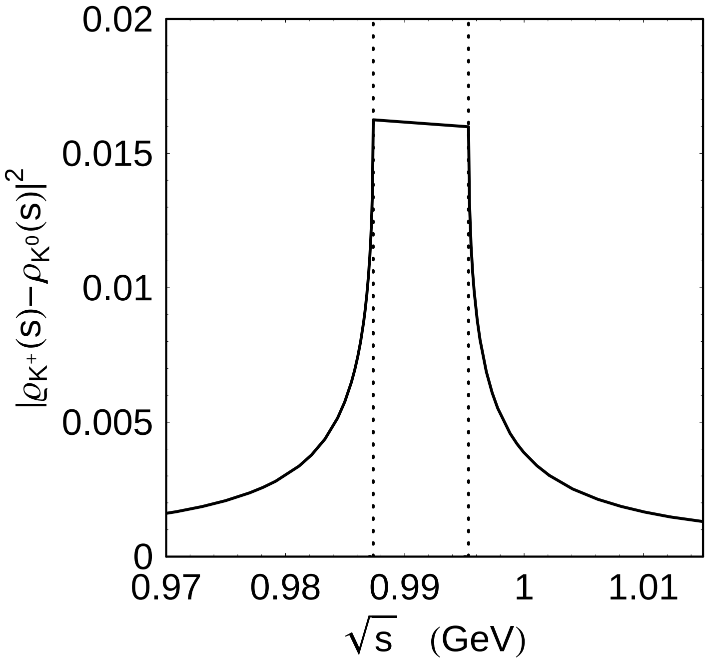
$\pi^0\eta$



$\pi^+\pi^-$.

As a result, the $a_0^0(980)-f_0(980)$ mixing cuts out from the a_0^0 and f_0 resonance peaks in $\eta\pi$ and $\pi\pi$ mass spectra the narrow resonance structure with a width of $2m_{K^0}-2m_{K^+}\approx 8$ MeV between the K^+K^- and $K^0\bar{K}^0$ thresholds (987.4 MeV and 995.2 MeV, respectively).

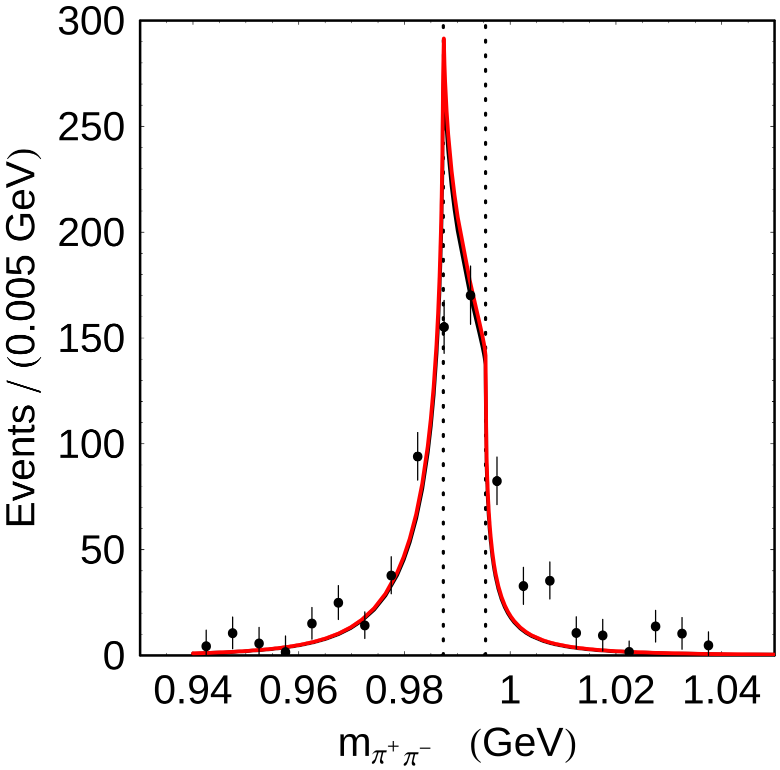
Introduction: $a_0^0(980) - f_0(980)$ mixing



The resonance-like energy behavior of the $a_0^0 - f_0$ transition amplitude.

In the the $K\bar{K}$ threshold region this amplitude is $\sim |\rho_{K^+}(s) - \rho_{K^0}(s)|$, where $\rho_K(s) = \sqrt{1 - 4m_K^2/s}$ is the K meson velocity. Between the $K\bar{K}$ thresholds, it turns out to be of the order of $\sqrt{(m_{K^0} - m_{K^+})/m_{K^0}}$, but not $(m_{K^0} - m_{K^+})/m_{K^0}$, i.e. by the order of magnitude greater than it could be expected from the naive considerations.

The BESIII data on the $J/\psi \to \gamma\eta(1405) \to \gamma\,\pi^+\pi^-\pi^0$ decay



For any production mechanism of the $I = 1\;(K\bar{K})_S$ system, the isospin violation amplitude in the $\pi^+\pi^-$ channel is $\sim|\rho_{K^+}(s) - \rho_{K^0}(s)|$ in the $K\bar{K}$ threshold region. All of the above data are consistent with such an expectation, in particular, the BESIII data on the $\pi^+\pi^-$ mass spectrum in the decay $\eta(1405) \to (K^+K^- + K^0\bar{K}^0)\pi^0 \to f_0(980)\pi^0 \to \pi^+\pi^-\pi^0$, shown in the figure.

The BESIII data on the $J/\psi \to \gamma\eta(1405) \to \gamma\,\pi^+\pi^-\pi^0$ decay

$$BR(J/\psi \to \gamma\eta(1405) \to \gamma f_0(980)\pi^0 \to \gamma\pi^+\pi^-\pi^0)$$
$$= (1.50 \pm 0.11 \pm 0.11) \cdot 10^{-5}\,.$$

Comparing this BR with the result of Particle Data Group (PDG) on

$$BR(J/\psi \to \gamma\eta(1405/1475) \to \gamma K\bar{K}\pi)$$
$$= (2.8 \pm 0.6) \cdot 10^{-3}\,,$$ one gets

$$\frac{BR(J/\psi \to \gamma\eta(1405) \to \gamma f_0(980)\pi^0 \to \gamma\pi^+\pi^-\pi^0)}{BR(J/\psi \to \gamma\eta(1405/1475) \to \gamma K\bar{K}\pi)}$$
$$= (0.53 \pm 0.13)\%\,.$$

The magnitude of this ratio tells about very large breaking of the isotopic invariance in the decay $\eta(1405) \to f_0(980)\pi^0$. Guided by naive considerations, this ratio is expected to be at the level of $[(m_{K^0} - m_{K^+})/m_{K^0}]^2 \lesssim 10^{-4}$.

In what follows we use also the short notation ι for $\eta(1405)$.

Decay $\iota \longrightarrow K^*\bar{K} + \bar{K}^*K \longrightarrow K\bar{K}\pi^0 \longrightarrow f_0(980)\pi^0 \longrightarrow \pi^+\pi^-\pi^0$

Since the rate of the $\iota \longrightarrow K\bar{K}\pi$ decay is much larger than that of $\iota \longrightarrow \eta\pi\pi$, there are attempts in the literature to explain the decay $\iota \longrightarrow \pi^+\pi^-\pi^0$ as being due to the mechanism which includes the logarithmic (triangle) singularities, i.e., due to the transition $\iota \longrightarrow K^*\bar{K} + \bar{K}^*K \longrightarrow (K^+K^- + K^0\bar{K}^0)\pi^0$ $\longrightarrow f_0(980)\pi^0 \longrightarrow \pi^+\pi^-\pi^0$ via the $K^*\bar{K} + \bar{K}^*K$ intermediate states,

J. J. Wu, X. H. Liu, Q. Zhao, and B. S. Zou, Phys. Rev. Lett. 108, 081803 (2012);
F. Aceti, W. H. Liang, E. Oset, J. J. Wu, and B. S. Zou, Phys. Rev. D 86, 114007 (2012);
X. G. Wu, J. J. Wu, Q. Zhao, and B. S. Zou, Phys. Rev. D 87, 014023 (2013). a)

The triangle diagram in question is

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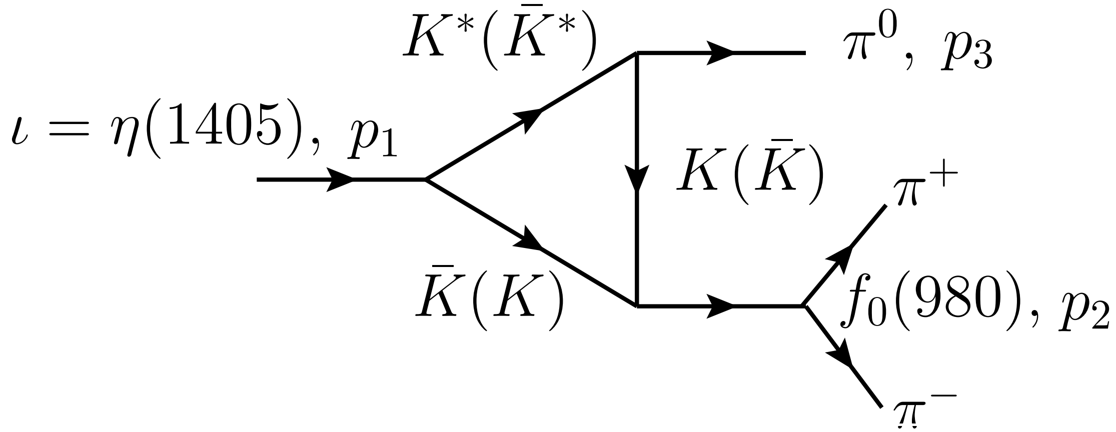
a) Here we pay attention to the fact that in these works the vector $K^*(892)$ meson in the intermediate state was considered to be stable.

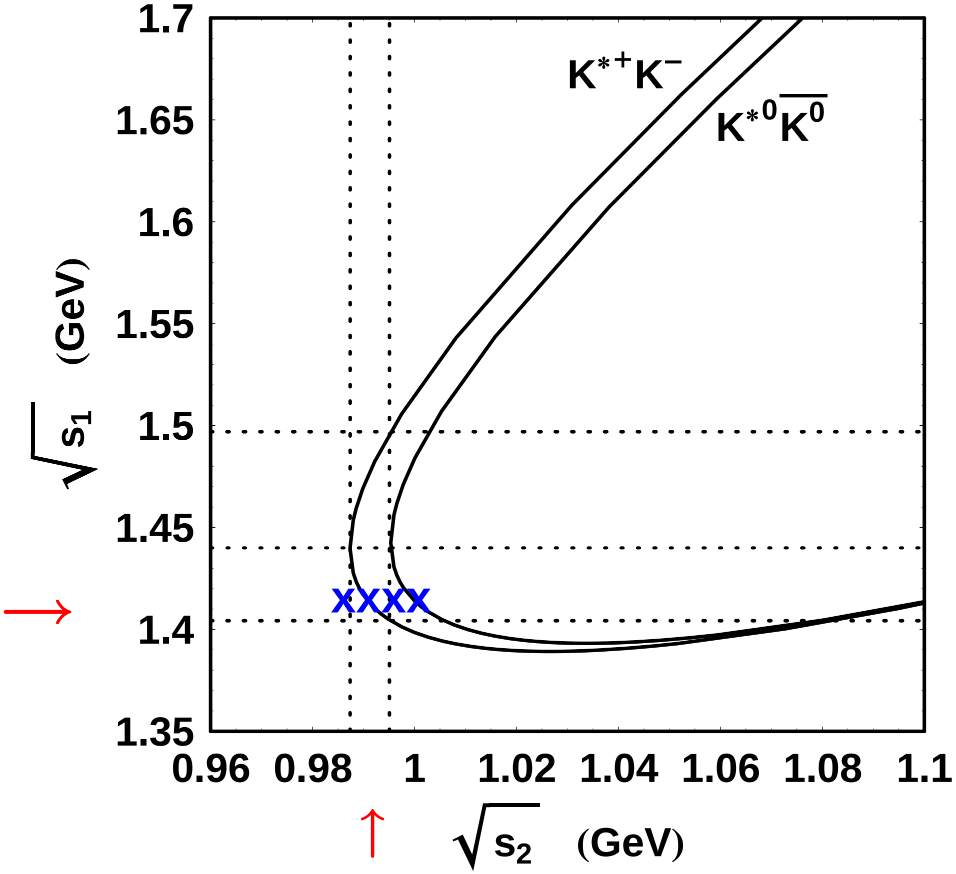
Decay $\iota \to K^*\bar{K} + \bar{K}^*K \to K\bar{K}\pi^0 \to f_0(980)\pi^0 \to \pi^+\pi^-\pi^0$



Here $p_1^2 = s_1$ is the invariant mass squared of the ι and $p_2^2 = s_2 = m^2_{\pi^+\pi^-}$ is the invariant mass squared of the $f_0(980)$ or of the final $\pi^+\pi^-$ system.

In the ι resonance region, all particles in the loop of triangle diagram, at the definite values of the kinematic variables $\sqrt{s_1}$ and $\sqrt{s_2}$, can lie on their mass shells. This means that in the hypothetic case of the stable K^* meson the logarithmic singularities appear in the imaginary part of the triangle diagram. In the ι resonance region these singularities are located in variable $\sqrt{s_2} = m_{\pi^+\pi^-}$ very close to the $K\bar{K}$ thresholds, as is clear from the next figure.

Decay $\iota \to K^*\bar{K} + \bar{K}^*K \to K\bar{K}\pi^0 \to f_0(980)\pi^0 \to \pi^+\pi^-\pi^0$



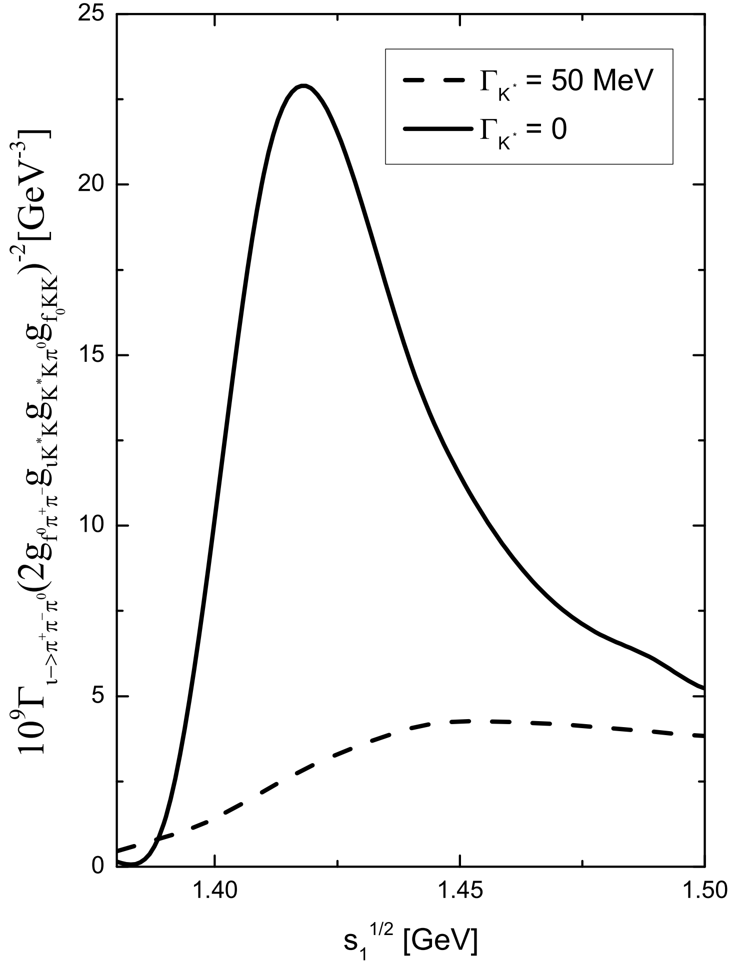
ι region →

Solid curves on the plane $\left(\sqrt{s_2} = m_\iota \,, \sqrt{s_1} = m_{\pi^+\pi^-}\right)$ show the location of the logarithmic singularity in the imaginary part (in the discontinuities on the $K^*\bar{K}$, $\bar{K}^*K$, and $K\bar{K}$ cuts) of the triangle diagram, in case of the $K^{*+}K^-$ and $K^{*0}\bar{K}^0$ intermediate states.

Decay $\iota \to K^* \bar{K} + \bar{K}^* K \to K\bar{K}\pi^0 \to f_0(980)\pi^0 \to \pi^+\pi^-\pi^0$

Since the located at different positions singularities from the charged and neutral intermediate states do not compensate each other, then the considered mechanism may seem to result in a catastrophic violation of isotopic symmetry in the decay $\iota \to \pi^+\pi^-\pi^0$. However, the accounting of the finite width of the K^* resonance, i.e., the averaging of the triangle amplitude over the resonance Breit – Wigner distribution in accord with the spectral Källén – Lehmann representation for the propagator of the unstable K^* meson, as it had been done by N. N. Achasov and A. A. Kozhevnikov, Z. Phys. C 48, 121 (1990); Phys. Lett. B 260, 425 (1991); Phys. Rev. D 49, 275 (1994), smoothes the logarithmic singularities of the amplitude and hence makes the compensation of the contributions of the $K^{*+}K^- + K^{*-}K^+$ and $K^{*0}\bar{K}^0 + \bar{K}^{*0}K^0$ intermediate states more strong. Namely, we show that the account of the finite width of K^*, $\Gamma_{K^*} \approx \Gamma_{K^* \to K\pi} \approx 50$ MeV, smoothes the logarithmic singularities in the amplitude resulting in the suppression of the calculated width of the decay $\iota \to f_0(980)\pi^0 \to 3\pi$ in the region 1.400 GeV $< \sqrt{s_1} <$ 1.425 GeV by the factor of $6 - 8$ in comparison with the case of $\Gamma_{K^*} = 0$.

Decay $\iota \to K^*\bar{K} + \bar{K}^*K \to K\bar{K}\pi^0 \to f_0(980)\pi^0 \to \pi^+\pi^-\pi^0$

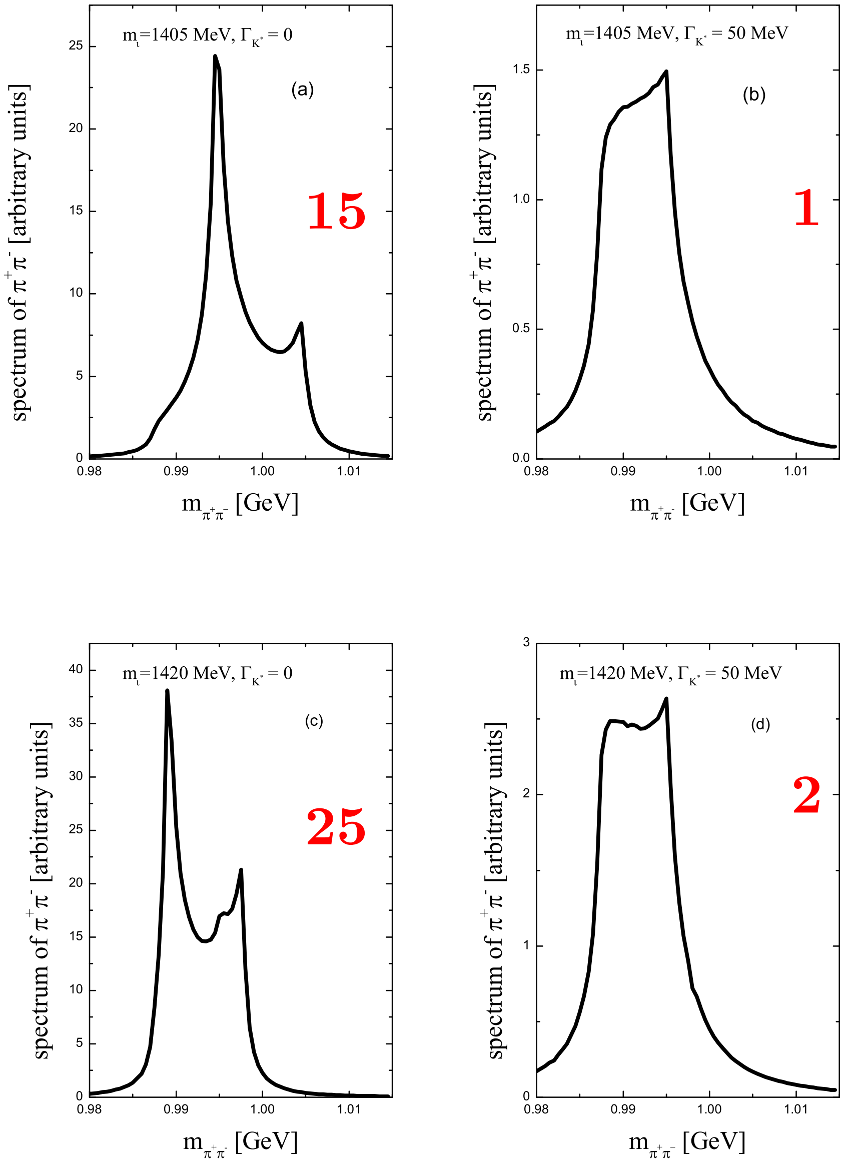


The influence of instability of the intermediate K^* meson on the calculated width of the decay $\iota \to (K^*\bar{K} + \bar{K}^*K) \to K\bar{K}\pi^0 \to f_0(980)\pi^0 \to \pi^+\pi^-\pi^0$ shown as a function of m_ι ($\sqrt{s_1}$).

Decay $\iota \to K^*\bar{K} + \bar{K}^*K \to K\bar{K}\pi^0 \to f_0(980)\pi^0 \to \pi^+\pi^-\pi^0$

$m_\iota = 1405$ MeV,
$\Gamma_{K^*} = 0$

$m_\iota = 1405$ MeV,
$\Gamma_{K^*} = 50$ MeV



$m_\iota = 1420$ MeV,
$\Gamma_{K^*} = 0$

$m_\iota = 1420$ MeV,
$\Gamma_{K^*} = 50$ MeV

The influence of instability of the K^* meson on the $\pi^+\pi^-$ mass spectra.

Decay $\iota \to K^*\bar{K} + \bar{K}^*K \to K\bar{K}\pi^0 \to f_0(980)\pi^0 \to \pi^+\pi^-\pi^0$

Thus we calculate the triangle diagram with $\Gamma_{K^*} = 50$ MeV and as a result estimate the ratio of the width $\Gamma_{\iota\to\pi^+\pi^-\pi^0}$ and $\Gamma_{\iota\to K\bar{K}\pi}$ averaged over the region 1.400 GeV $< \sqrt{s_1} <$ 1.425 GeV (where they rapidly change)

$$R = \frac{\overline{\Gamma}_{\iota\to\pi^+\pi^-\pi^0}}{\overline{\Gamma}_{\iota\to K\bar{K}\pi}} = \frac{\langle\Gamma_{\iota\to\pi^+\pi^-\pi^0}(s_1)\rangle}{\langle\Gamma_{\iota\to K\bar{K}\pi}(s_1)\rangle} \approx 4\cdot 10^{-3}\,.$$

This ratio is an important characteristic of the isospin violation in the considered model. It does not depend on the magnitude of the ι coupling with $K^*\bar{K}$, and its order of magnitude is controlled by the factor $[(m_{K^0} - m_{K^+})/m_{K^0}]$ $\times(g^2_{f_0K^+K^-}/g^2_{f_0\pi^+\pi^-})$ and decay kinematics. Then, using the PDG data for $BR(J/\psi \to \gamma\eta(1405/1475) \to \gamma K\bar{K}\pi) \approx 2.8\cdot 10^{-3}$, we estimate

$$BR(J/\psi \to \gamma\iota \to \gamma f_0(980)\pi^0 \to \gamma\pi^+\pi^-\pi^0)$$
$$\approx R\cdot BR(J/\psi \to \gamma\eta(1405/1475) \to \gamma K\bar{K}\pi) \approx 1.12\cdot 10^{-5},$$

in close agreement with the BESIII data $(1.50 \pm 0.11 \pm 0.11)\cdot 10^{-5}$.

Decay $\iota \longrightarrow K^*\bar{K} + \bar{K}^*K \longrightarrow K\bar{K}\pi^0 \longrightarrow f_0(980)\pi^0 \longrightarrow \pi^+\pi^-\pi^0$

But that is not the whole story. The fact is that the above estimate includes the assumption of dominance of the $\eta(1405/1475) \longrightarrow (K^*\bar{K} + \bar{K}^*K)$ $\longrightarrow K\bar{K}\pi$ mechanism in the decay $\eta(1405/1475) \longrightarrow K\bar{K}\pi$. The existing uncertainty is primary related to the scarcity of the data on the $K\pi$ and $K\bar{K}$ mass spectra in the $\eta(1405/1475) \longrightarrow K\bar{K}\pi$ decay.

Moreover, in view of the absence of the detailed data, one forcedly assumes that ι (or $\eta(1405)$, or $\eta(1440)$) and the resonance complex $\eta(1405/1475)$ constitute the single object looking differently in various channels.

Hence, the magnitude of $BR(J/\psi \longrightarrow \gamma\iota \longrightarrow \gamma f_0(980)\pi^0 \longrightarrow \gamma\pi^+\pi^-\pi^0)$, obtained in the present model, should be considered only as the upper estimate.

Decay $\iota \to K^*\bar{K} + \bar{K}^*K \to K\bar{K}\pi$

In connection with the question of the $\iota \to (K^*\bar{K} + \bar{K}^*K) \to K\bar{K}\pi$ decay dominance we would like to pay attention to the difficulty of using the simplest Breit – Wigner distribution

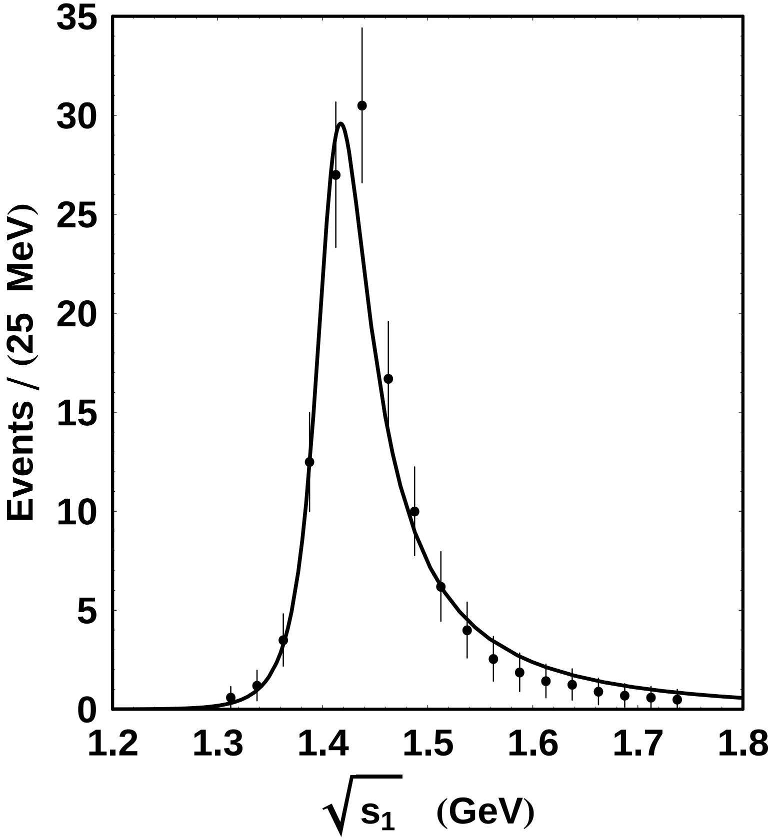
$$BR(\iota \to K\bar{K}\pi; m) = \frac{2m}{\pi} \frac{m\Gamma_{\iota\to K\bar{K}\pi}(m)}{|m_\iota^2 - m^2 - im\Gamma_\iota^{tot}(m)|^2},$$

where $\Gamma_\iota^{tot}(m) = \Gamma_{\iota\to K\bar{K}\pi}(m) = \Gamma_{\iota\to(K^*\bar{K}+\bar{K}^*K)\to K\bar{K}\pi}(m),$ for the description of the ι resonance. Really, in spite of a good description of the data on the $K\bar{K}\pi$ spectrum in the decay $J/\psi \to \gamma\eta(1440) \to \gamma K\bar{K}\pi$ with the help of the above expression and the fitting scale factor, the evaluation of the total $\iota \to K\bar{K}\pi$ decay probability gives

$$BR(\iota \to K\bar{K}\pi) = \int\limits_{1.3\,GeV}^{3\,GeV} BR(\iota \to K\bar{K}\pi; m)\, dm \approx 0.34,$$

instead of expected value close to 1.

Decay $\iota \longrightarrow K^*\bar{K} + \bar{K}^*K \longrightarrow K\bar{K}\pi$



The $K\bar{K}\pi$ mass spectrum in the decay $J/\psi \longrightarrow \gamma\iota \longrightarrow \gamma K\bar{K}\pi$ as the function of the invariant mass of the ι resonance $\sqrt{s_1}$. Points with error bars are the BES (2000) data. The curve is obtained in the $\iota \longrightarrow (K^*\bar{K} + \bar{K}^*K) \longrightarrow K\bar{K}\pi$ decay model.

Decay $\iota \to K^*\bar{K} + \bar{K}^*K \to K\bar{K}\pi$

The reason for this violation of the normalization is the sharp P wave growth of $\Gamma_{\iota\to(K^*\bar{K}+\bar{K}^*K)\to K\bar{K}\pi}(m)$ with increasing m.

Remind that, for the scalar mesons $\sigma(600)$, $a_0(980)$, $f_0(980)$ and the $X(3872, 1^{++})$ meson, their propagators, obtained upon taking into account of the finite width corrections, satisfy the Källén – Lehmann representation and, due to this fact, preserve the total decay probability normalization to unity.

[N. N. Achasov, S. A. Devyanin, and G. N. Shestakov, Sov. J. Nucl. Phys. 32, 566 (1980); N. N. Achasov and A. V. Kiselev, Phys. Rev. D 70, 111901(R) (2004); N. N. Achasov and E. V. Rogozina, JETP Lett. 100, 227 (2014)].

Unfortunately, we have not yet succeeded in constructing the propagator for the ι resonance, providing the desired normalization to unity, as in the case of the light scalars and $X(3872, 1^{++})$ meson.

So, one can conclude that the fittings of the data on the ι resonance and the results of the determination of its parameters from seemingly natural expressions should be considered as tentative guesses.

Decay $\iota \to a_0^0(980)\pi^0 \to f_0(980)\pi^0 \to \pi^+\pi^-\pi^0$

The decay $\iota \to \pi^+\pi^-\pi^0$ can also proceed due to the $a_0^0(980) - f_0(980)$ mixing: $\iota \to a_0^0(980)\pi^0 \to f_0(980)\pi^0 \to \pi^+\pi^-\pi^0$.

However, it is difficult with the help of this mechanism to obtain the magnitude of $BR(J/\psi \to \gamma\iota \to \gamma\pi^+\pi^-\pi^0)$ close to the measured value $\approx 1.5 \cdot 10^{-5}$ because the rate of the ι decay into $\eta\pi\pi$ is about an order of magnitude smaller than that for the decay $\iota \to K\bar{K}\pi$.

In this connection, we note that the suppression of the decay $\iota \to \eta\pi\pi$ is not directly related with the smallness of the $\iota \to a_0(980)\pi \to \eta\pi\pi$ transition. The fact is that the $a_0(980)\pi$ intermediate state in the $\iota \to \eta\pi\pi$ channel can be hidden due to the destructive interference with contributions from $\sigma(600)\eta \to \eta\pi\pi$ production and $\pi\pi$ final state interaction. If such a possibility is realized, it would mean that the $a_0^0(980) - f_0(980)$ mixing mechanism can provide up to 30% of the $\iota \to \pi^+\pi^-\pi^0$ decay amplitude.

The high statistics experimental investigations of the $\iota \to \eta\pi\pi$ decay channel are desirable to reveal the role of the $a_0(980)\pi$ intermediate state.

CONCLUSION

The phenomenon of the $a_0^0(980)-f_0(980)$ mixing gave an impetus to conduct experiments of VES on the decay $f_1(1285)\to\pi^+\pi^-\pi^0$ and BESIII on the decays $J/\psi\to\phi f_0(980)\to\phi a_0(980)\to\phi\eta\pi$,

$$\chi_{c1}\to a_0(980)\pi^0\to f_0(980)\pi^0\to\pi^+\pi^-\pi^0,$$

$$J/\psi\to\gamma\eta(1405)\to\gamma f_0(980)\pi^0\to\gamma 3\pi,$$

$$J/\psi\to\phi\pi^0 f_0(980)\ \text{with}\ f_0(980)\to\pi\pi,$$

$$J/\psi\to\phi f_1(1285)\to\phi\pi^0 f_0(980)\to\phi 3\pi.$$

We hope that the remarks presented here on the mechanisms of the isospin breaking in the decay $\eta(1405)\to 3\pi$ will stimulate both the further studies of this decay and the principal improvement of the data about $K\bar{K}$, $K\pi$, $\eta\pi$, and $\pi\pi$ mass spectra in the decays of the resonance structure $\eta(1405/1475)$ into $K\bar{K}\pi$ and $\eta\pi\pi$, and about the shape of these resonance peaks in the $K\bar{K}\pi$ and $\eta\pi\pi$ channels.

Thanks

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THANK YOU