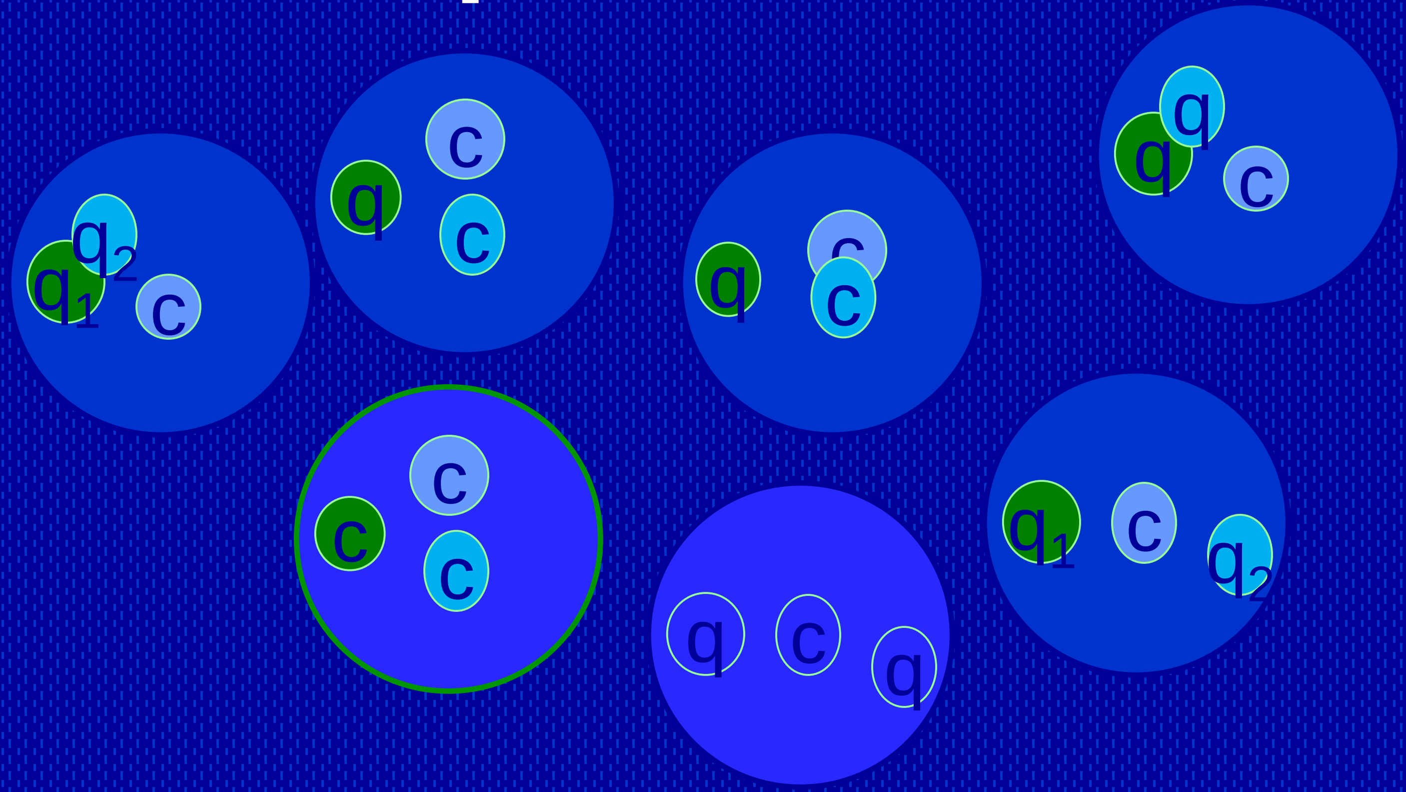
Theoretical Aspects of Charmed Baryons



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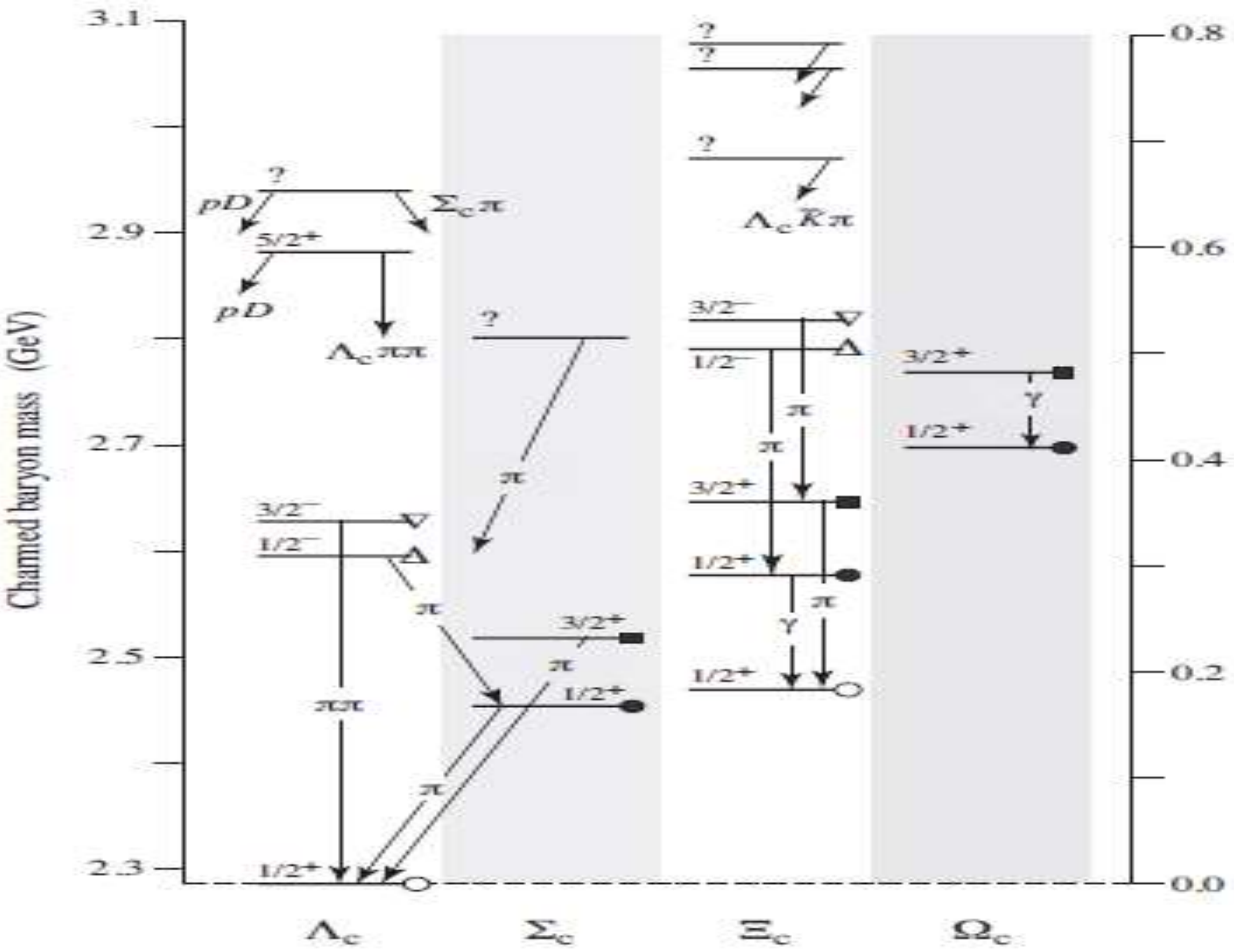
Hadron Spectrum Collaboration

Charmed baryons—why do we need to study them?

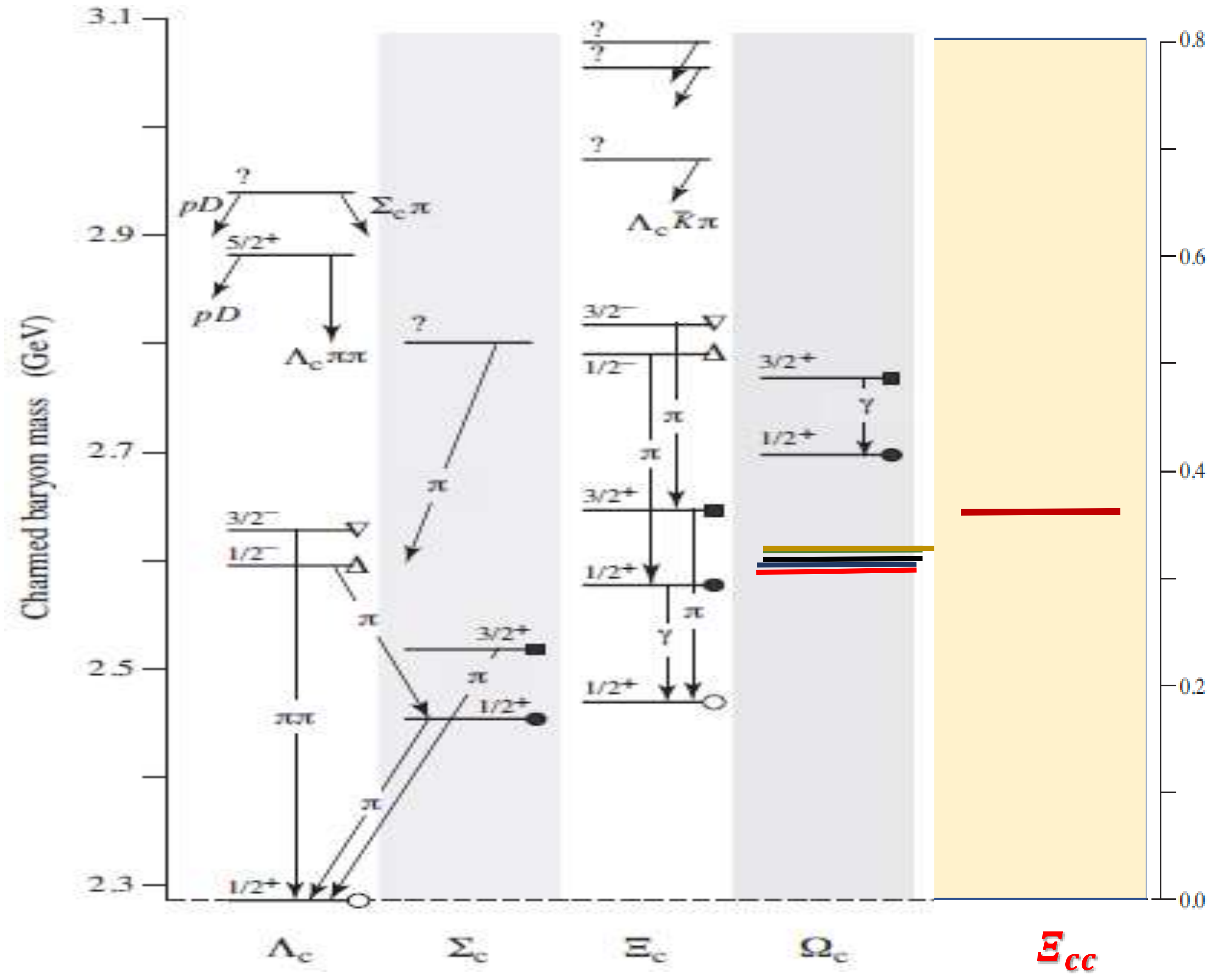
- Have not been studied (both experimentally and theoretically (to some extent)) in great details as charmed mesons even though they can provide similar or more information about the theory of strong interactions.

- **Singly charmed baryons :**
  - light quark dynamics in presence of one heavy quark.
  - Light diquarks in presence of heavy quark
  - Experimentally many more states may be observed (if not, then diquark picture).
- **Doubly charmed baryons :**
  - nature of strong force in the presence of slow relative motion of the heavy quarks along with the relativistic motion of a light quark.
  - Multiple scales
  - Is there any quark-diquark symmetry : $[QQ]q \sim Q'q$
  - Only one state has just been discovered. Should be many more..
- **Triply-charmed baryons :**
  - Charmonia analogues in baryons
  - Heavy quark-quark interaction
  - No experimental discovery yet

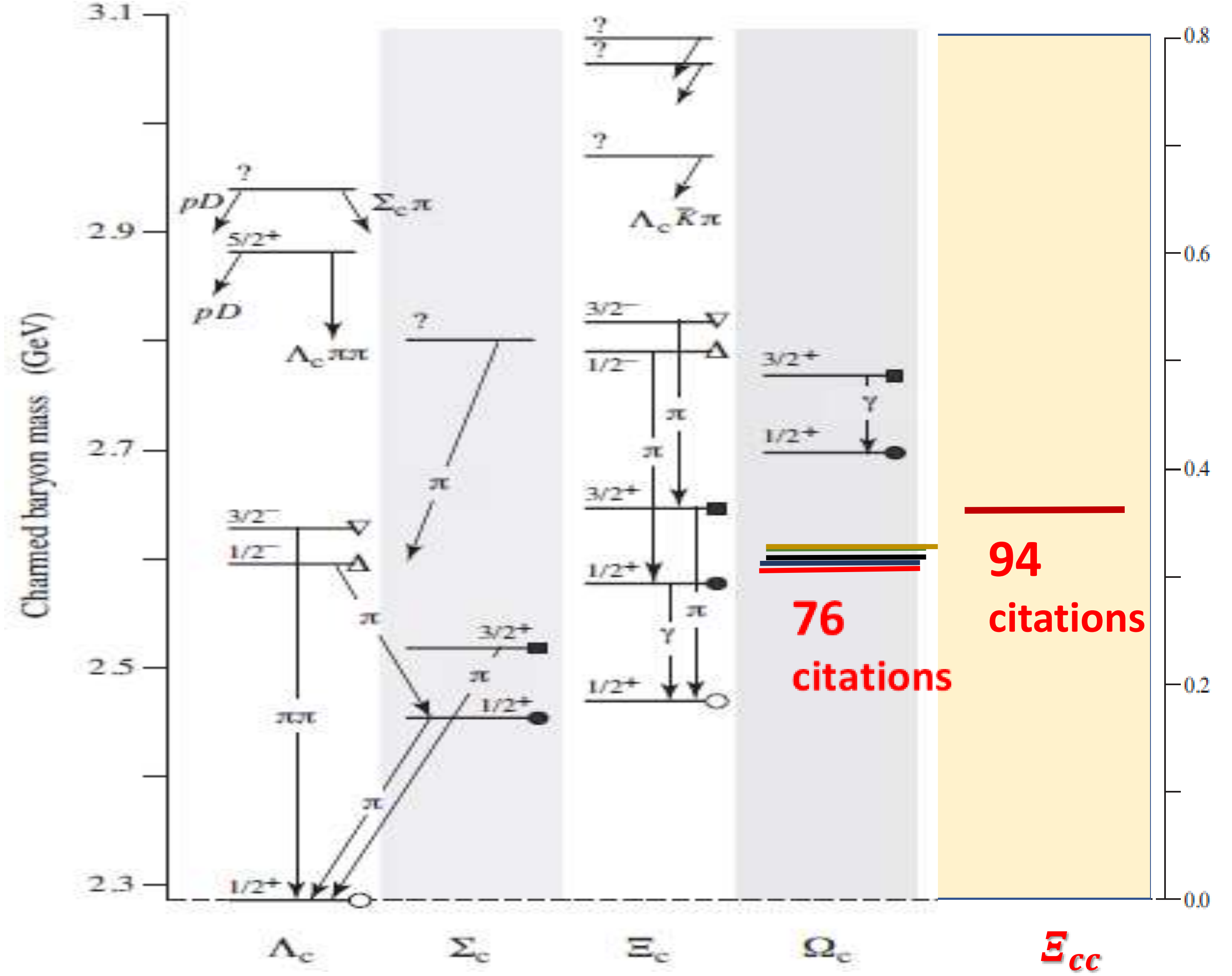
Charmed Baryons (PDG,2016)



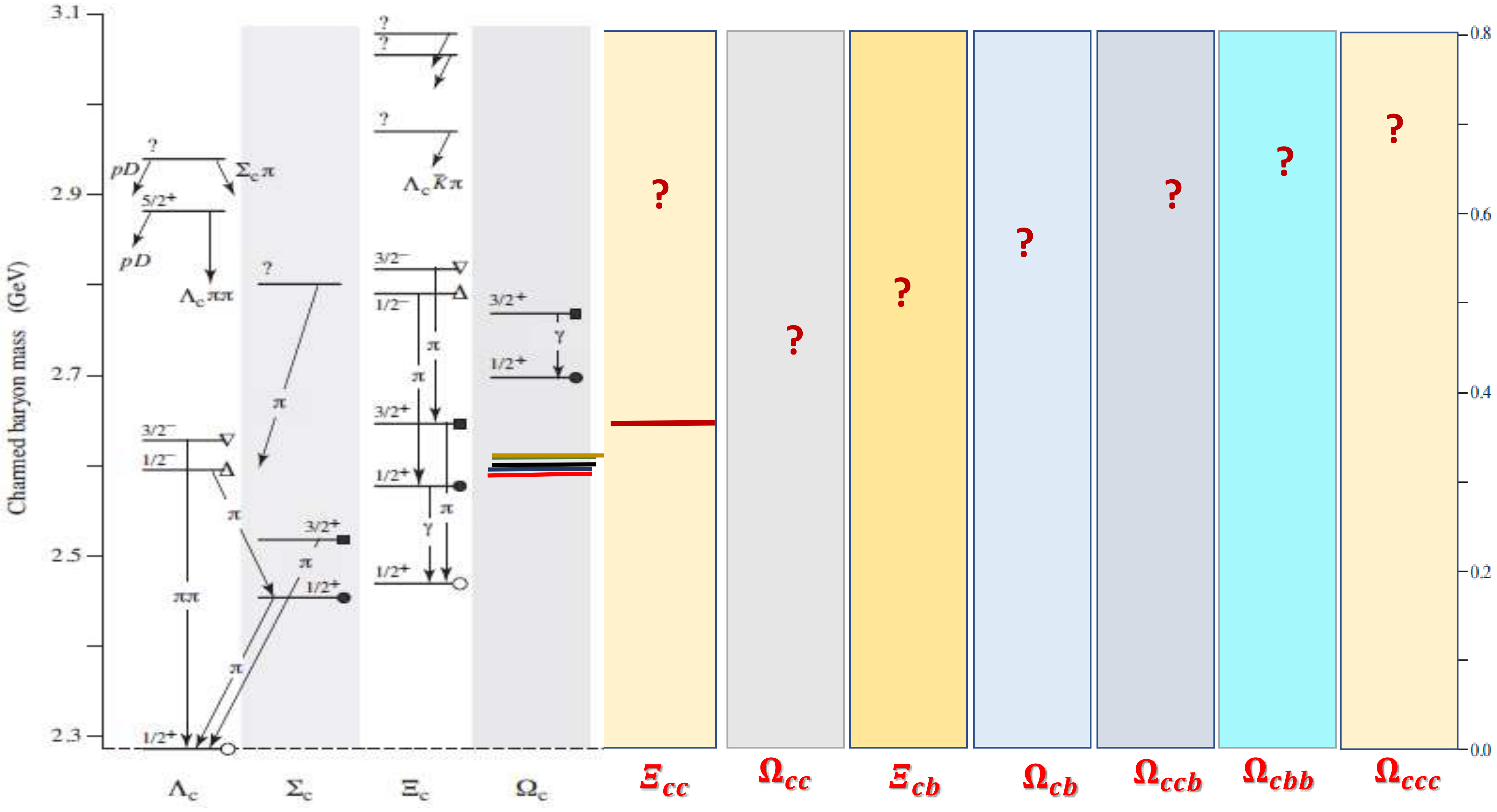
Charmed Baryons (2017)



Charmed Baryons (2017)



Charmed Baryons (2017)



Theory of hadrons

- **QCD** : Theory of strong Interactions
  - So far, we are incapable in solving the full theory analytically in the non-perturbative regime
- Define theory in 4D Euclidean space-time grid and solve numerically
  : **Lattice QCD** (systematics needs to be in good control, the study of excited states with multi-hadron scatterings has made impressive progress but is still under development)
- **Models :**
  - Motivated from QCD or low energy limits of QCD
    - Use chiral Lagrangian rather than full QCD Lagrangian
      - Spin independent confining interaction (linear or HO)
      - Spin dependent hyperfine interactions: $V_{HF} \propto \sum_{i>j} (\vec{\sigma}\lambda_a)_i (\vec{\sigma}\lambda_a)_j$

        $V_{HF} \propto \sum_{i>j} V(\vec{r}_{ij}) \lambda_i^F . \lambda_j^F \vec{\sigma}_i . \vec{\sigma}_j$
      - Spin-orbit and tensor interactions
      - Flavour dependent short range quark force
      - Flavour symmetric spin-spin interaction

Theory of Hadrons

- **Models :**
  - Quark models (relativistic, non-relativistic, quark-diquarks, Regge trajectories, heavy quark spin symmetry, chiral quark model, Nambu-Jona-Lasinio model)
  - Effective field theories (HQET, NRQCD, pNRQCD, couple channel analysis)
  - QCD sum rules
  - Holographic QCD
  - and more

Quark Model

$$H = \sum_i K_i + \sum_{i<j} \left(V^{ij}_{\rm conf} + H^{ij}_{\rm hyp} \right)$$

- Kinetic Energy :

$$K_i = \left(m_i + \frac{p_i^2}{2m_i} \right)$$

Roberts et.al : Int.J.Mod.Phys. A23 (2008) 2817-2860

- Spin independent confining potential :

$$V^{ij}_{\rm conf} = \sum_{i<j=1}^{3} \left(\underbrace{\frac{b r_{ij}}{2}}_{\text{linear}} - \underbrace{\frac{2\alpha_{\rm Coul}}{3 r_{ij}}}_{\text{Coulomb}} \right)$$

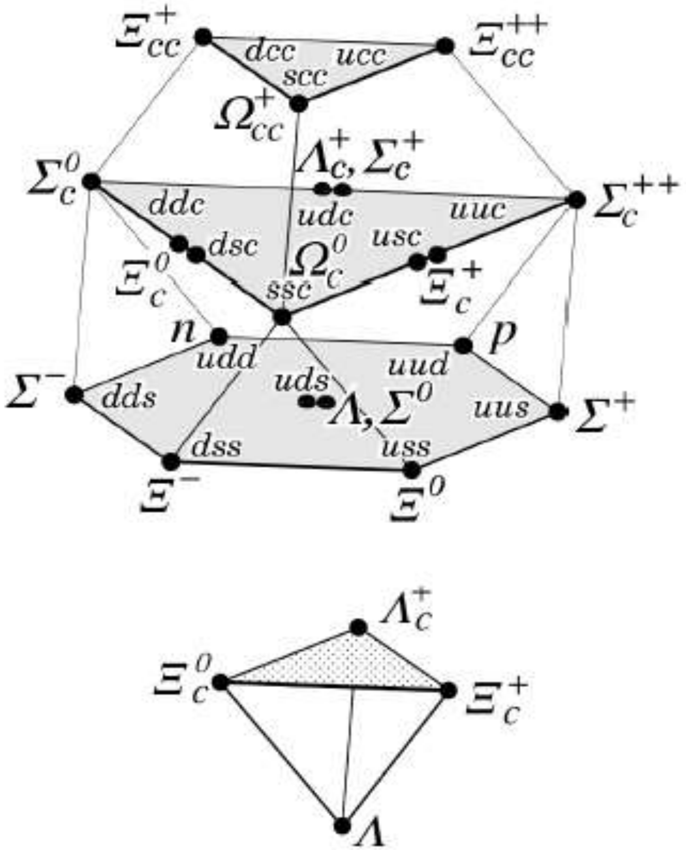
- Spin dependent hyperfine interaction :

$$H^{ij}_{\rm hyp} = \sum_{i<j=1}^{3} \left[\underbrace{\frac{2\alpha_{\rm con}}{3 m_i m_j} \frac{8\pi}{3} \mathbf{S}_i \cdot \mathbf{S}_j \delta^3(\mathbf{r}_{ij})}_{\text{Contact}} + \underbrace{\frac{2\alpha_{\rm ten}}{3 m_i m_j} \frac{1}{r^3_{ij}} \left(\frac{3 \mathbf{S}_i \cdot \mathbf{r}_{ij} \mathbf{S}_j \cdot \mathbf{r}_{ij}}{r^2_{ij}} - \mathbf{S}_i \cdot \mathbf{S}_j \right)}_{\text{tensor}} \right]$$

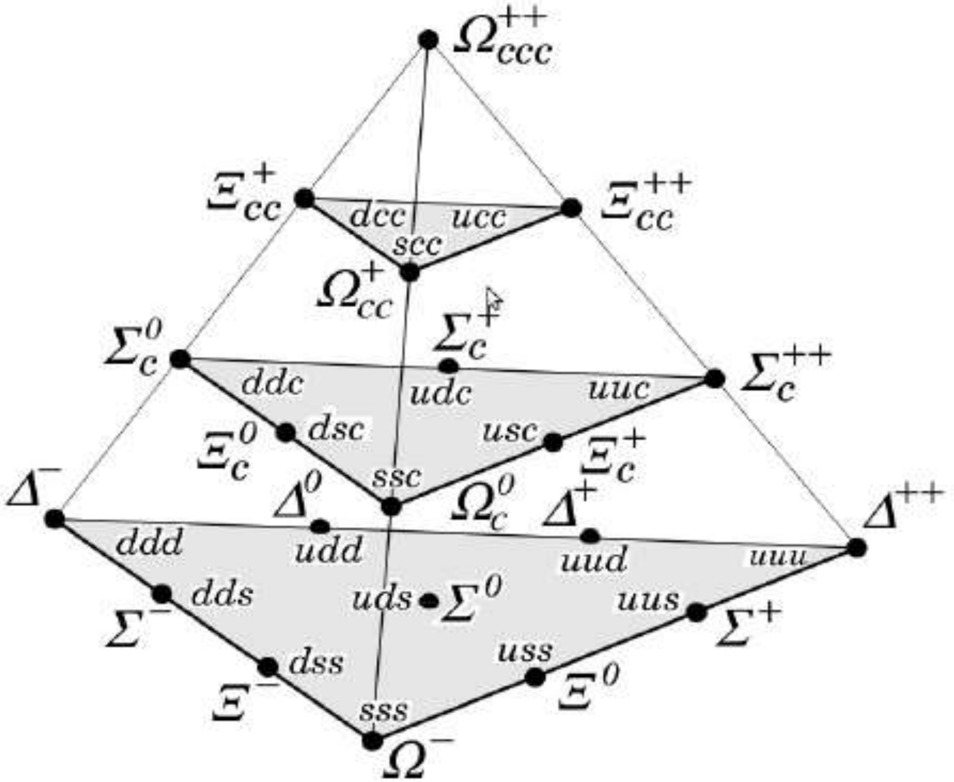
- Spin-orbit potential :

$$V_{\rm SO} = \frac{\alpha_{\rm SO}}{\rho^2 + \lambda^2} \frac{\mathbf{L} \cdot \mathbf{S}}{(m_1 + m_2 + m_3)^2}$$

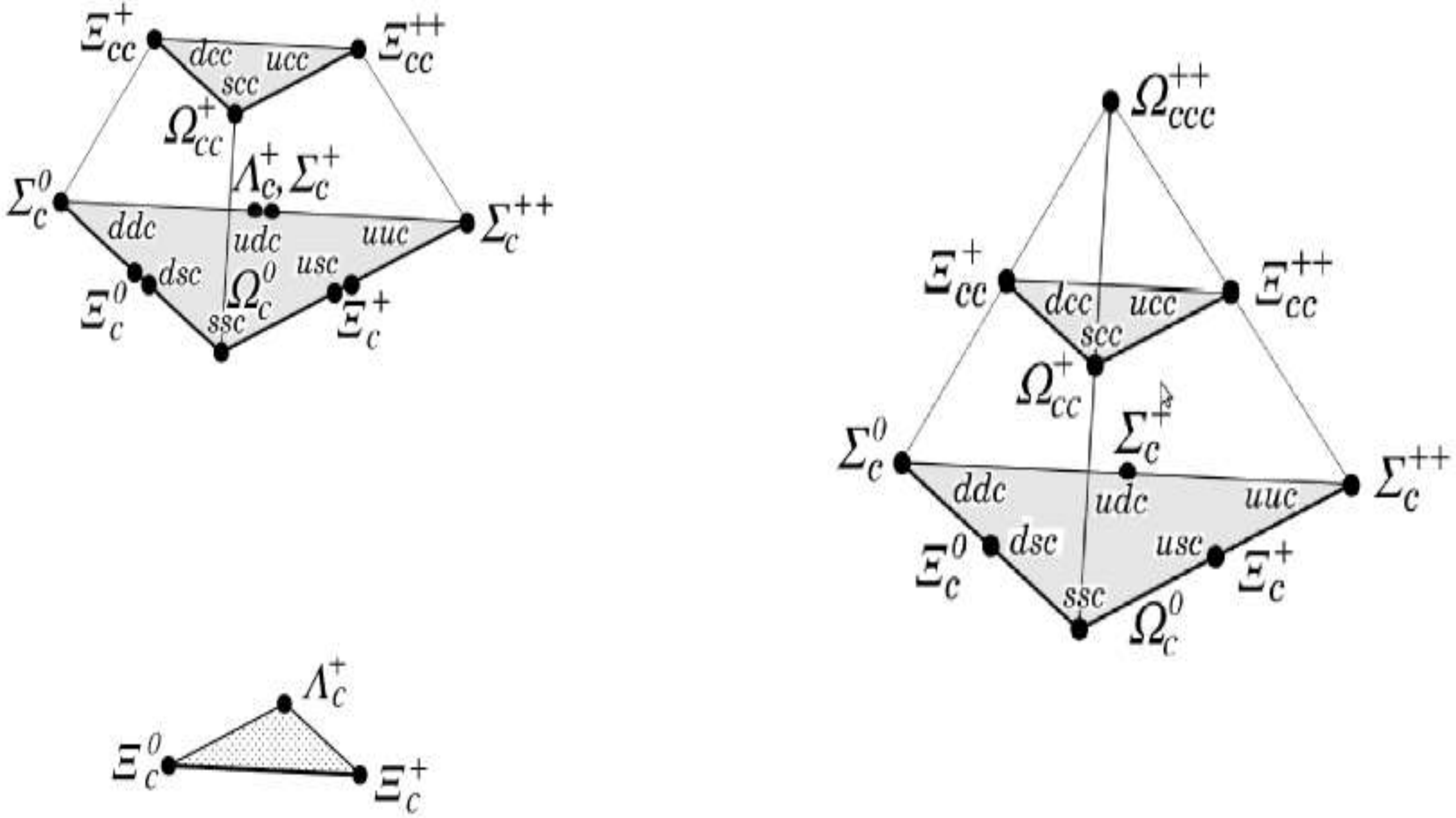
Octet



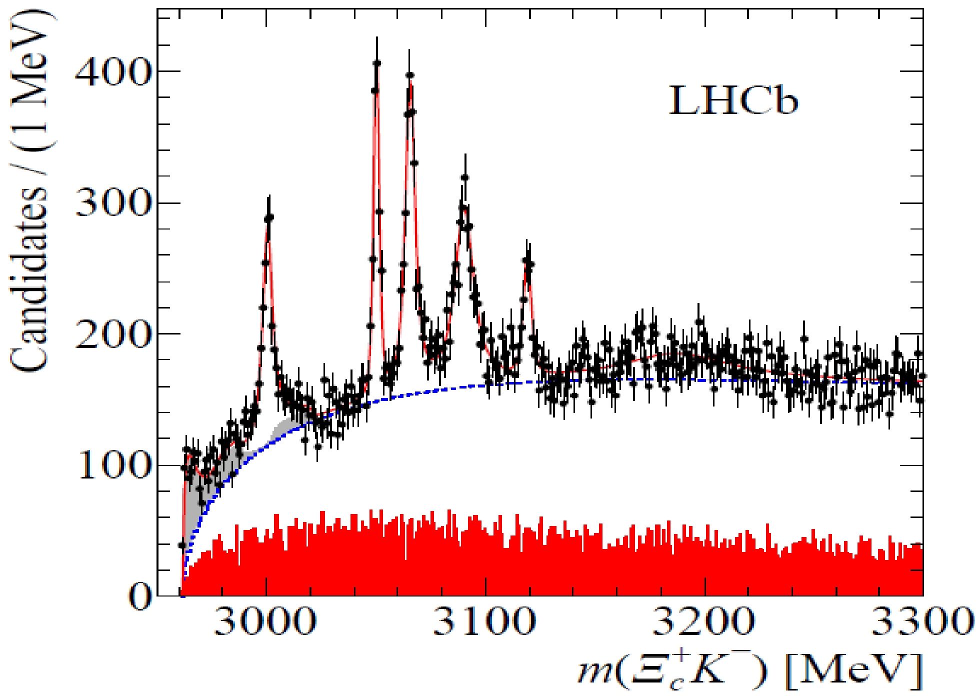
Decuplet



SU(4) multiplets


Ξ_{cc}^{+}
Ξ_{cc}^{++}
dcc
ucc
scc
Ω_{cc}^{+}
$\Lambda_c^{+}, \Sigma_c^{+}$
Σ_c^{0}
Σ_c^{++}
ddc
udc
uuc
dsc
usc
Ω_c^{0}
Ξ_c^{0}
Ξ_c^{+}
ssc
Λ_c^{+}
Ξ_c^{0}
Ξ_c^{+}
Ω_{ccc}^{++}
Ξ_{cc}^{+}
Ξ_{cc}^{++}
dcc
ucc
scc
Ω_{cc}^{+}
Σ_c^{+}
Σ_c^{0}
Σ_c^{++}
ddc
udc
uuc
dsc
usc
Ξ_c^{0}
Ξ_c^{+}
ssc
Ω_c^{0}

Ω_c^0 Baryons



LHCb Collaboration :
Phys. Rev. Lett. 118 (2017) no.18, 182001

Resonance	Mass (MeV)	Γ (MeV)	Yield	N_σ
$\Omega_c(3000)^0$	$3000.4 \pm 0.2 \pm 0.1^{+0.3}_{-0.5}$	$4.5 \pm 0.6 \pm 0.3$	$1300 \pm 100 \pm 80$	20.4
$\Omega_c(3050)^0$	$3050.2 \pm 0.1 \pm 0.1^{+0.3}_{-0.5}$	$0.8 \pm 0.2 \pm 0.1$	$970 \pm 60 \pm 20$	20.4
		< 1.2 MeV, 95% CL		
$\Omega_c(3066)^0$	$3065.6 \pm 0.1 \pm 0.3^{+0.3}_{-0.5}$	$3.5 \pm 0.4 \pm 0.2$	$1740 \pm 100 \pm 50$	23.9
$\Omega_c(3090)^0$	$3090.2 \pm 0.3 \pm 0.5^{+0.3}_{-0.5}$	$8.7 \pm 1.0 \pm 0.8$	$2000 \pm 140 \pm 130$	21.1
$\Omega_c(3119)^0$	$3119.1 \pm 0.3 \pm 0.9^{+0.3}_{-0.5}$	$1.1 \pm 0.8 \pm 0.4$	$480 \pm 70 \pm 30$	10.4
		< 2.6 MeV, 95% CL		
$\Omega_c(3188)^0$	$3188 \pm 5 \pm 13$	$60 \pm 15 \pm 11$	$1670 \pm 450 \pm 360$	
$\Omega_c(3066)^0_{\rm fd}$			$700 \pm 40 \pm 140$	
$\Omega_c(3090)^0_{\rm fd}$			$220 \pm 60 \pm 90$	
$\Omega_c(3119)^0_{\rm fd}$			$190 \pm 70 \pm 20$	

Structures and Quantum Numbers of Ω_c^0

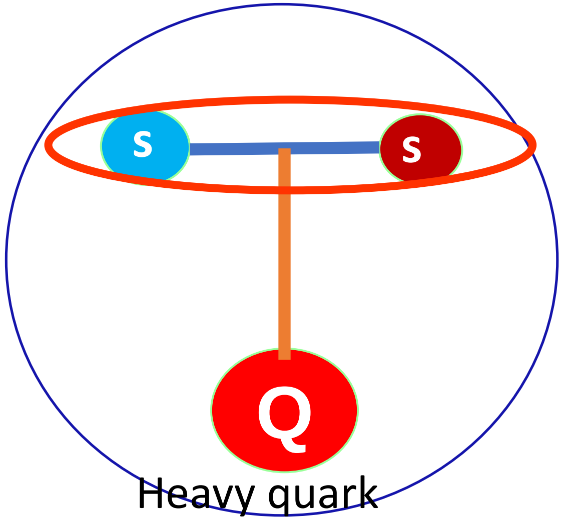
- Why are the new states narrower in widths?

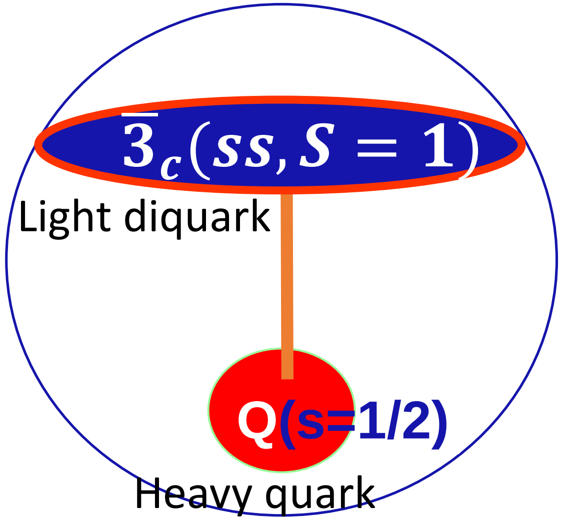
- Are those **c(*ss*)** systems? Is it thus difficult to pull apart two ***s*** quarks from a ***ss*** diquark? (The decay to $\boldsymbol{\Xi(uss)D(c\bar{u})}$ would have been favoured if it was kinemetically allowed.)

- Or, are these narrow pentaquark states?

- Why are there no peaks in $\Xi_c^+ K^+$?

Simple Picture (P wave states)

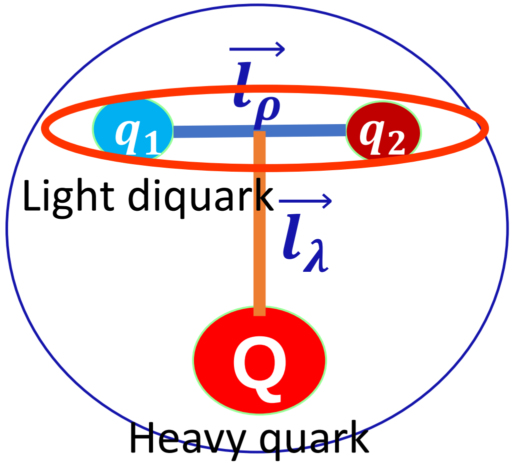


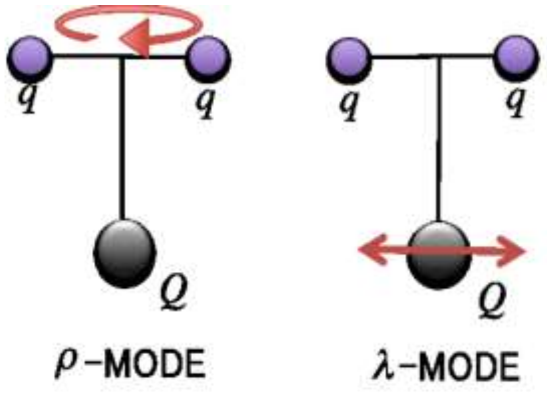


S : 1/2, 3/2

L(=1) + S(=1/2) : 1/2, 3/2

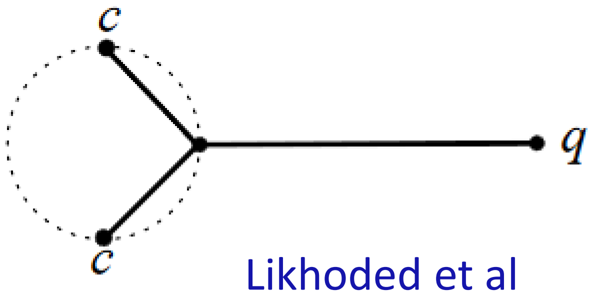
L(=1) + S(=3/2) : 1/2, 3/2, 5/2





Courtesy : M. Pappagallo

Yoshida et.al. Phys.Rev. D92 (2015) 114029



Likhoded et al

$\vec{l_\rho}$: Orbital angular momentum due to ρ-mode, that is between two light quarks which form a diquark system

$\vec{l_\lambda}$: Orbital angular momentum due to λ-mode, that is between heavy and light diquarks

$\vec{s}_{DQ} = \vec{s}_{q_1} + \vec{s}_{q_2}$: Spin of diquark (sum of light quark spins)

$\vec{s}_Q$: Spin of heavy quark

$\vec{L} = \vec{l}_\rho + \vec{l}_\lambda$: Total orbital angular momentum

$\vec{j}_{DQ} = \vec{L} + \vec{s}_{DQ}$: Angular momentum of the light diquark system

$\vec{J} = \vec{j}_{DQ} + \vec{s}_{DQ}$: Total angular momentum

$[H, s_Q] = 0$

Parity : $(-1)^{l_\rho + l_\lambda}$

The structure of pentaquarks Ω_c^0 in the chiral quark model

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Recently, the experimental results of LHCb Collaboration suggested the existence of five new excited states of Ω_c^0, $\Omega_c(3000)^0$, $\Omega_c(3050)^0$, $\Omega_c(3066)^0$, $\Omega_c(3090)^0$ and $\Omega_c(3119)^0$, the quantum numbers of these new particles are not determined now. To understand the nature of the states, a dynamical calculation of 5-quark systems with quantum numbers $IJ^P = 0(\frac{1}{2})^-$, $0(\frac{3}{2})^-$ and $0(\frac{5}{2})^-$ is performed in the framework of chiral quark model with the help of gaussian expansion method. The results show the $\Xi\bar{D}$, $\Xi_c\bar{K}$ and $\Xi_c^*\bar{K}$ are possible the candidates of these new particles. The distances between quark pairs suggest that the nature of pentaquark states.

The observed Ω_c^0 resonances as pentaquark states

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(Dated: July 27, 2017)

Abstract

In present work, we investigate the spectrum of several low-lying $sscq\bar{q}$ pentaquark configurations employing the constituent quark model, within which the hyperfine interaction between quarks is taken to be mediated by Goldstone boson exchange. Our numerical results show that four $sscq\bar{q}$ configurations with $J^P = 1/2^-$ or $J^P = 3/2^-$ lie at energies very close to the recently observed five Ω_c^0 states by LHCb collaboration, this indicates that the $sscq\bar{q}$ pentaquark configurations may form sizable components of the observed Ω_c^0 resonances.

Narrow pentaquarks as diquark-diquark-antiquark systems

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September 1, 2017

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Abstract

The diquark-diquark-antiquark model describes pentaquark states both in terms of quarks and hadrons. The latest LHCb data for pentaquarks with open charm emphasize the importance of hadron components in the structure of pentaquarks. We discuss pentaquark states with hidden charm $P(\bar{c}cuud)$ and those with open charm $P(\bar{u}ussc)$ which were discovered recently in LHCb data ($J/\Psi p$ and $\Xi_c^+ K^-$ spectra correspondingly). Considering the observed states as members of the lowest (s-wave) multiplet, we discuss the mass splitting of states and the dumping of their widths.

EXOTIC INTERPRETATION OF Ω_c EXCITED STATES

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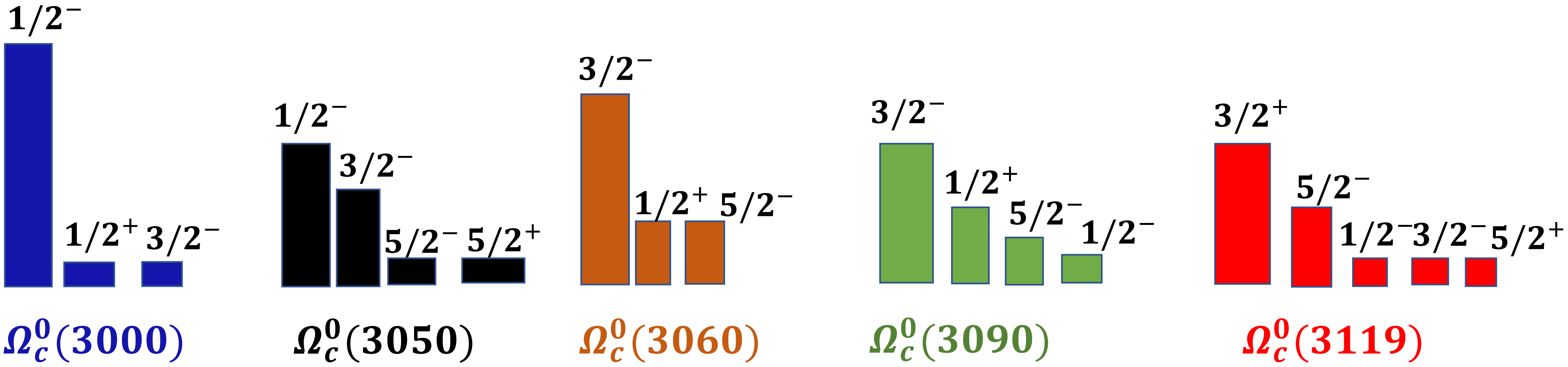
We use the chiral quark-soliton model to interpret five excited Ω_c states recently reported by the LHCb collaboration and confirmed by Belle. We briefly recapitulate the model and its application to light baryons. We then show how the model can be extended to the case of baryons with one heavy quark. We test the model against ground state heavy baryons and then examine possible excitations. We argue that it is not possible to accommodate all five Ω_c's within five parity minus excitations predicetd by the model and propose to interpret two narrowest states split by 70 MeV as pentaquarks belonging to the SU(3) representation $\overline{15}$.

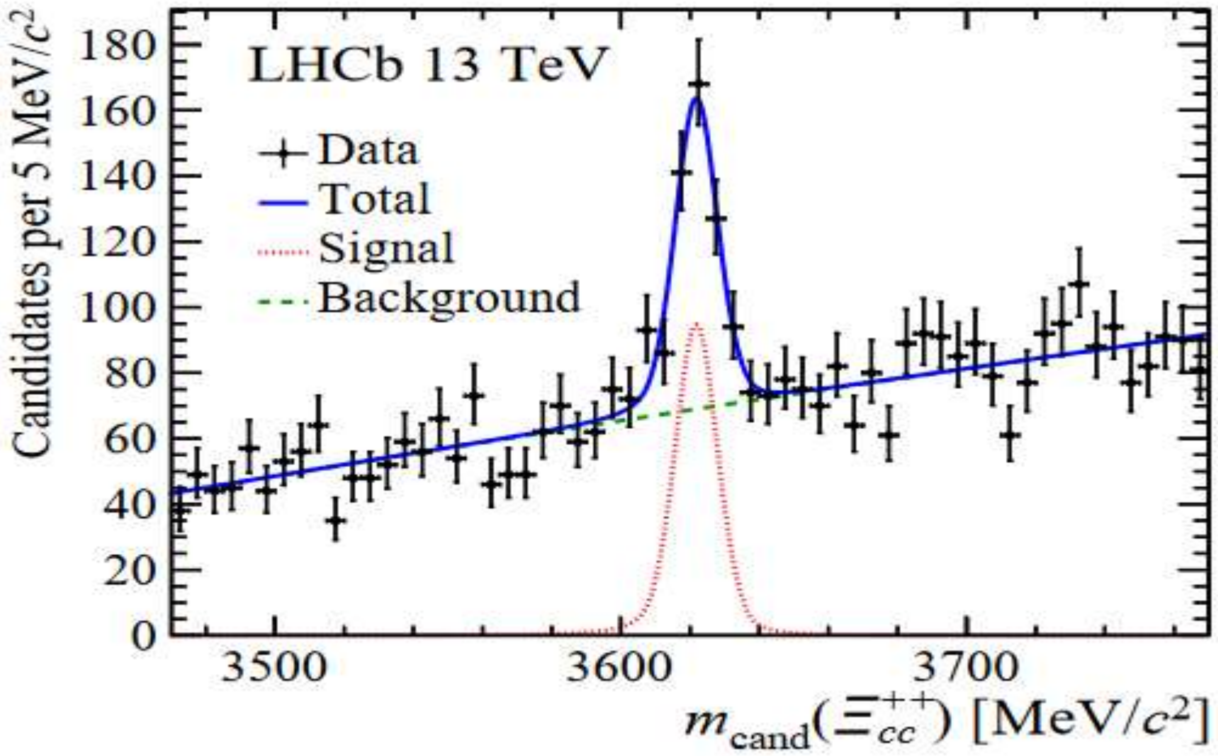
Structures and Quantum Numbers of Ω_c^0

- Are they orbitally excited (L=1) states? Or radial excitations?
- Or are they Pentaquarks? Lack of signal in $\Xi_c^+ K^+$ spectrum?
- Mixtures of pentaquarks and baryons?

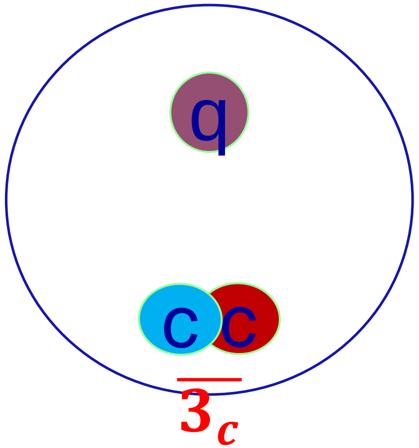


Most, though not all, of the studies favoured p-wave baryons

Discovery of doubly-charmed baryons



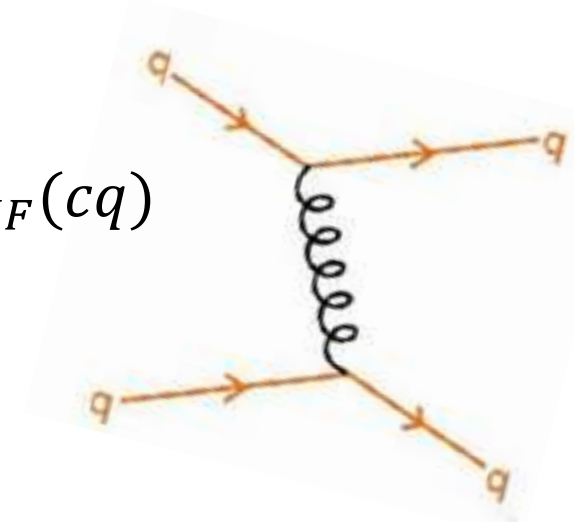
LHCb : **Phys. Rev. Lett. 119 (2017) no.11, 112001**



Phenomenological Quark Model

Karliner and Rosner arXiv:1408.5877

$$H \equiv 2m_c + m_q + V_{cc}(\overline{3}_c) + V_{HF}(cc) + V_{HF}(cq)$$



One gluon exchange :
Weak coupling approximation

State	Color	$\langle T_1 \cdot T_2 \rangle$
$Q\bar{Q}$	1	$-4/3$
$Q\bar{Q}$	8	$1/6$
QQ	3^*	$-2/3$
QQ	6	$1/3$

- V_{cc} from $V_{c\bar{c}}$:

$$V_{c\bar{c}} = \overline{M}(c\bar{c}{:}\,1S) - 2m_c^m = -258\ MeV$$

$$V_{cc} = \frac{1}{2} V_{c\bar{c}} = -129\ \text{MeV}$$

- $V_{HF}(cc)$ from $V_{HF}(c\bar{c})$:

$$V_{HF}(c\bar{c}) = \frac{4a_{c\bar{c}}}{(m_c^M)^2} = M(J/\psi) - M(\eta_c) = 113.2\ MeV$$

$$V_{HF}(cc) = \frac{a_{cc}}{(m_c^B)^2} = \frac{1}{2}\frac{a_{c\bar{c}}}{(m_c^B)^2} = 14.2\ MeV$$

Contribution	Value (MeV)
$2m_c^b + m_q^b$	3783.9
cc binding	-129.0
$a_{cc}/(m_c^b)^2$	14.2
$-4a/m_q^b m_c^b$	-42.4
Total	3627 ± 12

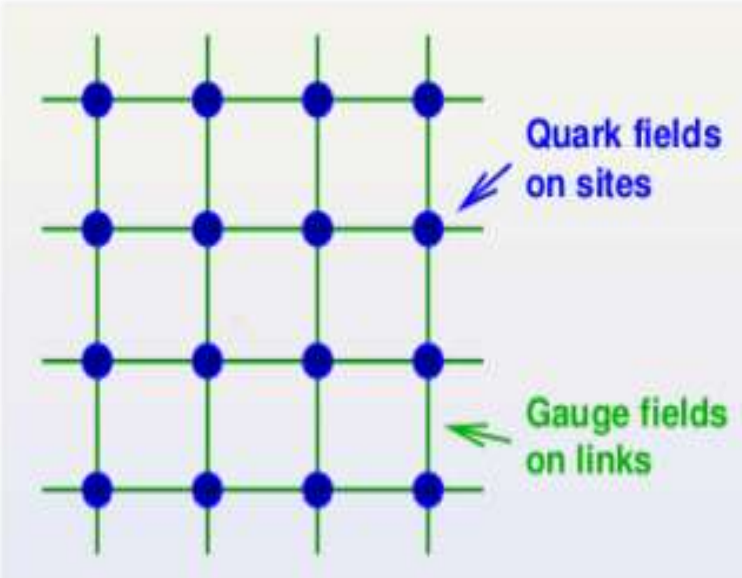
Karliner and Rosner arXiv:1408.5877

State	Quark content	$M(J=1/2)$	$M(J=3/2)$
$\Xi_{cc}^{(*)}$	ccq	3627 ± 12	3690 ± 12
$\Xi_{bc}^{(*)}$	$b[cq]$	6914 ± 13	6969 ± 14
Ξ_{bc}'	$b(cq)$	6933 ± 12	–
$\Xi_{bb}^{(*)}$	bbq	10162 ± 12	10184 ± 12

Baryon	This work	[28]	[51]	[71]	[72]
$\Xi_{cc}^{++} = ccu$	185	430 ± 100	460 ± 50	500	~ 200
$\Xi_{cc}^{+} = ccd$	53	120 ± 100	160 ± 50	150	~ 100
$\Xi_{bc}^{+} = bcu$	244	330 ± 80	300 ± 30	200	–
$\Xi_{bc}^{0} = bcd$	93	280 ± 70	270 ± 30	150	–
$\Xi_{bb}^{0} = bbu$	370	–	790 ± 20	–	–
$\Xi_{bb}^{-} = bbd$	370	–	800 ± 20	–	–

LQCD : A non-perturbative, gauge invariant regulator for the QCD path integrals.

- Quark fields lives on sites
- Gauge fields lives on links
- Lattice spacing : UV cut off
- Lattice size : IR cut off



Discretization $\Rightarrow$ Finite number of degrees of freedom
$\Rightarrow$ Infinte dimensional path integrals $\rightarrow$ finite dimensional integrals.

Employ Monte Carlo importance sampling methods on Euclidean metric for numerical studies.

LQCD for heavy quark physics

$$L_q = \overline{\psi}(\not{D} + m)\psi \rightarrow \overline{\psi}(\gamma \cdot \Delta + ma)\psi$$

Source of discretization error (need improved discretization method preserving continuum symmetries)

Λ_{QCD}

m_Q **?**

$$E = E_{a=0}(1 + A(m_Q a)^2 + B(m_Q a)^3 + \ldots)$$

***ma* << 1**

LQCD for heavy quark physics

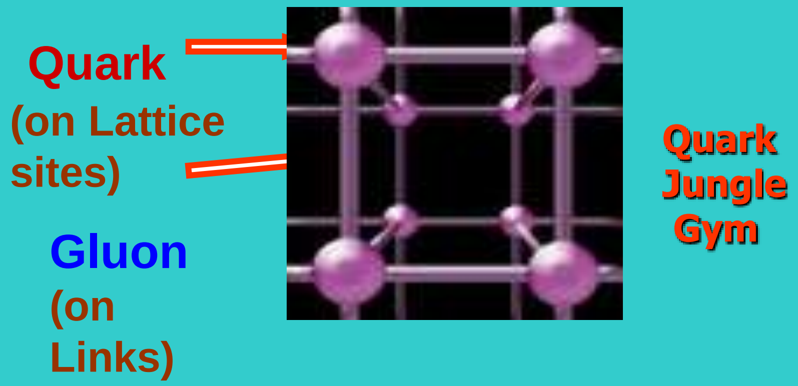
$$ma << 1$$

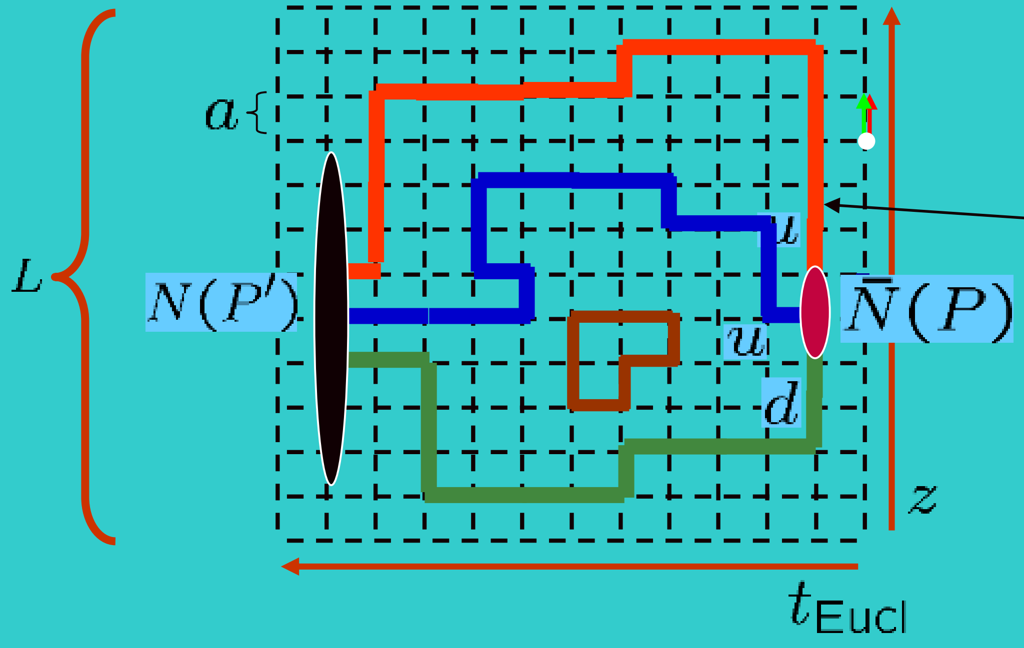
- **Charm :** ma = 1.275 GeV,
  ma = 0.5 → a ~ 0.075 fm
  ma = 0.3 → a ~ 0.046 fm
- **Bottom :** ma = 4.66 GeV
  ma = 0.5 → a = 0.021 fm
  ma = 0.3 → a = 0.013 fm

Being heavy lattice correlation functions for heavy quarks decay rapidly.

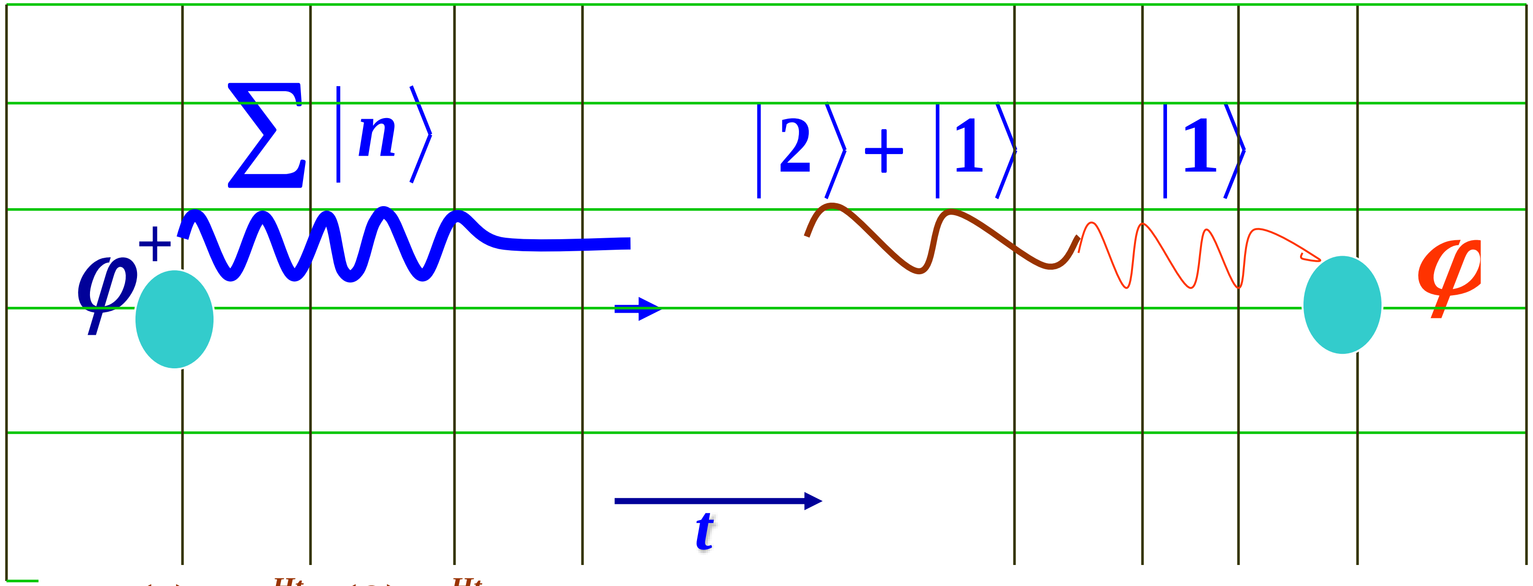
Relativistic charm quark calculations are now possible.

However, relativistic bottom-quark is still prohibitively costly.





quark propagators :
Inverse of very large matrix of space-time, spin and color



$$\varphi(t) = e^{Ht}\varphi(0)e^{-Ht}$$

$$G(t,\vec{p}) = \sum_{\vec{x}} e^{-i\vec{p}.(\vec{x}-\vec{x}_0)} \sum_{n,\vec{q}} \langle 0|\varphi(x)|n,\vec{q}\rangle\langle n,\vec{q}|\varphi(x_0)|0\rangle$$

$$= \sum_{\vec{x}} e^{-i\vec{p}.(\vec{x}-\vec{x}_0)} \sum_{n,\vec{q}} \langle 0|e^{H(t-t_0)-i\vec{p}'.(\vec{x}-\vec{x}_0)}\varphi(x_0)e^{-H(t-t_0)+i\vec{p}'.(\vec{x}-\vec{x}_0)}|n,\vec{q}\rangle\langle n,\vec{q}|\varphi(x_0)|0\rangle$$

$$= \sum_{\vec{x}} e^{-i\vec{p}.(\vec{x}-\vec{x}_0)} \sum_{n,\vec{q}} e^{i\vec{q}.(\vec{x}-\vec{x}_0)-E_q^n(t-t_0)} \langle 0|\varphi(x_0)|n,\vec{q}\rangle\langle n,\vec{q}|\varphi(x_0)|0\rangle$$

$$\approx \sum_{n,\vec{q}} \delta(\vec{p}-\vec{q}) e^{i(\vec{p}-\vec{q}).\vec{x}_0-E_q^n(t-t_0)} \langle 0|\varphi(x_0)|n,\vec{q}\rangle\langle n,\vec{q}|\varphi(x_0)|0\rangle$$

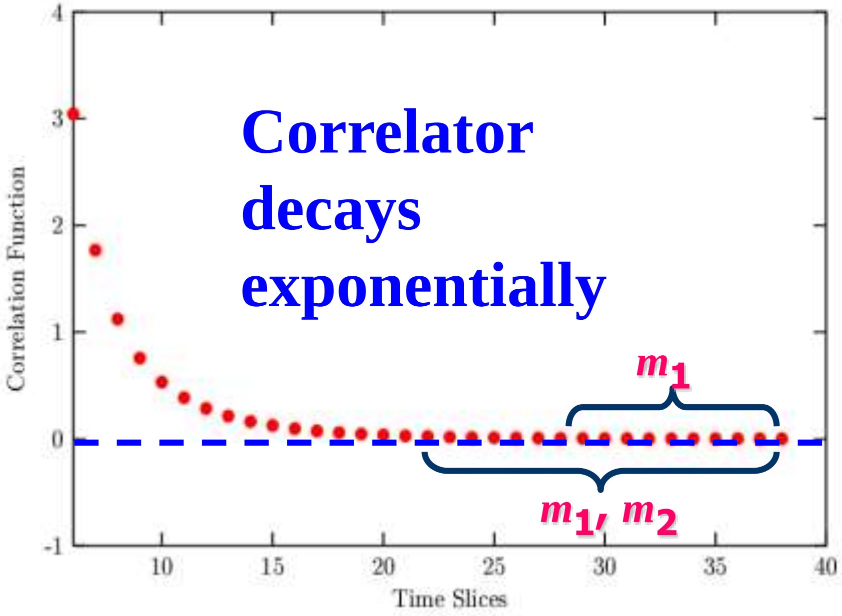
$$= \sum_{n} e^{-E_p^n(t-t_0)} \left|\langle 0|\varphi(x_0)|n,\vec{p}\rangle\right|^2$$

Determines how effectively this operator interpolates states 'n' from the vacuum

$$= \sum_{n} W_n e^{-E_p^n(t-t_0)} \xrightarrow[t\to\infty]{} W_1 e^{-E_1^n(t-t_0)}$$

Analysis (Extraction of Mass)

$$G(\tau) = \sum_{i=1}^{N} W_i\, e^{-m_i \tau} \underset{\tau\to\infty}{\approx} W_1\, e^{-m_1 \tau}$$



Effective mass :

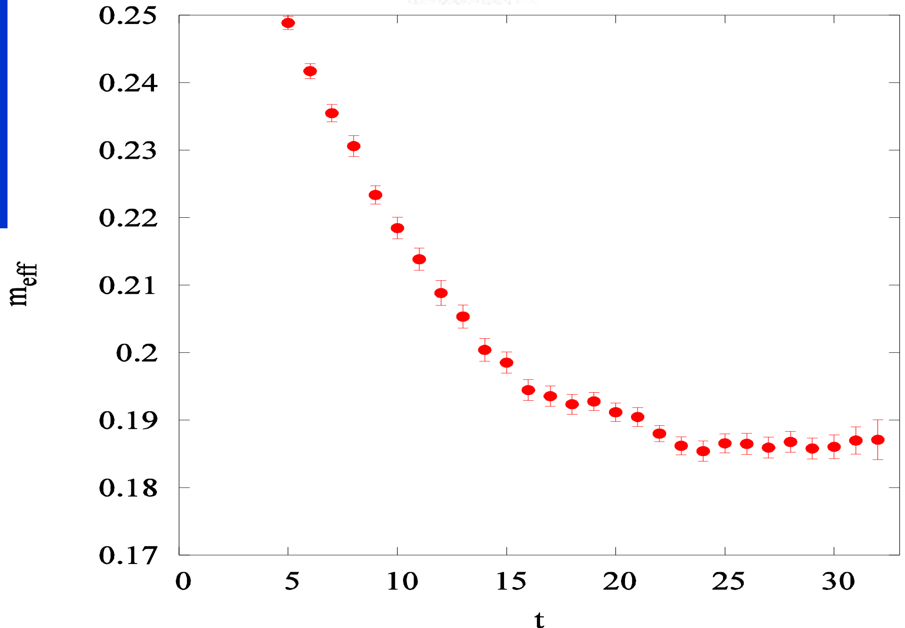
$$\frac{G(\tau)}{G(\tau+1)} = e^{-m_1\tau + m_1(\tau+1)}$$

$$m(\tau) = \ln\left[\frac{G(\tau)}{G(\tau+1)}\right]$$

$$\chi^2 = \sum_{i=1}^{N}\left[\frac{f(t_i) - \langle G(t_i)\rangle}{\varepsilon(t_i)}\right]$$

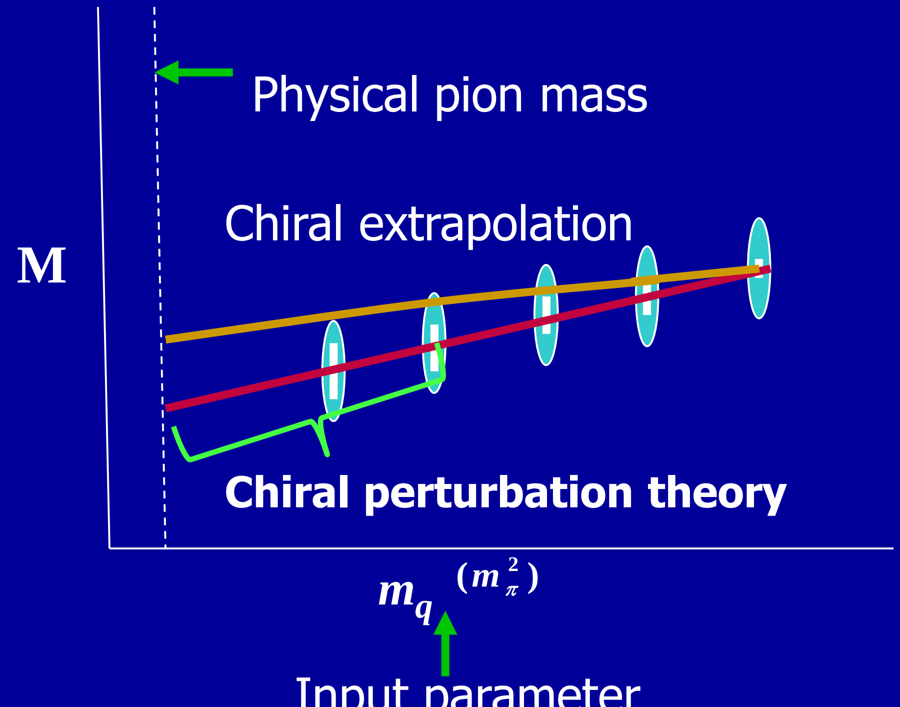
$$e^{-mt} = e^{-(ma)(t/a)}$$

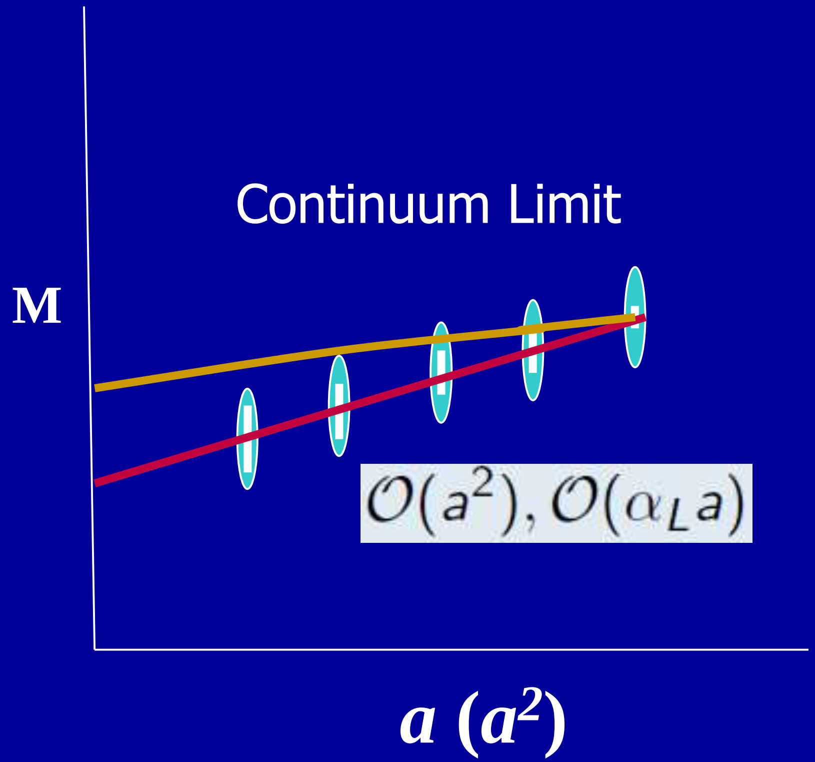
dimensionless mass integer timeslices

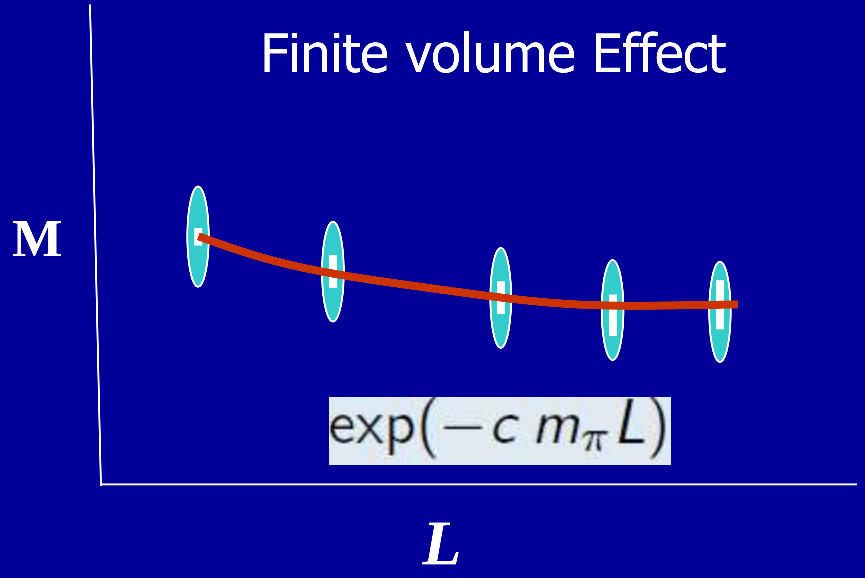


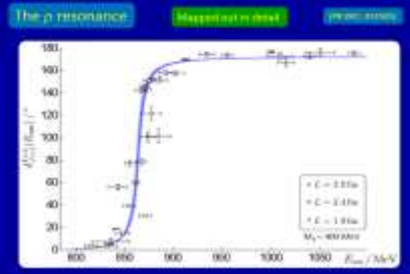
Determine *a* by measuring some physical quantity and compare that to expt, like parameter tuning in any renormalized field theory

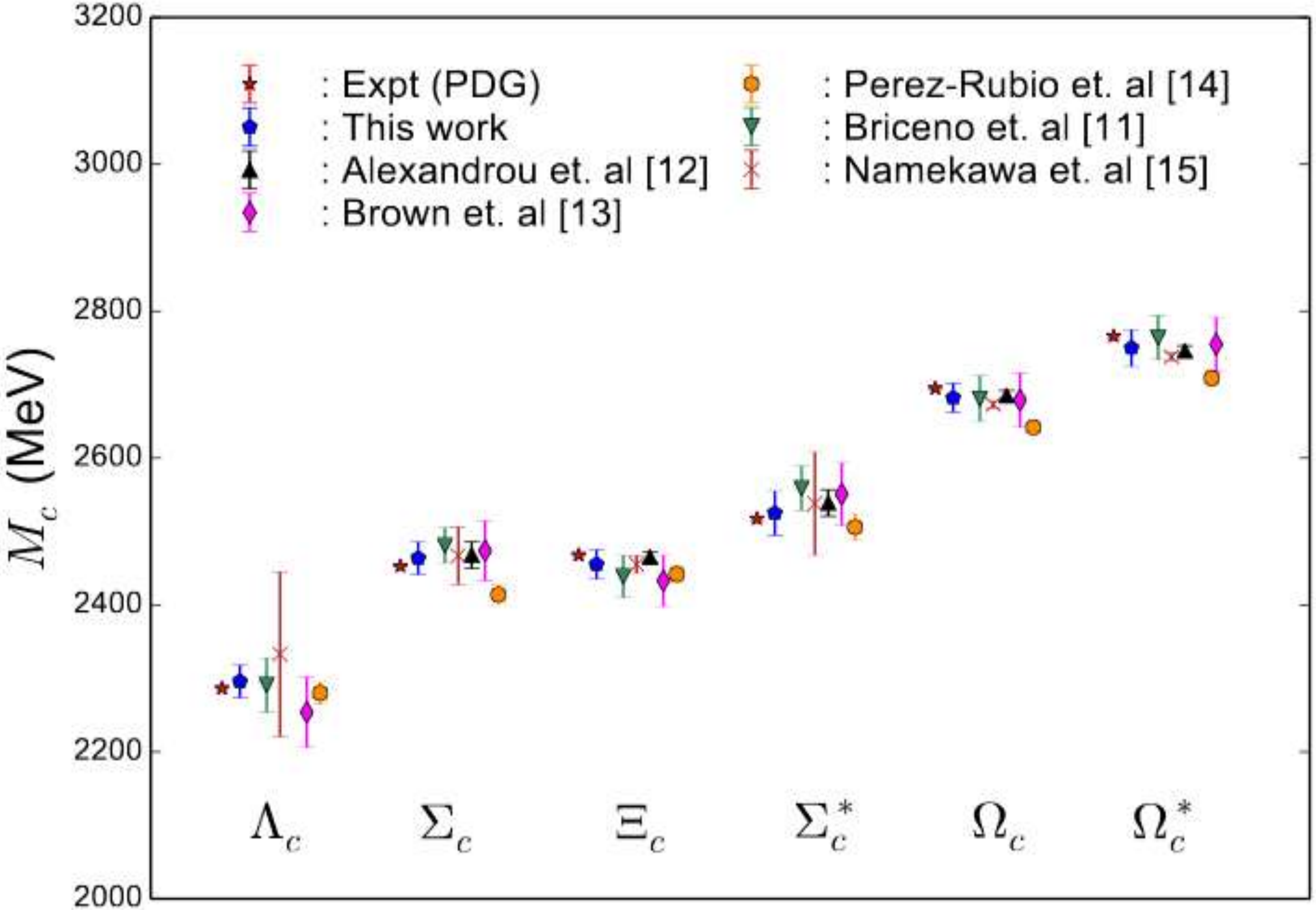
Control of Sytemetics



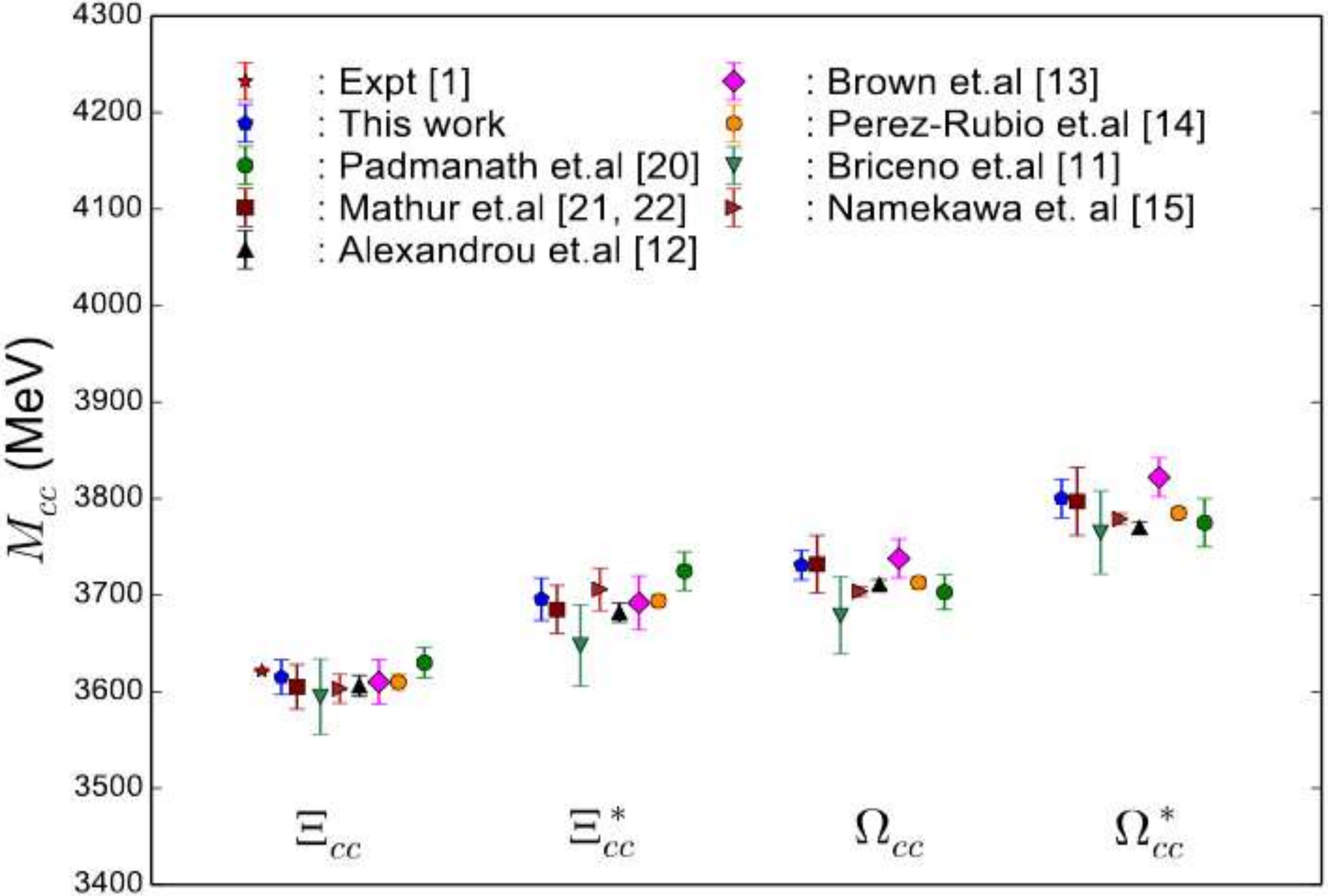




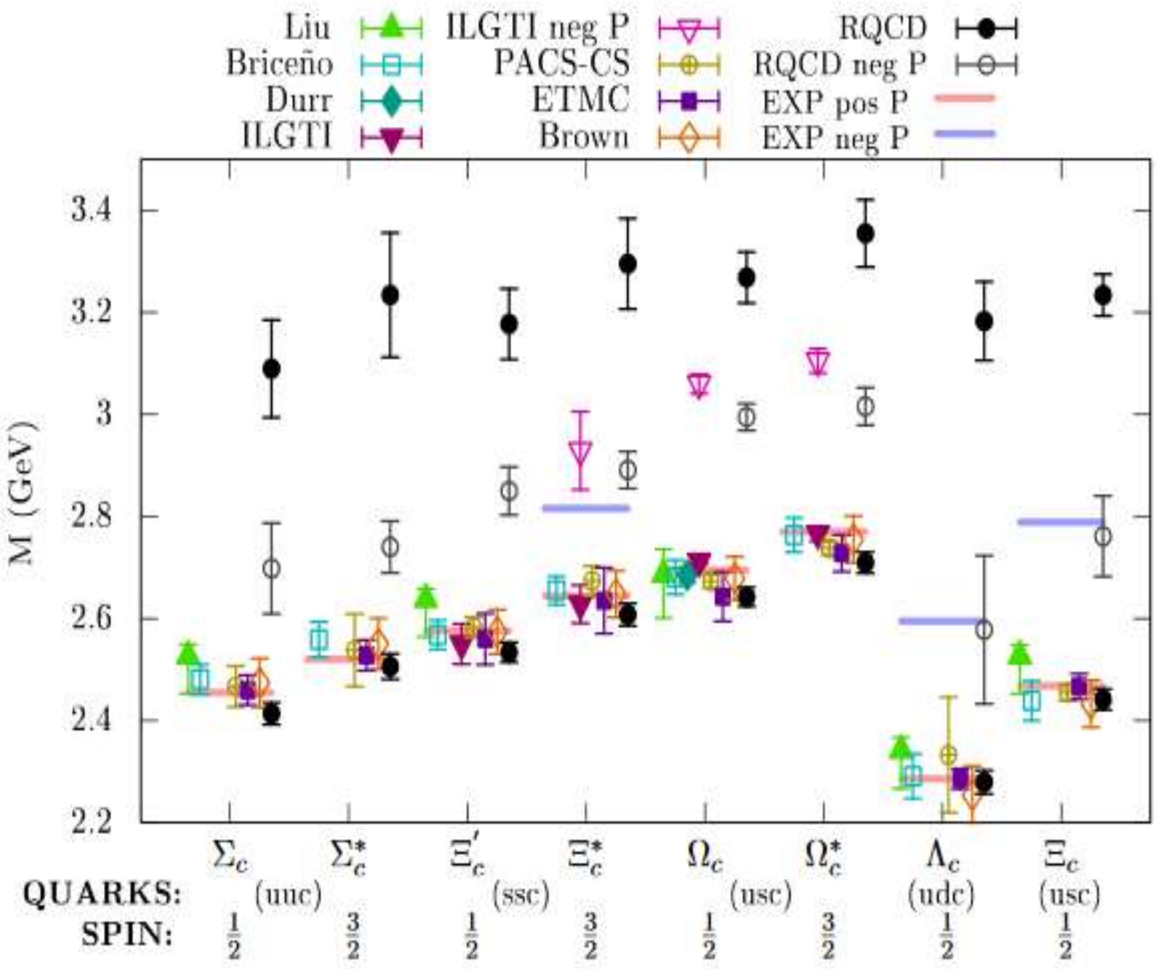


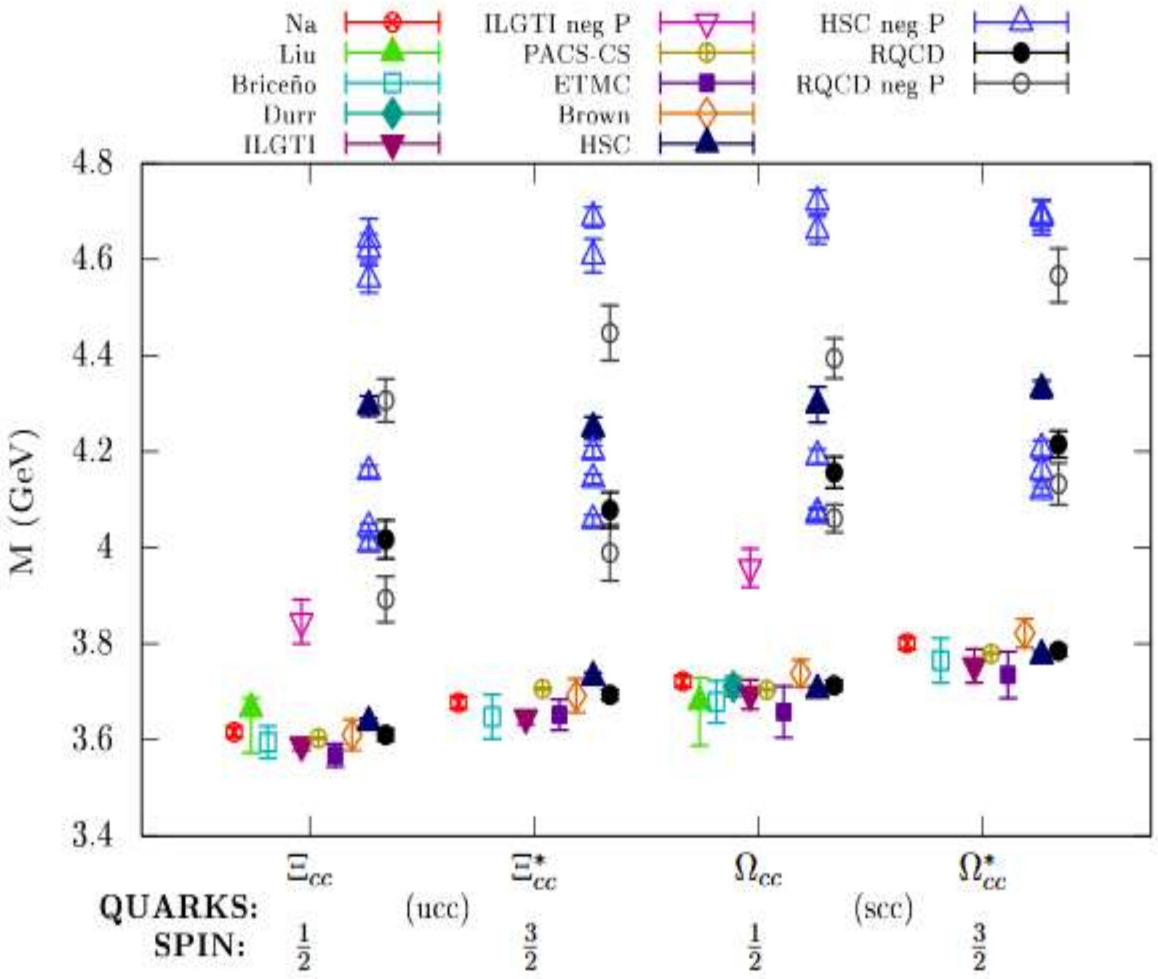


3200
3000
2800
2600
2400
2200
2000
M_c (MeV)
: Expt (PDG)
: This work
: Alexandrou et. al [12]
: Brown et. al [13]
: Perez-Rubio et. al [14]
: Briceno et. al [11]
: Namekawa et. al [15]
Λ_c
Σ_c
Ξ_c
Σ_c^*
Ω_c
Ω_c^*

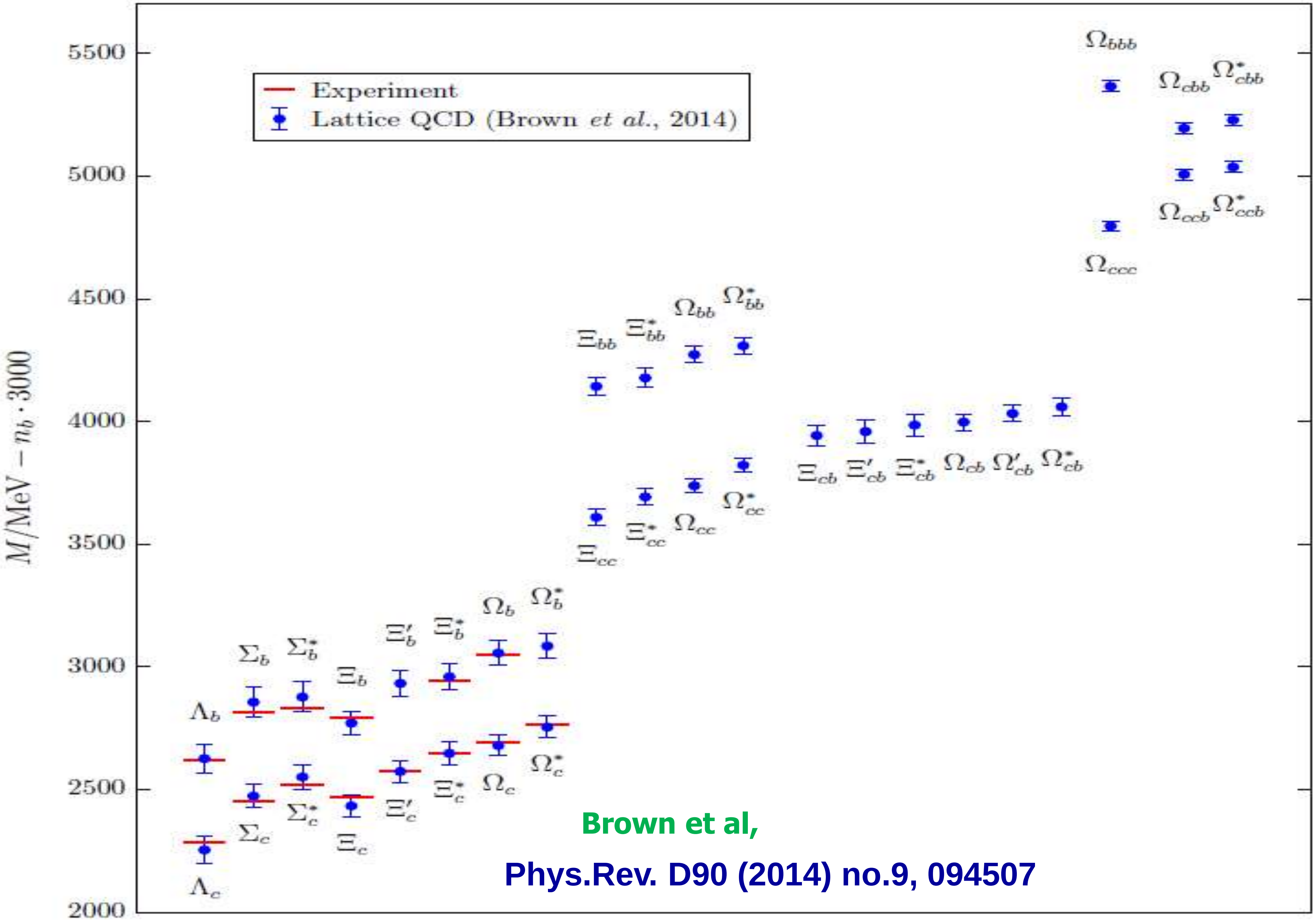


: Expt [1]
: This work
: Padmanath et.al [20]
: Mathur et.al [21, 22]
: Alexandrou et.al [12]
: Brown et.al [13]
: Perez-Rubio et.al [14]
: Briceno et.al [11]
: Namekawa et. al [15]
M_{cc} (MeV)
4300
4200
4100
4000
3900
3800
3700
3600
3500
3400
Ξ_{cc}
Ξ_{cc}^*
Ω_{cc}
Ω_{cc}^*





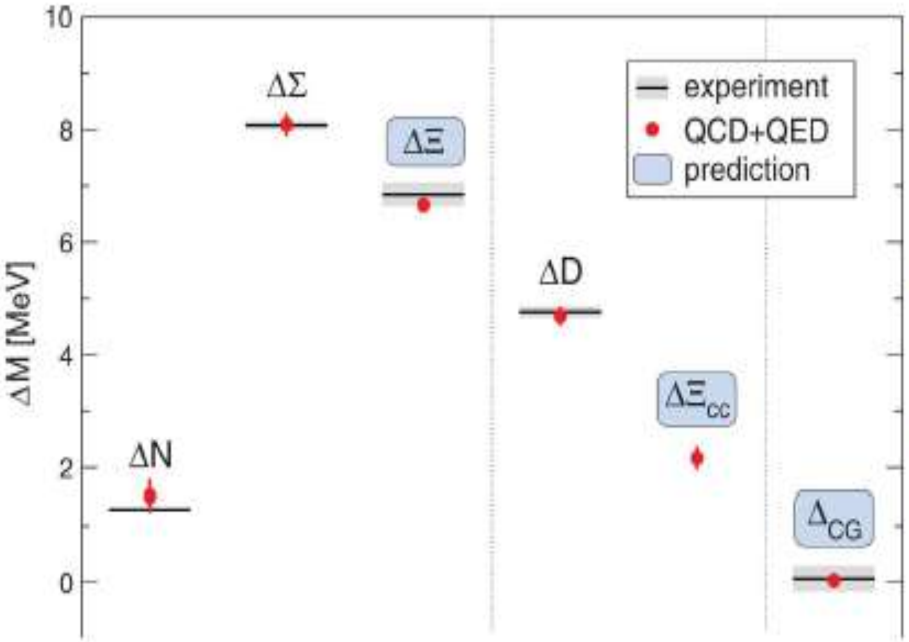
P. Perez-Rubio et al, Phys. Rev. D92 (2015) no.3, 034504



Brown et al,

Phys.Rev. D90 (2014) no.9, 094507

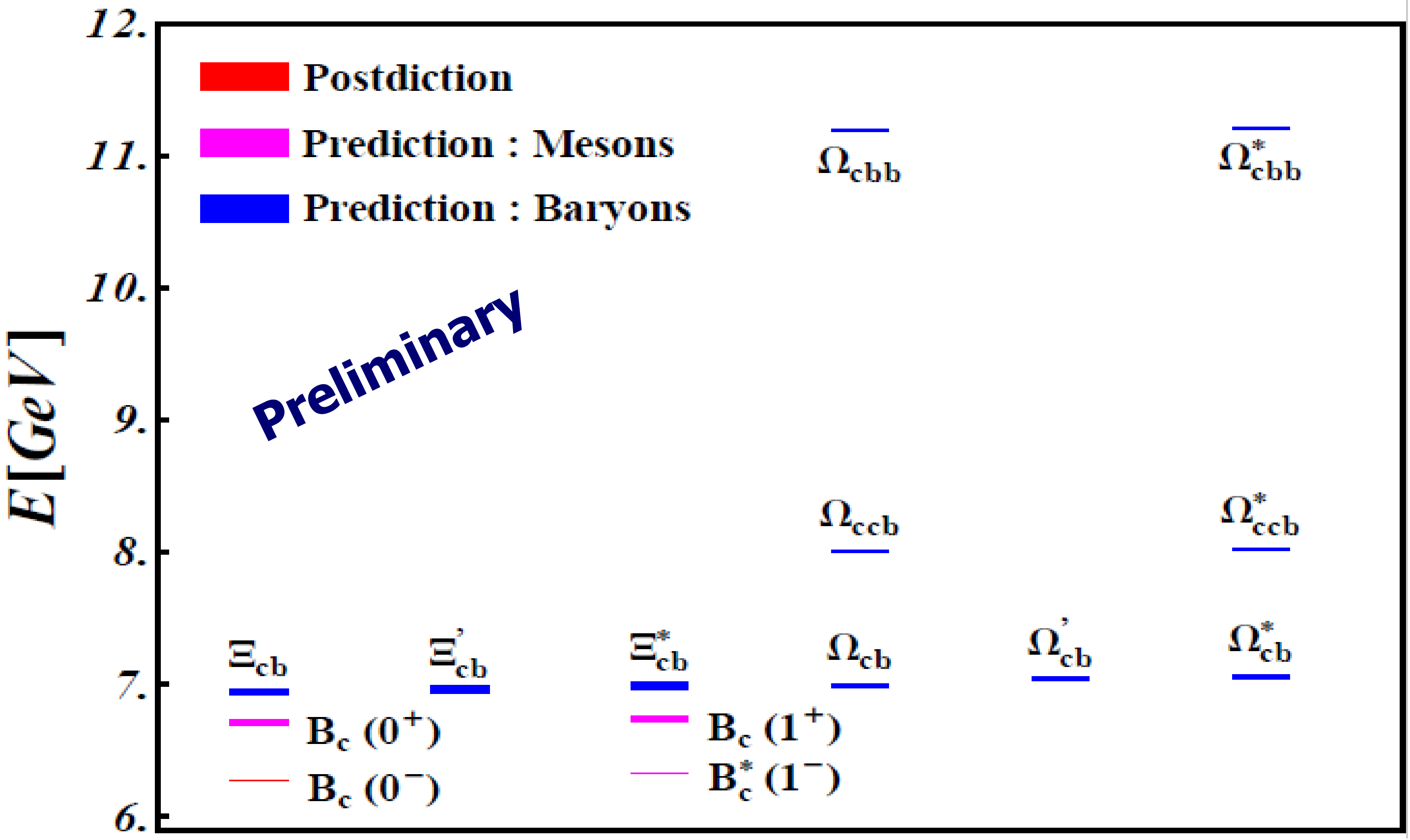
Isospin splittings in baryons



- Fully controlled ab initio calculation with 1+1+1+1 flavor QCD+QED with clover improved Wilson quarks.
- Precision of low energy description is down to per mil level.
- Precision at a level of challenging the experimental numbers.

Borsanyi, *et. al.*, Science Vol. 347 no. 6229 pp. 1452-1455

BC Spectra



@ILGTI NM and Padmanath

Challenges

- Charm quarks being heavy $\Rightarrow$ The discretization errors (ma) are generally very large.
- The exponential decay is very rapid.
  Rapid degradation of SNR for highly excited states.

Solution : **Anisotropic lattices**

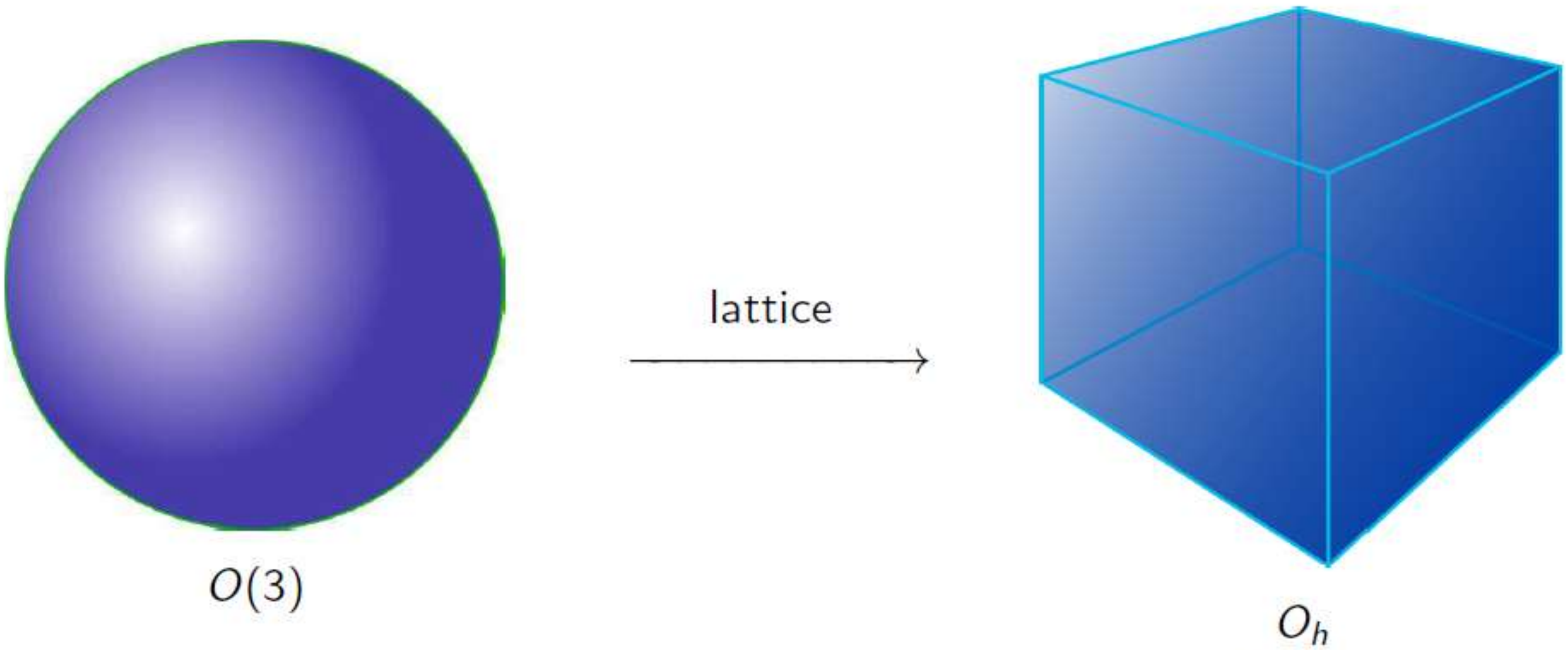
- Multiple excited state extraction : Multi parameter fit.
  Extremely cumbersome.

Solution : **A large basis of interpolating operators**

- A good analysis procedure for extraction of energy of physical states.
- Spin identification : Highly non-trivial

Solution : **Variational fitting method**

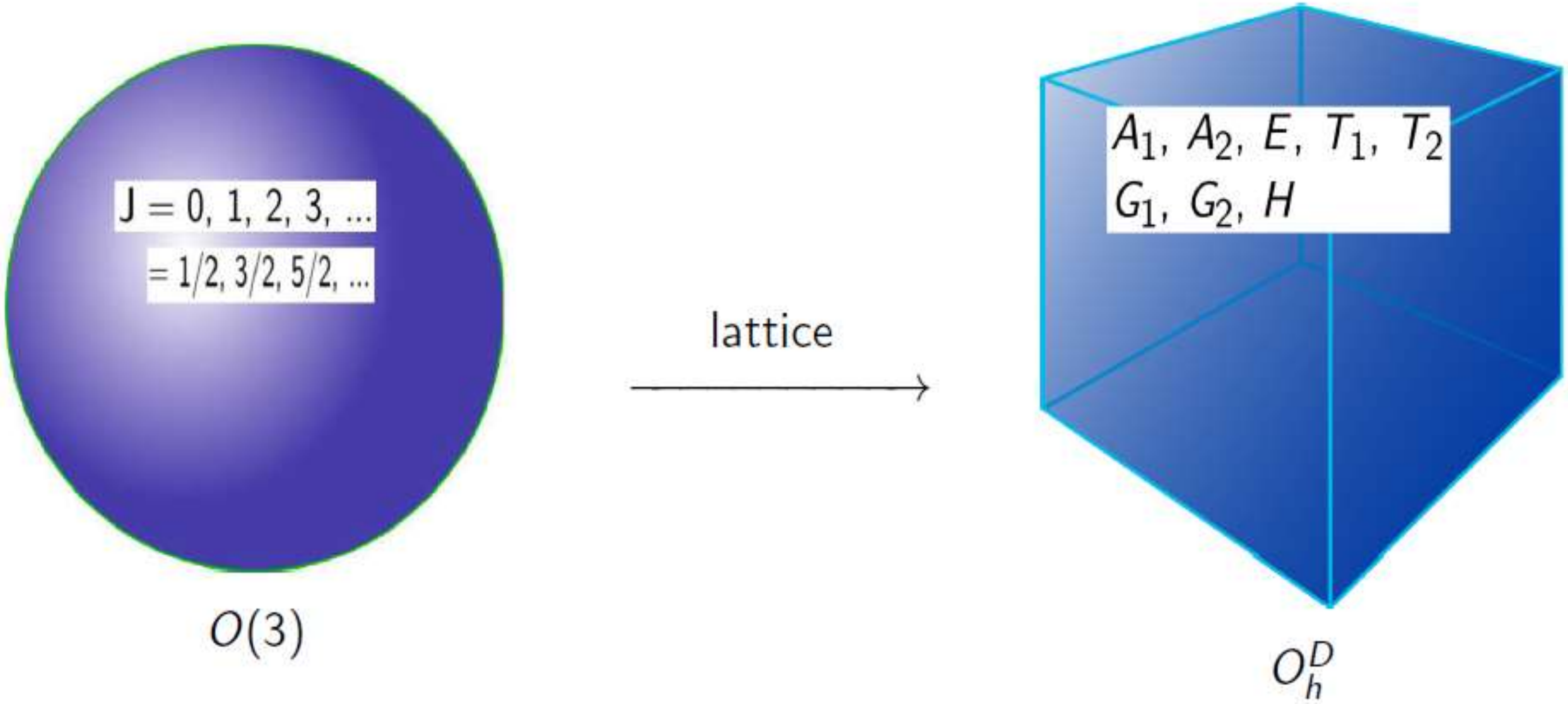
Continuum $\rightarrow$ Lattice : Symmetries



- Eigenstates of lattice Hamiltonian transform under irreps, Λ^n, of O_h.
- Continuum states with same J^P but different J_z : seperated across different lattice irreps.
- Subduce the continuum operators into the irreps of O_h.

R. C. Johnson, Phys. Lett. B114 (1982)

Continuum → Lattice : Irreps (2)



- Half-integer spin objects see an O_h^D symmetry on lattice.

Phys. Rev. D72 (2005) 094506
Phys. Rev. D72 (2005) 074501
Phys.Rev. D77 (2008) 034501
Phys.Rev. D82 (2010) 034508
Phys.Rev. D84 (2011) 074508

Spectroscopy : baryon operator construction

- Aim : Extraction of highly excited states.
  Local operators → low lying states.
  Extended operators → States with radial and orbital excitations.
- Proceeds in two steps
  Construct continuum operators with well defined quantum nos.
  Reduce/subduce into the irreps of the reduced symmetry.
- Used set of baryon continuum operators of the form
  $\Gamma^{\alpha\beta\gamma} q^\alpha q^\beta q^\gamma$, $\Gamma^{\alpha\beta\gamma} q^\alpha q^\beta (D_i q^\gamma)$ and $\Gamma^{\alpha\beta\gamma} q^\alpha q^\beta (D_i D_j q^\gamma)$
- Excluding the color part, the flavor-spin-spatial structure

$$O^{[J^P]} = [\mathcal{F}_{\Sigma_F} \otimes \mathcal{S}_{\Sigma_S} \otimes \mathcal{D}_{\Sigma_D}]^{J^P}.$$

HSC : Phys. Rev. D72 (2005) 074501

Phys.Rev. D84 (2011) 074508

- γ-matrix convention : $\gamma_4 = \text{diag}[1,1,-1,-1]$;
  Non-relativistic → purely based on the upper two component of q.
  Relativistic → All operators except non-relativistic ones.
- Subset of $D_i D_j$ operators that include $[D_i, D_j] \sim F_{ij}$ → hybrid.

Phys.Rev. D87 (2013) no.5, 054506

No. of interpolating operators

Ω_{ccc}

	G_1		H		G_2	
	g	u	g	u	g	u
Total	20	20	33	33	12	12
Hybrid	4	4	5	5	1	1
NR	4	1	8	1	3	0

Λ_{cdu}

	G_1		H		G_2	
	g	u	g	u	g	u
Total	53	53	86	86	33	33
Hybrid	12	12	16	16	4	4
NR	10	3	17	4	7	1

Ω_{ccs}, Ξ_{ccu}, Ω_{css} and Σ_{cuu}.

	G_1		H		G_2	
	g	u	g	u	g	u
Total	55	55	90	90	35	35
Hybrid	12	12	16	16	4	4
NR	11	3	19	4	8	1

Ξ_{csu}

	G_1		H		G_2	
	g	u	g	u	g	u
Total	116	116	180	180	68	68
Hybrid	24	24	32	32	8	8
NR	23	6	37	10	15	2

HSC: Phys.Rev. D87 (2013) no.5, 054506 **Phys.Rev. D87 (2013) no.5, 054506**

PRD90, 074504(2014)

Variational Analysis

ψ_i : gauge invariant fields on a timeslice t that corresponds to Hilbert space operator ψ_j whose quantum numbers are also carried by the states **|n>.**
Construct a matrix

$$C(t) = \begin{bmatrix} \langle 0|\phi_1(t)\phi_1^+(0)|0\rangle & \langle 0|\phi_1(t)\phi_2^+(0)|0\rangle & \ldots & \ldots \\ \langle 0|\phi_2(t)\phi_1^+(0)|0\rangle & \langle 0|\phi_2(t)\phi_2^+(0)|0\rangle & \ldots & \ldots \\ \ldots\ldots & \ldots\ldots\ldots & & \ldots\ldots \\ \ldots\ldots & \ldots\ldots & & \ldots\ldots \end{bmatrix}$$

- Need to find out variational coefficient $\{v_\alpha^{(m)}, \alpha = 1,2,\ldots,n\}$ such that the overlap to a state is maximum

$$\Phi^{(m)}(t)|0\rangle = \sum_{\alpha}^{N} v_\alpha^{(m)}\phi_\alpha(t)|0\rangle$$
$$= (1-\varepsilon_m)e^{-\hat{H}t}|m\rangle + \sum_{n\neq m}\varepsilon_n e^{-\hat{H}t}|n\rangle \quad \text{with } \varepsilon_n << 1$$

- Variational solution → Generalized eigenvalue problem :

$$C(t)v^n(t,t_0) = \lambda_n(t,t_0)C(t_0)v^n(t,t_0)$$

C. Michael, Nucl. Phys. B 259, 58, (1985)
M. Luscher and U. Wolff, Nucl. Phys. B 339, 222 (1990)

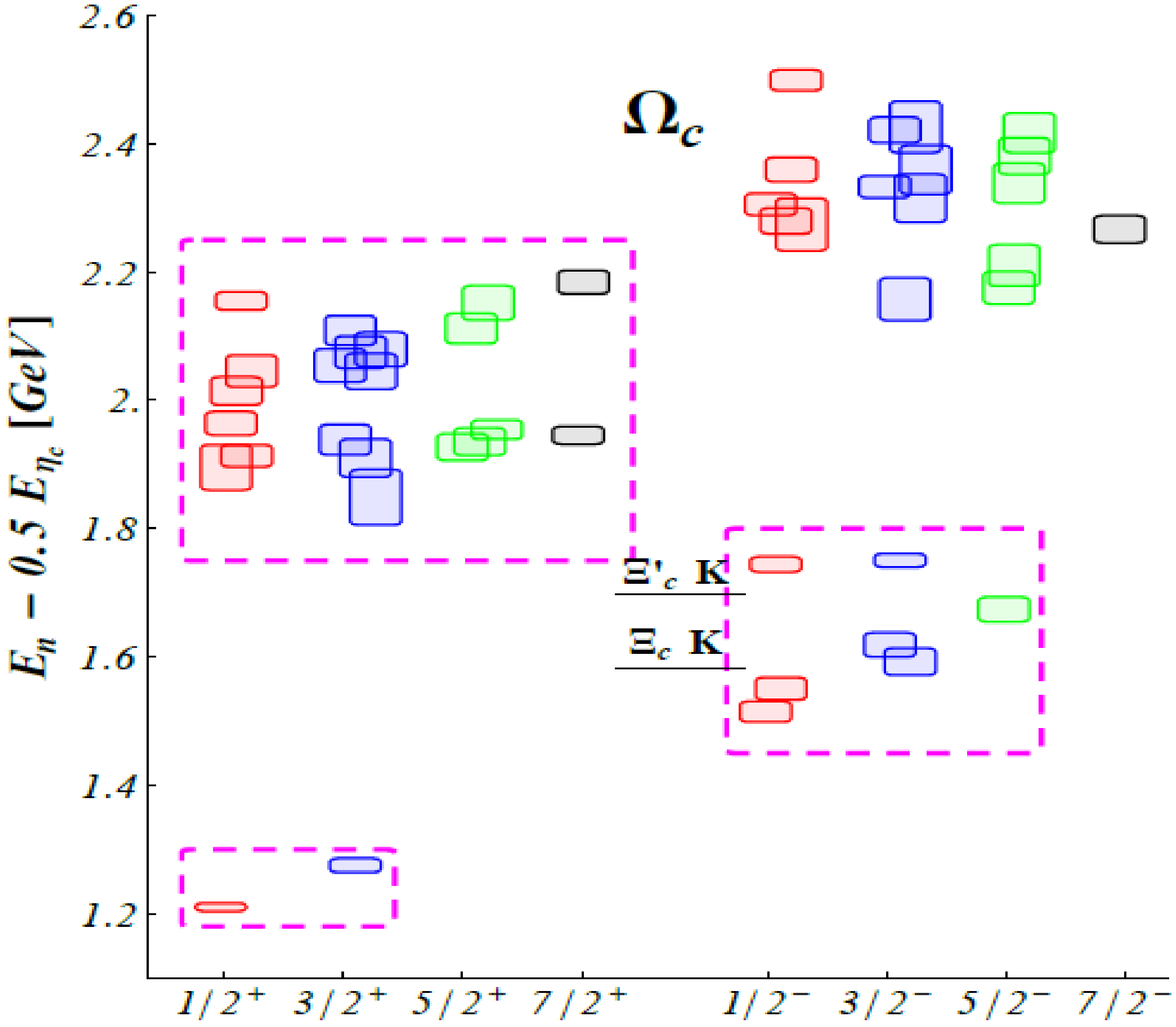
- Eigenvalues give spectrum :

$$\lim_{t\to\infty}\lambda_n(t,t_0) = e^{-(t-t_0)E_n}(1+e^{-t\Delta E_n})$$

Dudek, Edwards, Mathur, Richards, Phys.Rev. D77 (2008) 034501

- Eigenvectors give the optimal operator :

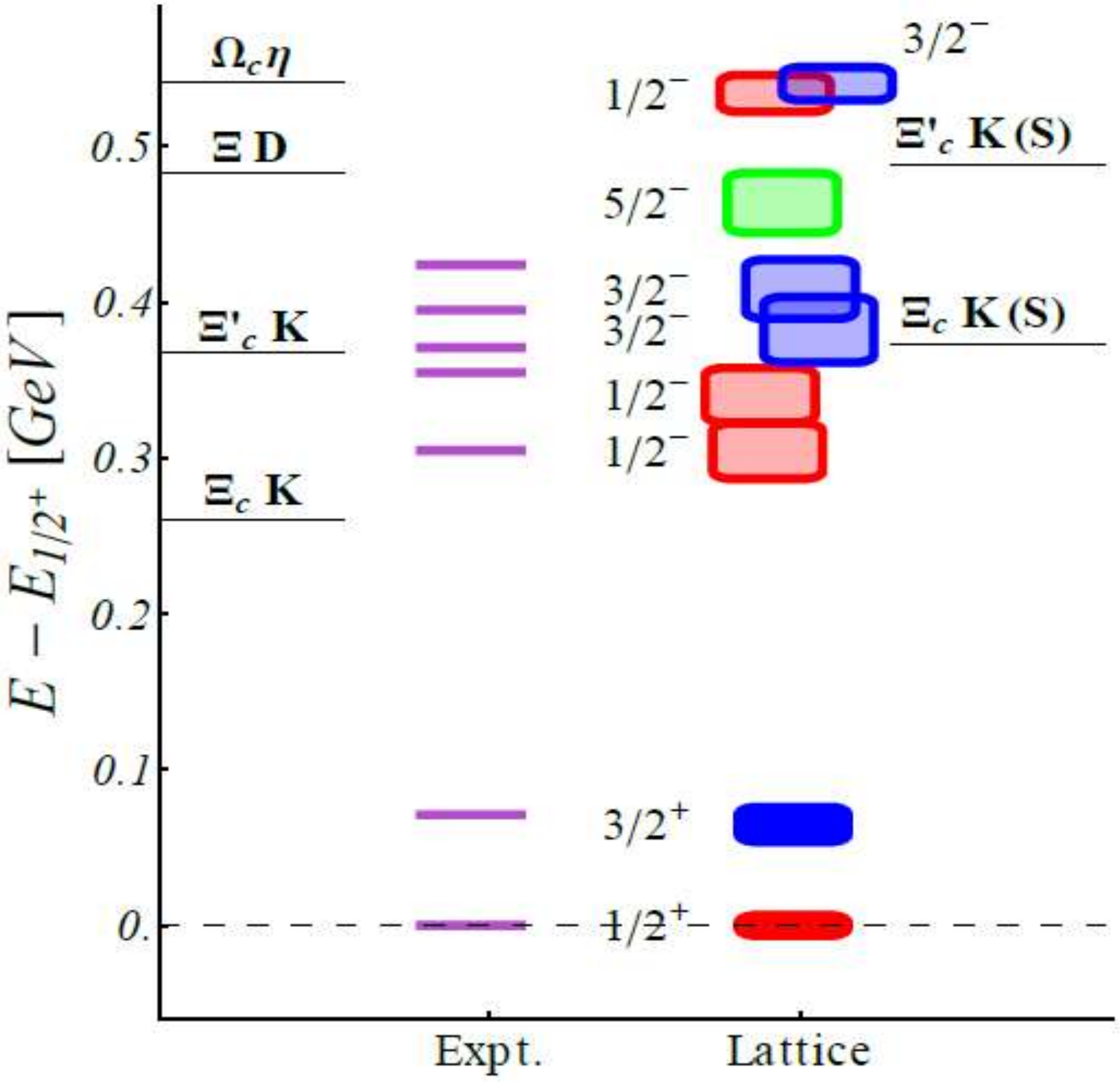
$$\Phi^m(t) = v_1^m\phi_1(t) + v_2^m\phi_2(t) + \ldots$$



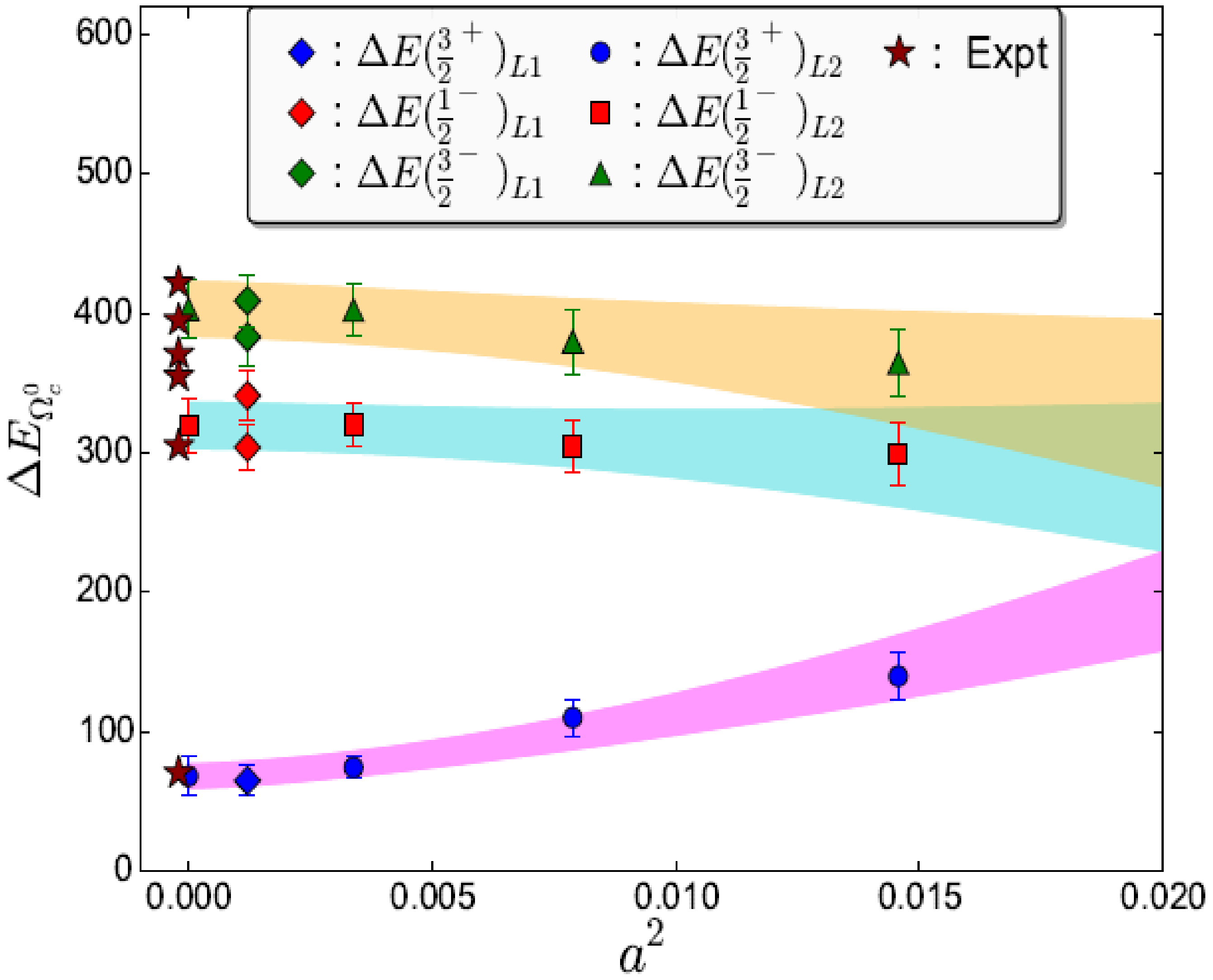
HSC : Padmanath et al, 1311.4806, Charm 2013 and 2015

Padmanath, TIFR thesis (2014)

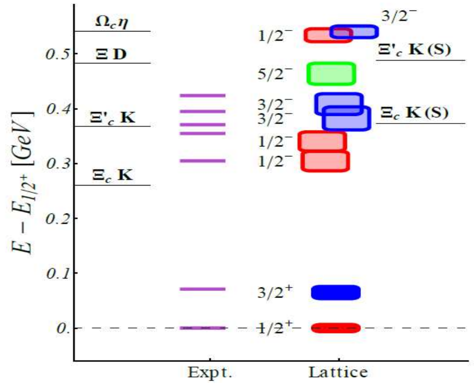
Padmanath and NM : Phys.Rev.Lett. 119 (2017) no.4, 042001



Padmanath and NM, **Phys.Rev.Lett. 119 (2017) no.4, 042001**



Padmanath and NM, Phys.Rev.Lett. 119 (2017) no.4, 042001

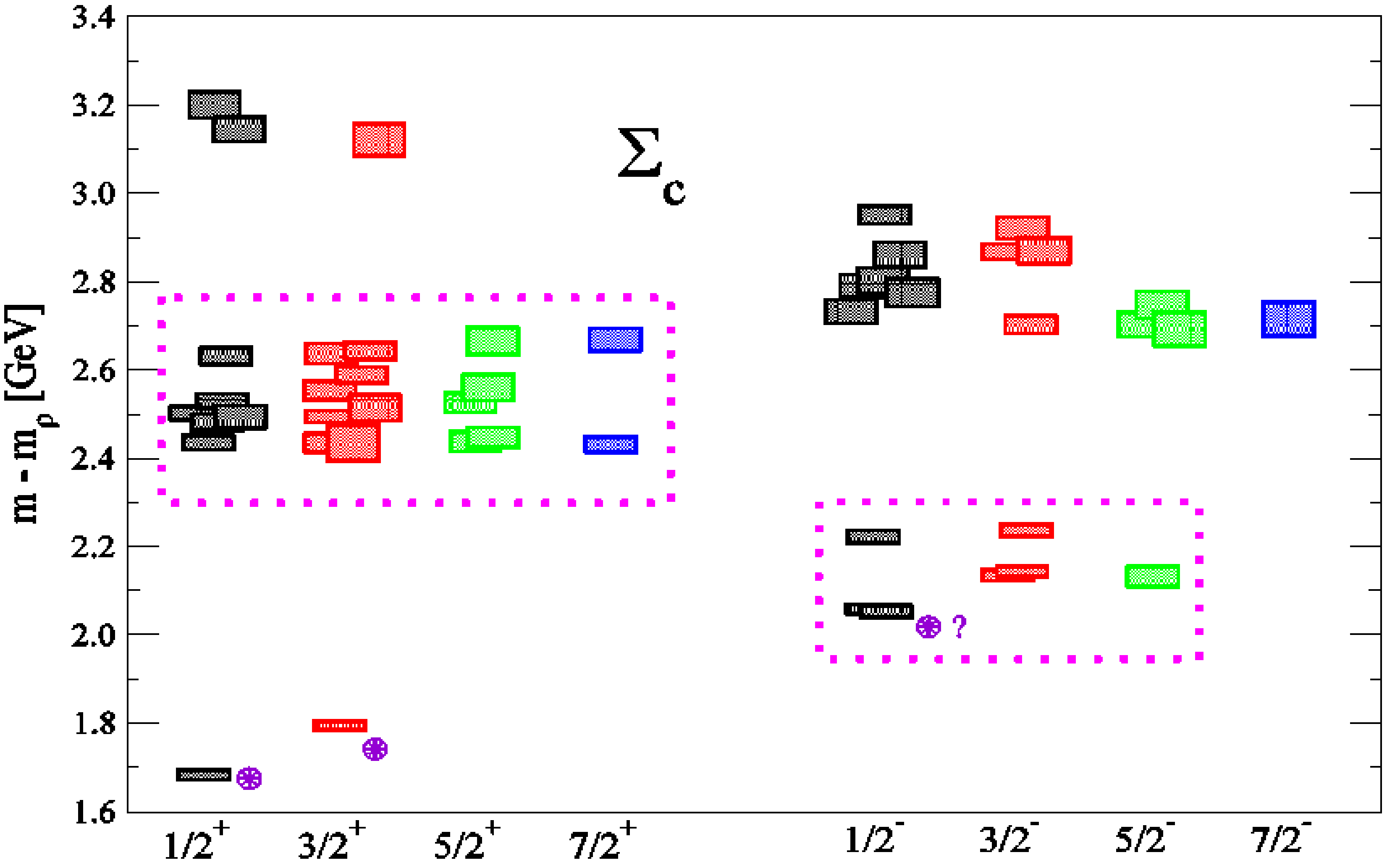


Energy Splittings (ΔE)	Experiment		Lattice	
	ΔE (MeV)	J^P (PDG)	ΔE (MeV)	J^P
$E_{\Omega_c^0} - \frac{1}{2}E_{\eta_c}$	1203(2)	$1/2^+$	1209(7)	$1/2^+$
$\Delta E_{\Omega_c^0(2770)}$	70.7(1)	$3/2^+$	65(11)	$3/2^+$
$\Delta E_{\Omega_c^0(3000)}$	305(1)	?	304(17)	$1/2^-$
$\Delta E_{\Omega_c^0(3050)}$	355(1)	?	341(18)	$1/2^-$
$\Delta E_{\Omega_c^0(3066)}$	371(1)	?	383(21)	$3/2^-$
$\Delta E_{\Omega_c^0(3090)}$	395(1)	?	409(19)	$3/2^-$
$\Delta E_{\Omega_c^0(3119)}$	422(1)	?	464(20)	$5/2^-$

Here $\Delta E^n = E^n - E^0$.
The new states correspond to the excited p-wave states.

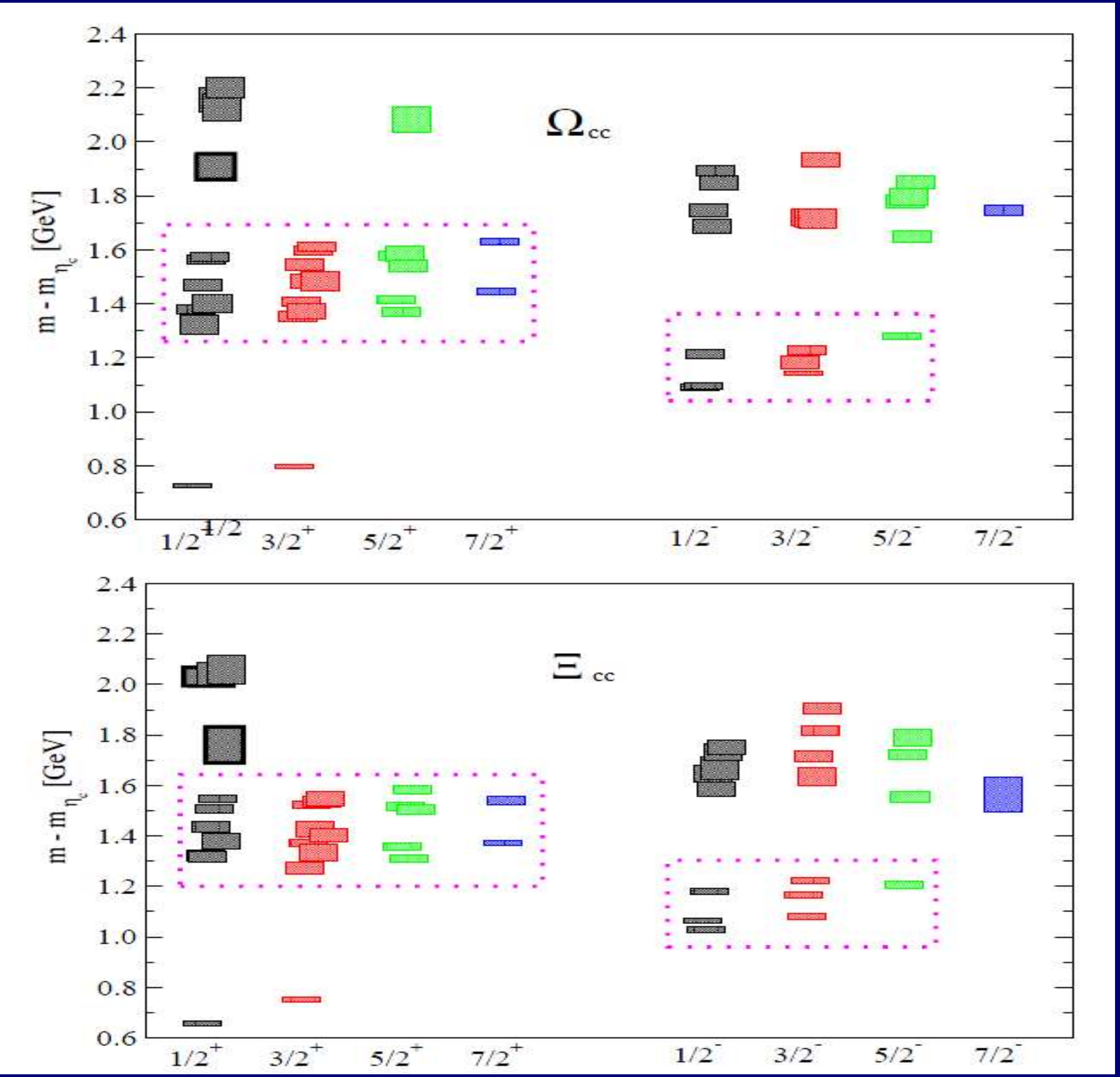
Padmanath and NM, **Phys.Rev.Lett. 119 (2017) no.4, 042001**

Singly Charm baryons



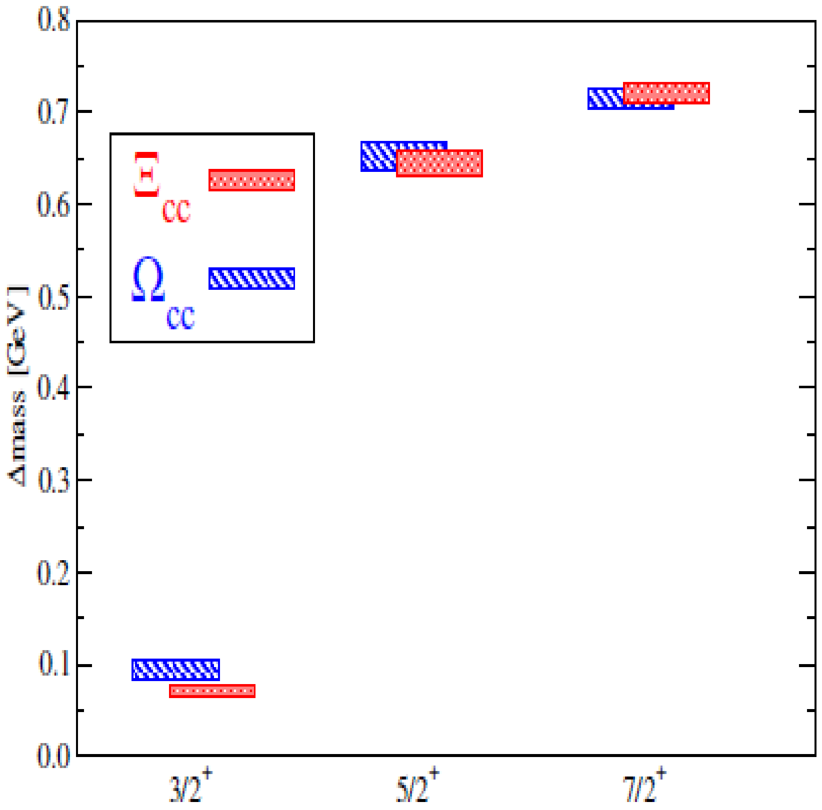
HSC : Padmanath et al, 1311.4806
Padmanath, TIFR thesis 2014

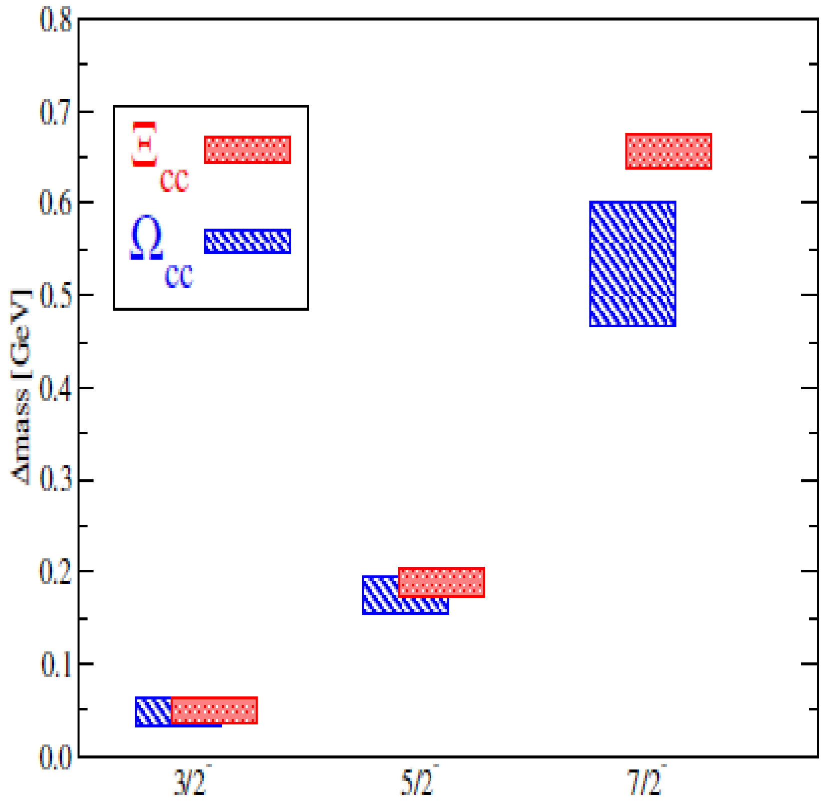
DOUBLY CHARM



BARYONS

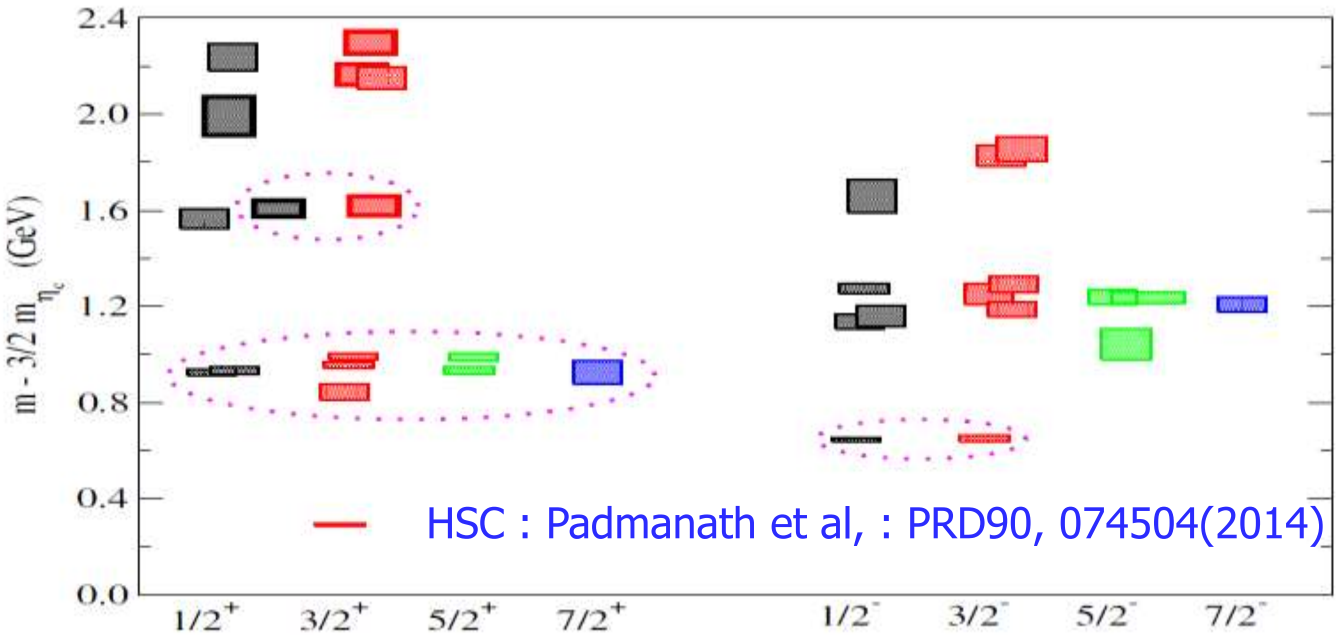
HSC : Padmanath et al, 1311.4354

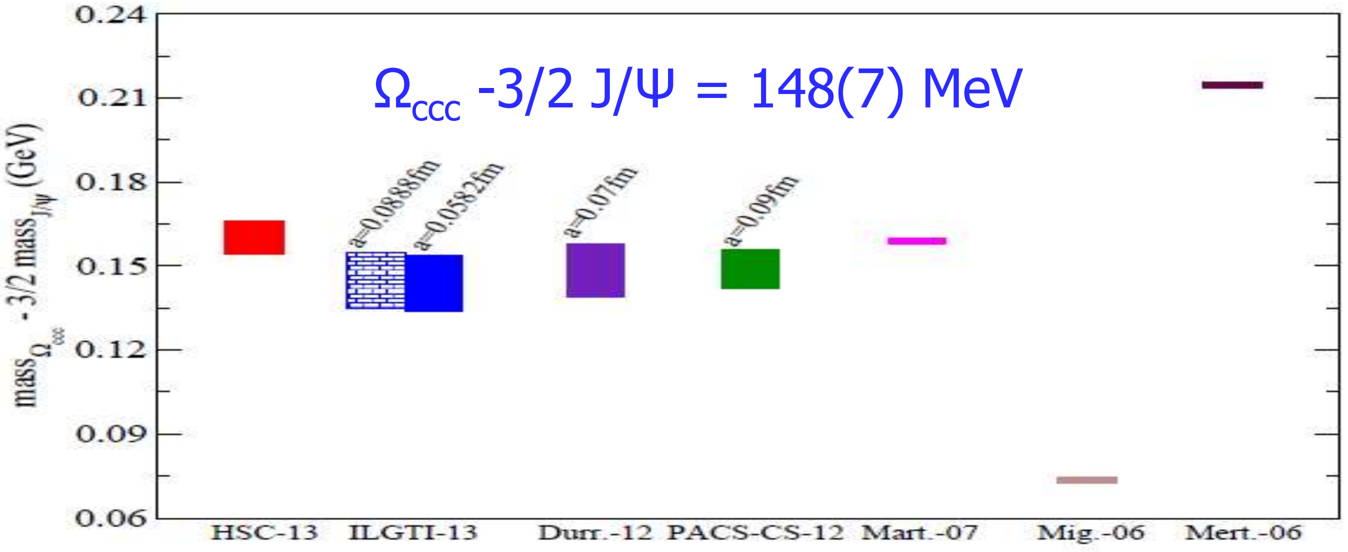




HSC : Padmanath et al, 1311.4354

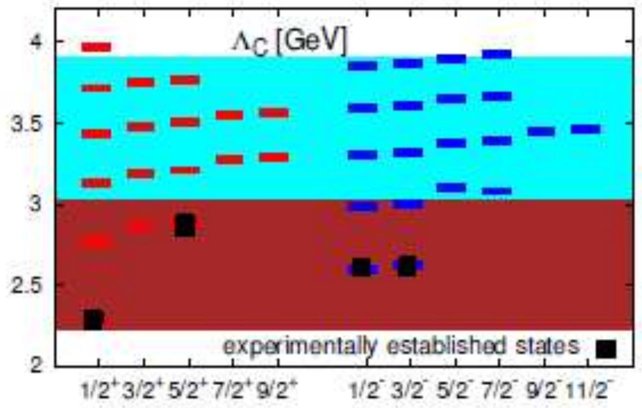
Triply charmed baryons





NM and Padmanath et al, arXiv:1311.4806

Ebert *et al.*, PRD, 84, 014025, 2011



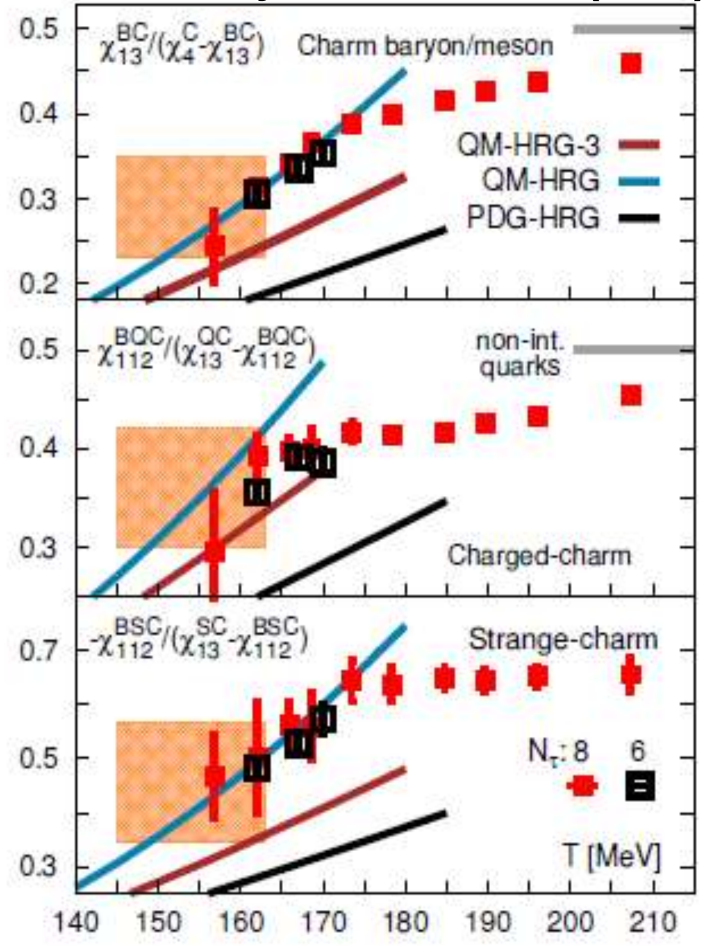
- **Charm hadron pressure (HRG) :**

$$P(\hat{\mu}_C, \hat{\mu}_B) = P_M cosh(\hat{\mu}_C) + P_{B,C=1} cosh(\hat{\mu}_C + \hat{\mu}_B)$$

$$\chi^{BC}_{kl} = \frac{\partial^{(k+l)}[P(\hat{\mu}_C, \hat{\mu}_B)/T^4]}{\partial \hat{\mu}^k_B \partial \hat{\mu}^l_C}$$

Bazavov *et al.*, PLB, 737, 210, 2014

Phys.Rev.Lett. 113 (2014) no.7, 072001



- Expt : Morning sessions
- Theory :
  - B. Zou : Charm Pentaquarks
  - YU, Fu-Sheng : Doubly charmed baryons
  - A. Likhoded : Doubly heavy baryons
  - DU, Meng-Lin : Heavy light baryons
  - MA, Li : Doubly heavy tri-hadron
  - SHAH, Zalak : bc baryons

Conclusions and Outlooks

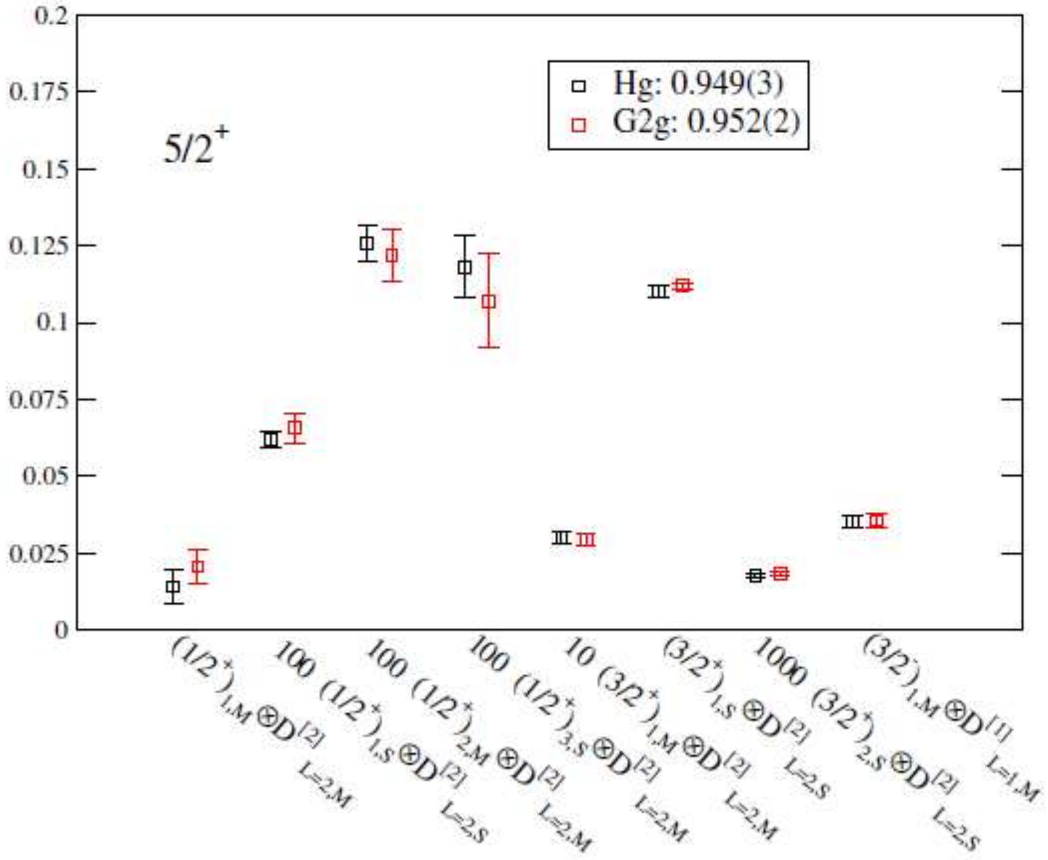
- There is a tremendous resurgence of interest in the study of bound states with heavy quark(s).
- Charmed baryons are contributing to that too.
- New discoveries of charmed baryons happened recently.
- Theory predicts many more including exotics.
- LQCD results with more control over systematics are coming.
- It is strongly envisaged that experimentalists will find many more soon.
- That will tremendously improve our knowledge about strong interactions and how to model `QCD more effectively if not solve fully.
- Will look forward to new discoveries.

20_M					
	I	I_z	S	$\mathcal{F}_{MS}$	$\mathcal{F}_{MA}$
Λ_c^+	0	0	0	$\frac{1}{\sqrt{2}}(\|cud\rangle_{MS} - \|udc\rangle_{MS})$	$\frac{1}{\sqrt{2}}(\|cud\rangle_{MA} - \|udc\rangle_{MA})$
Σ_c^{++}	1	+1	0	$\|uuc\rangle_{MS}$	$\|uuc\rangle_{MA}$
Σ_c^+	1	0	0	$\|ucd\rangle_{MS}$	$\|ucd\rangle_{MA}$
Σ_c^0	1	−1	0	$\|ddc\rangle_{MS}$	$\|ddc\rangle_{MA}$
$\Xi_c^{\prime +}$	$\frac{1}{2}$	$+\frac{1}{2}$	−1	$\|ucs\rangle_{MS}$	$\|ucs\rangle_{MA}$
$\Xi_c^{\prime 0}$	$\frac{1}{2}$	$-\frac{1}{2}$	−1	$\|dcs\rangle_{MS}$	$\|dcs\rangle_{MA}$
Ξ_c^+	$\frac{1}{2}$	$+\frac{1}{2}$	−1	$\frac{1}{\sqrt{2}}(\|cus\rangle_{MS} - \|usc\rangle_{MS})$	$\frac{1}{\sqrt{2}}(\|cus\rangle_{MA} - \|usc\rangle_{MA})$
Ξ_c^0	$\frac{1}{2}$	$-\frac{1}{2}$	−1	$\frac{1}{\sqrt{2}}(\|cds\rangle_{MS} - \|dsc\rangle_{MS})$	$\frac{1}{\sqrt{2}}(\|cds\rangle_{MA} - \|dsc\rangle_{MA})$
Ω_c^0	0	0	−2	$\|scs\rangle_{MS}$	$\|scs\rangle_{MA}$
Ξ_{cc}^{++}	$\frac{1}{2}$	$+\frac{1}{2}$	0	$\|ccu\rangle_{MS}$	$\|ccu\rangle_{MA}$
Ξ_{cc}^+	$\frac{1}{2}$	$-\frac{1}{2}$	0	$\|ccd\rangle_{MS}$	$\|ccd\rangle_{MA}$
Ω_{cc}^+	0	0	−1	$\|ccs\rangle_{MS}$	$\|ccs\rangle_{MA}$

20_S				
	I	I_z	S	$\mathcal{F}_S$
Σ_c^{++}	1	+1	0	$\|uuc\rangle_S$
Σ_c^+	1	0	0	$\|ucd\rangle_S$
Σ_c^0	1	−1	0	$\|ddc\rangle_S$
Ξ_c^+	$\frac{1}{2}$	$+\frac{1}{2}$	−1	$\|ucs\rangle_S$
Ξ_c^0	$\frac{1}{2}$	$-\frac{1}{2}$	−1	$\|dcs\rangle_S$
Ω_c^0	0	0	−2	$\|ssc\rangle_S$
Ξ_{cc}^{++}	$\frac{1}{2}$	$+\frac{1}{2}$	0	$\|ccu\rangle_S$
Ξ_{cc}^+	$\frac{1}{2}$	$-\frac{1}{2}$	0	$\|ccd\rangle_S$
Ω_{cc}^+	0	0	−1	$\|ccs\rangle_S$
Ω_{ccc}^{++}	0	0	0	$\|ccc\rangle_S$

4_A				
	I	I_z	S	ϕ_A
Λ_c^+	0	0	0	$\|udc\rangle_A$
Ξ_c^+	$\frac{1}{2}$	$+\frac{1}{2}$	−1	$\|ucs\rangle_A$
Ξ_c^0	$\frac{1}{2}$	$-\frac{1}{2}$	−1	$\|dcs\rangle_A$

Overlap factors (Z) across multiple irreps : $5/2^+$



Spin identification : $J > \frac{3}{2}$

- For example, a continuum operator $O = [ccc \otimes (\frac{3}{2}^+)^1_S \otimes D^{[2]}_{L=2,S}]^{J=\frac{5}{2}}$.
  Projects on to $\frac{5}{2}^+$.
- In the continuum, $\langle 0|O|\frac{5}{2}^+\rangle = Z$.
- On lattice, O gets subduced over two lattice irreps H_g and G_{2g}.
- Then

  $$\langle 0|O_{H_g}|\tfrac{5}{2}^+\rangle = Z_1\alpha \qquad \& \qquad \langle 0|O_{G_{2g}}|\tfrac{5}{2}^+\rangle = Z_2\beta$$

  where α and β are the Clebsch-Gordan coefficients.
- If "close" to the continuum, then $Z \sim Z_1 \sim Z_2$.