

CHARM 2018

# Review of recent developments on leptonic and semileptonic charm decays from lattice QCD



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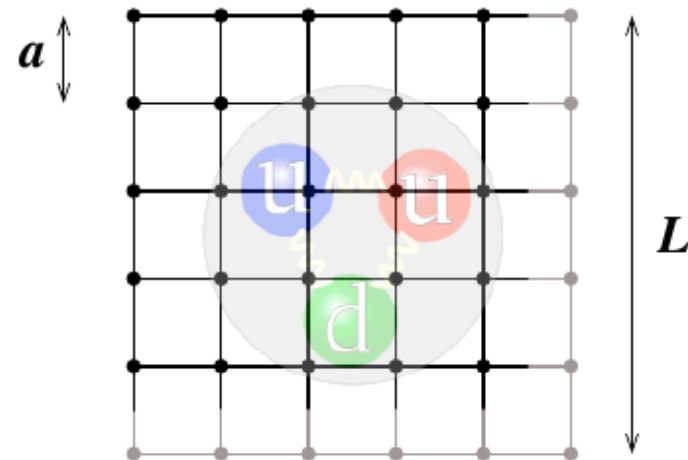
# Lattice QCD

$$\mathcal{L}_{\text{QCD}} = \sum_f \bar{\psi}_f (\not{D} + m_f) \psi_f + \frac{1}{4} \text{tr} F_{\mu\nu} F^{\mu\nu}$$

Non-perturbative QCD contributions from first principles



Control all systematic uncertainties



- ◆ Discrete Euclidean Space-Time
- ◆ Finite spatial volume and time extent
- ◆ Path integrals rigorously defined and computed via Monte Carlo methods

# Lattice QCD

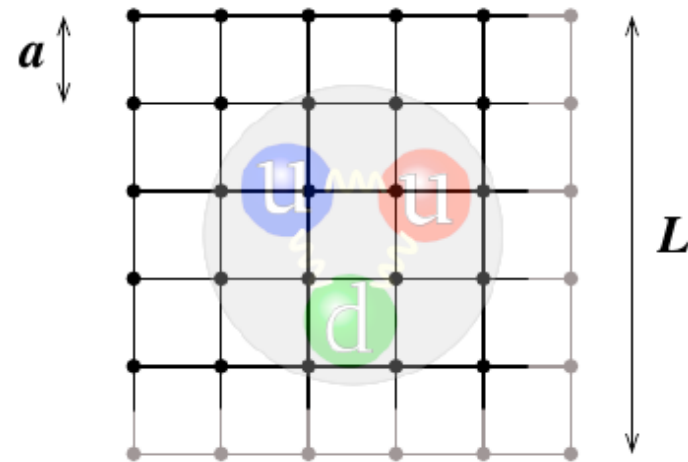
$$\mathcal{L}_{\text{QCD}} = \sum_f \bar{\psi}_f (\not{D} + m_f) \psi_f + \frac{1}{4} \text{tr} F_{\mu\nu} F^{\mu\nu}$$

Non-perturbative QCD contributions from first principles



Control all systematic uncertainties

Parameters in a simulation:



- ✦ Lattice spacing:  $a$
- ✦ Finite volume and time:  $L, T$
- ✦ Quark masses:  $m_{ud}, m_s, m_c, m_b$
- ✦ # sea quarks:  $N_f = 2, 2 + 1, 2 + 1 + 1, \dots$

# Lattice QCD

$$\mathcal{L}_{\text{QCD}} = \sum_f \bar{\psi}_f (\not{D} + m_f) \psi_f + \frac{1}{4} \text{tr} F_{\mu\nu} F^{\mu\nu}$$

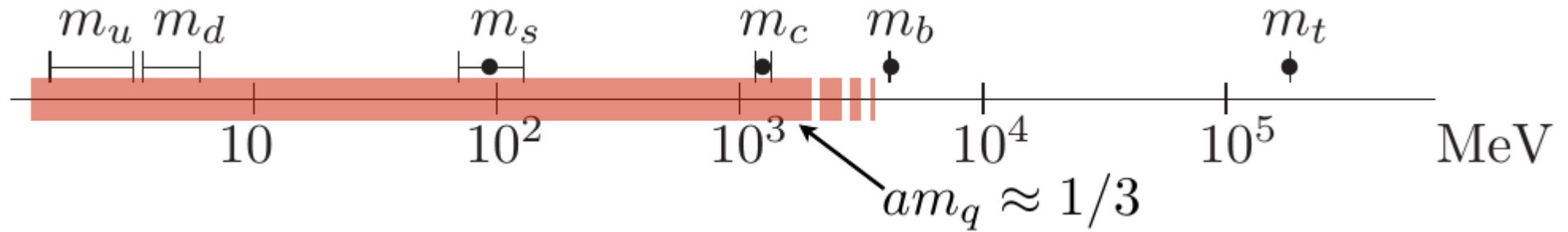
QCD recovered by removing cutoffs

- ♦ Continuum limit:  $a \rightarrow 0$
- ♦ Infinite volume limit:  $L, T \rightarrow \infty$
- ♦ Interpolation/Extrapolation to physical quark masses:  $m_f \rightarrow m_f^{\text{phys}}$

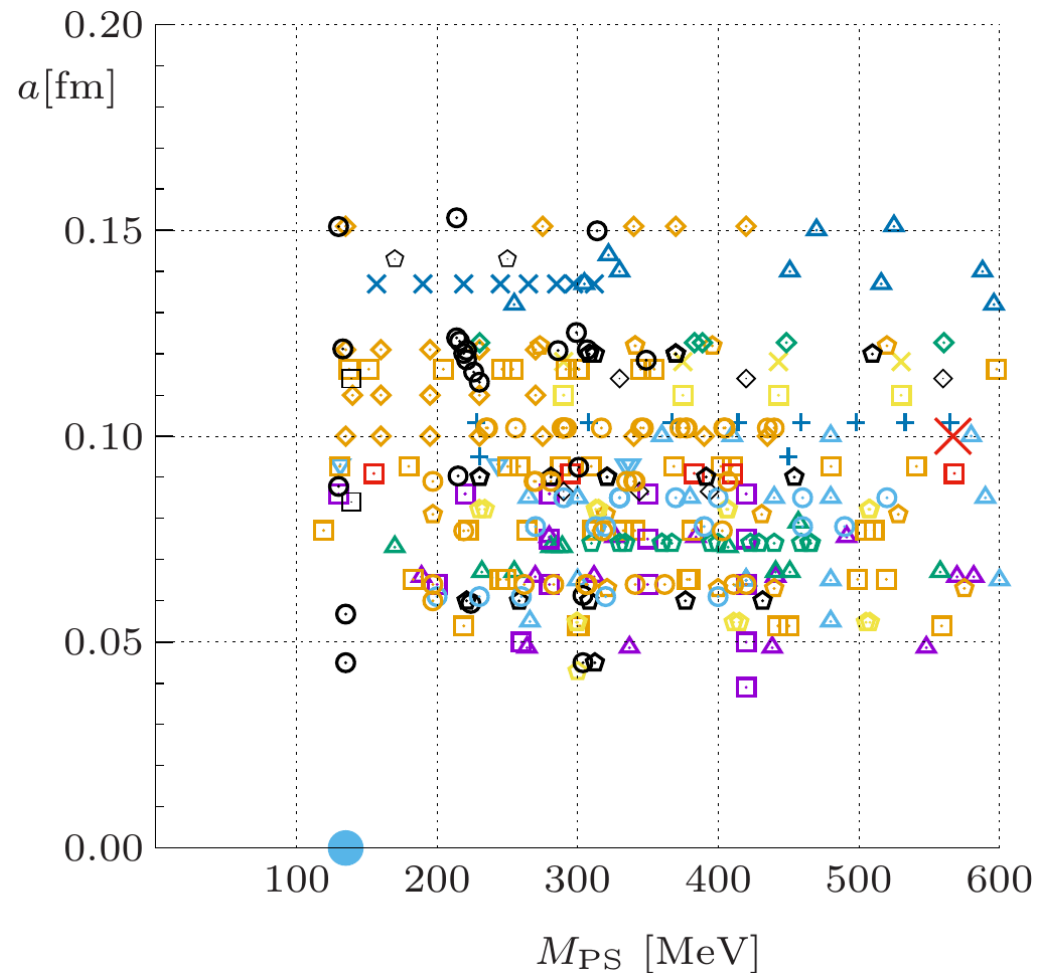
To reliably estimate systematic errors:

repeat the calculation for several lattice spacing, volumes and sea-quark masses

# Lattice QCD - State of the art

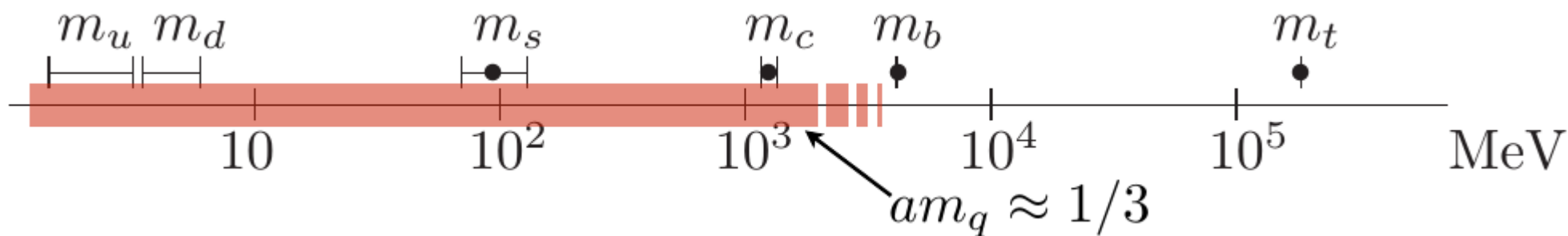


CLS	$N_f = 2$	▲
ETMC	$N_f = 2$	▲
(clover) ETMC	$N_f = 2$	▼
QCDSF	$N_f = 2$	▲
BGR	$N_f = 2$	▲
JLQCD	$N_f = 2$	×
(plaq) TWQCD	$N_f = 2$	+
(Iwa) TWQCD	$N_f = 2$	×
(HEX) BMW	$N_f = 2 + 1$	◻
(stout) BMW	$N_f = 2 + 1$	◻
(stout-stag) BMW	$N_f = 2 + 1$	◻
CLS	$N_f = 2 + 1$	◻
HSC	$N_f = 2 + 1$	◻
PACS-CS	$N_f = 2 + 1$	◻
QCDSF	$N_f = 2 + 1$	◻
JLQCD	$N_f = 2 + 1$	◻
(Möbius) JLQCD	$N_f = 2 + 1$	◻
RBC-UKQCD	$N_f = 2 + 1$	◻
(DSDR) RBC-UKQCD	$N_f = 2 + 1$	◻
(Möbius) RBC-UKQCD	$N_f = 2 + 1$	◻
MILC	$N_f = 2 + 1$	◻
MILC	$N_f = 2 + 1 + 1$	◻
ETMC	$N_f = 2 + 1 + 1$	◻
BMW	$N_f = 1 + 1 + 1 + 1$	◻
LQCD/CP-PACS (2001)	$N_f = 2$	×
$M_\pi$ (experiment)		●



[G. Herdoiza summer 2015 + (partial) updates, C. Pena]

# Quark masses on the lattice



**Light quarks:** discretization errors  $\sim (a\Lambda_{QCD})^n$

finite size effects  $\sim \exp[-M_\pi L]$

Extrapolation in  $m_{u,d}$  often necessary (ChPT)

**Heavy quarks:** discretization errors  $\sim (am_h)^n$

- Charm quarks:  $am_c \sim 0.3$

directly accessible on the lattice

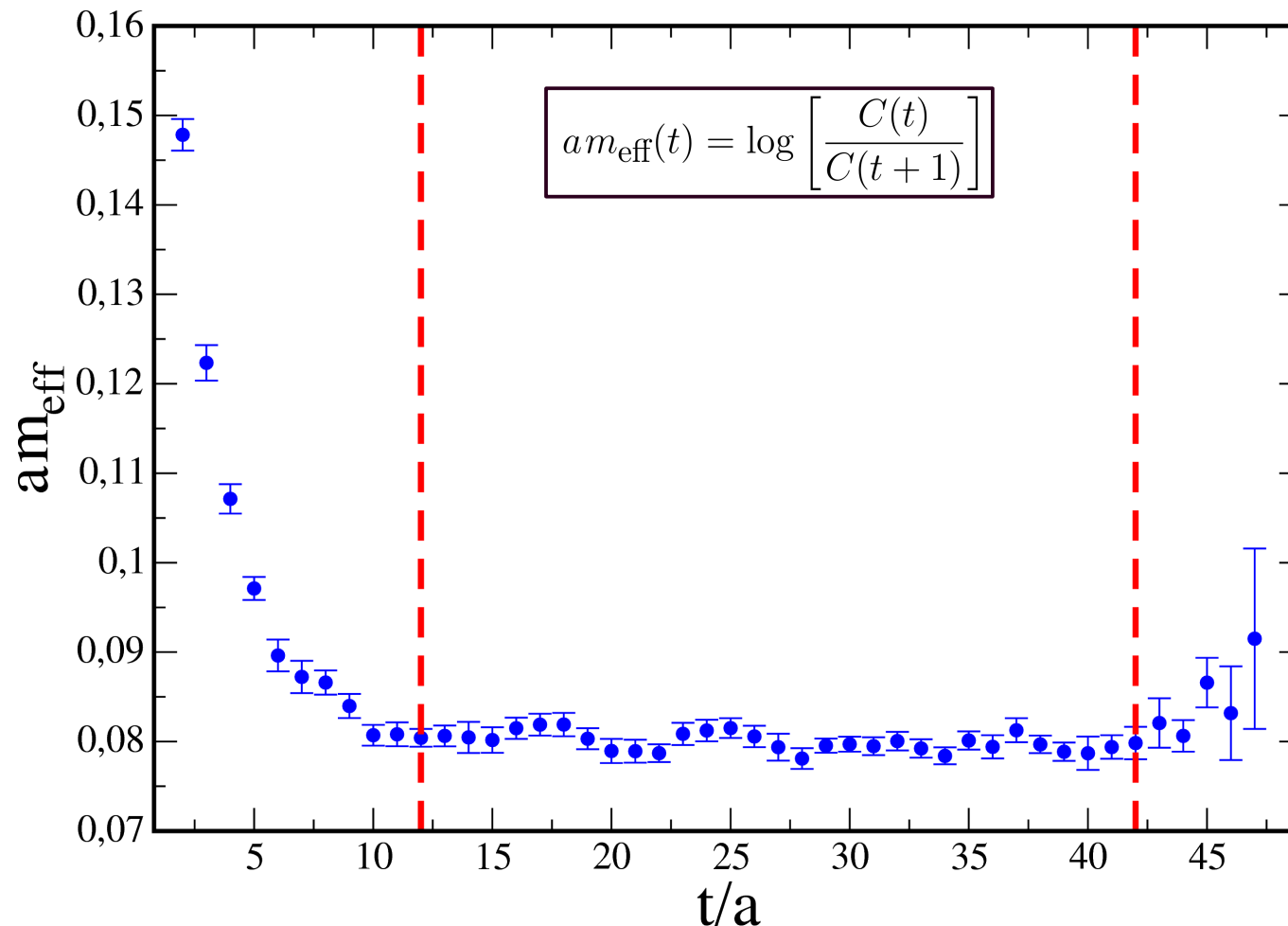
- Bottom quarks:  $am_b \gtrsim 1$

extrapolation or an effective theory (HQET, NRQCD, ...) is needed

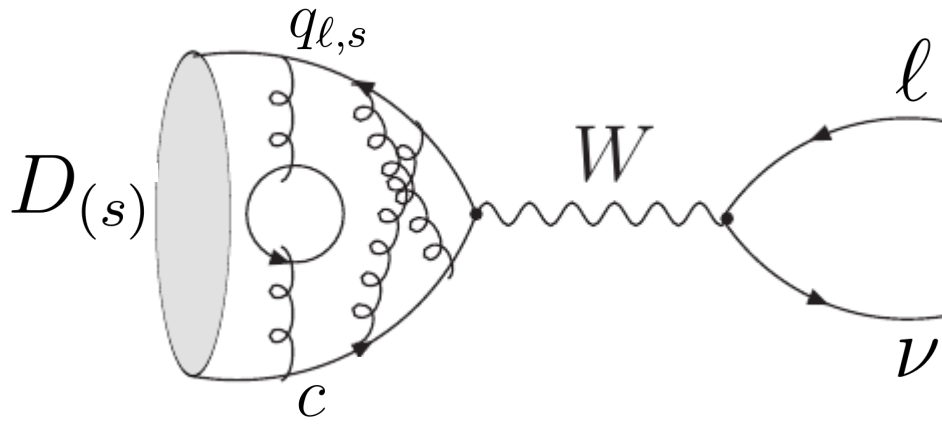
# Euclidean Correlators

Masses and hadronic matrix elements  
are extracted from Euclidean Correlators

$$C(t) \equiv \langle O_\pi(t) O_\pi^\dagger(0) \rangle \approx \sum_n \frac{|\langle 0 | O_\pi(0) | n \rangle|^2}{2E_n} \exp[-E_n t] \xrightarrow{t \rightarrow \infty} \frac{|\langle 0 | O_\pi(0) | \pi \rangle|^2}{2M_\pi} \exp[-M_\pi t]$$



# Leptonic decays



$$D_{(s)} \rightarrow \ell \nu$$

Decay constants  
(Lattice QCD)

$$\langle 0 | A_{cq}^\mu | D_q(p) \rangle = i f_{D_q} p_{D_q}^\mu$$

$$A_{cq}^\mu = \bar{c} \gamma_\mu \gamma_5 q$$

$$\Gamma(D_{(s)} \rightarrow \ell \nu) = |V_{cd(s)}|^2 \times f_{D_{(s)}}^2 \times \frac{G_F^2}{8\pi} m_\ell^2 m_{D_{(s)}} (1 - m_\ell^2/m_{D_{(s)}}^2)^2$$

Decay rate  
(Experiments)

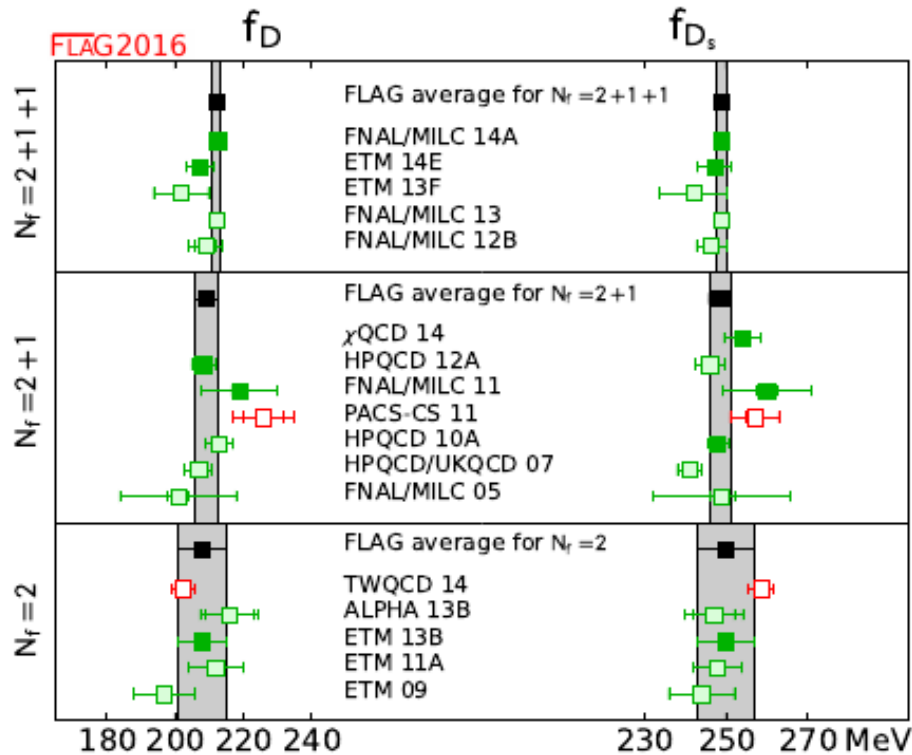
CKM matrix elements

- Experimental + LQCD inputs for the determination of CKM elements
- Systematic and statistical errors cancellation in the SU(3) ratio  $f_{D_s}/f_D$

# Leptonic decays - FLAG

## FLAG2016

EPJC77 no.2, 112



### FNAL/MILC 14A

PRD90, 074509

- Improved action
- Physical mass:  $ud$ ,  $s$ ,  $c$
- $a \sim 0.06 fm \div 0.15 fm$

0.5%

### ETM 14E

PRD91, 054507

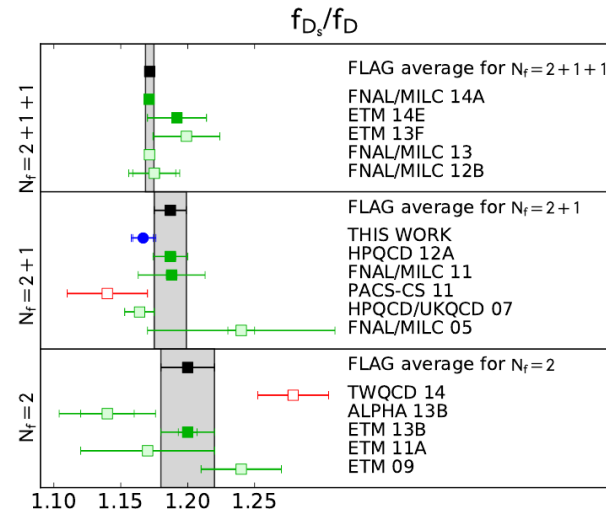
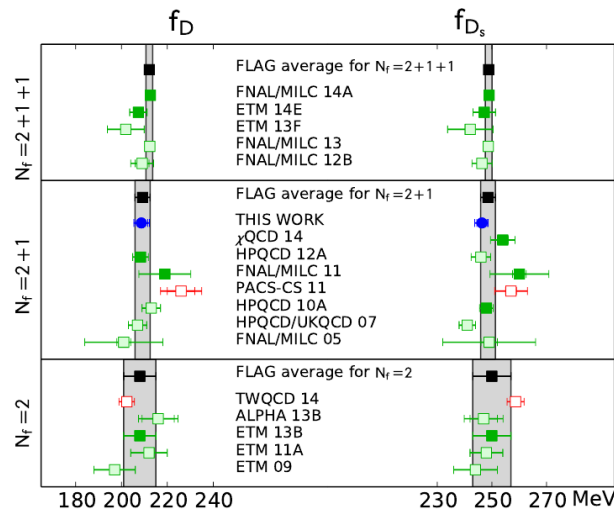
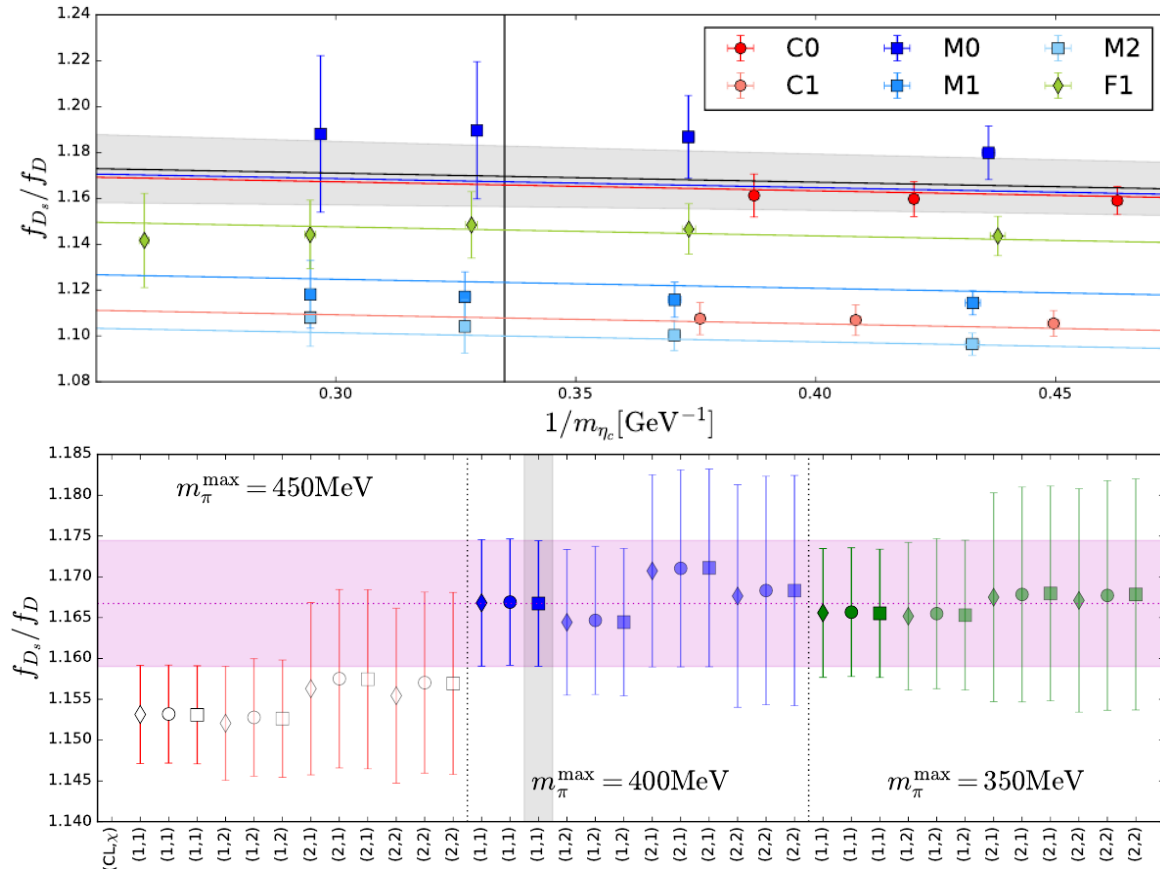
- Improved action
- Physical mass:  $s$ ,  $c$
- $a \sim 0.06 fm \div 0.09 fm$

1.5%

# Leptonic decays – New Results

RBC/UKQCD JHEP12 008

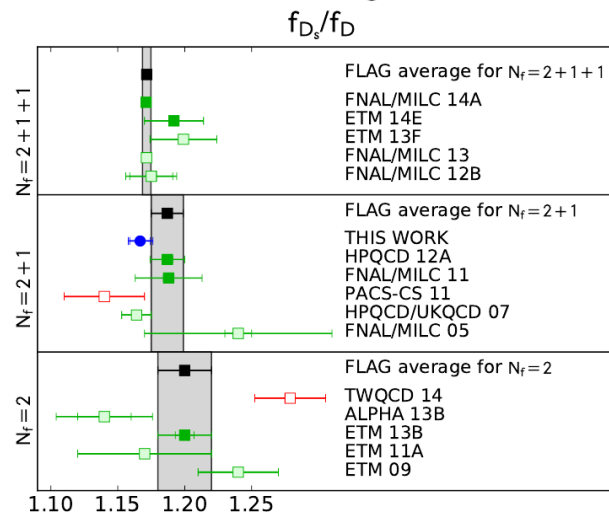
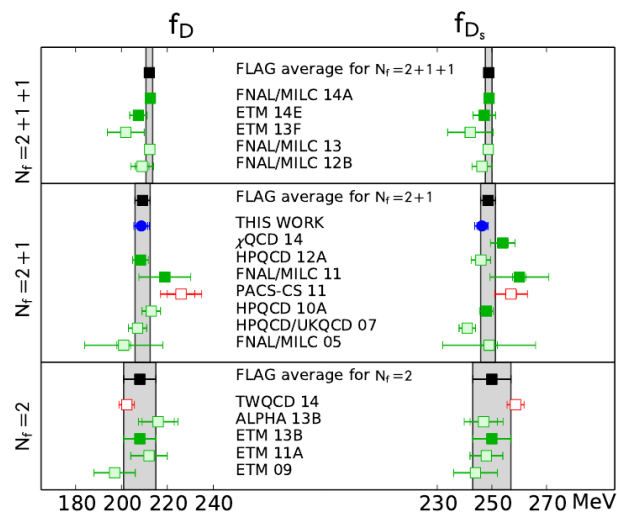
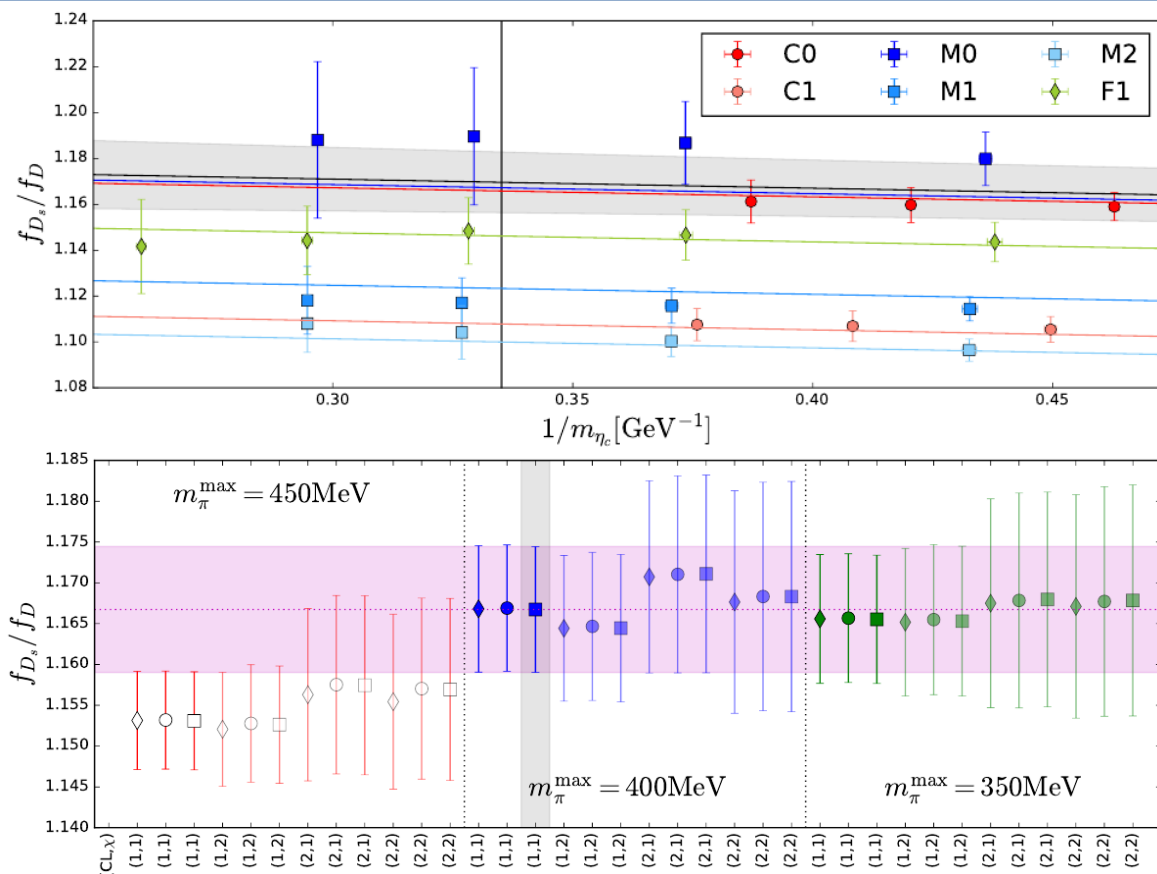
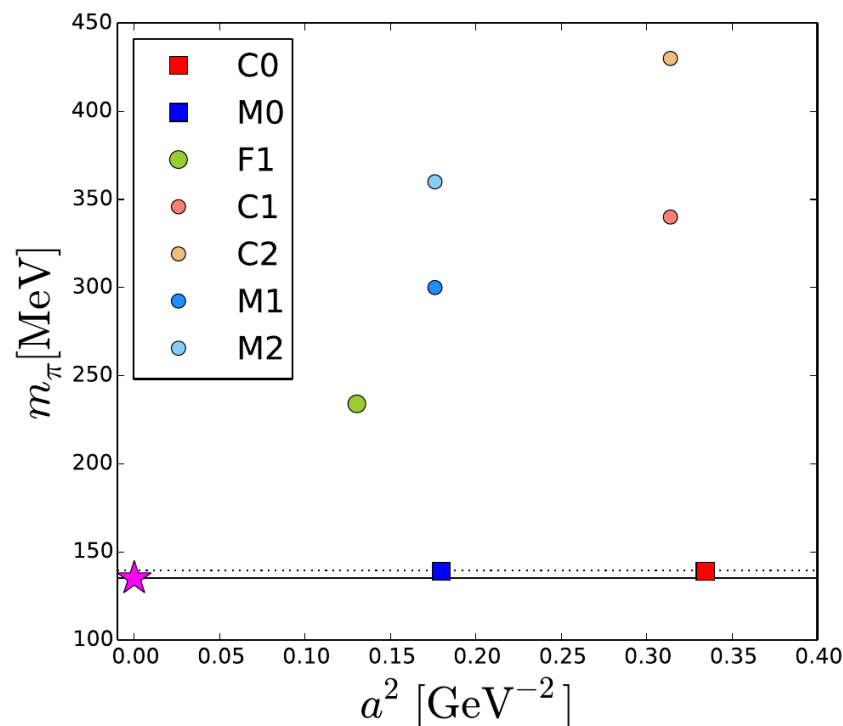
- $N_f = 2 + 1$  DW fermions
- Physical mass:  $ud, s, c$
- $a \sim 0.11 fm \div 0.07 fm$
- $am_c \lesssim 0.4$



$$f_D = 208.7(2.8)_{\text{stat}}(+2.1)_{\text{sys}} \text{ MeV}$$

$$f_{D_s} = 246.4(1.9)_{\text{stat}}(+1.3)_{\text{sys}} \text{ MeV}$$

## JHEP12 008



$$f_{D_s} = 246.4(1.9)_{\text{stat}}({}^{+1.3}_{-1.9})_{\text{sys}} \text{ MeV}$$

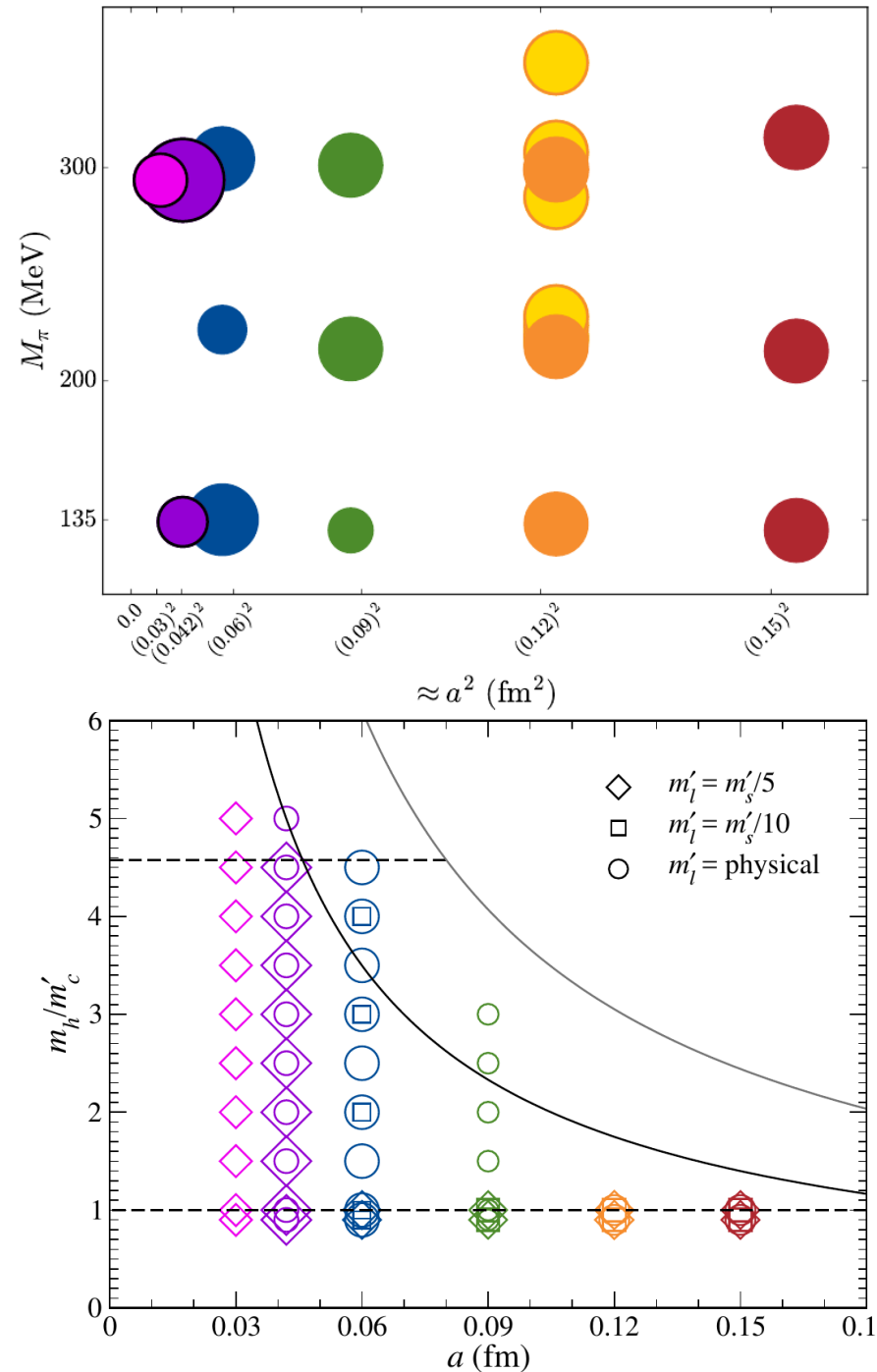
# Leptonic decays – New Results

FNAL/MILC

arXiv:1712.09262

- $N_f = 2 + 1 + 1$
- 20 ensembles
- Light-quark mass down to  $\frac{1}{2}(m_u + m_d)$
- down to  $a \sim 0.03 fm$
- Very high statistics (1000x4 samples)
- Big volumes (up to:  $144^3 \times 288$ ,  $6 fm$ )

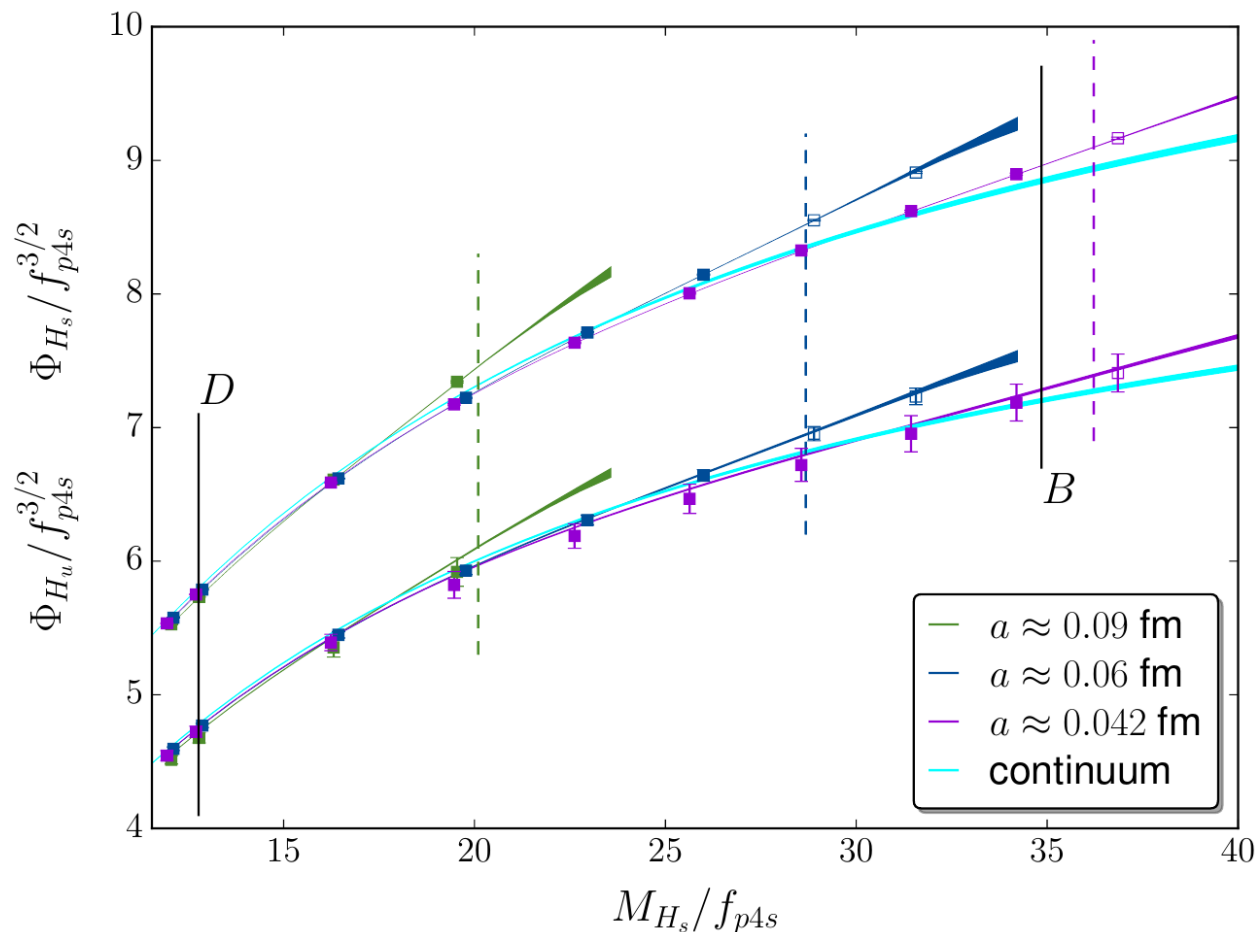
- always  $0.9m_c, m_c$ ;
- omit  $am_c \geq 0.9$   
from heavy-quark  
fits (need  $< \pi/2$ );



# Leptonic decays – New Results

## FNAL/MILC

- 492 data pts;
- 60 parameters;
- $\chi^2/\text{dof} = 466/432$ ;



$$f_D = 212.1(0.3)_{\text{stat}}(0.3)_{\text{syst}}(0.2)f_{\pi,\text{PDG}} \text{ MeV}$$

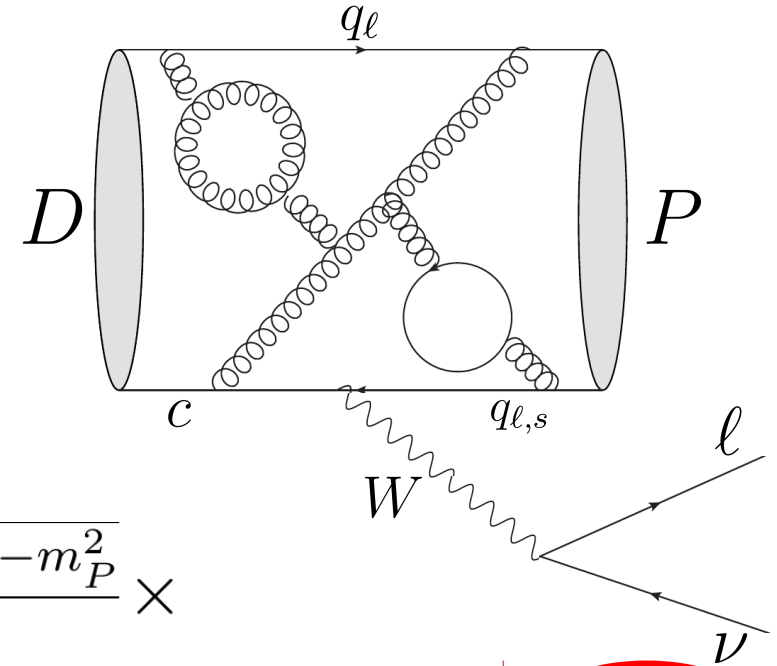
$$f_{D_s} = 249.8(0.3)_{\text{stat}}(0.3)_{\text{syst}}(0.2)f_{\pi,\text{PDG}} \text{ MeV}$$

0.2%

# Semileptonic decays

$$D \rightarrow \pi(K)\ell\nu$$

CKM matrix elements



$$\frac{d\Gamma}{dq^2}(D \rightarrow P\ell\nu) = \frac{G_F^2}{24\pi^3} |V_{cd(s)}|^2 \frac{(q^2 - m_\ell^2)^2 \sqrt{E_P^2 - m_P^2}}{q^4 m_D^2} \times$$

$$\times \left[ \left(1 + \frac{m_\ell^2}{2q^2}\right) m_D^2 (E_P^2 - m_P^2) |f_+(q^2)|^2 + \frac{3m_\ell^2}{8q^2} (m_D^2 - m_P^2)^2 |f_0(q^2)|^2 \right]$$

Form factors  
(Lattice QCD)

- Experimental + LQCD inputs for the determination of CKM elements
- Determination of the forms factors in all the physical  $q^2$  range
- Momentum dependence of the tensor form factor for BSM analysis

# Semileptonic decays - FLAG

# FLAG 2016

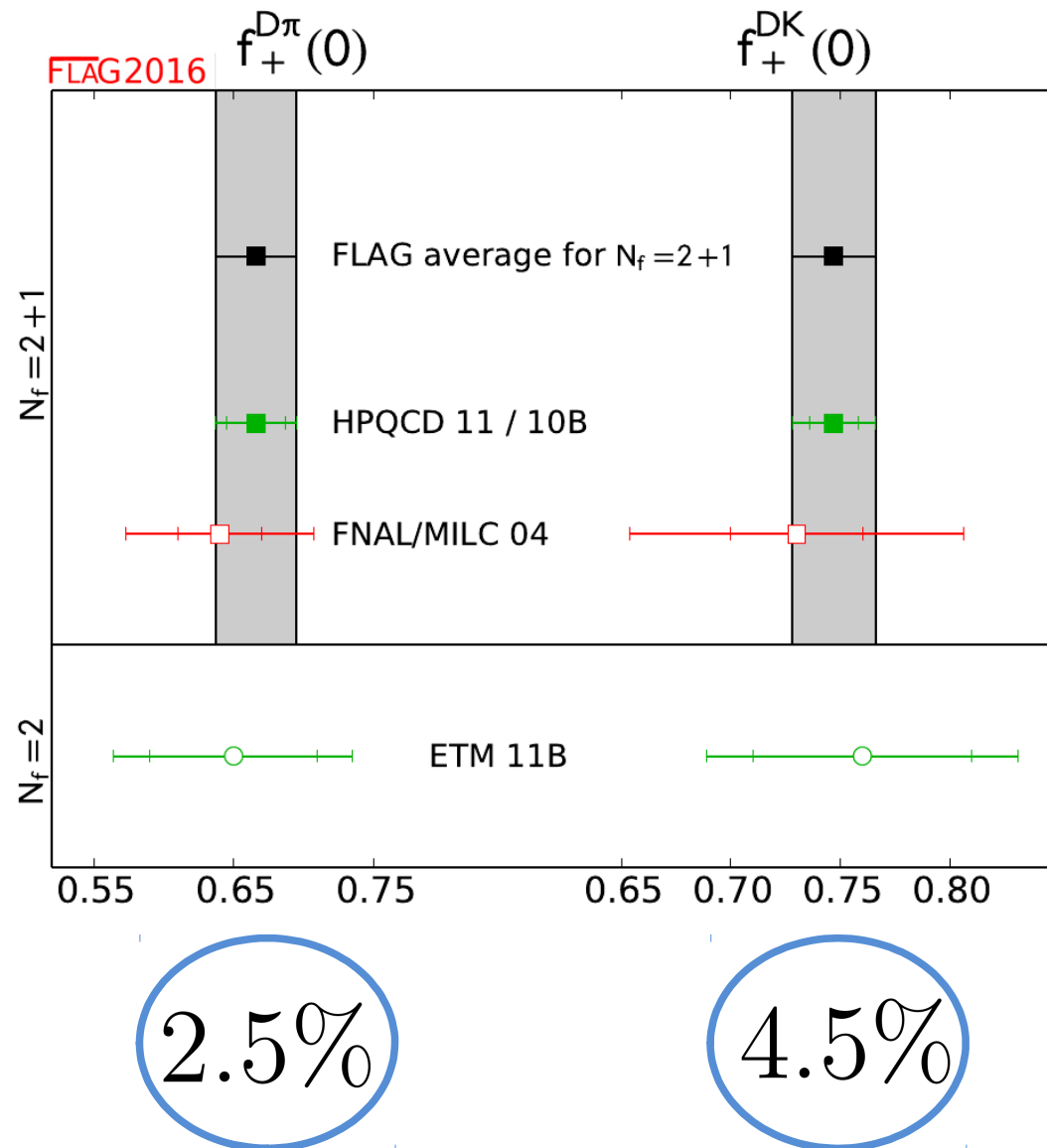
EPJC77 no.2, 112

## HPQCD

PRD82, 114506

PRD84, 114505

- $N_f = 2 + 1$  flavors of HISQ
- Improved action
- Physical mass:  $s, c$
- $a \sim 0.09 fm \div 0.12 fm$
- No renormalization
- D at rest frame
- Modified z-expansion
- $f_0$  from the Scalar matrix element



# Semileptonic decays - New Results

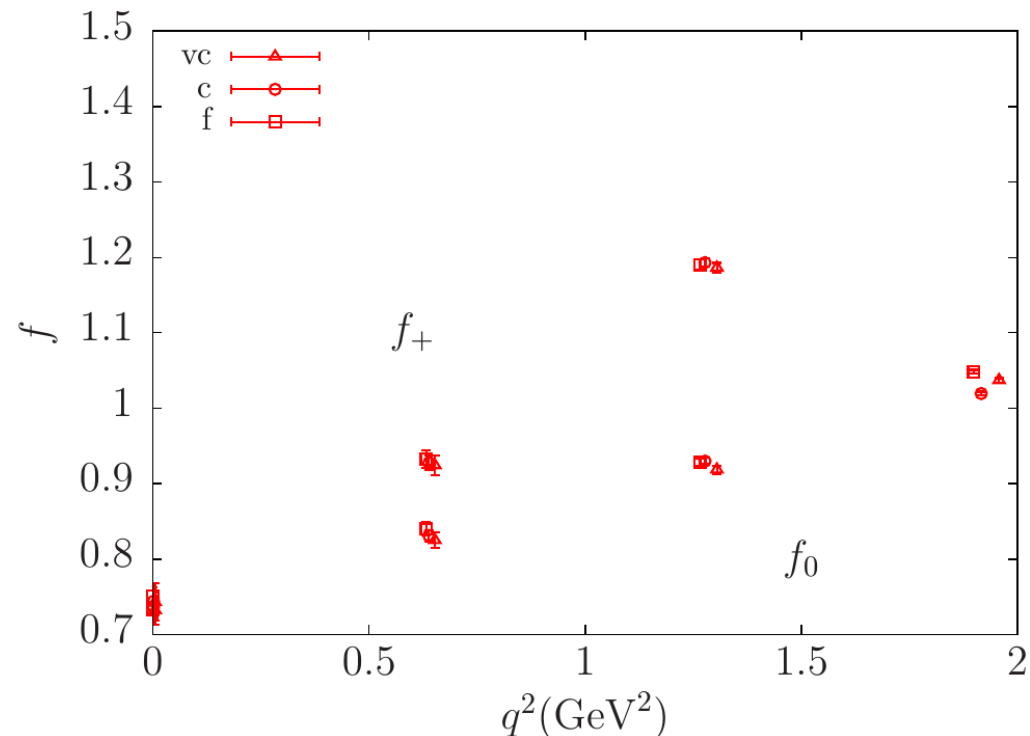
New (Preliminary)  
(B. Chakraborty) @ LATTICE 2017

## HPQCD

PRD82, 114506  
PRD84, 114505

- $N_f = 2 + 1$  flavors of HISQ
- Improved action
- Physical mass:  $s, c$
- $a \sim 0.09 fm \div 0.12 fm$
- No renormalization
- D at rest frame
- Modified z-expansion
- $f_0$  from the Scalar matrix element

- $N_f = 2 + 1 + 1$
- Physical u/d quarks
- Both  $f_+$  and  $f_0$  over whole  $q^2$ -range



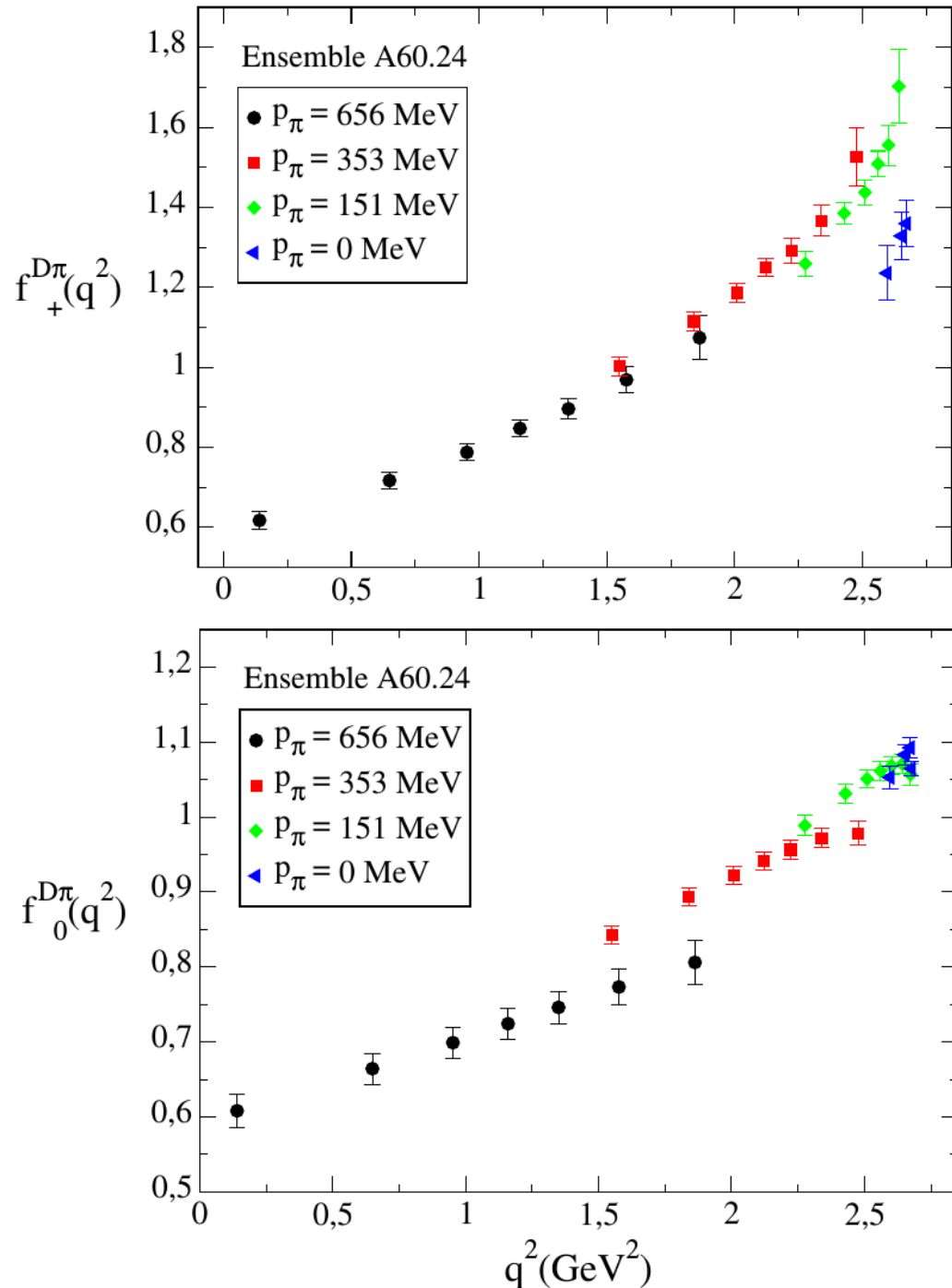
# Semileptonic decays - New Results

ETMC

PRD96 no.5, 054514

arXiv:1803.04807

- $N_f = 2 + 1 + 1$  flavors of tmW
- Improved action
- Physical mass:  $s, c$
- $a \sim 0.06 fm \div 0.09 fm$
- No renormalization
- $f_{+,0}$  and  $f_T$  over whole  $q^2$ -range
- Hypercubic discretization effects
- Modified z-expansion
- Vector & Scalar matrix elements

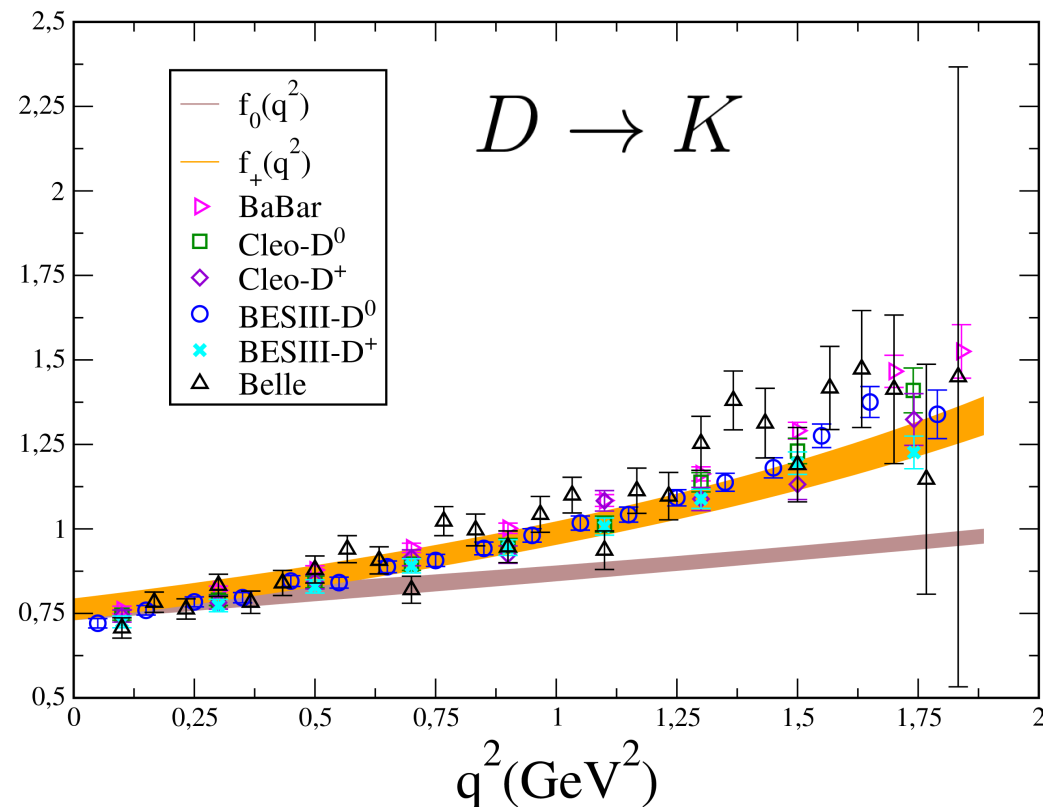
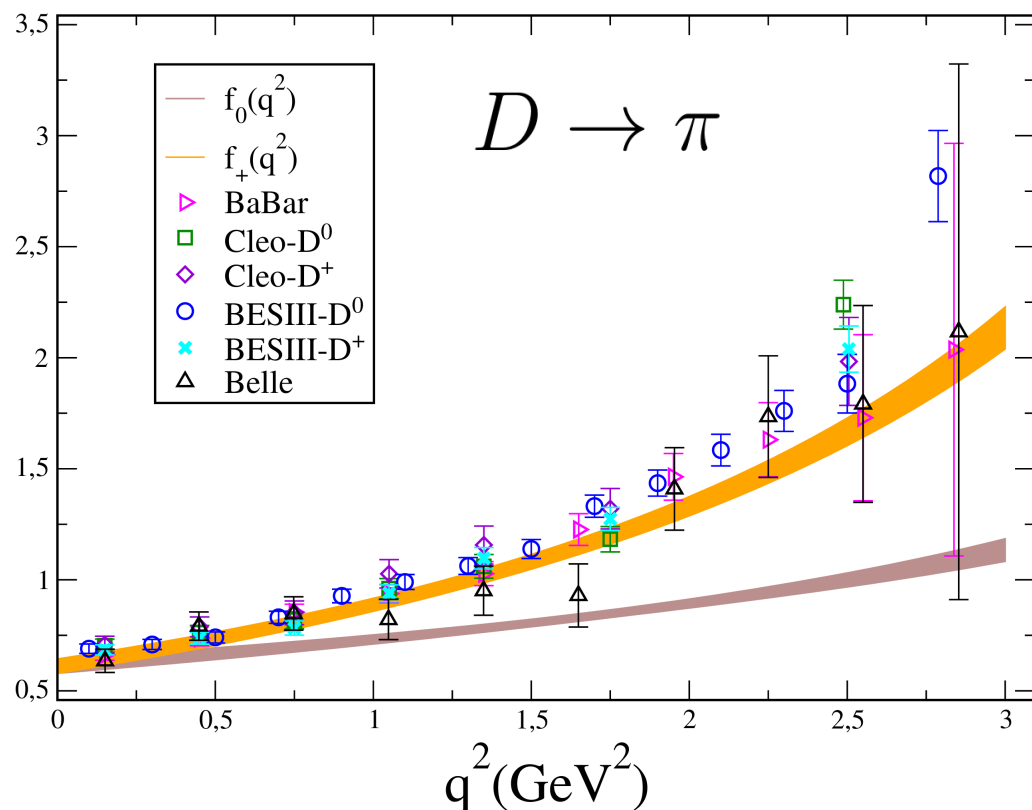


# Semileptonic decays - New Results

$f_0(q^2)$  and  $f_+(q^2)$  at the physical point

ETMC

$$\chi^2/d.o.f. = 1.2 \quad d.o.f. \approx 1100$$



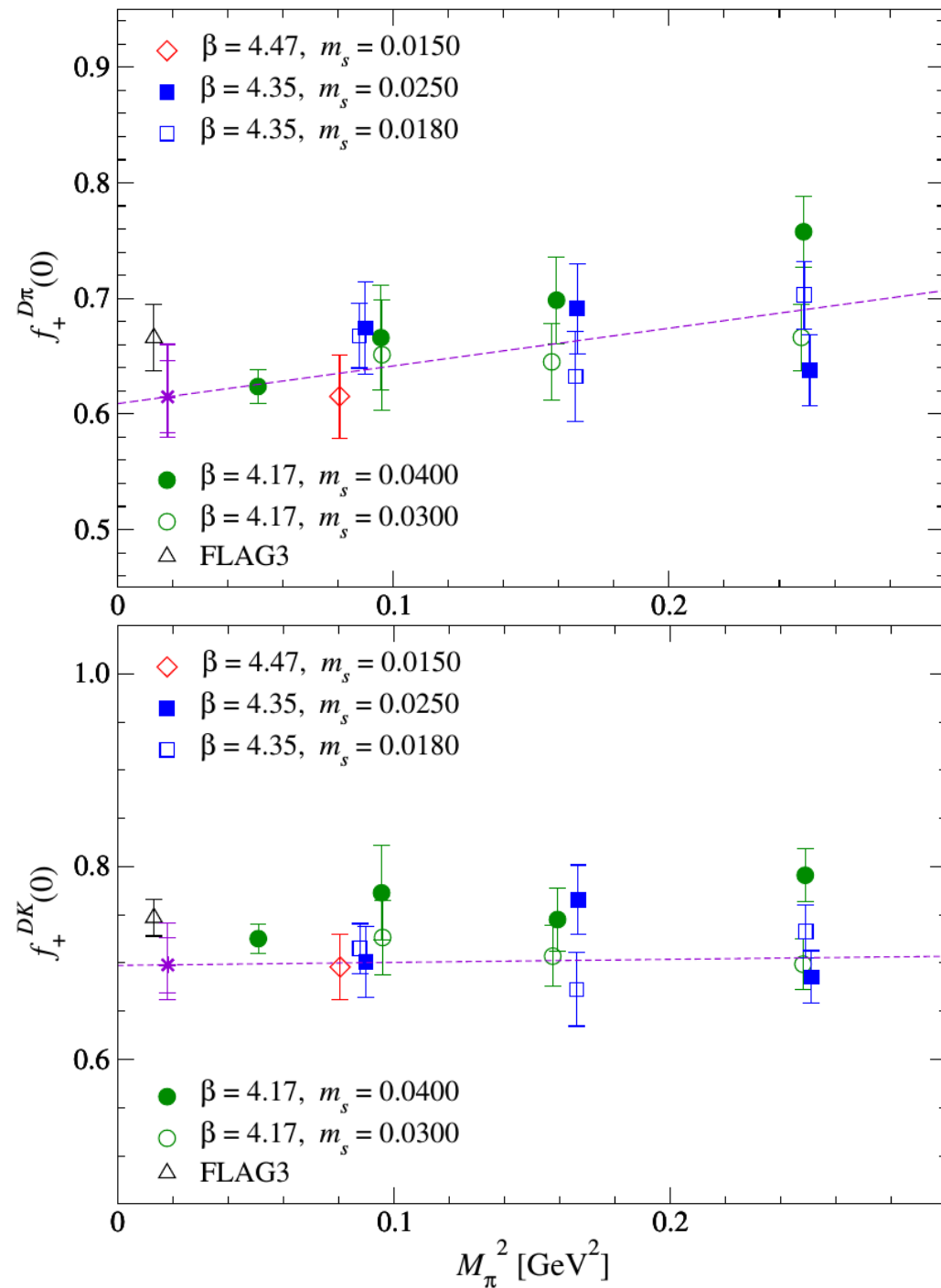
$$f_+^{D \rightarrow \pi}(0) = 0.612 (35)_{\text{Stat+fit}} (4)_{\text{Ch}} (7)_{\text{FSE}} (1)_{\text{Disc}} = 0.612 (36)$$

$$f_+^{D \rightarrow K}(0) = 0.765 (29)_{\text{Stat+fit}} (11)_{\text{Ch}} (1)_{\text{Disc}} = 0.765 (31)$$

# Semileptonic decays - New Results

JLQCD (T. Kaneko) @ LATTICE 2017  
Preliminary

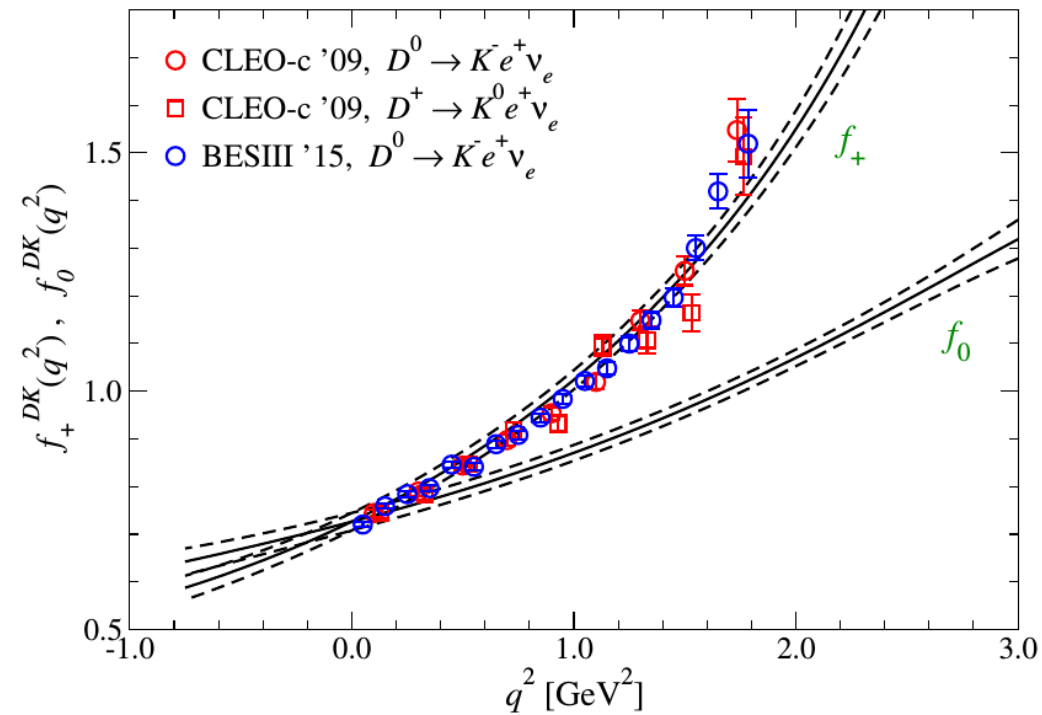
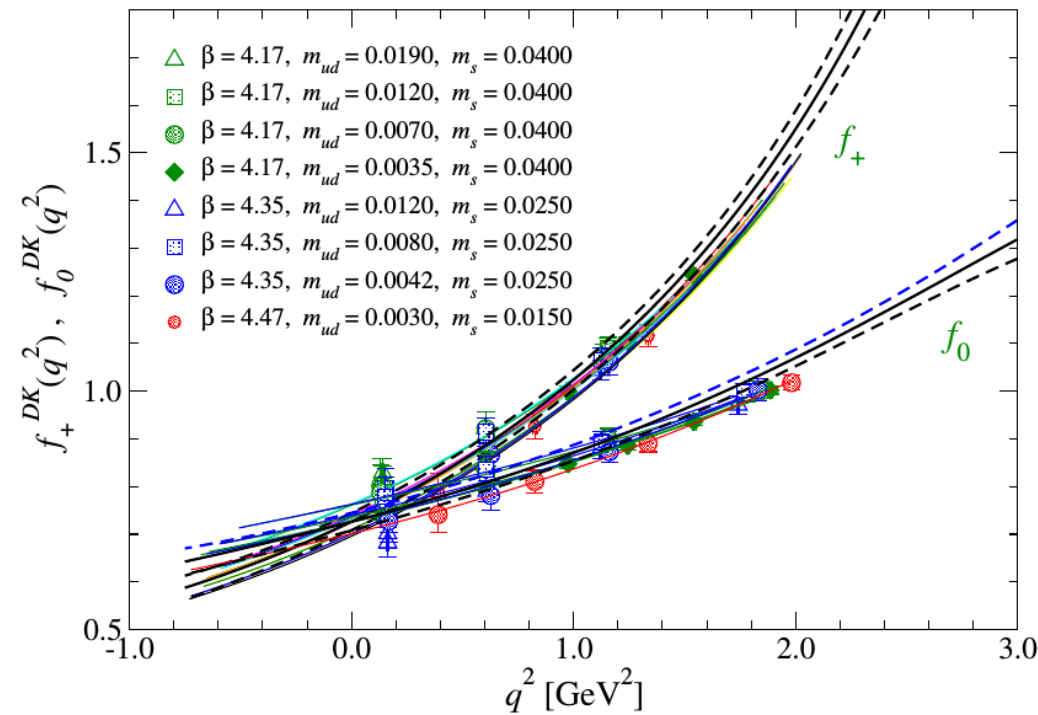
- $N_f = 2 + 1$  of DW fermions
- New ensembles: fine lattice spacing
- Improved action
- Physical mass:  $s, c$
- $a \sim 0.05 fm \div 0.09 fm$
- Both  $f_+$  and  $f_0$  over whole  $q^2$ -range
- D at rest frame
- $q^2=0$  through BCL z-expansion



# Semileptonic decays - New Results

JLQCD (T. Kaneko) @ LATTICE 2017

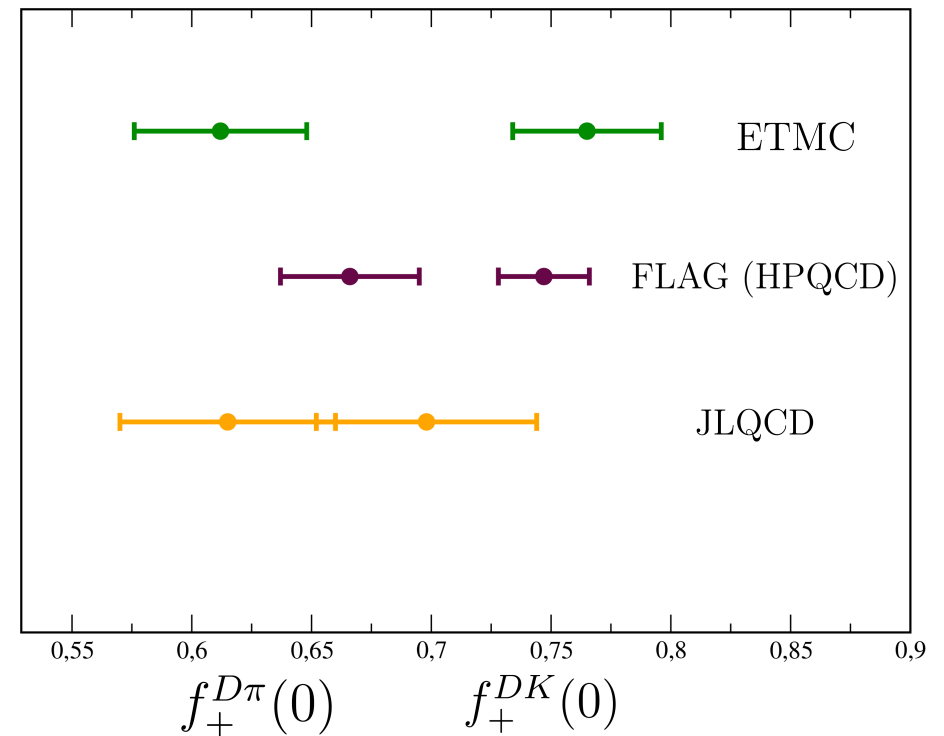
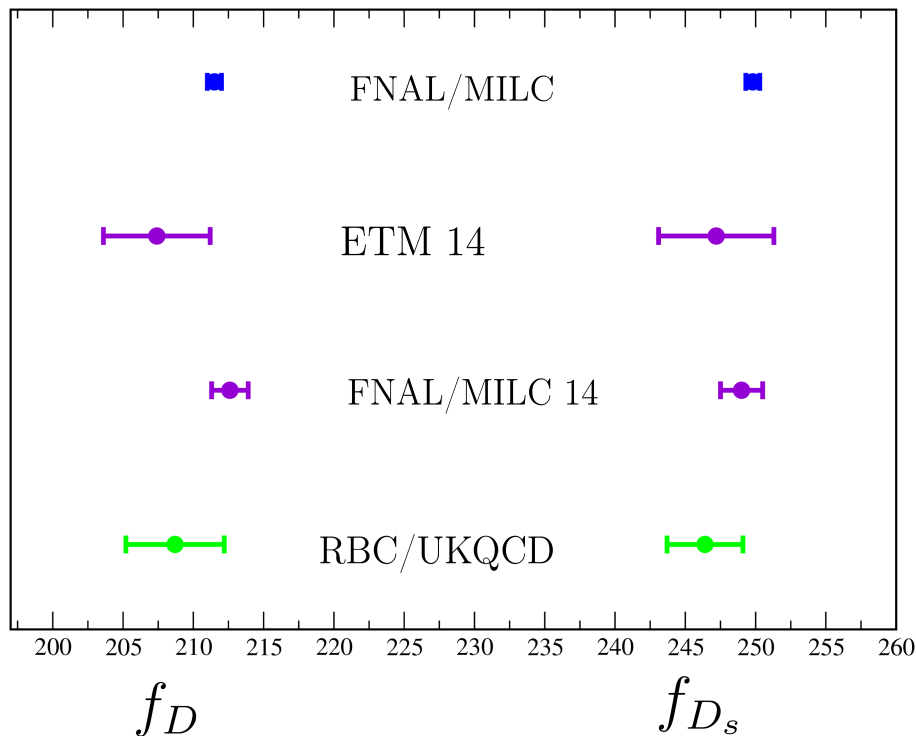
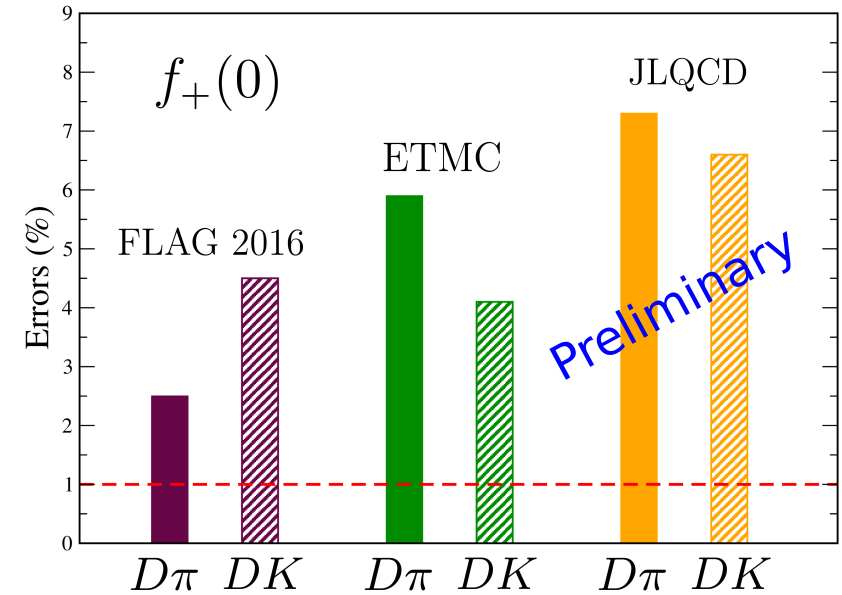
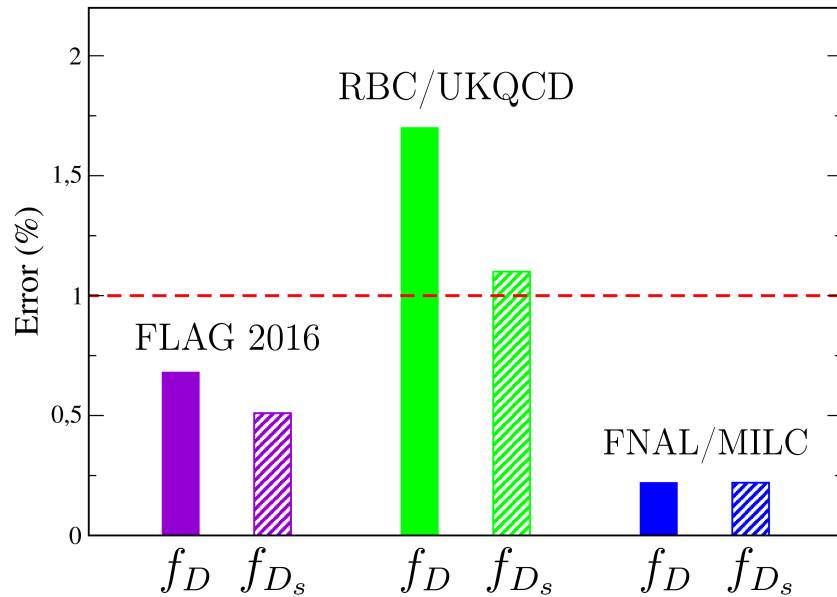
Preliminary



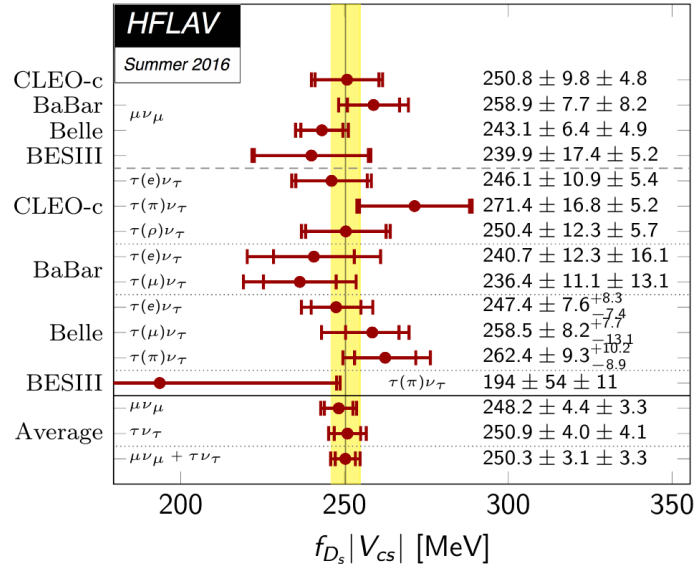
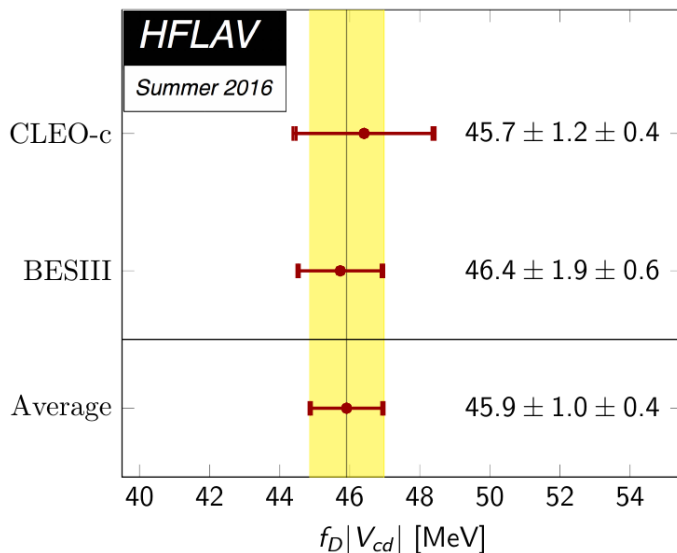
$$f_+^{D\pi}(0) = 0.615(31)_{\text{stat}} \left( {}^{+17}_{-16} \right)_{q^2 \rightarrow 0} \left( {}^{+28}_{-7} \right)_{a \rightarrow 0, \text{chiral}}$$

$$f_+^{DK}(0) = 0.698(29)_{\text{stat}} (-18)_{q^2 \rightarrow 0} \left( {}^{+32}_{-12} \right)_{a \rightarrow 0, \text{chiral}}$$

# Summary of results



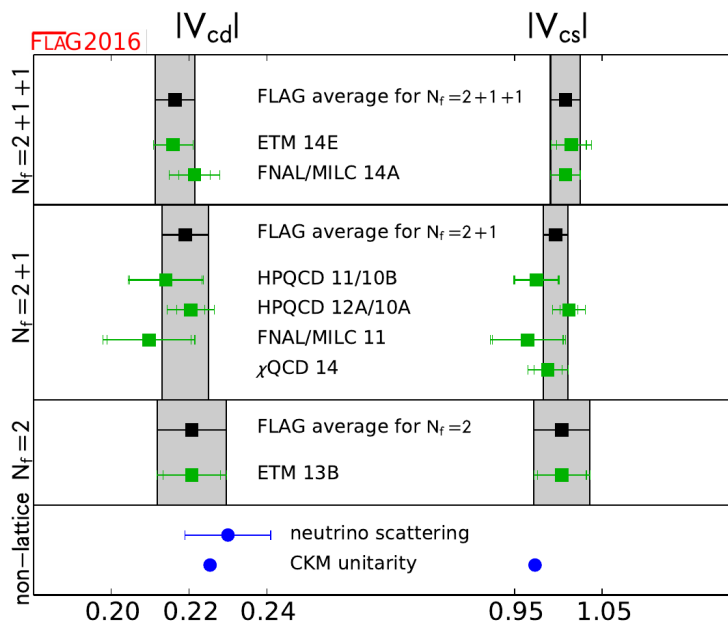
# Determination of $V_{cd}$ and $V_{cs}$



Expt. $D \rightarrow K \ell \nu_\ell$	Mode	$ V_{cs}  f_+^K(0)$
BES III (tagged)	$(D^0)$	$0.7195(35)(43)$
CLEO-c (tagged)	$(D^0, D^+)$	$0.7189(64)(48)$
CLEO-c (untagged)	$(D^0, D^+)$	$0.7436(76)(79)$
BABAR	$(D^0)$	$0.7241(64)(60)$
Belle	$(D^0)$	$0.700(19)$
FOCUS and others		$0.724(29)$
<b>Combined</b>	$(D^0, D^+)$	<b><math>0.7226(22)(26)</math></b>

Expt. $D \rightarrow \pi \ell \nu_\ell$	mode	$ V_{cd}  f_+^\pi(0)$
BES III (tagged)	$(D^0)$	$0.1422(25)(10)$
CLEO-c (tagged)	$(D^0, D^+)$	$0.1507(42)(11)$
CLEO-c (untagged)	$(D^0, D^+)$	$0.1394(58)(25)$
BABAR	$(D^0)$	$0.1381(36)(22)$
Belle	$(D^0)$	$0.142(11)$
<b>Combined</b>	$(D^0, D^+)$	<b><math>0.1426(17)(8)</math></b>



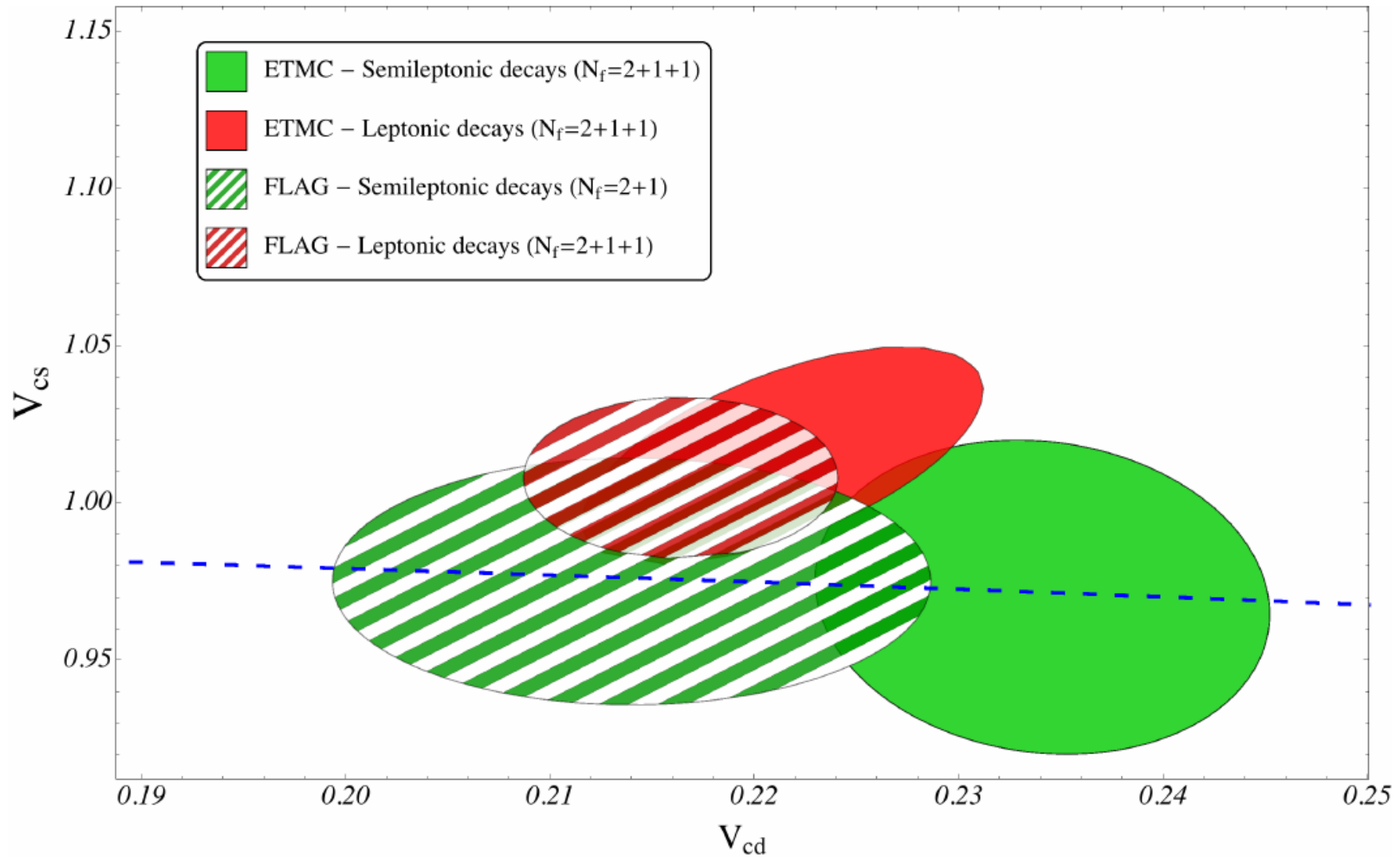
$V_{cd(s)}$  from semileptonic decays  
(errors dominated by theoretical inputs)

	$V_{cd}$	$V_{cs}$
FLAG 2016	$0.2140(93)_{\text{lat}}(29)_{\text{exp}}$	$0.975(25)_{\text{lat}}(7)_{\text{exp}}$
ETMC	$0.2330(133)_{\text{lat}}(31)_{\text{exp}}$	$0.945(38)_{\text{lat}}(4)_{\text{exp}}$
JLQCD	$0.232(17)_{\text{lat}}(3)_{\text{exp}}$	$1.035(64)_{\text{lat}}(5)_{\text{exp}}$

$V_{cd(s)}$  from leptonic decays  
(errors dominated by experimental inputs)

	$V_{cd}$	$V_{cs}$
FLAG 2016	$0.2164(14)_{\text{LQCD}}(49)_{\text{exp}}$	$1.008(5)_{\text{LQCD}}(16)_{\text{exp}}$
FNAL/MILC	$0.2144(5)_{\text{LQCD}}(49)_{\text{exp}}(13)_{\text{EM}}$	$0.997(2)_{\text{LQCD}}(16)_{\text{exp}}(6)_{\text{EM}}$
RBC/UKQCD	$0.2185(37)_{\text{LQCD}}(50)_{\text{exp}}$	$1.011(11)_{\text{LQCD}}(16)_{\text{exp}}$


# CKM 2nd-row Unitarity



$$|V_{cd}|^2 + |V_{cs}|^2 + |V_{cb}|^2 - 1 = 0.06(3) \quad N_f = 2 + 1 + 1$$

$$|V_{cd}|^2 + |V_{cs}|^2 + |V_{cb}|^2 - 1 = 0.04(3) \quad N_f = 2 + 1$$

# Conclusions & Outlooks

- ✦ LQCD simulations with very small lattice spacings and light sea quarks at their physical masses are being performed
  - No need (reduced) of chiral extrapolation  better precision
- ✦ LQCD predictions of the decay constants of the D and D<sub>s</sub> mesons are very precise (errors below 1%, now ~0.2%). Uncertainties in CKM matrix elements dominated by experimental inputs
  - New determinations are required to confirm systematic errors are under control
- ✦ Determinations of D<sub>(s)</sub> semileptonic form factors still need to be improved. Uncertainties in CKM matrix elements dominated by LQCD inputs
  - $f_{+,0}$  over whole  $q^2$ -range
  - $f_T$  over whole  $q^2$ -range

Thank you for the attention

# Other results

## ETMC

PRD96, 034524

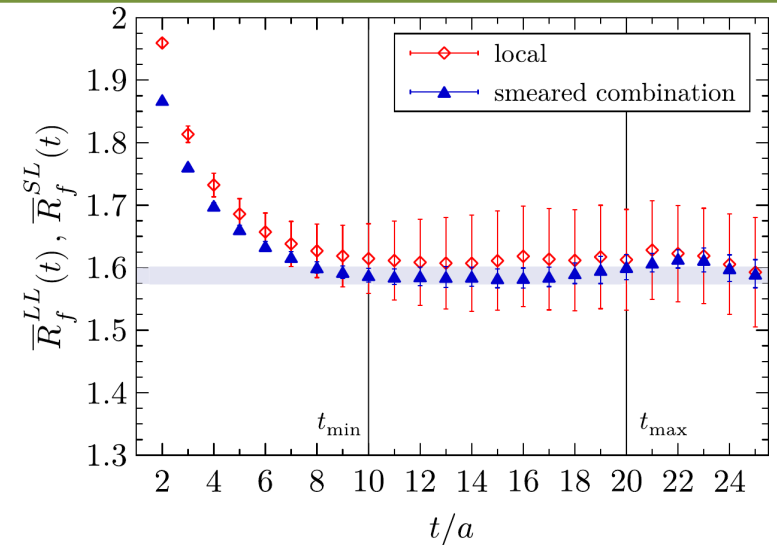
$$N_f = 2 + 1 + 1$$

$$\langle 0 | \hat{V}_\mu | H_\ell^*(\vec{p}, \lambda) \rangle = f_{H_\ell^*} M_{H_\ell^*} \epsilon_\mu^\lambda$$

$$(m_h + m_\ell) \langle 0 | \hat{P} | H_\ell(\vec{p}) \rangle = f_{H_\ell} M_{H_\ell}^2$$

$$f_{D^*}/f_D = 1.078(31)_{\text{stat}}(5)_{\text{input}}(6)_{t_{\min}}(8)_{\text{disc}}(9)_{\text{chir}}[36]$$

$$f_{D_s^*}/f_{D_s} = 1.087(16)_{\text{stat}}(6)_{\text{input}}(6)_{t_{\min}}(7)_{\text{disc}}(5)_{\text{chir}}[20]$$



## CLS ensembles

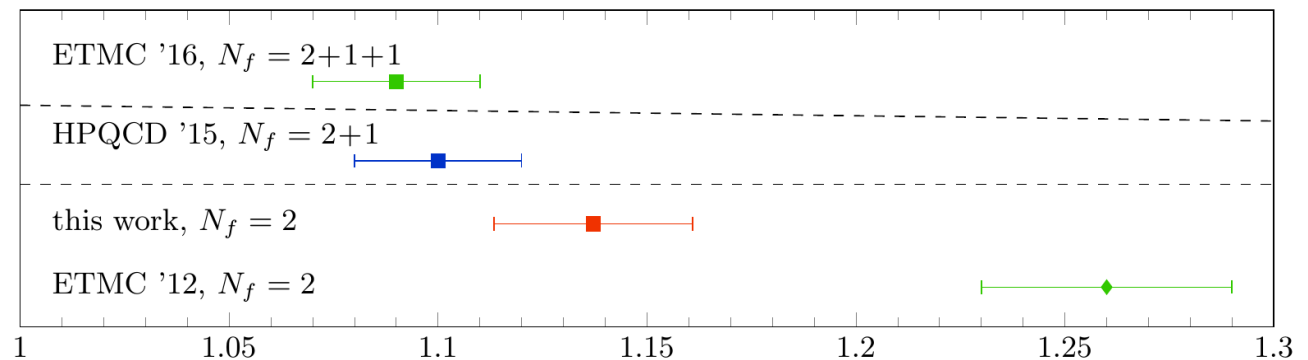
$$N_f = 2$$

arXiv:1803.03065

B. Blossier, J. Heitger, M. Post

id	$\beta$	$(L/a)^3 \times (T/a)$	$\kappa_{\text{sea}}$	$a$ [fm]	$m_\pi$ [MeV]	$Lm_\pi$	# cfgs	$\kappa_s$	$\kappa_c$
E5	5.3	$32^3 \times 64$	0.13625	0.0653	439	4.7	200	0.135777	0.12724
F6		$48^3 \times 96$	0.13635		313	5	120	0.135741	0.12713
F7		$48^3 \times 96$	0.13638		268	4.3	200	0.135730	0.12713
G8		$64^3 \times 128$	0.13642		194	4.1	176	0.135705	0.12710
N6	5.5	$48^3 \times 96$	0.13667	0.0483	341	4	192	0.136250	0.13026
O7		$64^3 \times 128$	0.13671		269	4.2	160	0.136243	0.13022

$$f_{D_s^*}/f_{D_s} = 1.14(2)$$



# Extraction of the form factors

The two semileptonic form factors  $f_0$  and  $f_+$  can be determined from the matrix element of the vector current

$$f_+(q^2) = \frac{(E_D - E_P) \langle \hat{V}_i \rangle - (p_{Di} - p_{Pi}) \langle \hat{V}_0 \rangle}{2E_D p_{Pi} - 2E_P p_{Di}}$$

$$f_-(q^2) = \frac{(p_{Di} + p_{Pi}) \langle \hat{V}_0 \rangle - (E_D + E_P) \langle \hat{V}_i \rangle}{2E_D p_{Pi} - 2E_P p_{Di}}$$

$$f_0(q^2) = f_+(q^2) + \frac{q^2}{M_D^2 - M_P^2} f_-(q^2)$$

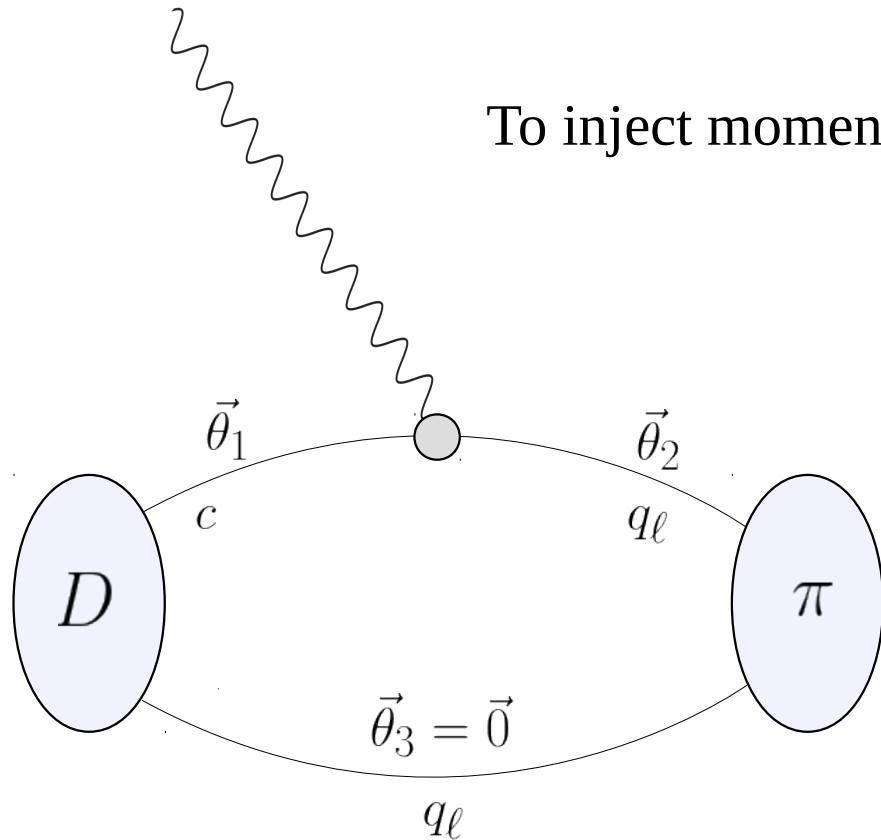
$$\begin{aligned} V_\mu &= \bar{c} \gamma_\mu d \\ S &= \bar{c} d \end{aligned}$$

An alternative way to determine  $f_0$  is to use the scalar density

$$f_0(q^2) = \frac{\mu_c - \mu_q}{M_D^2 - M_P^2} \langle P(p_P) | S | D(p_D) \rangle$$

# Simulation Details

To inject momenta we used non-periodic boundary conditions



Both the D and the  $\pi(K)$  mesons can have non-zero momentum

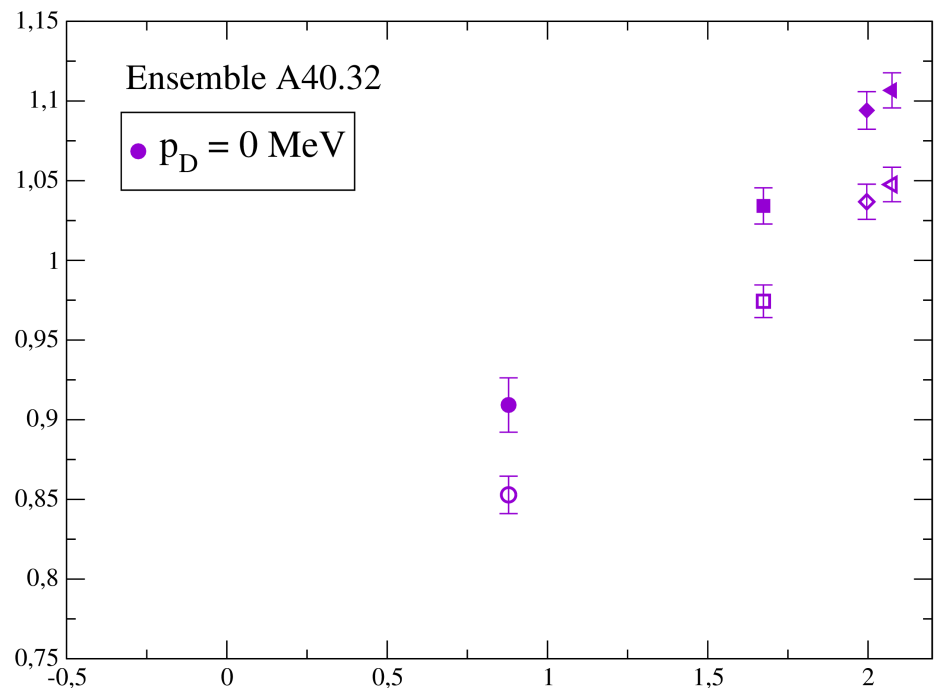
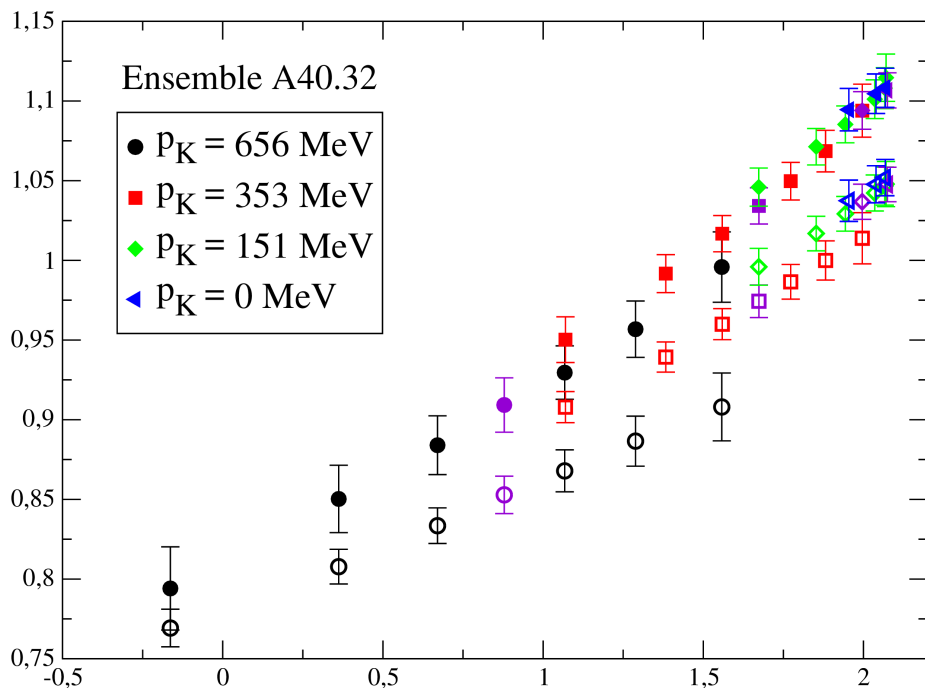
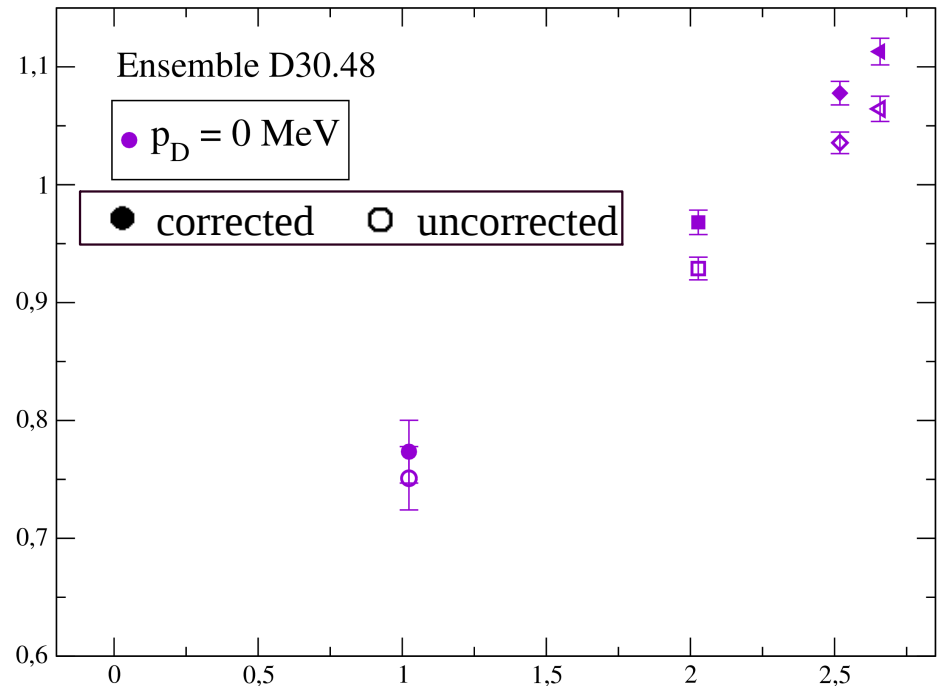
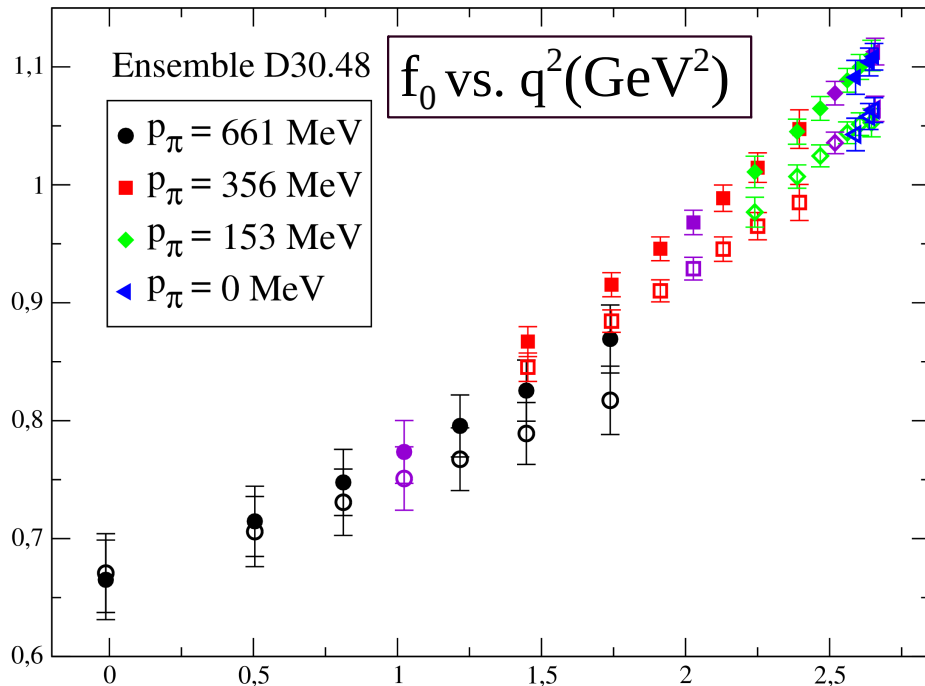
$\beta$	$V/a^4$	$\theta$	
1.90	$32^3 \times 64$	0.0,	$\pm 0.200,$ $\pm 0.467,$ $\pm 0.867$
	$24^3 \times 48$	0.0,	$\pm 0.150,$ $\pm 0.350,$ $\pm 0.650$
1.95	$32^3 \times 64$	0.0,	$\pm 0.183,$ $\pm 0.427,$ $\pm 0.794$
	$24^3 \times 48$	0.0,	$\pm 0.138,$ $\pm 0.321,$ $\pm 0.596$
2.10	$48^3 \times 96$	0.0,	$\pm 0.212,$ $\pm 0.493,$ $\pm 0.916$

$$\vec{p}_D = \frac{2\pi}{L} \vec{\theta}_1$$

$$\vec{p}_\pi = \frac{2\pi}{L} \vec{\theta}_2$$

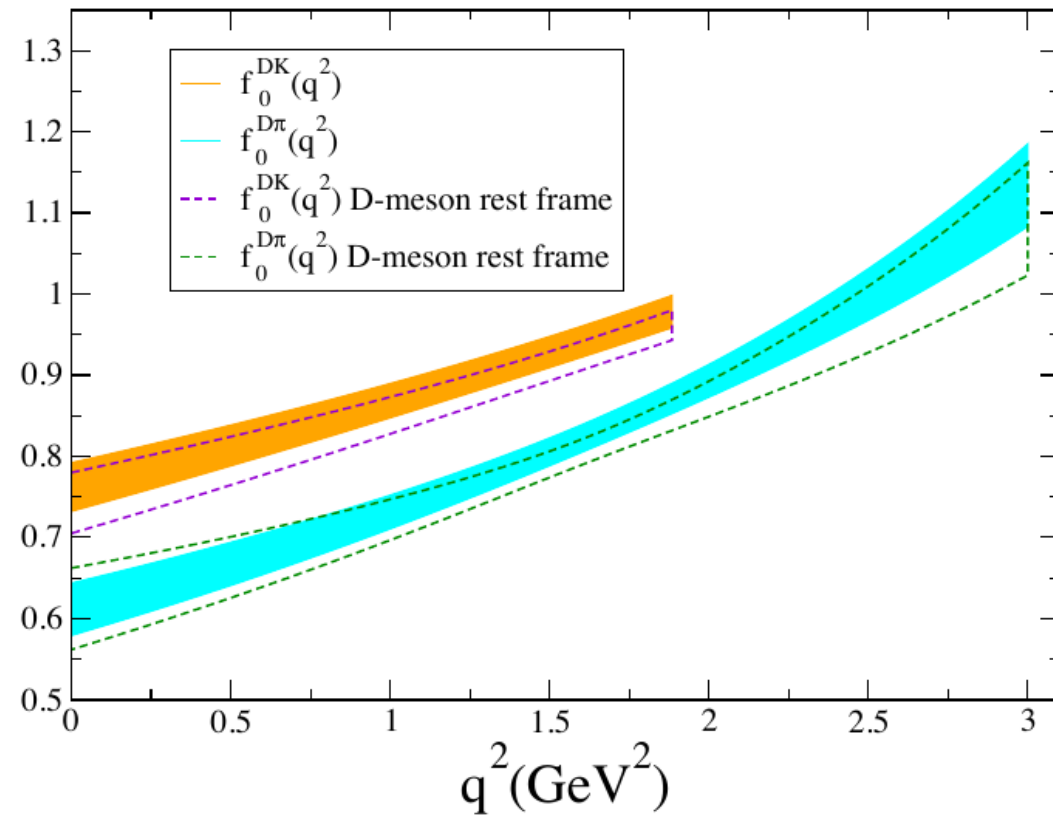
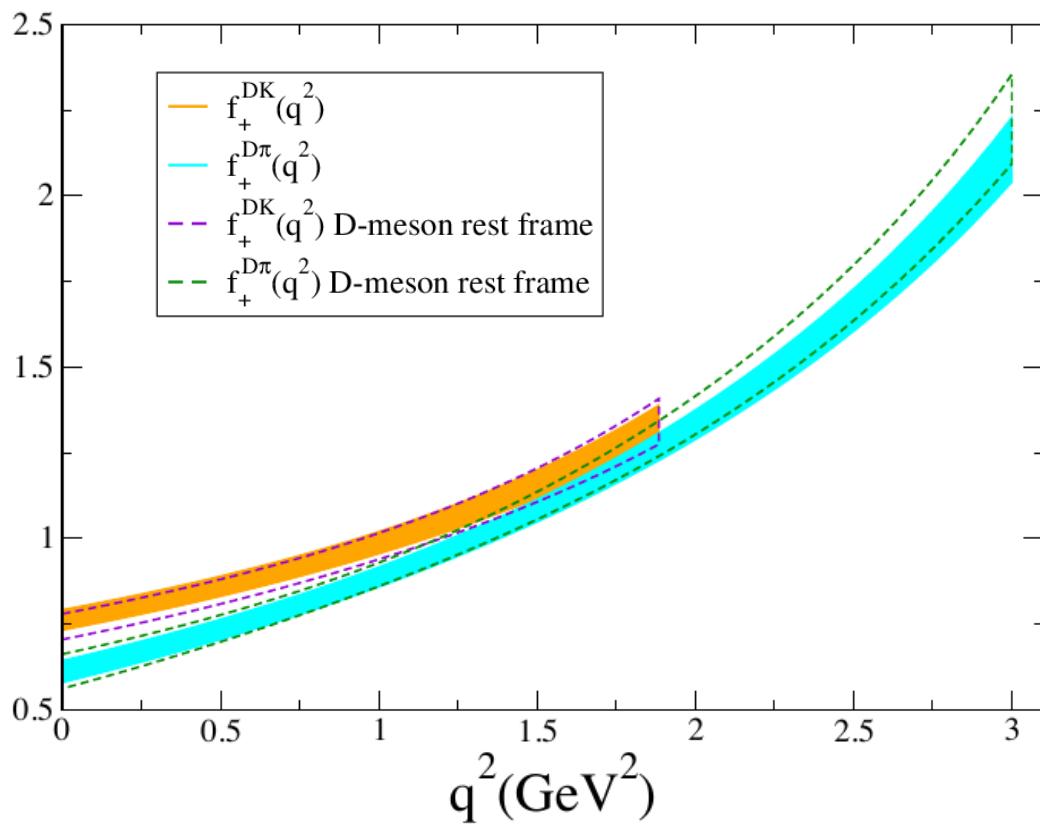
$$\vec{\theta} = \theta(1, 1, 1)$$

# Lorentz Symmetry Breaking



# Results

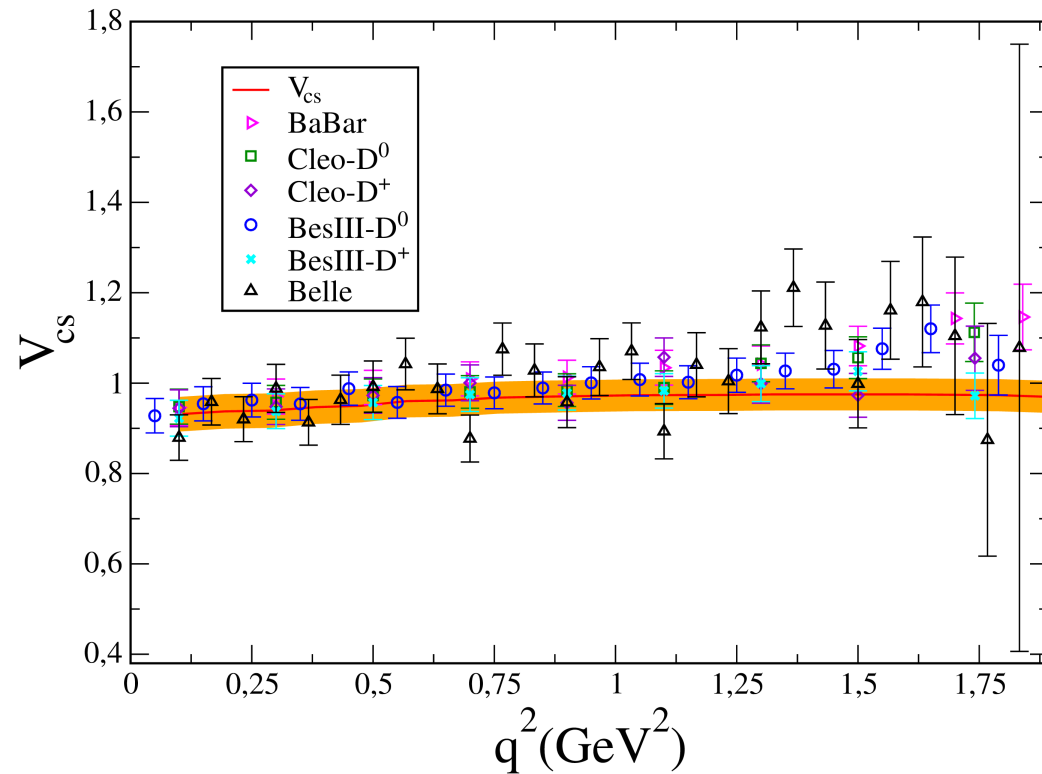
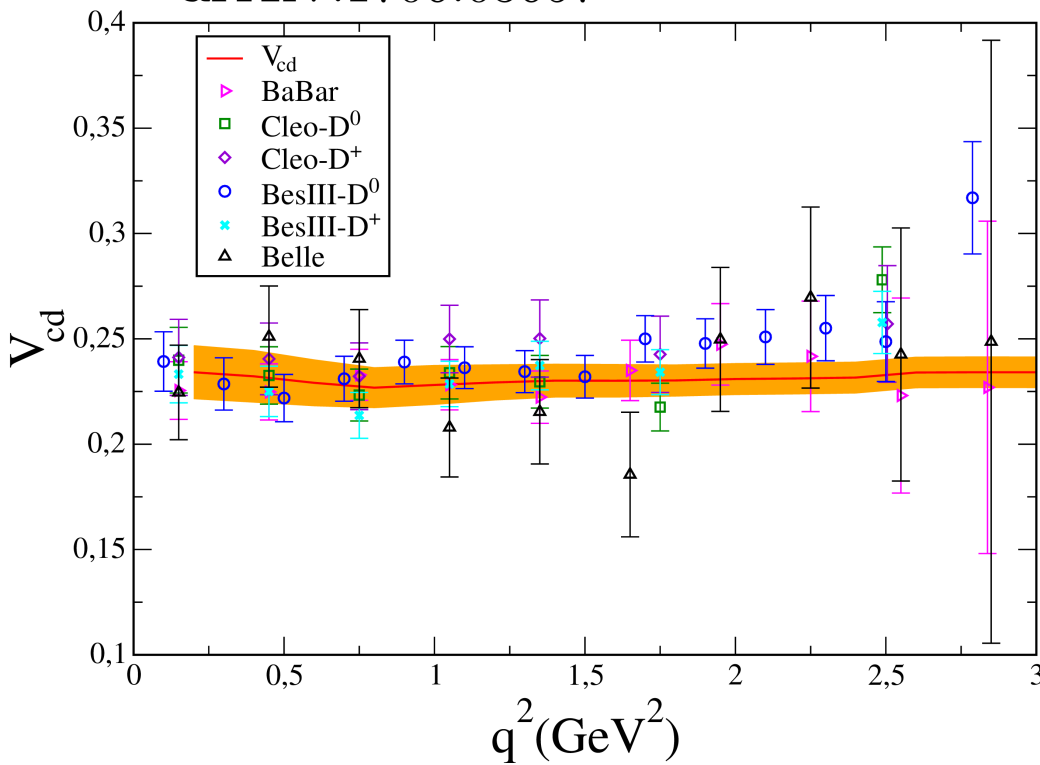
In the continuum limit



# Determination of $V_{cd}$ and $V_{cs}$

$$\frac{d\Gamma(D \rightarrow P\ell\nu)}{dq^2} = \frac{G_F^2 |V_{cx}|^2}{24\pi^3} |\vec{p}_P|^3 |f_+^{DP}(q^2)|^2 \quad \Rightarrow \quad V_{cx}(q_i^2) = \sqrt{\frac{24\pi^3}{G_F^2} \frac{\Delta\Gamma(q_i^2)}{I(q_i^2)}}$$

arXiv:1706.03657



$$q^2 \in [0, q_{\max}^2]$$

$$|V_{cd}| = 0.2341 \text{ (74)} \quad |V_{cs}| = 0.970 \text{ (33)}$$

$$q^2 = 0$$

$$|V_{cd}| = 0.2330 \text{ (137)}$$

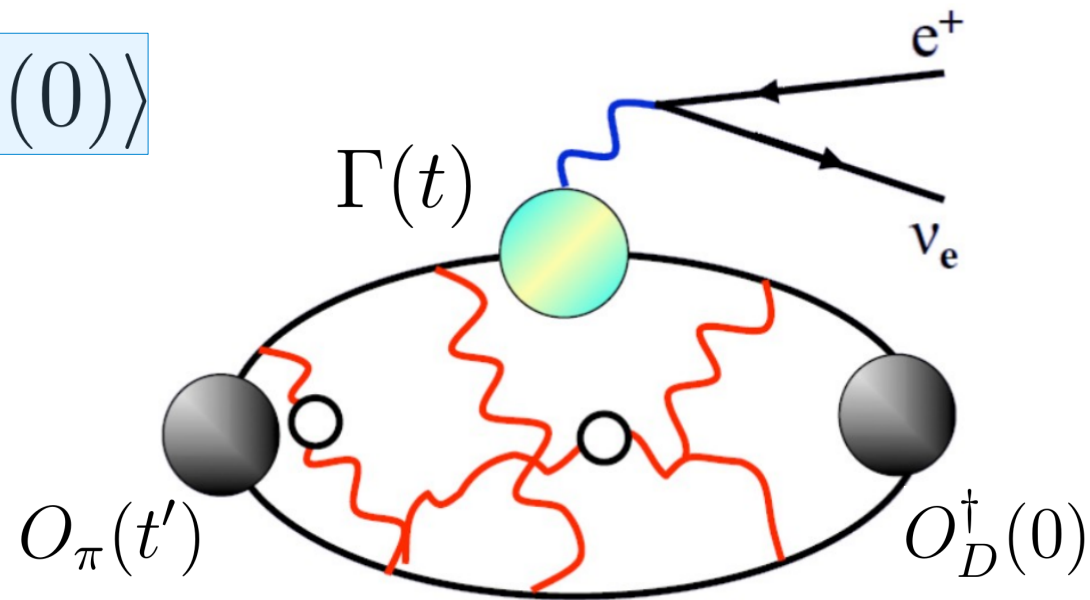
$$|V_{cs}| = 0.945 \text{ (38)}$$

# 3-points correlators

$$C_{\Gamma}^{DP}(t) \equiv \langle O_P(t') \Gamma(t) O_D^{\dagger}(0) \rangle$$

large  $t$

↓



$$\frac{\langle 0 | O_{\pi}(0) | \pi(\mathbf{p}_{\pi}) \rangle \langle D(\mathbf{p}_D) | O_D^{\dagger}(0) | 0 \rangle e^{-E_D t} e^{-E_{\pi}(t'-t)}}{(2E_D)(2E_{\pi})} \times \langle \pi(\mathbf{p}_{\pi}) | \Gamma(0) | D(\mathbf{p}_D) \rangle$$

From 2-point  
correlators

Form factors

# Euclidean Correlators

$$C(t) \equiv \langle O_\pi(t) O_\pi^\dagger(0) \rangle \approx \sum_n \frac{|\langle 0 | O_\pi(0) | n \rangle|^2}{2E_n} \boxed{\exp[-E_n t]} \xrightarrow{t \rightarrow \infty} \frac{|\langle 0 | O_\pi(0) | \pi \rangle|^2}{2M_\pi} \boxed{\exp[-M_\pi t]}$$



Calculated on the lattice (Importance sampling Monte Carlo)

$$\frac{1}{Z} \int \mathcal{D}[\phi] O_1(\phi) O_2(\phi) e^{-S_E[\phi]} \simeq \frac{1}{N_c} \sum_{c=1}^{N_c} O_1(\phi_c) O_2(\phi_c)$$

$$(\Delta O)^2 = \frac{1}{N_c} \sum_{c=1}^{N_c} (O(\phi_c) - \langle O \rangle)^2 \quad \Rightarrow \quad \text{Statistical error} \sim \frac{1}{\sqrt{N_c}}$$

# Quenching

$$\int \mathcal{D}[U] \mathcal{D}[\psi, \bar{\psi}] e^{-S_g[U] - \bar{\psi} M \psi} = \int \mathcal{D}[U] \det M e^{-S_g[U]}$$

Gaussian integral for anticommutative (Grassmann) variables

$\det M$  encodes the contribution of sea quark loops



Quenched



Unquenched

# Modified z-expansion

$$f_+^{D \rightarrow \pi}(q^2) = \frac{f^{D \rightarrow \pi}(0, a^2) + c_+(a^2) (z - z_0) \left(1 + \frac{z+z_0}{2}\right)}{1 - \frac{q^2}{M_V^2}}$$

$$f_0^{D \rightarrow \pi}(q^2) = \frac{f^{D \rightarrow \pi}(0, a^2) + c_0(a^2) (z - z_0) \left(1 + \frac{z+z_0}{2}\right)}{1 - \frac{q^2}{M_S^2} K_{FSE}^0(L)}$$

$$t_+ = (M_D + M_P)^2$$

$$t_0 = (M_D + M_P)(\sqrt{M_D} - \sqrt{M_P})^2$$

$$z = \frac{\sqrt{t_+ - q^2} - \sqrt{t_+ - t_0}}{\sqrt{t_+ - q^2} + \sqrt{t_+ - t_0}}$$

$$z_0 = z(q^2 = 0)$$

$$f_+^{D \rightarrow K}(q^2) = \frac{f^{D \rightarrow K}(0, a^2) + c_+(a^2) (z - z_0) \left(1 + \frac{z+z_0}{2}\right)}{1 - \frac{q^2}{M_{D_s^*}^2} (1 + P_+ a^2)}$$

$$f_0^{D \rightarrow K}(q^2) = f^{D \rightarrow K}(0, a^2) + c_0(a^2) (z - z_0) \left(1 + \frac{z+z_0}{2}\right)$$

$c_+(a^2)$  and  $c_0(a^2)$  have a polynomial dependence on  $a^2$

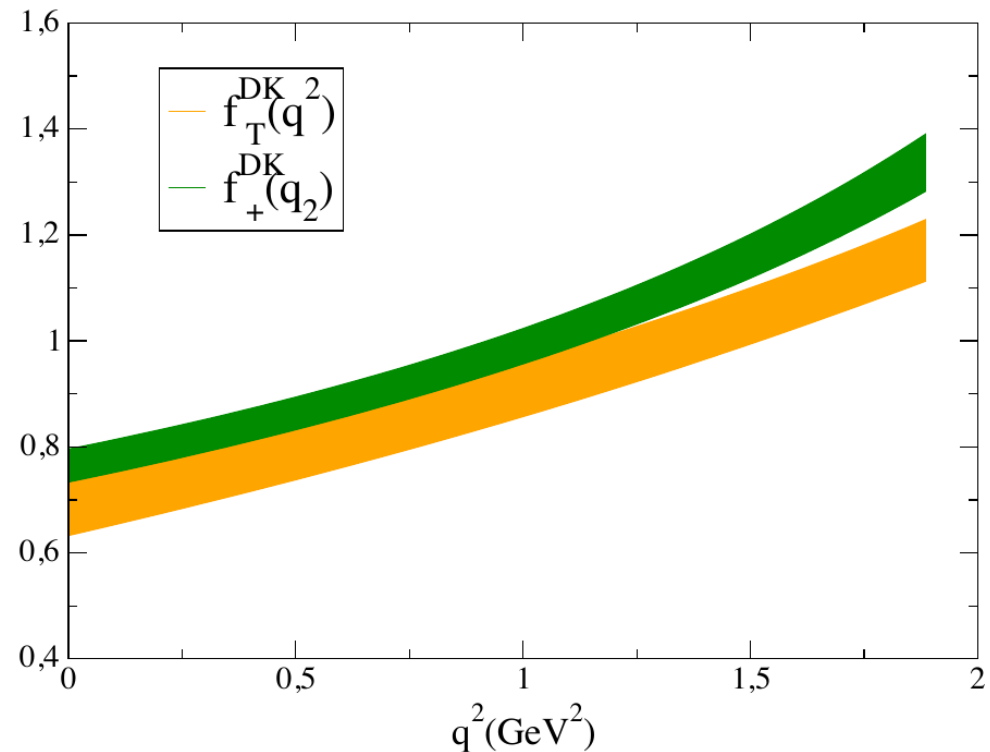
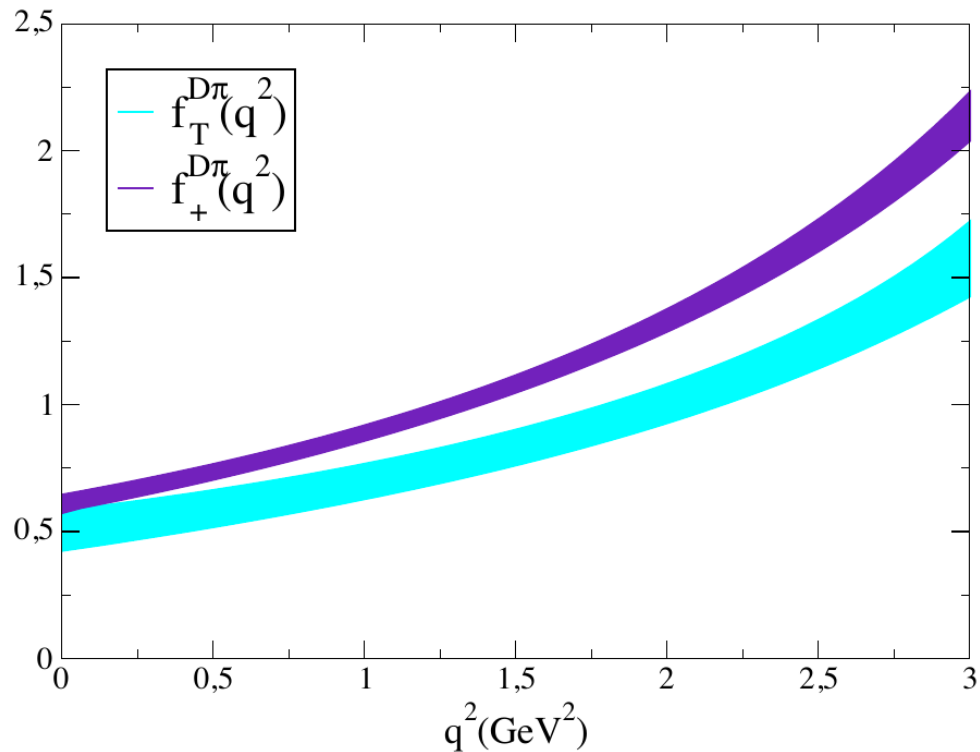
$M_V$  and  $M_S$  are parameters left free to move in the fit

$$f_+(0, a^2) = f_0(0, a^2) \equiv f(0, a^2)$$

# ETMC – Tensor form factor

$f_T(q^2)$  at the physical point

ETMC



$$f_T^{D\pi}(0) = 0.506 \text{ (79)}$$

$$f_T^{DK}(0) = 0.687 \text{ (54)}$$