Charmonia at nonzero temperature from high precision lattice QCD computations

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in collaboration with

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Outline

- Motivation
- Spectral functions of charmonia
  - Dissociation temperature
  - Heavy quark diffusion coefficient
- Screening masses of charmonia
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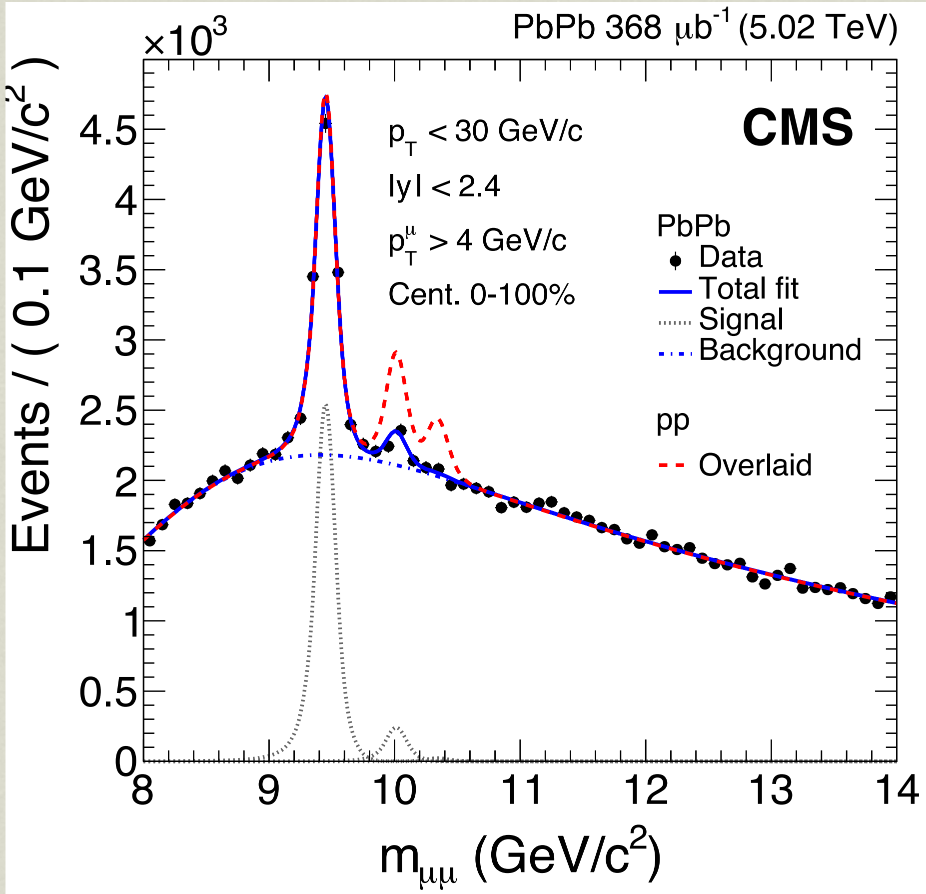
  - Dispersion relation
- Summary

Motivation

- Hadron spectral functions

—Carry all information about the in-medium properties of quarkonia

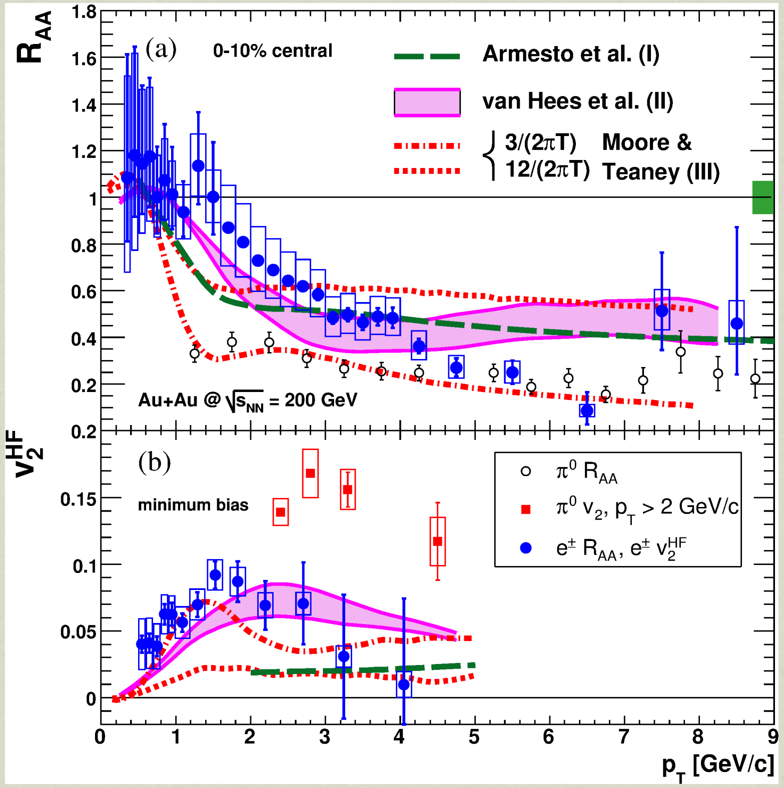
- Dissociation temperatures
- Thermal modifications when moving



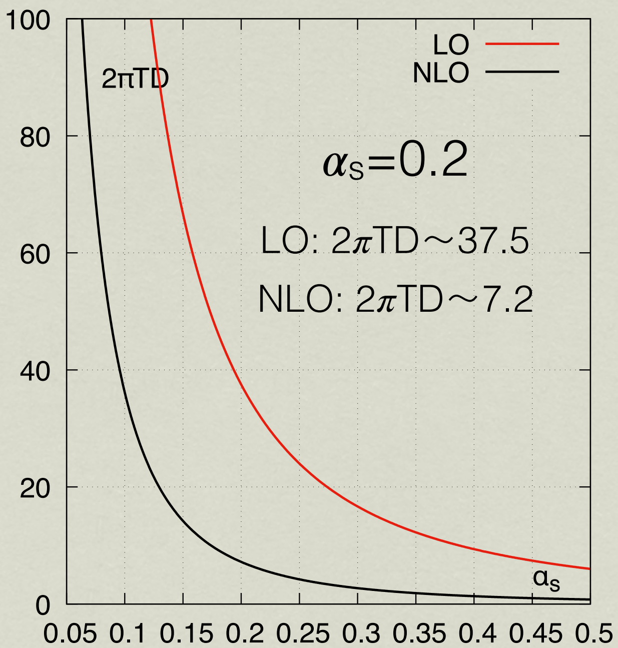
CMS, PRL120(2018) 142301

Heavy quark diffusion coefficient:

$$D = \frac{1}{6\chi_{00}} \lim_{\omega\to 0} \sum_{i=1}^{3} \frac{\rho_{ii}^{V}(\omega,\mathbf{0})}{\omega}$$



PHENIX, PRL98(2007)172301



Moore & Teaney, PRC71,064904

Caron-Huot & Moore, PRL100,052301

Spectral functions from LQCD

Temporal correlation function relates to spectral function:

discretized ~$O(10)$

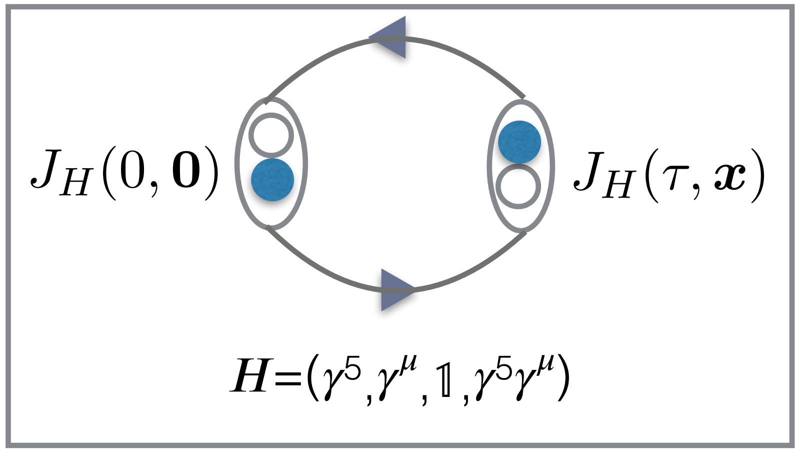
continuos ~$O(1000)$

$$K(\omega,\tau,T) \equiv \frac{\cosh\left(\omega(\tau T - 1/2)\right)}{\sinh(\omega/2T)}$$

$$G_H(\tau,\vec{p}) = \sum_{x,y,z} \exp(-i\,\vec{p}\cdot\vec{x})\langle J_H(0,\vec{0})J_H^{\dagger}(\tau,\vec{x})\rangle = \int \frac{d\omega}{2\pi} K(\omega,\tau,T)\rho_H(\omega,\vec{p},T)$$

ill-posed!

- ✳ New Bayesian Method Y. Burnier and A. Rothkopf, PRL 111,18,182003
- ✳ Backus-Gilbert Method B. B. Brandt, et al., PRD93, 054510(2016)
- ✳ Stochastic Approaches H.-T. Ding, et al., PRD97, 094503
- ✳ **Maximum Entropy Method** M. Asakawa, et al., PPNP. 46(2001) 445-508



Inversion Methods: Maximum Entropy Method

A method based on Bayesian theorem to obtain the **most probable** solution

- Maximize the probability of having ρ given $\boldsymbol{G}$&$\boldsymbol{D}$:

$$P[\rho|G,D,\alpha] = \frac{P[G|\rho,D,\alpha]\ P[\rho|D,\alpha]}{P[G|D,\alpha]}$$

ρ : spectral function
G : correlation function
D : default model

- Ingredients: $P[G|\rho,D,\alpha] \propto \exp(-\chi^2/2)$: likelihood function

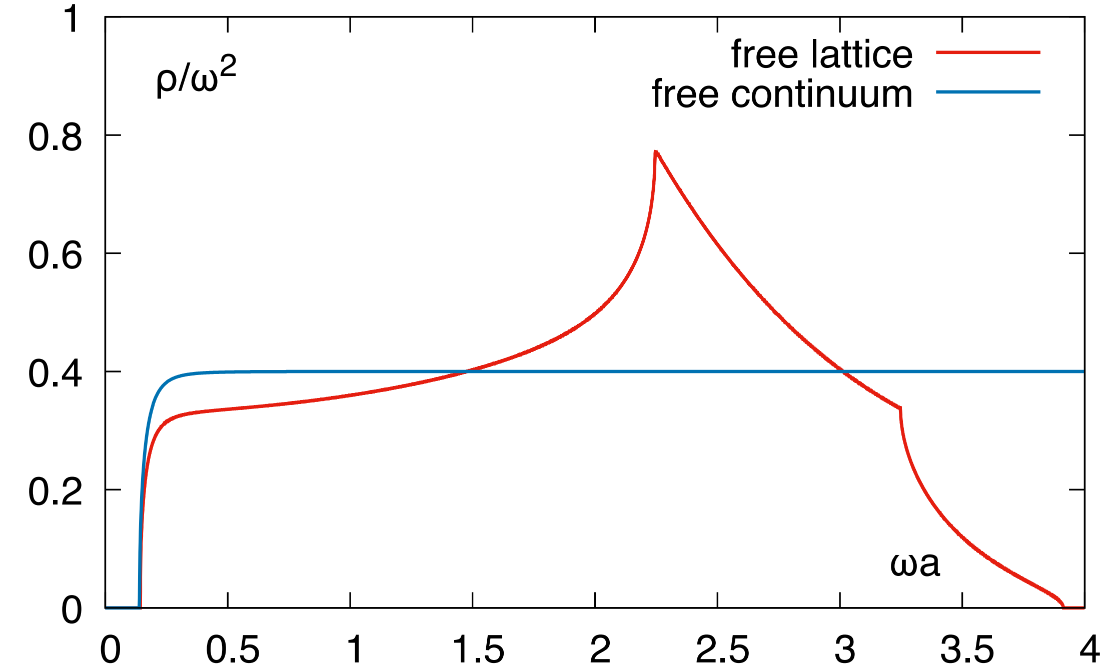
$P[\rho|D,\alpha] \propto \exp(\alpha S)$: prior probability

Shannon-Jaynes entropy: $S[\rho] = \int \mathrm{d}\omega[\rho(\omega) - D(\omega) - \rho(\omega)\ln(\frac{\rho(\omega)}{D(\omega)})]$

- Check the dependence on N_τ & default model (DM)

Prior information in the default model

- High frequency of the SPF :
  - ✶Free continuum SPF
  - ✶Free lattice SPF H.-T. Ding, et al, arXiv:0910.3098



- Low frequency of the SPF:
  - ✶Non-interacting: F. Karsch et al., PRD68, 014504; G. Aarts et al, NPB726, 93

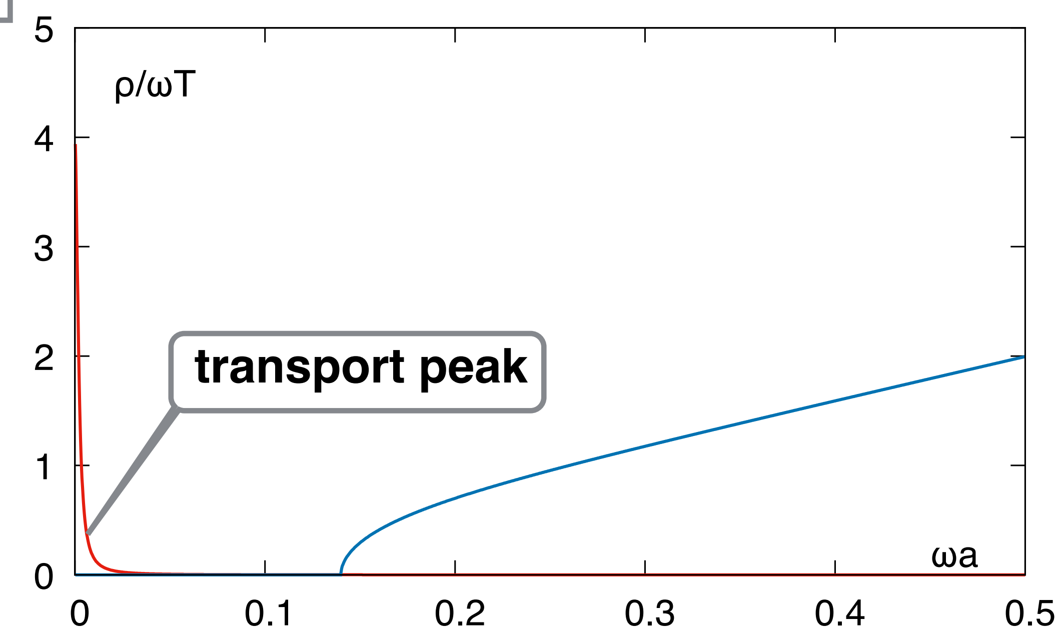
$$\rho_H(\omega) = N_c\big[(a_H^{(1)} + a_H^{(3)})I_1 + (a_H^{(2)} + a_H^{(3)})I_2\big]\omega\delta(\omega) \Longrightarrow D = \frac{1}{6\chi_{00}} \lim_{\omega\to 0}\sum_{i=1}^{3}\frac{\rho_{ii}^V(\omega,\mathbf{0})}{\omega} = \infty$$

$\omega\delta(\omega)$ gives infinite quark diffusion coefficient

  - ✶Interacting: P. Petreczky and D. Teaney, PRD73,014508

$$\delta(\omega) \to \frac{1}{\pi}\frac{\eta}{\omega^2+\eta^2} \Longrightarrow D \propto 1/\eta$$

$\delta(\omega)$ is smeared into Breit-Wigner form at ω~0

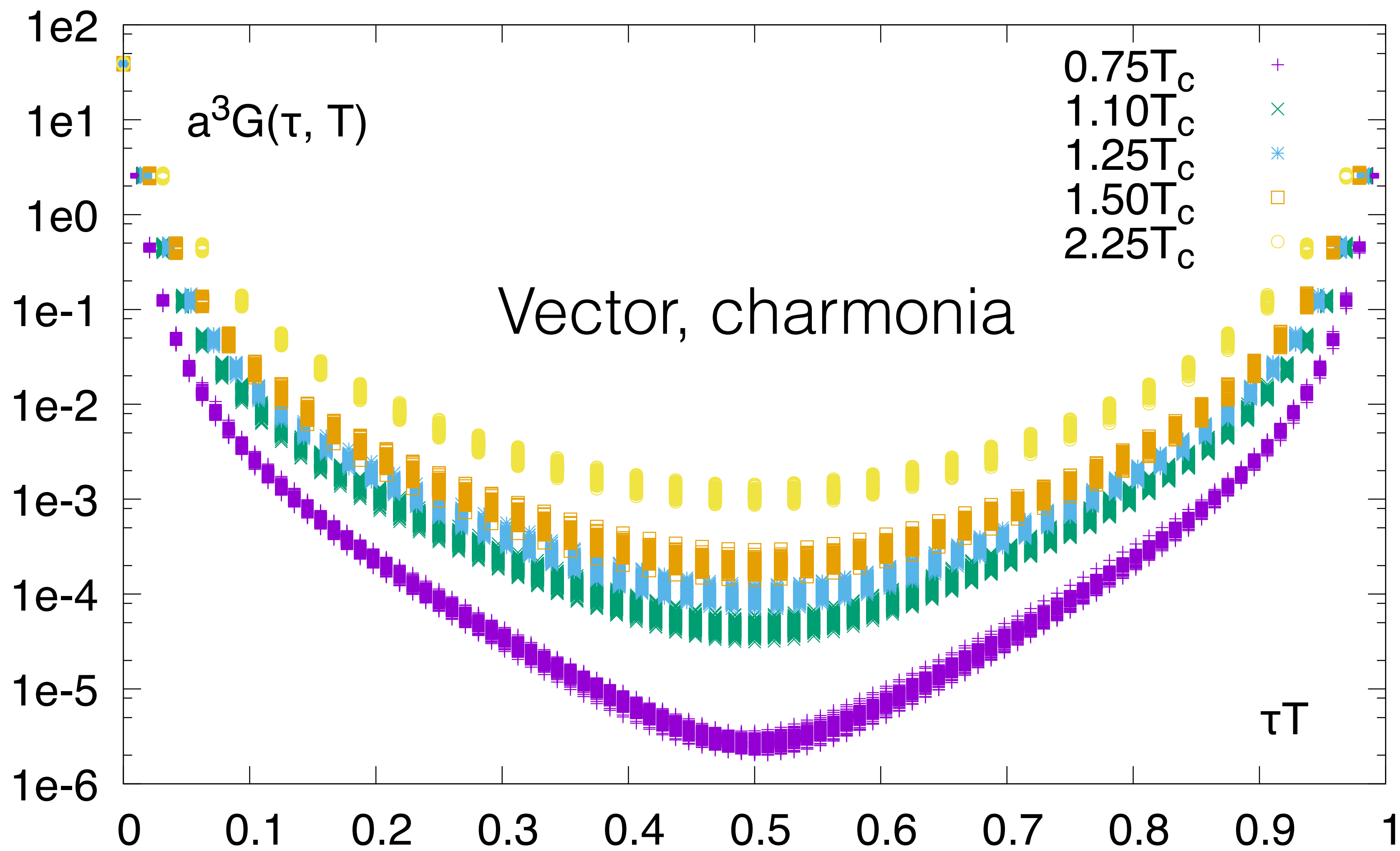


Simulation details

- Large quenched QCD on isotropic lattices close to continuum
- Relativistic treatment of heavy quarks (aM_Q<<1)
- M_Q tuned to reproduce nearly physical J/ψ mass
- Relative error: ~0.3%

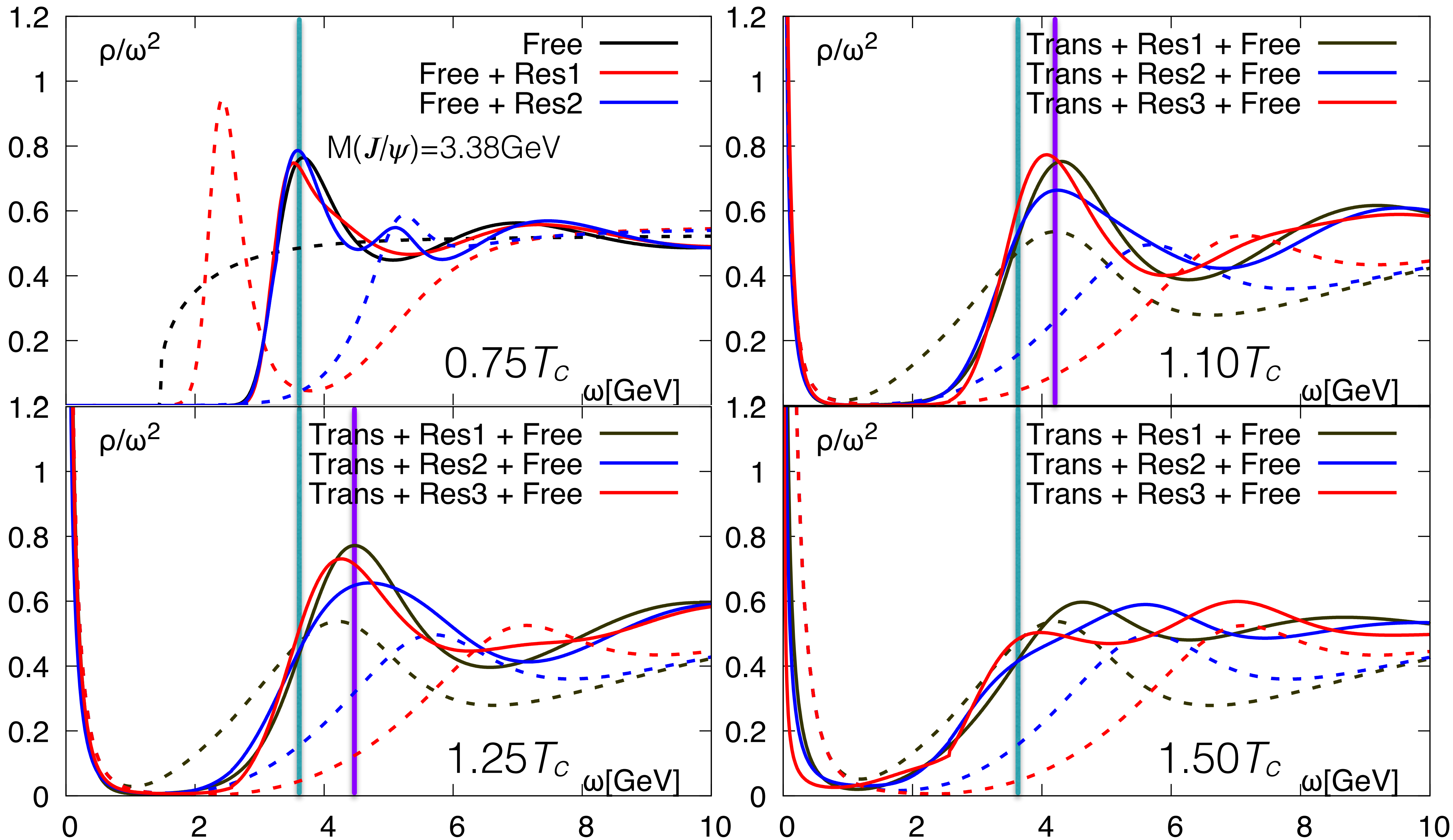
β	a[fm]	M_V [GeV]	N_σ	N_τ	T/T_c	$\delta G/G$
7.793	0.009fm	3.38(J/ψ)	192	96	0.75	0.32%
				64	1.10	0.21%
				56	1.20	0.23%
				48	1.50	0.17%
				32	2.25	0.39%

Temporal correlation functions



- Symmetric around the middle point
- Almost exponential decaying
- Clear temperature dependence

DM dep: charmonia SPFs in VC channel at all T

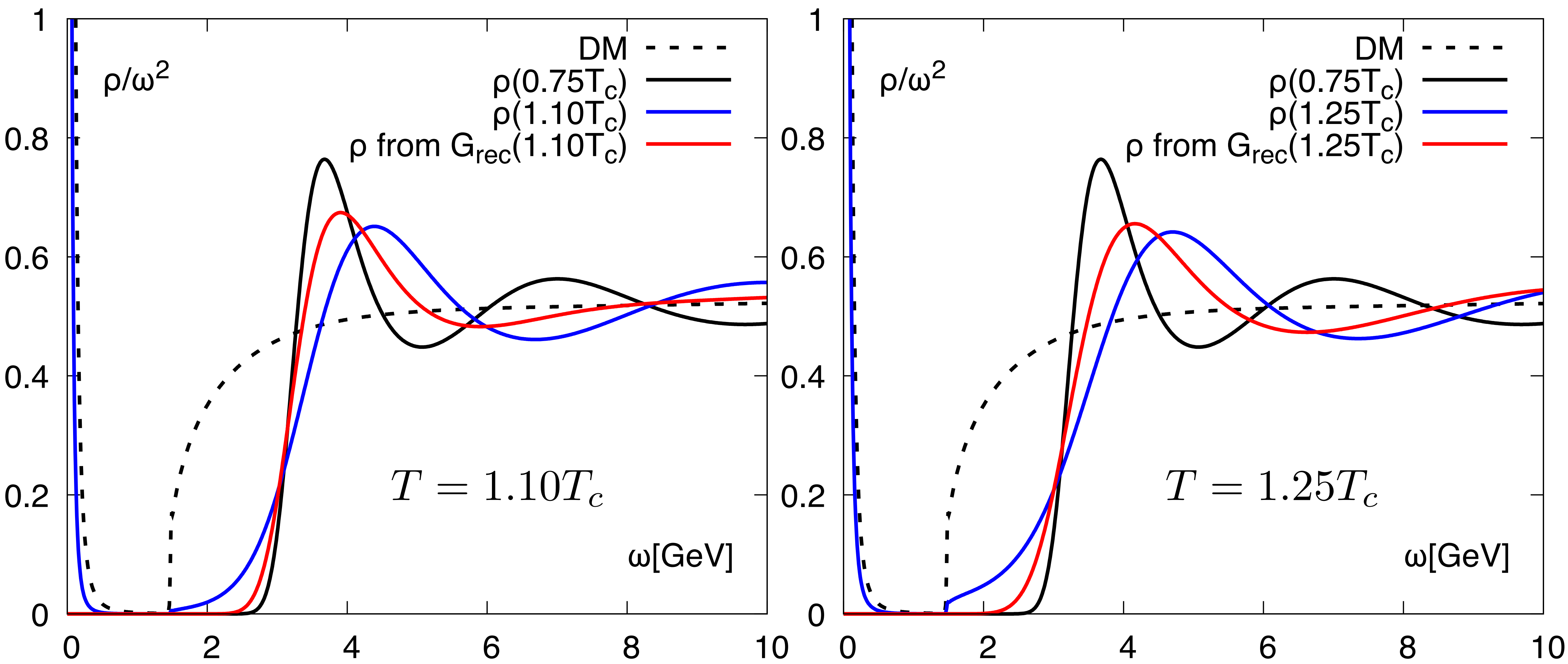


- First low-lying peaks are stable at 0.75, 1.10&1.25 T_c
- A shift of the first peak location(~600MeV) is observed at 1.10 &1.25 T_c
- SPFs become flat at 1.50 T_c

$N\tau$ dep: charmonia SPFs in the VC channel at $T>T_c$

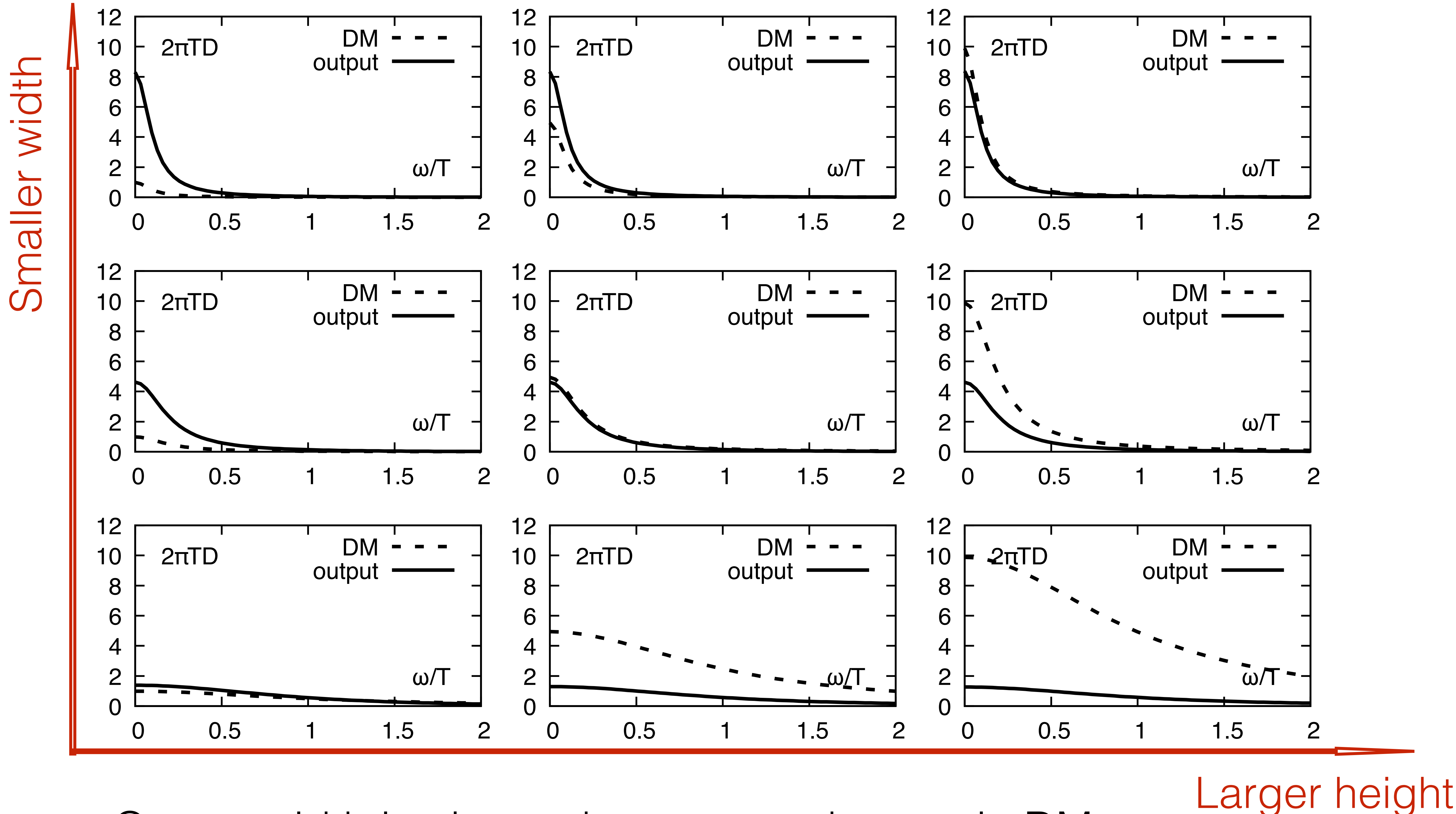
Reconstructed correlation functions:

$$G_{rec}(\tau, T; T') = \int d\omega\ \rho(\omega, T') K(\omega, \tau, T) \quad T' = 0.75T_c$$



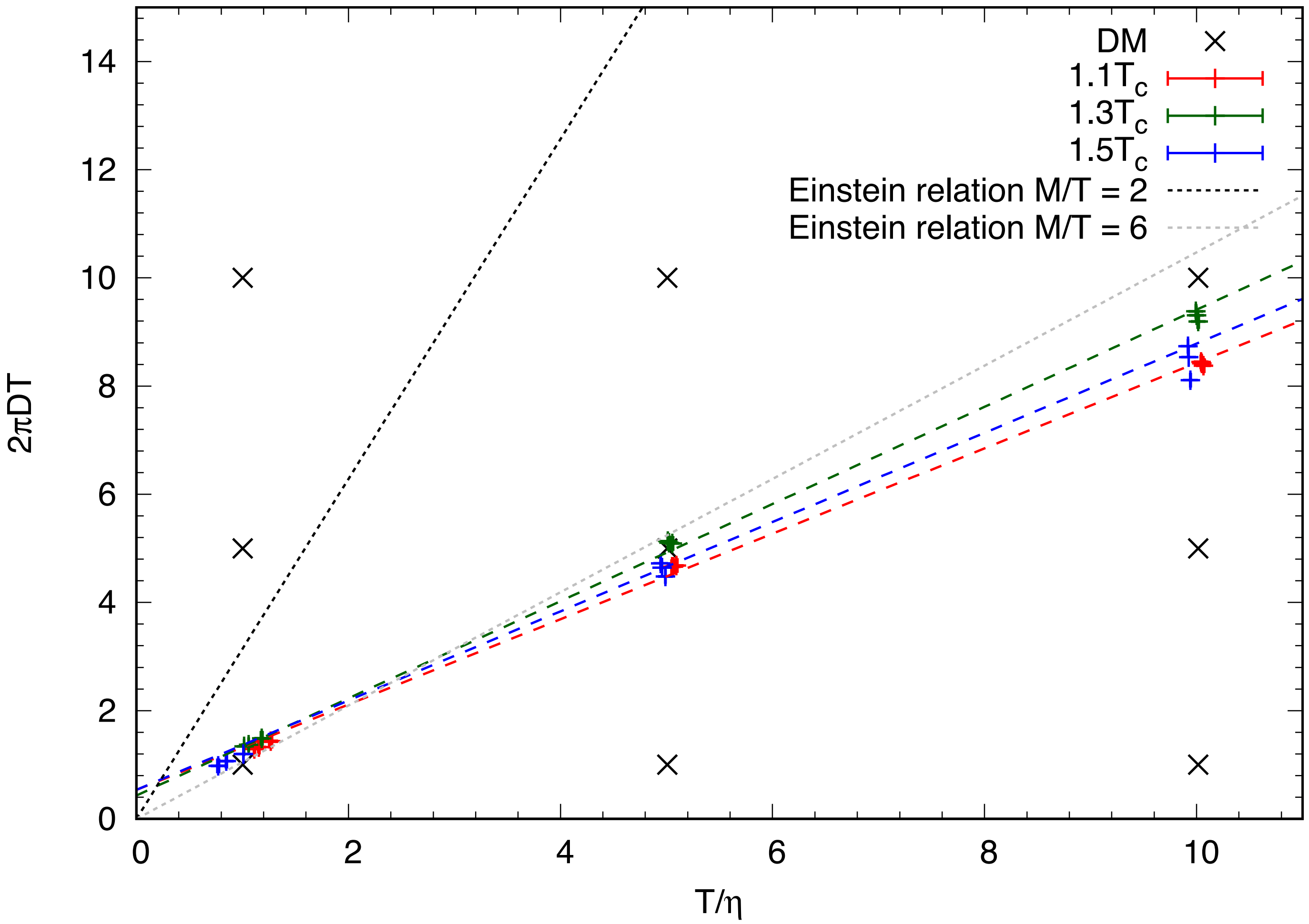
- After getting rid of $N\tau$ dependence the shift of first peak location becomes smaller at $1.10T_c$ & $1.25T_c$

Charm transport peak at 1.10T_c



- Output width is almost the same to the one in DM
- Output $2\pi TD$ varies as the width in DM changes

Charm quark diffusion coefficient at different T



Breit-Wigner ansatz for the transport peak in DM leads to fixed $2\pi TD \cdot \eta/T$:

$$D = \frac{1}{6\chi_{00}} \lim_{\omega\to 0} c \, \frac{\eta}{\omega^2+\eta^2} \Longrightarrow c \propto D\eta$$

$$G(N_\tau/2) = \int \frac{d\omega}{2\pi} \, c \, \frac{\omega\eta}{\omega^2+\eta^2} \frac{1}{\sinh(\omega/2T)}$$

$$\approx \frac{c}{2N_\tau} (\omega \ll T)$$

MEM can only determine the coefficient c or $D\eta$

→ The diffusion coefficient can be determined once η/T is fixed

$2\pi TD$ = 0.789(11) T/η + 0.53(8) at 1.10 T_c

$2\pi TD$ = 0.898(8) T/η + 0.43(2) at 1.25 T_c

$2\pi TD$ = 0.825(9) T/η + 0.54(7) at 1.50 T_c

Spatial correlation function & Screening mass

- Spatial correlation function: sum over space (x,y,τ)

$$G_H(z,\boldsymbol{p}_\perp,\omega_n) = \sum_{x,y,\tau} \exp(-i\ \tilde{\boldsymbol{p}}\cdot\tilde{\boldsymbol{x}})\langle J_H(0,\mathbf{0})J_H^\dagger(\tau,\boldsymbol{x})\rangle$$

- Relation to the spectral function

$$G_H(z,\boldsymbol{p}_\perp,\omega_n) = \int_{-\infty}^{\infty}\frac{\mathrm{d}p_z}{2\pi}\exp(ip_z z)\int_0^\infty \frac{\mathrm{d}\omega}{\pi}\rho_H(\omega,\boldsymbol{p},T)\frac{\omega}{\omega^2+\omega_n^2}$$

- Long distance behavior: exponential decay in *z*

$$G_H(z,\boldsymbol{p}_\perp,\omega_n) \sim \exp(-zE_{scr}) \qquad E_{scr}{}^2 = \vec{p}^2 + M^2 + \Pi(\vec{p},T)$$

at p=0: E_{scr}=screening mass
at p=0,T=0: E_{scr}=pole mass

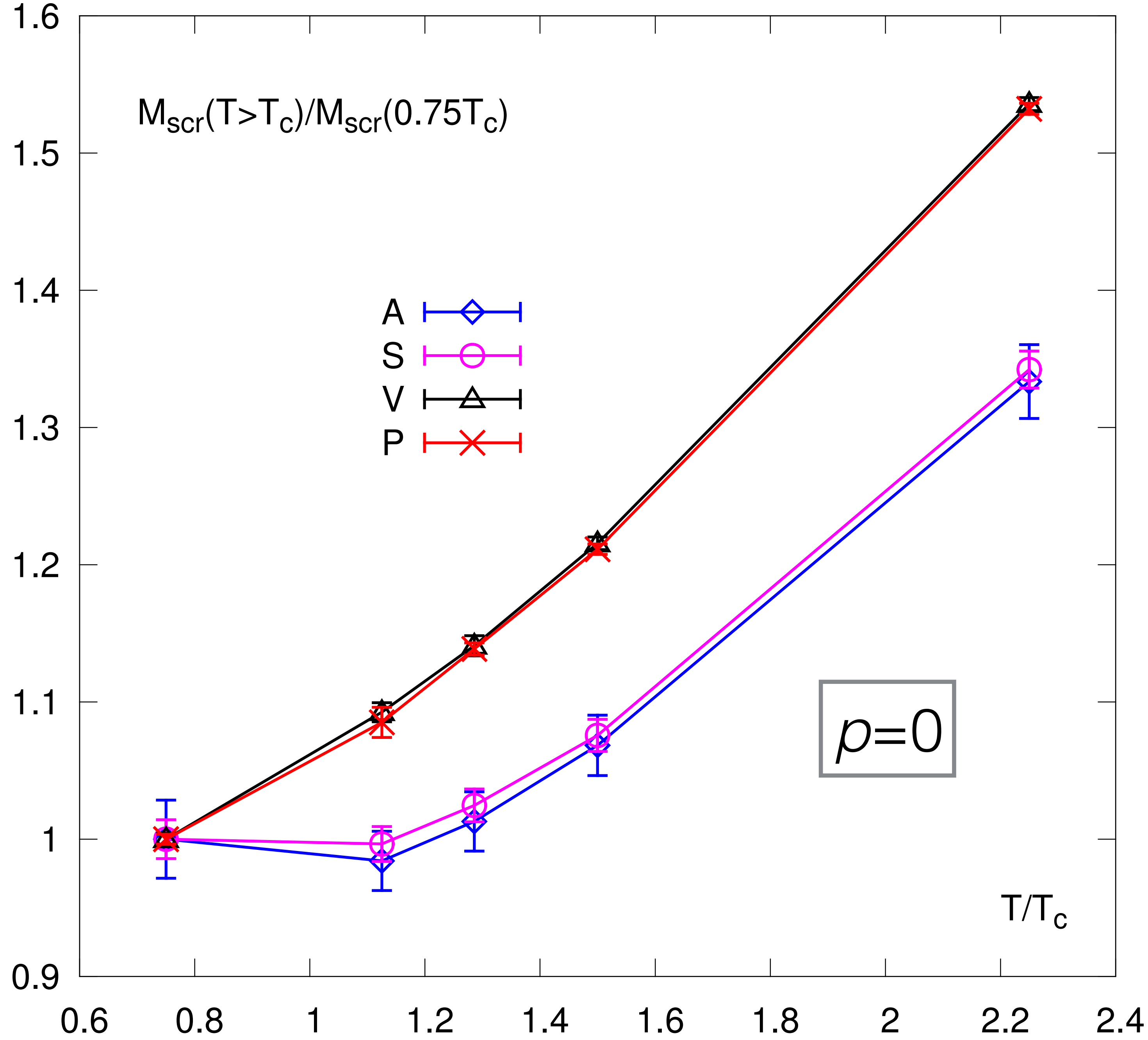
Absorb thermal effects into *M(T)* and *A(T)*

$$E_{scr}{}^2 = A(T)\vec{p}^2 + M^2(T)$$

- Non-interacting limit

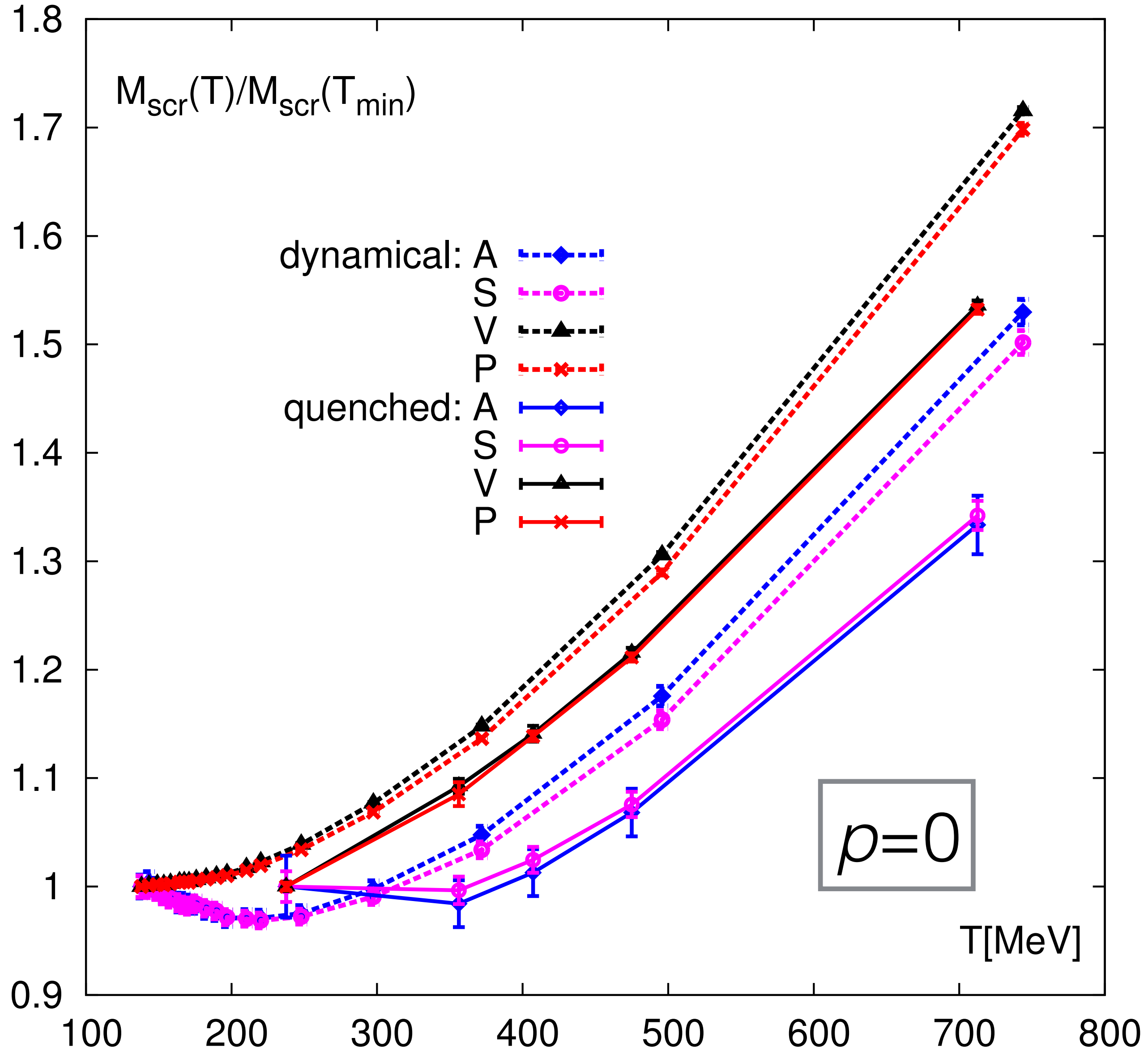
$$E_{free} = 2\sqrt{(\pi T)^2 + m_q^2}$$

Screening masses of charmonia



- M_{scr} of S-wave states increases in *T*
- M_{scr} of P-wave states decrease first in *T* and then increase

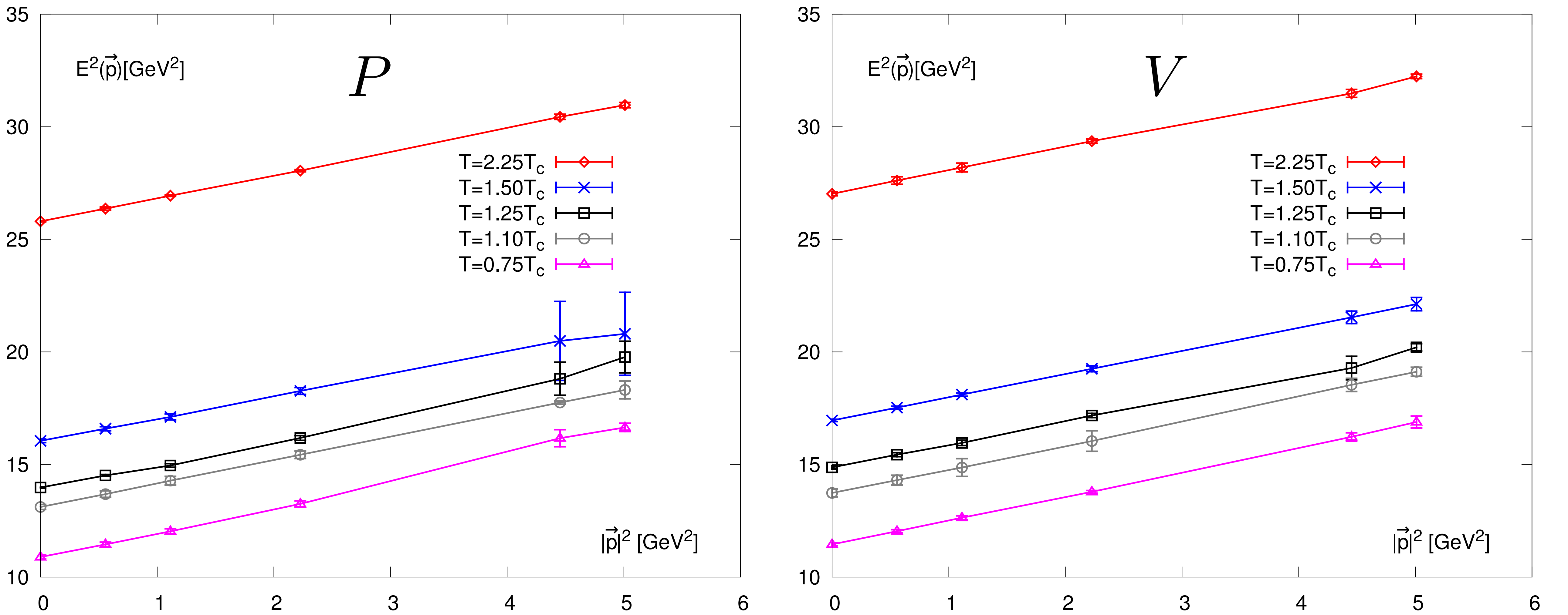
Screening mass—Quenched v.s. Dynamic QCD



Nf=2+1, HISQ : A. Bazavov, et al.,PRD91,054503

- Quenched and 2+1 HISQ : similar T-dependence
- Different dip location for p-waves: 1.10 T_c in quenched QCD, 1.43 T_c in dynamic QCD

Dispersion relation of charmonia



$$E_{scr}^{\;2} = A(T)\vec{p}^{\,2} + M^2(T)$$

- Dispersion relation for S-wave states remains unmodified for charmonia as *A(T)*~1 See also: A. Ikeda, et al., PRD95. 014504
- The reason could be that the largest momentum is still less than the masses of charmonia (~3.5GeV)

Summary

We have performed simulations on large quenched isotropic lattices to calculate the temporal & spatial correlation functions at both zero and nonzero p

- ✴ There exists a shift for the first resonance peak in vector charmonia SPF at $1.10T_c$&$1.25T_c$
- ✴ So far we observed a linear relation between $2\pi TD$ and T/η
- ✴ The screening mass of S-wave states increases in T while for P-wave states they decrease first and then increase
- ✴ Dispersion relation in our quenched simulations seems to be not modified in medium when $p < M_{src}$

Thanks for your attention!

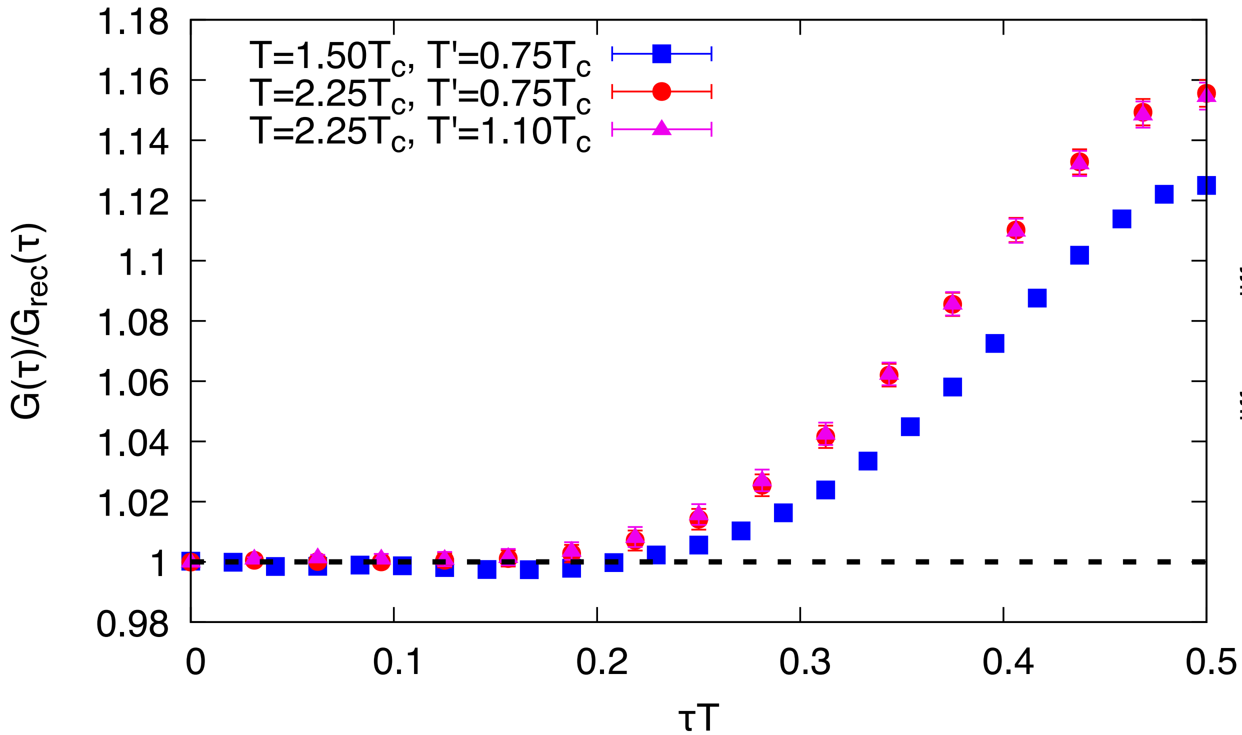
Back-up: Charmonia correlation functions in the VC channel

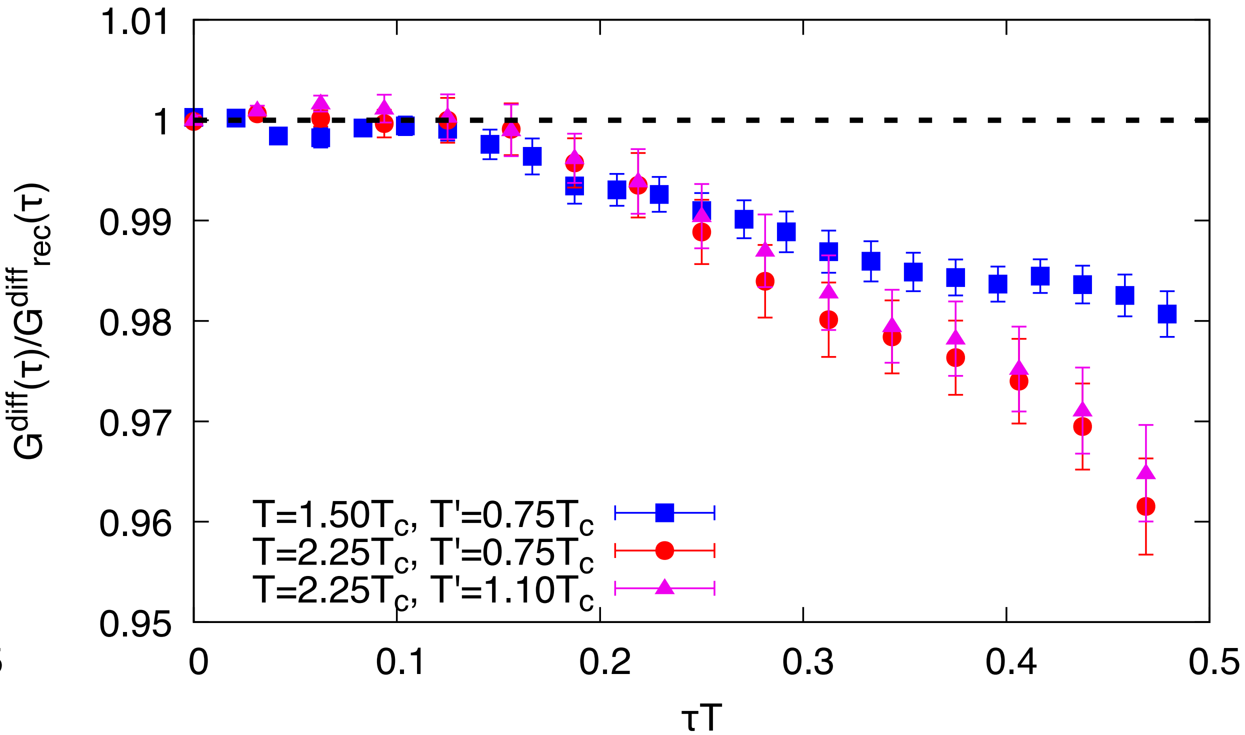
G/G_{rec} cancels out the trivial temperature dependence of $K(\omega,\tau,T)$:

$$\frac{G(\tau,T)}{G_{rec}(\tau,T;T')} = \frac{\int d\omega\ \rho(\omega,T) K(\omega,\tau,T)}{\int d\omega\ \rho(\omega,T') K(\omega,\tau,T)}$$

G_{diff} suppresses τ independent contributions, e.g. $\omega\delta(\omega)$ term in SPF:

$$\frac{G^{\rm diff}(\tau/a)}{G^{\rm diff}_{rec}(\tau/a)} = \frac{G(\tau/a) - G(\tau/a+1)}{G_{rec}(\tau/a) - G_{rec}(\tau/a+1)}$$





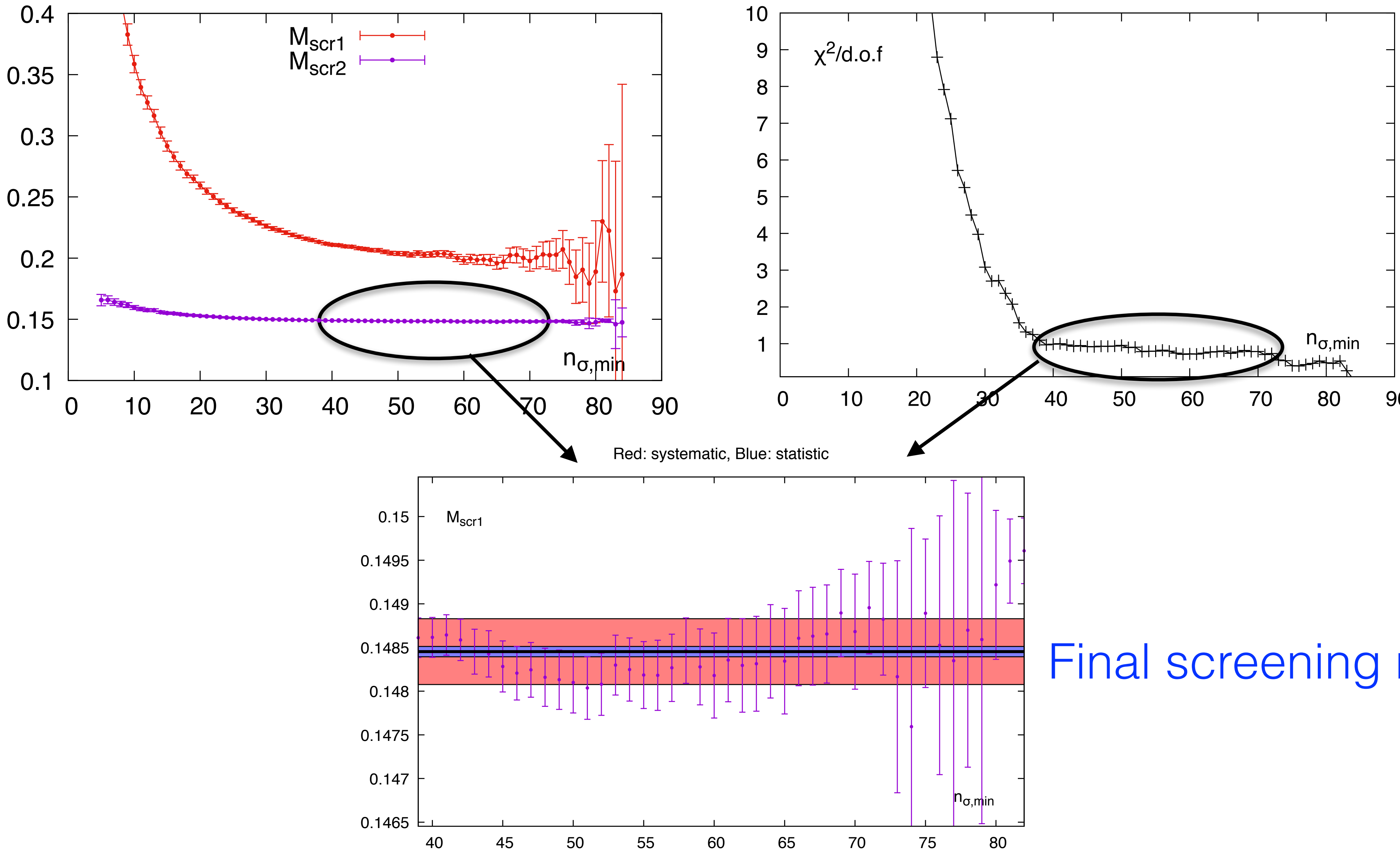
- Thermal modification includes two parts: transport peak & resonance part
- Large thermal modifications are observed to the low-lying states in charmonia correlation functions

Back-up: Fitting the correlation functions

Spatial correlators: $G(n_\sigma) = A_1 \cosh\left(M_{scr1}(n_\sigma - N_\sigma/2)\right) + A_2 \cosh\left(M_{scr2}(n_\sigma - N_\sigma/2)\right) + \cdots$

Two-state ansatz:

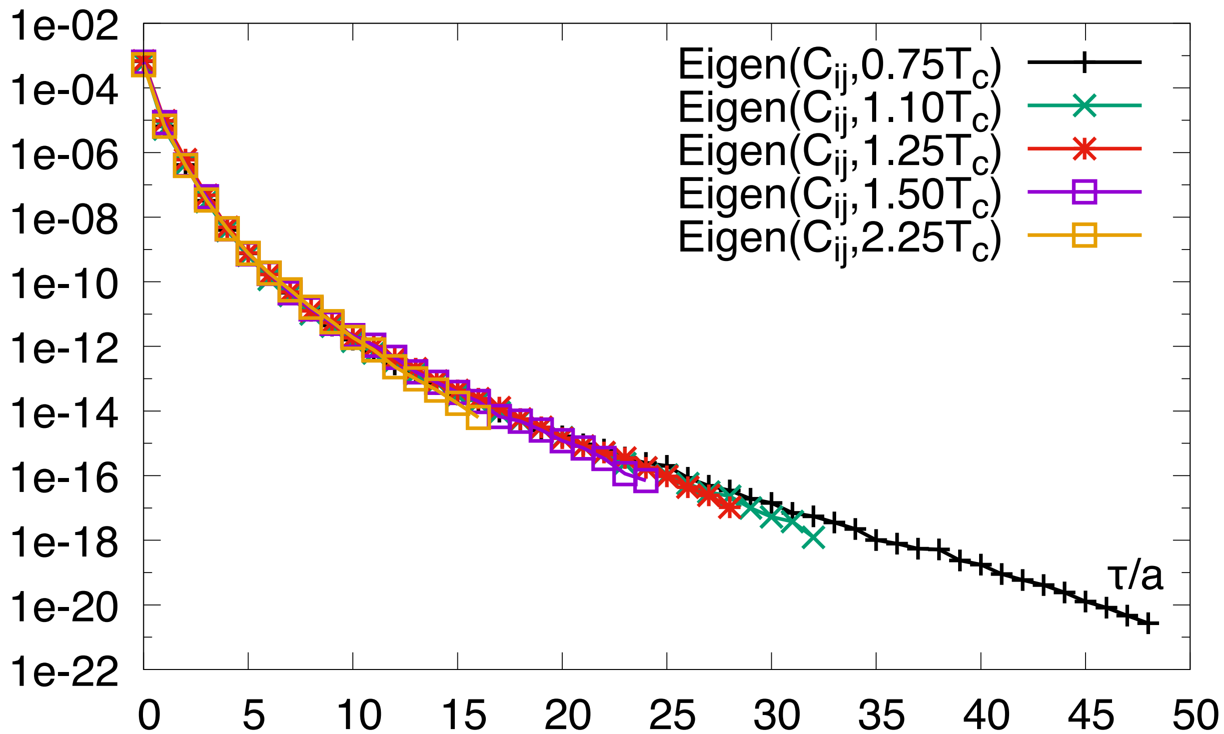
Plateau in χ^2/d.o.f:

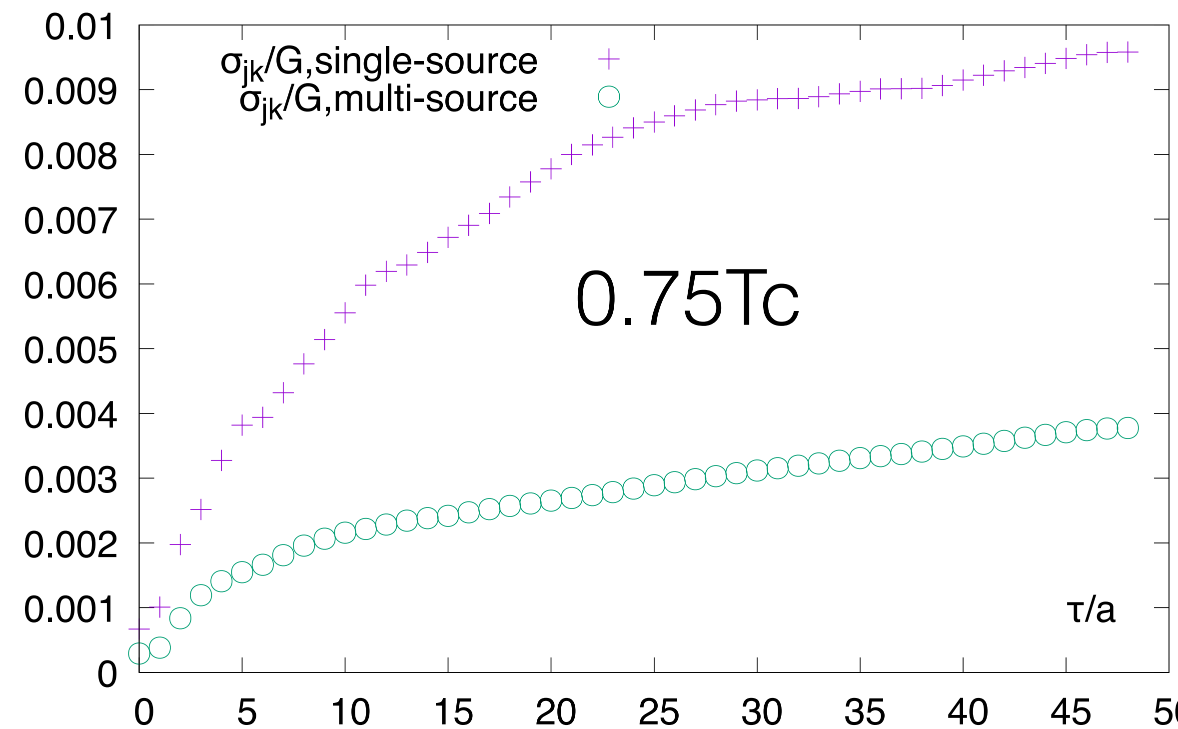


Final screening mass.

Back-up: Data quality examination

Vector, charmonia





F. Meyer et al, PRD94,034504

- Eigen values decrease monotonically with tau for all temperatures.

- $\delta G/\bar{G}$ ~2 times smaller for multi-source than single-source.

✴ No exceptional configurations.
✴ Small enough error.
✴ Good benefits from multi-source.