Heavy Quark Expansion for Inclusive Semileptonic Charm Decays

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HQE in $B \to X_c \ell \bar{\nu}$

$$d\Gamma = d\Gamma_0 + d\Gamma_2 \left(\frac{\Lambda_{\mathrm{QCD}}}{m_b}\right)^2 + d\Gamma_3 \left(\frac{\Lambda_{\mathrm{QCD}}}{m_b}\right)^3 + d\Gamma_4 \left(\frac{\Lambda_{\mathrm{QCD}}}{m_b}\right)^4 + d\Gamma_5 \left[a_0 \left(\frac{\Lambda_{\mathrm{QCD}}}{m_b}\right)^5 + a_1 \left(\frac{\Lambda_{\mathrm{QCD}}}{m_b}\right)^3 \left(\frac{\Lambda_{\mathrm{QCD}}}{m_c}\right)^2\right] + \cdots$$

Reviews:

D. Benson, I. Bigi, T. Mannel, N. Uraltsev, Nucl.Phys. B665 (2003) 367;

J. Dingfelder, T. Mannel, Rev.Mod.Phys. 88 (2016) 035008.

- Can we apply $B \to X_c \ell \bar{\nu}$ HQE to $D \to X_s \ell \bar{\nu}$ decays?

See also: P. Gambino, F. Kamenik, Nucl.Phys. B840 (2010) 424

- Can we apply $B \to X_c \ell \bar{\nu}$ HQE to $D \to X_s \ell \bar{\nu}$ decays?
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- Can we apply $B \to X_c \ell \bar{\nu}$ HQE to $D \to X_s \ell \bar{\nu}$ decays?
- In $D \to X_s \ell \bar{\nu}$ there are two expansion parameter: $\Lambda_{\rm QCD}/m_c$ and m_s/m_c.
- RGE origin of non-analytic term $\sim \log m_s$.

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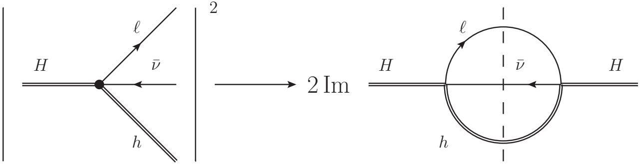
$$\Gamma(H \to X_q \ell \bar{\nu}) = \sum_h \Gamma(H \to h \ell \bar{\nu})$$



$$\Gamma \propto \sum_h \left| \langle h| \,\mathcal{H}_{\rm eff}(0) \,|H\rangle \right|^2 = 2\,\mathrm{Im} \int d^4x \, e^{-im_Q v\cdot x} \,\langle H| \; T\{\tilde{\mathcal{H}}_{\rm eff}(x), \tilde{\mathcal{H}}^\dagger_{\rm eff}(0)\} \,|H\rangle$$

- Meson momentum: $p_H = m_H v$
- Quark rephasing: $Q(x) = e^{-im_Q v\cdot x} Q_v(x)$, i.e. $p_Q = m_Q v + k$.

Operator Product Expansion

$$\int d^4x\, e^{-iq\cdot x}\, T\{\mathcal{O}_1(x), \mathcal{O}_2(0)\} = \sum_n \mathcal{C}_n\, \mathcal{O}_n(0)$$

- Matching scale μ.
- $\mathcal{C}_n(\mu)$: short distance (perturbative) effects.
- $\langle H|\, \mathcal{O}_n(0)\, |H\rangle_\mu$: large distance (non-perturbative) effects.

OPE for Semileptonic Decays

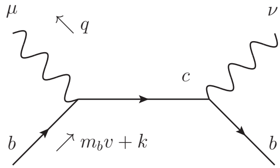
- The Effective Hamiltonian

$$\mathcal{H}_{\mathrm{eff}} = \frac{4 G_F V_{\mathrm{CKM}}}{\sqrt{2}} j_q^\mu L_\mu$$

- The decay rate $\Gamma(H \to X \ell \bar{\nu}) \propto L^{\mu\nu} W_{\mu\nu}$ with

$$W^{\mu\nu} = 2\mathrm{Im} \int d^4 x e^{-iq\cdot x} \langle H | \, T\{ j_q^{\dagger\mu}(x), j_q^\nu(0) \} \, | H \rangle$$

- Take matrix elements with free quark and gluon states:



$$= \bar{u}(p_b)\gamma^\mu P_L \left[\frac{1}{\not{P} + \not{k} - m_c}\right] \gamma^\nu P_L u(p_b)$$

with $P = m_b v - q$.

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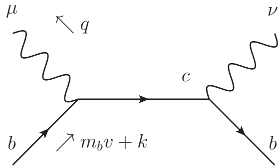

$$= \bar{u}(p_b)\gamma^\mu P_L \left[\frac{1}{\not{P} + \not{k} - m_c} \right] \gamma^\nu P_L u(p_b)$$

with $P = m_b v - q$.

Now expand ...

- $b \to c$ transitions:
  - large: $m_b v - q \sim m_b \sim m_c \gg \Lambda_{\rm QCD}$.
  - small: $k \sim \Lambda_{\rm QCD}$.

$$\frac{1}{\not{p} + \not{k} - m_c} = \frac{1}{\not{p} - m_c} - \frac{1}{\not{p} - m_c}\not{k}\frac{1}{\not{p} - m_c} + \frac{1}{\not{p} - m_c}\not{k}\frac{1}{\not{p} - m_c}\not{k}\frac{1}{\not{p} - m_c} + \dots$$

- $c \to s$ transitions:
  - large: $m_c v - q \sim m_c \gg \Lambda_{\rm QCD}$.
  - small: $k \sim m_s \sim \Lambda_{\rm QCD}$.

$$\frac{1}{\not{p} + \not{k} - m_s} = \frac{1}{\not{p}} - \frac{1}{\not{p}}(\not{k} - m_s)\frac{1}{\not{p}} + \frac{1}{\not{p}}(\not{k} - m_s)\frac{1}{\not{p}}(\not{k} - m_s)\frac{1}{\not{p}} + \dots$$

- $b \to c$ transitions:
  - large: $m_b v - q \sim m_b \sim m_c \gg \Lambda_{\mathrm{QCD}}$.
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$$\frac{1}{\not{p} + \not{k} - m_c} = \frac{1}{\not{p} - m_c} - \frac{1}{\not{p} - m_c} \not{k} \frac{1}{\not{p} - m_c} + \frac{1}{\not{p} - m_c} \not{k} \frac{1}{\not{p} - m_c} \not{k} \frac{1}{\not{p} - m_c} + \dots$$

- $c \to s$ transitions:
  - large: $m_c v - q \sim m_c \gg \Lambda_{\mathrm{QCD}}$.
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$$\frac{1}{\not{p} + \not{k} - m_s} = \frac{1}{\not{p}} - \frac{1}{\not{p}}(\not{k} - m_s)\frac{1}{\not{p}} + \frac{1}{\not{p}}(\not{k} - m_s)\frac{1}{\not{p}}(\not{k} - m_s)\frac{1}{\not{p}} + \dots$$

Operators in the HQET

$$\mathcal{O}_n(0) = \bar{Q}_v (iD_{\mu 1})(iD_{\mu 2}) \dots (iD_{\mu n}) \Gamma Q_v$$

HQE Parameters

- $1/m_Q^2$

$$\text{Kinetic energy:} \quad -2m_H\hat{\mu}_\pi^2 = \langle H| \, \bar{Q}_v (iD)^2 Q_v \, |H\rangle$$

$$\text{Chromomagnetic moment:} \quad -2m_H\hat{\mu}_G^2 = \langle H| \, \bar{Q}_v (iD_\mu)(iD_\nu)(i\sigma^{\mu\nu}) Q_v \, |H\rangle$$

- $1/m_Q^3$

$$\text{Darwin term:} \quad 2m_H\hat{\rho}_D^3 = \langle H| \, \bar{Q}_v (iD_\mu)(ivD)(iD^\mu) Q_v \, |H\rangle$$

$$\text{Spin-orbit:} \quad 2m_H\hat{\rho}_{LS}^3 = \langle H| \, \bar{Q}_v (iD_\mu)(ivD)(iD_\nu)(i\sigma^{\mu\nu}) Q_v \, |H\rangle$$

- $1/m_Q^4$: 4 parameters; $1/m_Q^5$: 18 parameters.
  T. Mannel, S. Turczyk, N. Uraltsev, JHEP 1011 (2010) 109;
  T. Mannel, K. Vos, hep-ph:1802.09409.

The general structure of the expansion for $D \to X_s \ell \bar{\nu}$:

$$d\Gamma = d\Gamma_0 + d\Gamma_{(2,1)} \left(\frac{\Lambda_{\mathrm{QCD}}}{m_c}\right)^2 + d\Gamma_{(2,2)} \left(\frac{m_s}{m_c}\right)^2 + d\Gamma_3 \left(\frac{\Lambda_{\mathrm{QCD}}}{m_c}\right)^3 + d\Gamma_{(4,1)} \left(\frac{\Lambda_{\mathrm{QCD}}}{m_c}\right)^4 + d\Gamma_{(4,2)} \left(\frac{m_s}{m_c}\right)^4 + \cdots$$

Non-analytic terms $b \to c$ decays

- Quark mass ratio: $\rho = (m_c/m_b)^2$.
- Partonic:

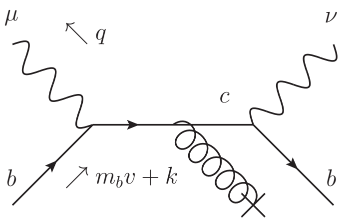
$$\Gamma\Big|_{\text{partonic}} = \frac{G_F^2 m_b^5}{192\pi^3}|V_{cb}|^2\{1 - 12\rho^2 \log\rho + \cdots\}$$

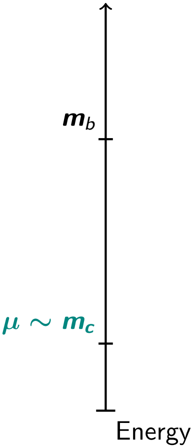
- Darwin term ρ_D^3:

$$\Gamma\Big|_{\rho_D^3} = \frac{G_F^2 m_b^5}{192\pi^3}|V_{cb}|^2 \frac{\rho_D^3}{m_b^3}\{\log\rho + \cdots\}$$

- ...

Full Theory





HQET v1:

- matching scale $\mu \sim m_c$
- Operators contain b_v and iD_μ

$$\Gamma\Big|_{\rho_D^3} = \frac{G_F^2 m_b^5}{192\pi^3} |V_{cb}|^2 \frac{\rho_D^3}{m_b^3} \{\log \rho + \cdots\}$$

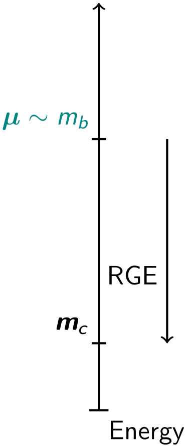
HQET v2:

- Operators:

$$\langle B|\, \bar{b}_v (iD_\mu)(ivD)(iD^\mu) b_v \,|B\rangle = 2M_B \rho_D^3(\mu)$$
$$\langle B|\, (\bar{b}_v \gamma^\nu P_L c)(\bar{c}\gamma^\mu P_L b_v) \,|B\rangle = 2M_B \left[T_1(\mu) g^{\mu\nu} + T_2(\mu) v^\mu v^\nu\right]$$

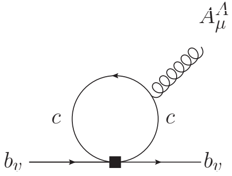
- Matching conditions:

$$\mathcal{C}_{\rho_D^3}(m_b) = \frac{17}{12} \qquad \mathcal{C}_{T_1}(m_b) = -3 \qquad \mathcal{C}_{T_2}(m_b) = 0$$



RGE evolution

$$\frac{d\vec{\mathcal{C}}(\mu)}{d\mu} = \gamma^T \vec{\mathcal{C}}(\mu)$$



- RGE solution:

$$\mathcal{C}_{T_i}(\mu) = \mathcal{C}_{T_i}(m_b)$$

$$\mathcal{C}_{\rho_D^3}(\mu) = \mathcal{C}_{\rho_D^3}(m_b) - \frac{1}{3}\log\left(\frac{\mu^2}{m_b^2}\right)\left[\mathcal{C}_{T_1}(m_b) - 2\mathcal{C}_{T_2}(m_b)\right]$$

- At the scale $\mu = m_c$

$$\mathcal{C}_{T_i}(m_c) = 0 \qquad\qquad \mathcal{C}_{\rho_D^3}(m_c) = \frac{17}{12} + \log\left(\frac{m_c^2}{m_b^2}\right)$$

See also: I. Bigi, N. Uraltsev, R. Zwicky, Eur.Phys.J. C50 (2007) 539; I. Bigi, T. Mannel, S. Turczyk, N. Uraltsev, JHEP 1004 (2010) 073; C. Breidenbach, T. Feldmann, T. Mannel, S. Turczyk, Phys.Rev. D78 (2008) 014022.

HQET v2:

- Operators:

$$\langle D| \, \bar{c}_v (iD_\mu)(ivD)(iD^\mu) c_v \, |D\rangle = 2 M_D \rho_D^3(\mu)$$
$$\langle D| \, (\bar{c}_v \gamma^\nu P_L s)(\bar{s} \gamma^\mu P_L c_v) \, |D\rangle = 2 M_D \left[T_1(\mu) g^{\mu\nu} + T_2(\mu) v^\mu v^\nu \right]$$

- Matching conditions:

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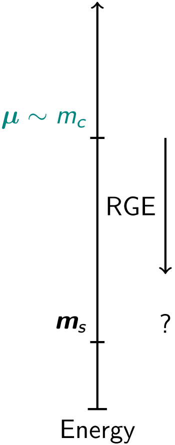

Can we evolve $\mu \to m_s$?

Define the RGE invariant:

$$R_D = \rho_D(\mu)\left[\frac{17}{12} + \log\left(\frac{\mu^2}{m_c^2}\right)\right] - 3\,T_1(\mu)$$

Then measure:

$$\Gamma\Big|_{\rho_D^3} = \frac{G_F^2 m_b^5}{24\pi^3}|V_{cs}|^2 \frac{1}{m_b^3} R_D$$

$m_s^4 \log m_s$

- Operator Mixing:

$$\langle D|\,(i\partial_\alpha \bar{c}_v \Gamma_\mu s)(\bar{s}\Gamma_\nu c_v)\,|D\rangle = 2M_D\Big[T_3(\mu)g_{\mu\nu}v_\alpha + T_4(\mu)g_{\mu\alpha}v_\nu + T_5(\mu)g_{\nu\alpha}v_\mu + T_6(\mu)v_\mu v_\nu v_\alpha + iT_7(\mu)\epsilon_{\mu\nu\alpha\beta}v^\mu\Big]$$

$$O_m = m_s^4(\mu)\bar{c}_v \not{v} c_v$$

- RGE evolution

$$C_m(\mu) = C_m(m_c) - \frac{1}{8}\Big[C_{T_3}(m_c) - C_{T_4}(m_c) - C_{T_5}(m_c) - C_{T_7}(m_c)\Big] = -12\log\left(\frac{\mu^2}{m_c^2}\right)$$

- RGE invariant:

$$M_D = C_m(\mu)m_c(\mu)^4 + \sum_{i=3}^{7} T_i(\mu)$$

See also: C. Bauer, A. Falk, M. Luke, Phys.Rev. D54 (1996) 2097

Outlook and Conclusions

- Systematic heavy quark expansion of $D \to X_s \ell \nu$ rate in $\Lambda_{\mathrm{QCD}}/m_c$ and m_s/m_c.
- In $c \to s \ell \bar{\nu}$ transition we cannot integrate out four-quark operators.
- We propose to employ HQE parameters which are RGE invariant.

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- In $c \to s \ell \bar{\nu}$ transition we cannot integrate out four-quark operators.
- We propose to employ HQE parameters which are RGE invariant.

* Please measure inclusive semileptonic $D \to X_s \ell \bar{\nu}$ rate, lepton's spectrum, moments ...
* Please calculate HQE parameters for $D \to X_s \ell \bar{\nu}$ on the lattice.

Outlook and Conclusions (To Do List)

- HQE parameters at $1/m_c^4$
- RGE origin of $\frac{1}{m_c^3}\frac{1}{m_s^2}$?
- Role α_s corrections.
- Clarify the m_c definition that must be employed.