Heavy hybrids an tetraquarks in EFT

Jaume Tarrús Castellà

Institut de Física d'Altes Energies (UAB)

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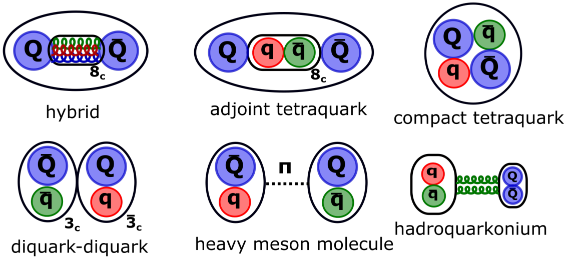
Institut de Física
d'Altes Energies

Exotic Quarkonia

- Exotic Quarkonia are candidates for **non traditional** hadronic states, including **four constituent quark** or an **excited gluon** constituent.
- Many pictures and corresponding models have been proposed...



- However a compelling, unified, understanding of these new states has not yet emerged.
- The objective is to connect the different pictures to QCD through **EFT and/or lattice**.

Scales in exotic quarkonium

Two distinct components

- Heavy quark Q, $\bar{Q}$ pair (heavy d.o.f) and a gluon and light-quark excitation (light d.o.f).

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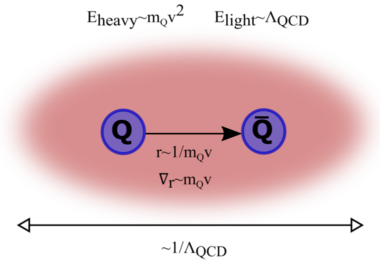


Characteristic Scales

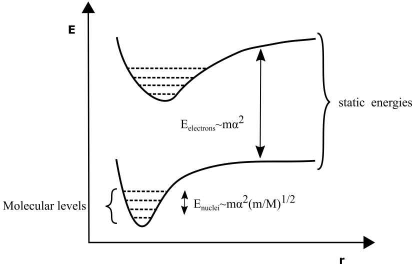
- Heavy-quarks are **non-relativistic** $m_Q \gg \Lambda_{\rm QCD}$.
- Two components with very different dynamical time scales $\Lambda_{\rm QCD} \gg m_Q v^2$.
  - Light d.o.f state $E_{light} \sim \Lambda_{\rm QCD}$.
  - Heavy-quark binding $E_Q \sim m_Q v^2$ ($v \ll 1$ relative velocity).
  - Adiabatic expansion, **Born-Oppenheimer** approximation in atomic physics. L. Griffiths, C. Michael, P. Rakow Phys.Lett.129B (1983); K.. Juge, J. Kuti, C. Morningstar Nucl.Phys.Proc.Suppl.63 (1998); E. Braaten, C. Langmack, D. Smith Phys.Rev.D90 (2014); C. Meyer, E. Swanson Prog.Part.Nucl.Phys.82 (2015)...

QCD analog to diatomic molecules

N. Brambilla, G. Krein, JTC, A. Vairo Phys.Rev.D97 (2018)

Quarkonium hybrids are a similar system to diatomic molecules

- Heavy d.o.f: Nuclei $\rightarrow$ Heavy Quark
- Light d.o.f: Electrons $\rightarrow$ Gluons&Light-quarks



- Separation between the two components energy scales: $E_{electrons} \gg E_{nuclei}$.
- **Static energies**: electronic eigenenergies for static nuclei separated by a distance r.
- Molecular energy levels: vibrations of the nuclei on the static energies minima.

Exotic quarkonium

- Study exotic quarkonium as states living on top of gluonic and light-quark static energies. N. Brambilla, A. Vairo, A. Polosa, J. Soto Nucl.Phys.Proc.Suppl.185 (2008);E. Braaten, C. Langmack, D. Smith Phys.Rev.D90 (2014)

Static energies in NRQCD

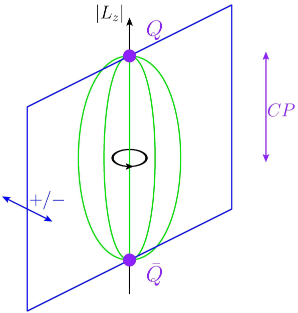
- The **static energies** of a $Q\bar{Q}$ system are defined as the energies of the eigenstates of NRQCD in the static limit.
- Nonperturbative quantity.

Static eigenstates quantum numbers:

* $Q\bar{Q}$ **distance** $\boldsymbol{r} = \boldsymbol{x}_1 - \boldsymbol{x}_2$.
* **Flavor** $q_1\bar{q}_2$: Isospin (I, I_3) and strangeness. In the isospin limit there are only four different configurations: $I = 0$, $I = 1$, $s\bar{q}$, $s\bar{s}$.
* Representation $\boldsymbol{\Lambda}_{\eta}^{\sigma}$ of $D_{\infty\,h}$.

Representations of $D_{\infty\,h}$

- $\Lambda = |\lambda|$ rotational quantum number $\lambda = \hat{\boldsymbol{r}} \cdot \boldsymbol{K} = 0, \pm 1, \pm 2 \ldots$ corresponds to $\Lambda = \Sigma, \Pi, \Delta \ldots$
- η eigenvalue of CP: $g \hat{=} +1$, $u \hat{=} -1$
- σ eigenvalue of reflections (only Σ states)



Static energies as Wilson loops

- The static energies, $E_n^0(r)$, can be obtained from a correlator of any given state $|X_n\rangle$ with a non-vanishing overlap with the desired static state:

$$\langle X_n(T)|X_n(0)\rangle = \langle X_n|e^{-iH^{(0)}_{\mathrm{NRQCD}}T}\sum_{n'}|n'\rangle\langle n'|X_n\rangle = \sum_{n'}|\langle X_n|n'\rangle|^2 e^{-iE^0_{n'}T}$$

- For large T the smallest static energy dominates

$$E_n^{(0)}(r) = \lim_{T\to\infty}\frac{i}{T}\log\langle X_n(T)|X_n(0)\rangle$$

- A convenient choice for these $|X_n\rangle$ gives the static energies in terms of **Wilson loops**

$$|X_n\rangle = \chi(\boldsymbol{x}_2)\phi(\boldsymbol{x}_2,\boldsymbol{R})H_n(\boldsymbol{R})\phi(\boldsymbol{R},\boldsymbol{x}_1)\psi^\dagger(\boldsymbol{x}_1)|vac\rangle$$

with $H_n(\boldsymbol{R})$ any convenient light d.o.f operator with the desired $n = q_1\bar{q}_2\Lambda^\sigma_\eta$.

The **Wilson loop** definition of the static energies allows to compute them on the **lattice**.

- Gluonic operators for quarkonium hybrids

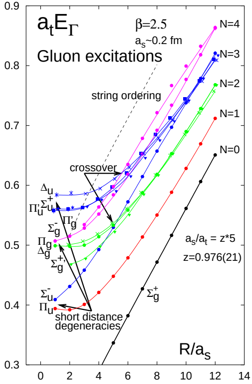
Λ_η^σ	K^{PC}	H
Σ_u^-	1^{+-}	$\hat{\mathbf{r}}\cdot\mathbf{B}\,,\hat{\mathbf{r}}\cdot(\mathbf{D}\times\mathbf{E})$
Π_u	1^{+-}	$\hat{\mathbf{r}}\times\mathbf{B}\,,\hat{\mathbf{r}}\times(\mathbf{D}\times\mathbf{E})$
$\Sigma_g^{+\prime}$	1^{--}	$\hat{\mathbf{r}}\cdot\mathbf{E}\,,\hat{\mathbf{r}}\cdot(\mathbf{D}\times\mathbf{B})$
Π_g	1^{--}	$\hat{\mathbf{r}}\times\mathbf{E}\,,\hat{\mathbf{r}}\times(\mathbf{D}\times\mathbf{B})$
Σ_g^-	2^{--}	$(\hat{\mathbf{r}}\cdot\mathbf{D})(\hat{\mathbf{r}}\cdot\mathbf{B})$
Π_g'	2^{--}	$\hat{\mathbf{r}}\times((\hat{\mathbf{r}}\cdot\mathbf{D})\mathbf{B}+\mathbf{D}(\hat{\mathbf{r}}\cdot\mathbf{B}))$
Δ_g	2^{--}	$(\hat{\mathbf{r}}\times\mathbf{D})^i(\hat{\mathbf{r}}\times\mathbf{B})^j+(\hat{\mathbf{r}}\times\mathbf{D})^j(\hat{\mathbf{r}}\times\mathbf{B})^i$
Σ_u^+	2^{+-}	$(\hat{\mathbf{r}}\cdot\mathbf{D})(\hat{\mathbf{r}}\cdot\mathbf{E})$
Π_u'	2^{+-}	$\hat{\mathbf{r}}\times((\hat{\mathbf{r}}\cdot\mathbf{D})\mathbf{E}+\mathbf{D}(\hat{\mathbf{r}}\cdot\mathbf{E}))$
Δ_u	2^{+-}	$(\hat{\mathbf{r}}\times\mathbf{D})^i(\hat{\mathbf{r}}\times\mathbf{E})^j+(\hat{\mathbf{r}}\times\mathbf{D})^j(\hat{\mathbf{r}}\times\mathbf{E})^i$

- Light-quark operators for tetraquarks $H = H^a T^a$ ($\boldsymbol{q} = (u,\ d)$, τ^i isospin Pauli matrices)

Λ_η^σ	K^{PC}	$H^a(I=0,\ I=1)$
Σ_g^+	0^{++}	$\bar{\boldsymbol{q}}T^a(\mathbb{1},\ \boldsymbol{\tau})\boldsymbol{q}$
Σ_u^+	0^{+-}	$\bar{\boldsymbol{q}}\gamma^0T^a(\mathbb{1},\ \boldsymbol{\tau})\boldsymbol{q}$
Σ_u^-	0^{-+}	$\bar{\boldsymbol{q}}\gamma^5T^a(\mathbb{1},\ \boldsymbol{\tau})\boldsymbol{q}$
Σ_g^+	1^{--}	$\bar{\boldsymbol{q}}\,(\hat{\boldsymbol{r}}\cdot\boldsymbol{\gamma})\,T^a(\mathbb{1},\ \boldsymbol{\tau})\boldsymbol{q}$
Π_g	1^{--}	$\bar{\boldsymbol{q}}\,(\hat{\boldsymbol{r}}\times\boldsymbol{\gamma})\,T^a(\mathbb{1},\ \boldsymbol{\tau})\boldsymbol{q}$
Σ_g^+	1^{++}	$\bar{\boldsymbol{q}}\,(\hat{\boldsymbol{r}}\cdot\boldsymbol{\gamma})\gamma^5T^a(\mathbb{1},\ \boldsymbol{\tau})\boldsymbol{q}$
Π_g	1^{++}	$\bar{\boldsymbol{q}}\,(\hat{\boldsymbol{r}}\times\boldsymbol{\gamma})\gamma^5T^a(\mathbb{1},\ \boldsymbol{\tau})\boldsymbol{q}$

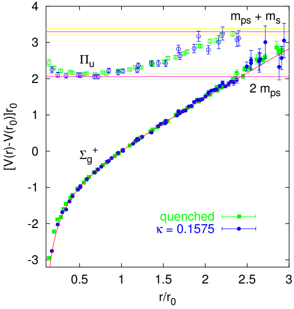
Lattice determinations of static energies for gluonic operators

- **Quenched** lattice NRQCD.



K. Juge, J. Kuti, C. Morningstar Phys.Rev.Lett.90 (2003)

- **Unquenched** lattice NRQCD.

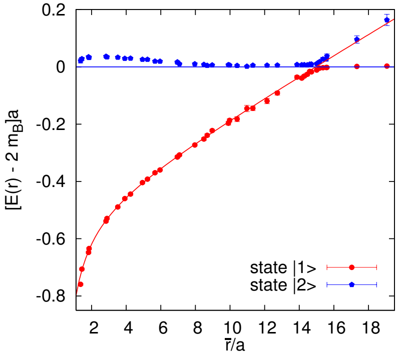


SESAM/TCL Col. G. Bali et al. Phys.Rev.D62 (2000)

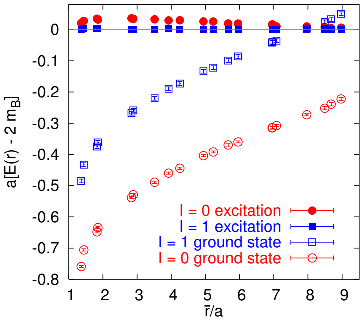
- Short distances degeneracy due to an enlargement of the symetries $D_{\infty\,h} \to O(3) \times C$.
  N. Brambilla, A. Pineda, J. Soto, A. Vairo Nucl.Phys.B566 (2000)
- Linear regime at long distances.
- Σ_g^+ and Π_u^- do not change much below string breaking distance.
- No information about other Λ_η^σ, but there is no reason to expect a different behavior.

Lattice determinations of static energies

- Once we turn on light-quarks we must consider isospin (I) as a quantum number.
- However new states appear: heavy meson pairs, pions, light-quark excitations.



SESAM/TCL Col. G. Bali et al. Phys.Rev.D71 (2005)

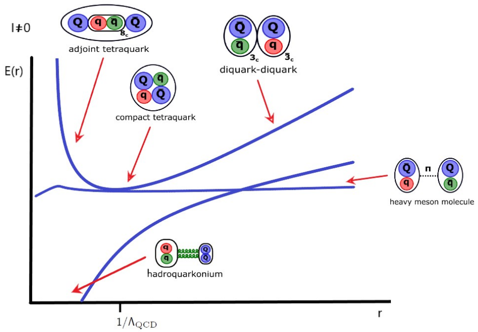


SESAM/TCL Col. G. Bali et al. Phys.Rev.D71 (2005)

- $I = 0$ ground state is just Σ_g^+ $\Rightarrow$ Hybrid static energies are just the $I = 0$ case?
- $I = 1$ ground state is Σ_g^+ shifted up by the pion mass (up to small corrections due to interaction).
- First excitations correspond to heavy meson thresholds.

Lattice determinations of static energies

- Are there further excited states with $I \neq 0$ equivalent to the excited gluonic static energies?
- Several models for exotic quarkonia can be accommodated within this picture. N. Brambilla, A. Vairo, A. Polosa, J. Soto Nucl.Phys.Proc.Suppl.185 (2008); E. Braaten, C. Langmack, D. Smith Phys.Rev.D90 (2014)



Questions:

∗ How (un)stable are the excited light-quark static energies?

∗ What is the long distance behavior?

- BB static energies studied in several channels. P. Bicudo, K. Cichy, A. Peters, B. Wagenbach M. Wagner, Phys.Rev.D92 (2015); P. Bicudo, K. Cichy, A. Peters and M. Wagner, Phys.Rev.D93 (2016)
- Pion-$Q\bar{Q}$ static energy studied in: M. Alberti et al. Phys.Rev. D95 (2017)
- Unfortunately no comprehensive studies of light-quark static energies are available.
- **Hard problem**: Not stable like gluonic static energies in quenched calculations.

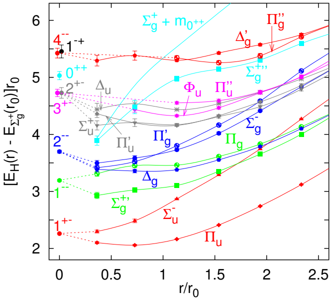
Born-Oppenheimer EFT for hybrids and tetraquarks

EFT approach: Exploit the hierarchy of scales at the Lagrangian level

* Integrate out m_Q modes: NRQCD W. Caswell, G. Lepage Phys.Lett.167B (1986); G. Bodwin, E. Braaten, G. Lepage Phys.Rev.D51 (1995)
* In the short distance regime $r \lesssim 1/\Lambda_{\rm QCD}$: integrate out $m_Q v \sim 1/r$ modes: (weakly-coupled) pNRQCD A. Pineda, J. Soto Nucl.Phys.Proc.Suppl.64 (1998); N. Brambilla, A. Pineda, J. Soto, A. Vairo Nucl.Phys.B566 (2000)
* Integrate out $\Lambda_{\rm QCD}$: Hybrid and tetraquarks EFT (BOEFT) at $E \sim mv^2$. M. Berwein, N. Brambilla, JTC, A. Vairo. Phys.Rev.D92 (2015); N. Brambilla, G. Krein, JTC, A. Vairo Phys.Rev.D97 (2018); R. Oncala, J. Soto Phys.Rev.D96 (2017)

Born-Oppenheimer EFT for hybrids and tetraquarks

- In the short distance limit the static energies are characterized by $O(3) \times C$ instead of $D_{\infty\,h}$.



M. Foster and C. Michael Phys.Rev. D59 (1999)

Light d.o.f Operators

- $G^{i\,a}_{\kappa}$ create a basis of color-octet eigenstates of $h_0(\boldsymbol{R})$ in the presence of a static, local, color-octet source O^a.

$$h_0(\boldsymbol{R})G^{i\,a}_{\kappa}(\boldsymbol{R})|0\rangle = \Lambda_\kappa G^{i\,a}_{\kappa}(\boldsymbol{R})|0\rangle$$

- The light d.o.f Hamiltonian density leading order in the multipole expansion.

$$h_0 = \frac{1}{2}\left(\boldsymbol{E}^2 - \boldsymbol{B}^2\right) - \sum_{j=1}^{n_f} \bar{q}_j i\boldsymbol{D}\cdot\boldsymbol{\gamma} q_j$$

- States are constrained to satisfy the Gauss law.

- BOEFT is formulated for the subspace spanned by

$$\int d^3r d^3R \sum_{\kappa} P^i_{\kappa\lambda} O^{a\,\dagger}(\boldsymbol{r},\boldsymbol{R})\, G^{i\,a}_{\kappa}(\boldsymbol{R})|0\rangle\, \Psi_{\kappa\lambda}(t,\,\boldsymbol{r},\,\boldsymbol{R})$$

- $\Psi_{\kappa\lambda}$ is the basic degree of freedom upon which we build the EFT.
- $P^i_{\kappa\lambda}$ projects $G^{i\,a}_{\kappa}$ into a representation of $D_{\infty\,h}$.

Born-Oppenheimer EFT for hybrids and tetraquarks

- After projecting and integrating out Λ_{QCD}:

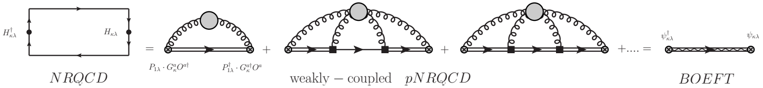
$$\mathcal{L}_{BOEFT} = \int d^3r \sum_{\kappa} \sum_{\lambda\lambda'} \Psi^{\dagger}_{\kappa\lambda}(t,\, \boldsymbol{r},\, \boldsymbol{R}) \left\{ \delta_{\lambda\lambda'} i\partial_t - V_{\kappa\lambda\lambda'}(r) - P^{i\,\dagger}_{\kappa\lambda} \frac{\boldsymbol{\nabla}^2_r}{m_Q} P^{i}_{\kappa\lambda'} \right\} \Psi_{\kappa\lambda'}(t,\, \boldsymbol{r},\, \boldsymbol{R}) + \dots$$

  - The potential $V_{\kappa\lambda\lambda'}$ can be organized into an expansion in $1/m_Q$

$$V_{\kappa\lambda\lambda'}(r) = V^{(0)}_{\kappa\lambda}(r)\delta_{\lambda\lambda'} + \frac{V^{(1)}_{\kappa\lambda\lambda'}(r)}{m_Q} + \frac{V^{(2)}_{\kappa\lambda\lambda'}(r)}{m_Q^2} + \dots$$

  - The static potential, $V^{(0)}_{\kappa\lambda}$, can be matched to the lattice NRQCD static energies and a short distance weak-couplig pNRQCD description:

$$E^{(0)}_{|\lambda|^{\sigma}_{\eta}}(r) = V_o(r) + \Lambda_\kappa + b_{\kappa\lambda} r^2 + \dots = V^{(0)}_{\kappa\lambda}(r)$$



  - The nonadiabatic coupling mixes states which are different projections of the same light d.o.f operator.

$$P^{i\,\dagger}_{\kappa\lambda} \left[\frac{\boldsymbol{\nabla}^2_r}{m_Q}, P^{i}_{\kappa\lambda'} \right] = P^{i\,\dagger}_{\kappa\lambda} \frac{\boldsymbol{\nabla}^2_r}{m_Q} P^{i}_{\kappa\lambda'} - \frac{\boldsymbol{\nabla}^2_r}{m_Q}$$

Hybrid spectrum for $\kappa = 1^{+-} \to \Lambda^{\sigma}_{\eta} = \Sigma_u^-, \Pi_u$

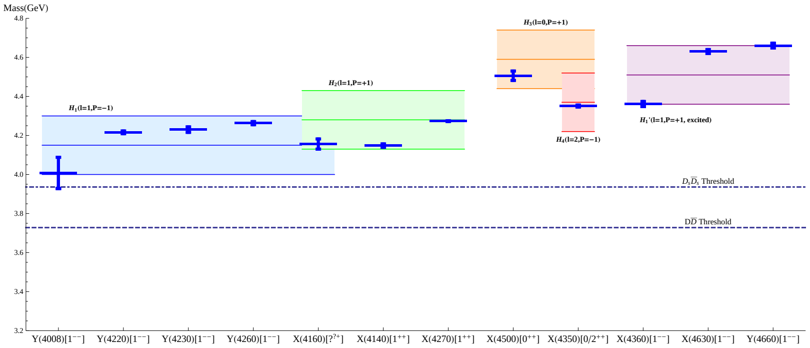
- Lowest lying Gluelump, $\Lambda^{RS}_{1^{+-}} = 0.87 \pm 0.15$GeV. G. Bali, A. Pineda Phys.Rev.D69 (2004)

Several authors have computed the hybrid spectrum

- Single channel, i.e ignoring off-diagonal terms in λ in the nonadiavatic coupling: K.. Juge, J. Kuti, C. Morningstar Nucl.Phys.Proc.Suppl.63 (1998); E. Braaten, C. Langmack, D. Smith Phys.Rev.D90 (2014)
- Coupled channel Σ_u^--Π_u. M. Berwein, N. Brambilla, JTC, A. Vairo. Phys.Rev.D92 (2015)
  - Breaking of degeneracy between parity doublet hybrid states (Λ-doubling in molecular physics).
- Mixing with standard quarkonium (Σ_g^+-Σ_u^--Π_u). R. Oncala, J. Soto Phys.Rev.D96 (2017)
  - Significant hybrid-quarkonium mixing in some state which provides a source of spin-symmetry violation.

Charmonium Hybrid spectrum for $\kappa = 1^{+-}$

M. Berwein, N. Brambilla, JTC, A. Vairo. Phys.Rev.D92 (2015)



1. Solid blue bars: Neutral exotic charmonium states (Belle, CDF, BESIII, Babar, LHCb).
2. Bands: Predicted masses for hybrid spin-symmetry multiplets $\pm$ uncertainty of $\Lambda_{1^{+-}}$.

- Spin-symmetry multiplets

	l	J^{PC}	
H_1	1	$\{1^{--}, (0,1,2)^{-+}\}$	Σ_u^-, Π_u
H_2	1	$\{1^{++}, (0,1,2)^{+-}\}$	Π_u
H_3	0	$\{0^{++}, 1^{+-}\}$	Σ_u^-
H_4	2	$\{2^{++}, (1,2,3)^{+-}\}$	Σ_u^-, Π_u

Tetraquarks

- The lowest masss adjoint light-quark operators have been studied in quenched lattice QCD M. Foster and C. Michael Phys.Rev. D59 (1999)
- No lattice determination of the static energies.

Light-quark operators masses

[MeV]	K^{PC}	$q\bar{q}(I=0)$	$s\bar{s}$
$\Lambda_\kappa - \Lambda_{1^{+-}}$	1^{--}	47 ± 90	120 ± 70
	0^{-+}	91 ± 216	170 ± 99

Tetraquark static energies

- **Short distance:** $V_o + \Lambda_\kappa + \ldots$
- **Long distance:** Unknown, String picture suggests $\sigma r +$ const.

Speculative assignments E.Braaten, C.Langmack, D.Smith Phys.Rev.D90 (2014)

K^{PC}	Λ_η^σ	l	$S=0$	$S=1$	isospin-1	isospin-0	$s\bar{s}$
1^{--}	$\Sigma_g^+ - \Pi_g$	1	1^{+-}	$(0,1,2)^{++}$	$Z_c^+(4023)$	$X(3918)$	$Y(4145)$
		2	2^{-+}	$(1,2,3)^{++}$	4100	4327	4205
	Π_g	1	$\mathbf{1}^{-+}$	$(\mathbf{0},1,2)^{--}$	$Z_c^+(3898)$	(3793)	(4020)
		2	$\mathbf{2}^{+-}$	$(1,2,3)^{++}$	4080	3975	4201
	Σ_g^+	0	0^{-+}	1^{--}	(4368)	$Y(4263)$	(4490)
0^{-+}	Σ_u^-	0	0^{++}	1^{+-}			
		1	1^{--}	$(0,\mathbf{1},2)^{-+}$			

Spin-dependent operators in BOEFT for hybrids

- Spin-dependent operators have been worked out for spin $\kappa = 1$ gluelump operator.
- Work extendable to tetraquarks with $\kappa = 1$ light-quark operators.
- Unlike standard quarkonium they appear at $1/m_Q$ instead of $1/m_Q^2$:

$$V^{(1)}_{1\lambda\lambda'\,SD}(r) = V_{1\,SK}(r)\left(P^{i\dagger}_{1\lambda}\boldsymbol{K}^{ij}_1 P^{j}_{1\lambda'}\right)\cdot\boldsymbol{S} + \dots\,,$$

$$\begin{aligned} V^{(2)}_{1\lambda\lambda'\,SD}(r) = {} & V_{1\,LSa}(r)\left(P^{i\dagger}_{1\lambda}\boldsymbol{L}_{Q\bar{Q}}P^{i}_{1\lambda'}\right)\cdot\boldsymbol{S} + V_{1\,LSb}P^{i\dagger}_{1\lambda}(r)\left(\boldsymbol{L}^{i}_{Q\bar{Q}}\boldsymbol{S}^{j} + \boldsymbol{S}^{i}\boldsymbol{L}^{j}_{Q\bar{Q}}\right)P^{j}_{1\lambda'} \\ & + V_{1\,S^2}(r)\boldsymbol{S}^2\delta_{\lambda\lambda'} + V_{1\,S_{12}a}(r)\boldsymbol{S}_{12}\delta_{\lambda\lambda'} + V_{1\,S_{12}b}(r)P^{i\dagger}_{1\lambda}P^{j}_{1\lambda'}\left(\boldsymbol{S}^{i}_1\boldsymbol{S}^{j}_2 + \boldsymbol{S}^{i}_2\boldsymbol{S}^{j}_1\right) + \dots \end{aligned}$$

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$$+ V_{1\,S^2}(r)\boldsymbol{S}^2\delta_{\lambda\lambda'} + V_{1\,S_{12}a}(r)\boldsymbol{S}_{12}\delta_{\lambda\lambda'} + \boxed{V_{1\,S_{12}b}(r)P^{i\dagger}_{1\lambda}P^j_{1\lambda'}\left(\boldsymbol{S}^i_1\boldsymbol{S}^j_2 + \boldsymbol{S}^i_2\boldsymbol{S}^j_1\right) + \ldots}$$

New operators not present in standard Quarkonium.

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$$+ V_{1\,S^2}(r)\boldsymbol{S}^2\delta_{\lambda\lambda'} + V_{1\,S_{12}a}(r)\boldsymbol{S}_{12}\delta_{\lambda\lambda'} + V_{1\,S_{12}b}(r)P^{i\dagger}_{1\lambda}P^j_{1\lambda'}\left(\boldsymbol{S}^i_1\boldsymbol{S}^j_2 + \boldsymbol{S}^i_2\boldsymbol{S}^j_1\right) + \dots$$

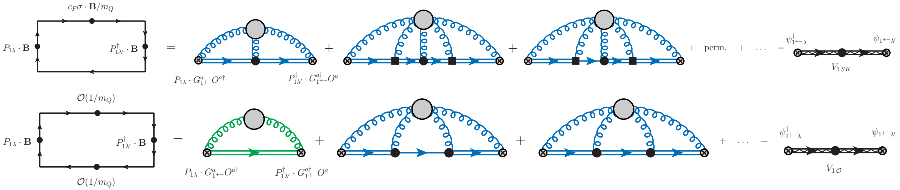
$\mathcal{O}(1/m_Q)$ **J. Soto arXiv:1709.08038 [hep-ph]**

- Relations independent of $V_{1\,SK}(r)$ among states $(M_{S\,J})$ of the same multiplet have been derived

$$\frac{M_{1\,J+1} - M_{0\,J}}{M_{1\,J} - M_{0\,J}} = -J\,, \quad \frac{M_{1\,J-1} - M_{0\,J}}{M_{1\,J} - M_{0\,J}} = J+1$$

$$H_1: M_{2-+} + M_{0-+} = M_{1-+} + M_{1--}$$
$$H_2: M_{2+-} + M_{0+-} = M_{1+-} + M_{1++}$$
$$H_4: M_{3+-} + M_{1+-} = M_{2+-} + M_{2++}$$
$$H_5: M_{3+-} + M_{1+-} = M_{2+-} + M_{2--}$$

Spin-dependent operators in BOEFT for hybrids



- Unfortunately no lattice determinations of the spin-dependent potentials are available.
- In the short distance regime we can use the weakly-coupled pNRQCD description (up to LO in the multipole and $1/m_Q$ expansions)

$$V_{1\,SK} = V_{SK}^{np\,(0)} + V_{SK}^{np\,(1)} r^2 + \dots\,,$$

$$V_{1\,\mathcal{O}} = V_{o\,\mathcal{O}}(r) + V_{\mathcal{O}}^{np\,(0)} + \dots\,, \quad \mathcal{O} = SLa,\, SLb,\, S^2,\, S_{12},\, S_{12}b$$

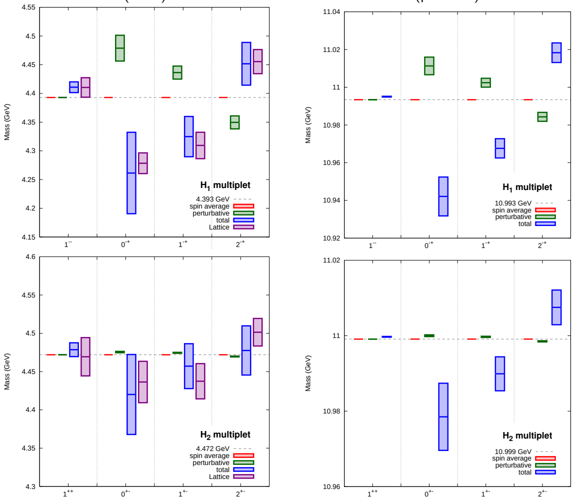
- The V^{np} depend on nonperturbative gluon correlators (unknown) and gluon-octet field couplings (known) which may carry heavy flavour dependence.

Hybrids Spin splittings

Lattice data: Hadron Spectrum col. JHEP 1612 (2016)

N. Brambilla, W-K. Lai, J. Segovia JTC, A. Vairo in preparation

$c\bar{c}$ (fitted) $b\bar{b}$ (predicted)



- H_3 and H_4 multiplets spin splittings also computed.

Semi-inclusive decays of hybrid to low-lying quarkonia

R. Oncala, J. Soto Phys.Rev.D96 (2017)

- If the energy gap ΔE to lowest lying quarkonium is $\Delta E \geq 1$ GeV $\Rightarrow$ a perturbative **estimate** makes sense.

$$\Gamma(\Psi_m \to S_n) = \frac{4}{3}\frac{\alpha_s T_F}{N_c}|\langle\Psi_m|\boldsymbol{r}|S_n\rangle|^2(\Delta E_{mn})^3$$

- **Semi-inclusive**: decay width include all possible decay products.

charm				
$NL \to N'L'$	ΔE	$\langle r\rangle_{mn}$	$\|\Delta E\langle r\rangle_{mn}\|$	Γ [MeV]
$H_3(1s) \to 2s$	808	0.40	0.32	7.5(7.4)
$H_1'(2p) \to 1p$	861	0.63	0.54	22(19)
bottom				
$NL \to N'L'$	ΔE	$\langle r\rangle_{mn}$	$\|\Delta E\langle r\rangle_{mn}\|$	Γ [MeV]
$H_3(1s) \to 1s$	1569	-0.42	0.65	44(23)
$H_3(1s) \to 2s$	1002	0.43	0.43	15(9)
$H_3'(2s) \to 2s$	1290	-0.14	0.18	2.9(1.3)
$H_1'(1p) \to 1p$	977	0.47	0.46	17(8)

- Multiplet selection rules: $\Delta\ell = 0$.
- State selection rules: $\Delta S = 0$, $\Delta J = 0$.

Outlook and Conclusions

- Conclusions:
  - Exotic quarkonia can be understood as states living on gluon-light quark static energies.
  - EFT with lattice inputs can be used to study exotic quarkonia in a largely model independent way.
  - Lattice NRCQD determinations of gluonic static energies has been used to obtain the hybrid spectrum including some (but not all!) multi-channel effects.
  - Progress in hybrid quarkonia:
    - Spin-dependent potentials have been incorporated into the determination of the hybrid spectrum.
    - First steps towards made towards the description of hybrids decays.
  - Light-quark static energies are sorely needed for a meaningful analysis of tetraquark states.
- Outlook:
  - Hybrid transitions into quarkonia with pion emission
  - Extension of the EFT to $I \neq 0$ sectors including transition between sectors.
  - Effective string models for $Q\bar{Q}$ potentials in light-quark backgrounds.

Thank you for your attention