Soft gluon factorization for heavy quarkonium production

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NRQCD Factorization

➢ Factorization formula

Bodwin, Braaten, Lepage, 9407339

$$(2\pi)^3 2P_H^0 \frac{d\sigma_H}{d^3 P_H} = \sum_n d\hat{\sigma}_n(P_H)\langle \mathcal{O}_n^H \rangle$$

Production of a heavy quark pair

Hadronization (LDMEs)

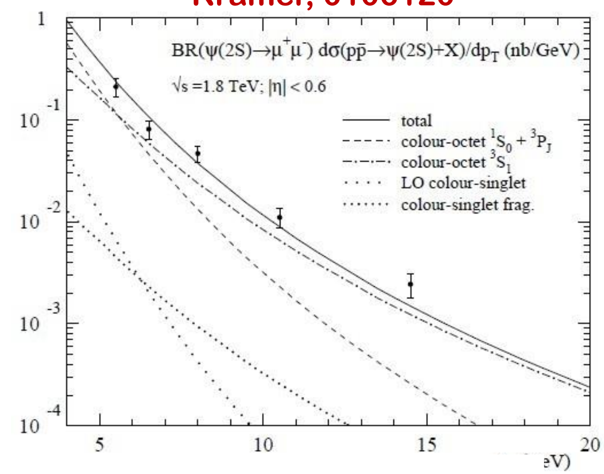
- n: quantum numbers of the pair: color, spin, orbital angular momentum, total angular momentum, spectroscopic notation ${}^{2S+1}L_J^{[c]}$

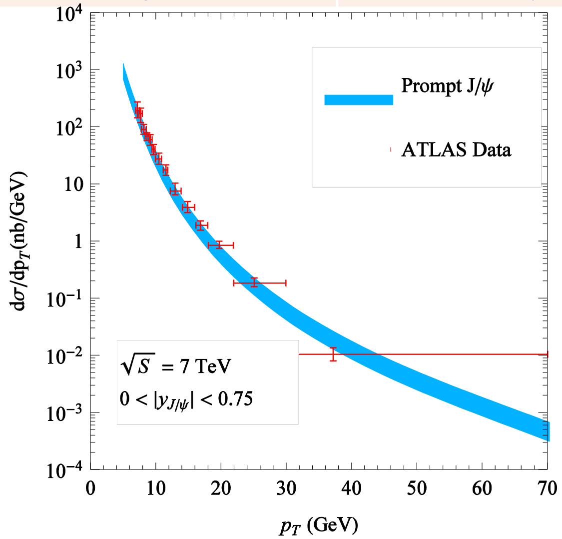
Achievement: explain ψ' surplus

- **Nicely explain ψ' surplus by CO contributions**

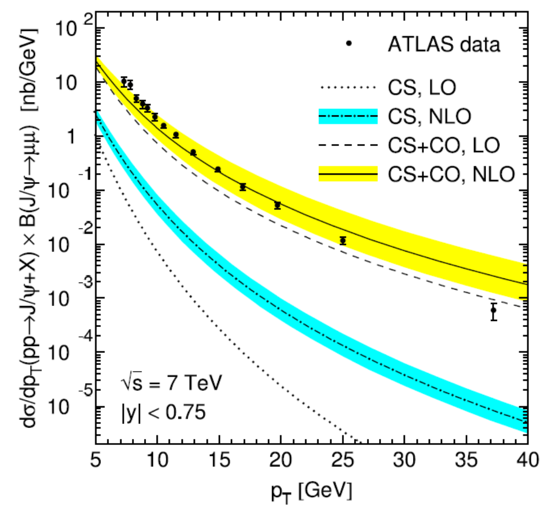
States	p_T behavior at LO
${}^3S_1^{[1]}$	p_T^{-8}
${}^3S_1^{[8]}$	p_T^{-4}
${}^1S_0^{[8]}$	p_T^{-6}
${}^3P_J^{[8]}$	p_T^{-6}

Kramer, 0106120





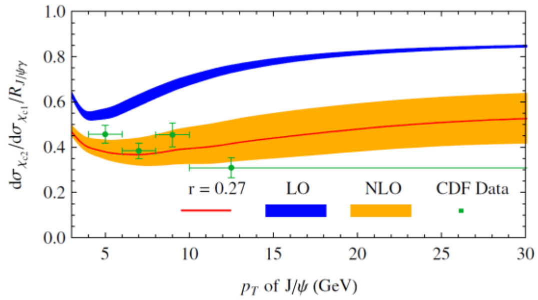
YQM, Wang, Chao, 1012.1030



Butenschoen, Kniehl, 1105.0820

Achievement: prediction for χ_{cJ} production

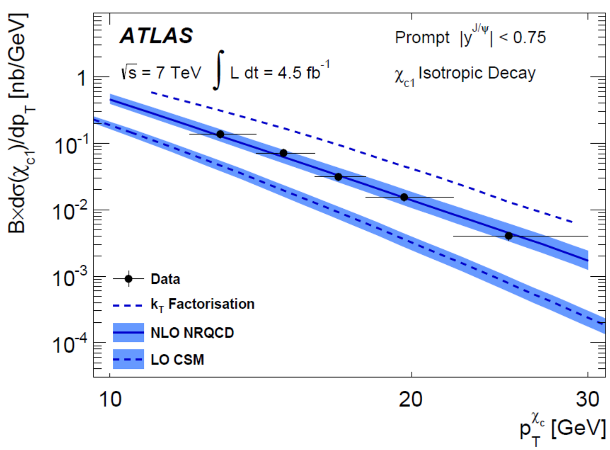
- χ_{cJ} production: $d\sigma_{\chi_{cJ}} \approx d\hat{\sigma}_{{}^3P_J^{[1]}} \langle O({}^3P_0^{[1]}) \rangle + (2J+1) d\hat{\sigma}_{{}^3S_1^{[8]}} \langle O({}^3S_1^{[8]}) \rangle$

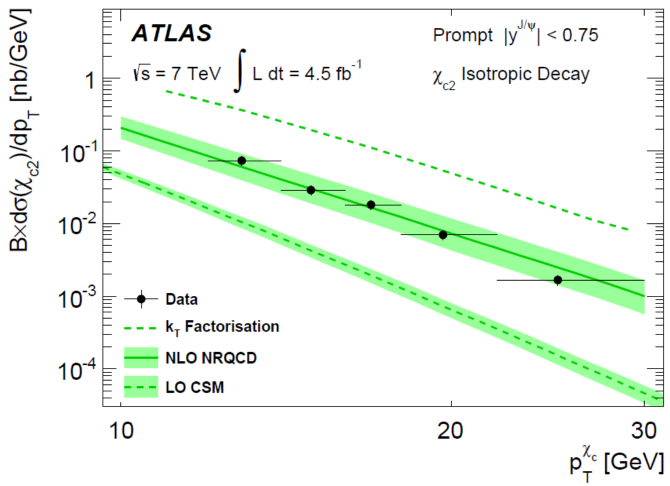


YQM, Wang, Chao, 1002.3987

- Agree with new data

ATLAS, 1404.7035





Difficulty: polarization puzzle

➢ LO NRQCD

- Dominated by ${}^3S_1^{[8]}$, LO NRQCD predicts transversely polarized $\psi(\mathrm{nS})$, contradicts with CDF data

CDF, 0704.0638

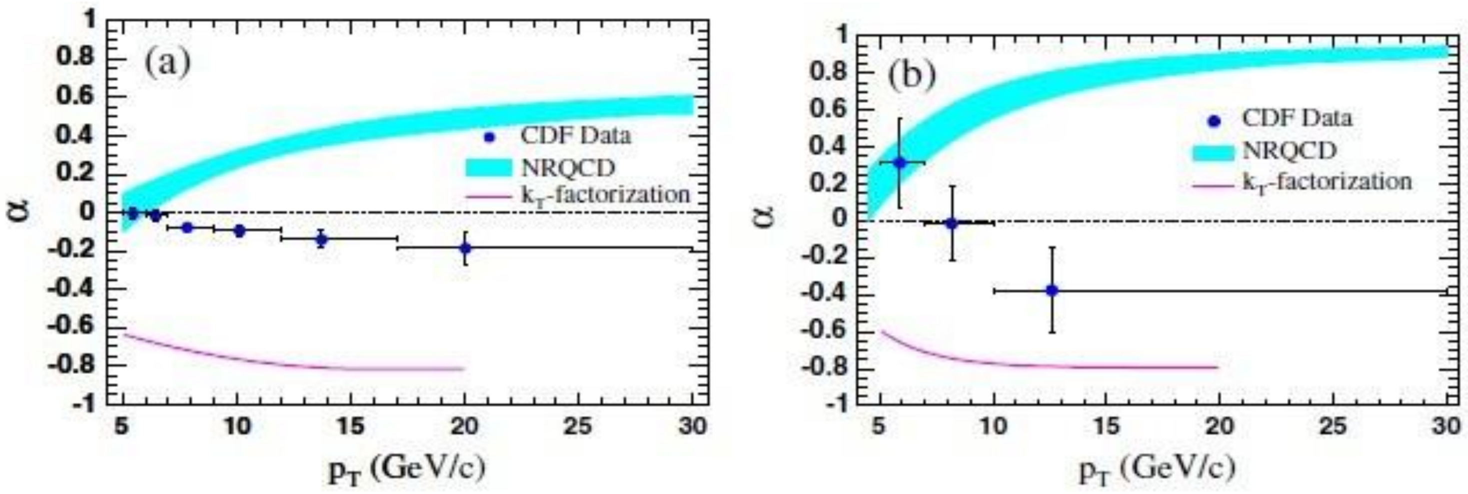
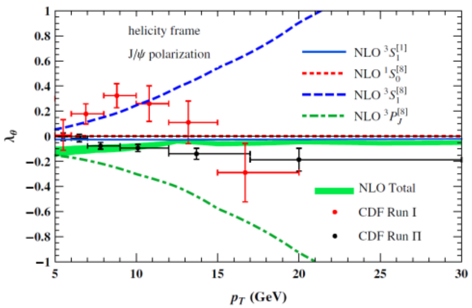


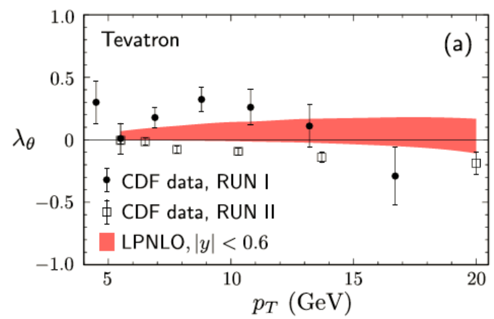
FIG. 4 (color online). Prompt polarizations as functions of p_T: (a) J/ψ and (b) $\psi(2S)$. The band (line) is the prediction from NRQCD [4] (the k_T-factorization model [9]).

Difficulty: polarization puzzle

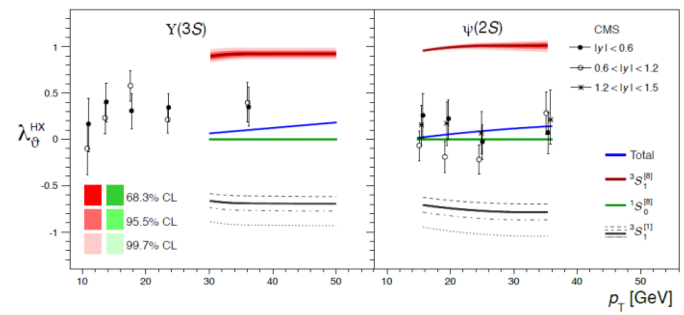
- J/ψ: transverse polarization cancelled between $^3S_1^{[8]}$ and $^3P_J^{[8]}$ channel, $^1S_0^{[8]}$ may dominate



Chao,YQM,Shao,Wang, Zhang,1201.2675



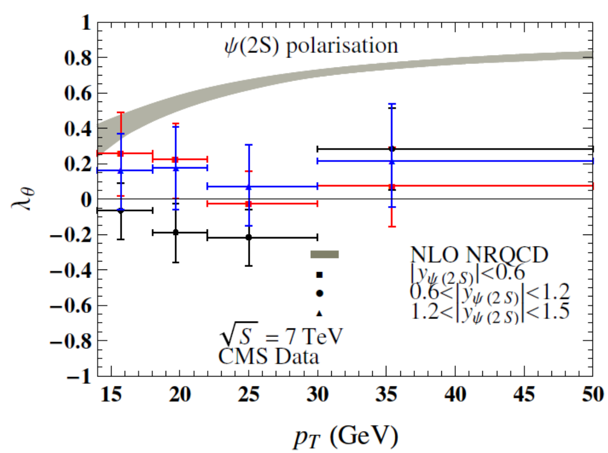
Bodwin, Chung, Kim, Lee, 1403.3612



Faccioli,Knunz,Lourenco, Seixas,Wohri,1403.3970

- $\psi(2S)$: still hard to understand

Shao, Han, YQM, Meng, Zhang, Chao, 1411.3300



Difficulty: hierarchy problem

- **Best fit of J/ψ yield data at high p_T**

YQM, Wang, Chao, 1009.3655

$$\mathrm{M}_0 = \langle O\left({}^1\mathrm{S}_0^{[8]}\right)\rangle + 3.9\langle O\left({}^3\boldsymbol{P}_0^{[8]}\right)\rangle/m_c^2 \approx 0.074\ \mathbf{GeV}^3$$

$$\mathrm{M}_1 = \langle O\left({}^3\mathrm{S}_1^{[8]}\right)\rangle - 0.56\langle O\left({}^3\boldsymbol{P}_0^{[8]}\right)\rangle/m_c^2 \approx 0.0005\ \mathbf{GeV}^3$$

- **Velocity scaling rule of NRQCD**

$$\langle O\left({}^1\mathrm{S}_0^{[8]}\right)\rangle \sim \langle O\left({}^3\mathrm{S}_1^{[8]}\right)\rangle \sim \langle O\left({}^3\boldsymbol{P}_0^{[8]}\right)\rangle/\mathrm{m}_\mathrm{c}^2$$

Thus

$$M_0 \sim M_1$$

- **Two orders difference: unnatural**

Difficulty: universality problem

- **Necessary condition for NRQCD**
  - LDMEs, like M_0 and M_1, are process independent
- **Upper bound of M_0 set by e^+e^- collision**

Zhang, YQM, Wang, Chao, 0911.2166

$$M_0 < 0.02\text{GeV}^3$$

  - Comparing with $\mathrm{M}_0 \approx 0.074$ GeV^3 from pp collison
- **Global fit of LDMEs**

Butenschoen, Kniehl, 1105.0820

$$\chi^2_{\text{d.o.f.}} = 725/194 = 3.74$$

  - Data cannot be described consistently!

Rigorousness of NRQCD

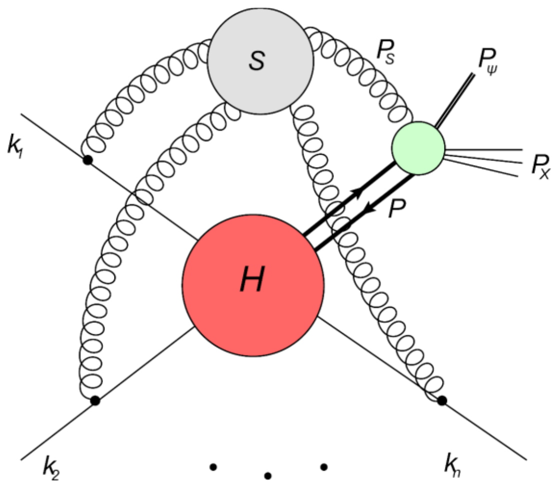
➢ **Looks like a rigorous theory**

- EFT of QCD
- Factorization has been tested to NNLO   Nayak, Qiu, Sterman, 0509021

But why does not it work for quarkonium production?

An important effect: overlooked

- **Soft gluon emission in the hadronization process**
  - P_ψ **is different from** P, $P = P_\psi[1 + O(\lambda)]$
  - **NRQCD approximate** P **by** P_ψ



- **An over simplified model of NRQCD expansion**
  - **Cross section approximately** $\propto P^{-4} = P_\psi^{-4}[1 + O(\lambda)]^{-4}$

$$\int_{-1}^{1} \frac{d\cos\theta}{2(1+\lambda+\lambda\cos\theta)^4} = 0.42$$

With $\lambda \approx v^2 \approx 0.3$

$$= 1 - 4\lambda + 40/3\lambda^2 - 40\lambda^3 + \cdots$$

$$= 1 - 1.2 + 1.2 - 1.08 + 0.91 - 0.73 + \cdots$$

Soft gluon factorization (SGF)

➢ SGF for quarkonium H production:

YQM, Chao, 1703.08402

$$(2\pi)^3 2P_H^0 \frac{d\sigma_H}{d^3P_H} = \sum_n \int \frac{d^4P}{(2\pi)^4} d\hat{\sigma}_n(P) F_n^H(P, P_H) \qquad n = {}^{2S+1}L_J^{[c]}$$

- $d\hat{\sigma}$: perturbatively calculable hard part
- F_n^H: nonperturbabtive soft gluon distribution
- UV renormalization scale is suppressed

➢ Keep momentum difference between $Q\bar{Q}$ and H

- Expect no further large relativistic corrections

Soft gluon distributions (SGDs)

➢ Operator definition

- Expectation values of bilocal operators in QCD vacuum

$$F_n^H(P,P_H)=\int d^4x e^{iP\cdot x}\, S_l(x)\langle 0|\bar{\psi}(0)\Gamma'_n\Phi_l^\dagger(0)\psi(0)a_H^\dagger a_H\bar{\psi}(x)\Gamma_n\Phi_l(x)\psi(x)|0\rangle$$

- Gauge links to ensure gauge invariance

$$\Phi_l(x)=\mathcal{P}\exp\left\{-ig_s\int_0^\infty d\lambda\, l\cdot A(x+\lambda l)\right\}$$

- Soft factor $S_l(x)$ to absorb additional IR divergences
- A different choice of l: $S_l(x)$ will be changed by a gauge invariant Wilson loop

➢ Set $P \approx P_H$ in hard part: "reproduce" NRQCD

$$(2\pi)^3 2P_H^0\frac{d\sigma_H}{d^3P_H}\approx\sum_n d\hat{\sigma}_n(P_H)\langle\mathcal{O}_n^H\rangle_l\, S_l(0)$$

CO → TMD v.s. NRQCD → SGF

➢ CO factorization v.s. TMD factorization

- TMD factorization: both longitudinal-momentum dependence and transverse-momentum dependence
- CO factorization: integrated out transverse momentum, and leaving only longitudinal-momentum dependence

➢ NRQCD v.s. SGF

- SGF: have relative-momentum dependence between momentum of $Q\bar{Q}$ pair and that of quarkonium
- NRQCD: integrated out relative momentum, no momentum dependence

➢ Implication

SGF is a "TMD version" of NRQCD

TMD v.s. SGF

- **Similarity: gauge links, soft factors**
  - Study of TMD can help to understand SGF, and vice versa
  - Soft factor vanishes in CO factorization; does it vanish in NRQCD factorization?
- **When to use TMD?**
  - In small p_T region where higher-twist contributions are significant
  - TMD resums a series of higher-twist contributions in CO factorization
- **When to use SGF?**
  - In any region where relativistic corrections are significant: all p_T regions in hadroproduction, Xsection changes fast
  - SGF resums a series of relativistic contributions in NRQCD

Simplification: 1d form

- **SGF-4d hard to use in practice**
  - Hard to extract four-dimensional SGDs
  - Hard to do perturbative calculation
- **Property of SGDs**
  - At the rest frame of H, dominant region (with $P^2 = M^2$)

$$P^{\mu}_{rest} = (M + O(\lambda^2), O(\lambda), O(\lambda), O(\lambda))$$

- **Expanding $O(\lambda)$ terms in hard part: SGF-1d**

$$(2\pi)^3 2P_H^0 \frac{d\sigma_H}{d^3P_H} \approx \sum_n \int dz\, d\hat{\sigma}_n(P_H/z) F_n^H(z) \quad \text{with} \quad z = \frac{m_H}{M}$$

Very similar to CO factorization

- **Comparison:** relating momentum of χ_{cJ} and its decaying J/ψ

YQM, Wang, Chao, 1002.3987

$$p_{J/\psi} \approx \frac{m_{J/\psi}}{m_{\chi_{cJ}}} p_{\chi_{cJ}}$$

- Deviation less than 8%

The over simplified model

- **"SGF-1d expansion"**

$$\int_{-1}^{1}\frac{d\cos\theta}{2(1+\lambda+\lambda\cos\theta)^4}=0.42$$

With $\lambda = 0.3$

$$=\frac{1}{(1+\lambda)^4}\left(1+\frac{10}{3}\lambda^2-\frac{20}{3}\lambda^3+17\lambda^4+\cdots\right)$$

$$=0.350+0.105-0.063+0.048-0.035+\cdots$$

- **Comparing with "NRQCD expansion"**

$$\int_{-1}^{1}\frac{d\cos\theta}{2(1+\lambda+\lambda\cos\theta)^4}=0.42$$

$$=1-4\lambda+40/3\lambda^2-40\lambda^3+\cdots$$

$$=1-1.2+1.2-1.08+0.91-0.73+\cdots$$

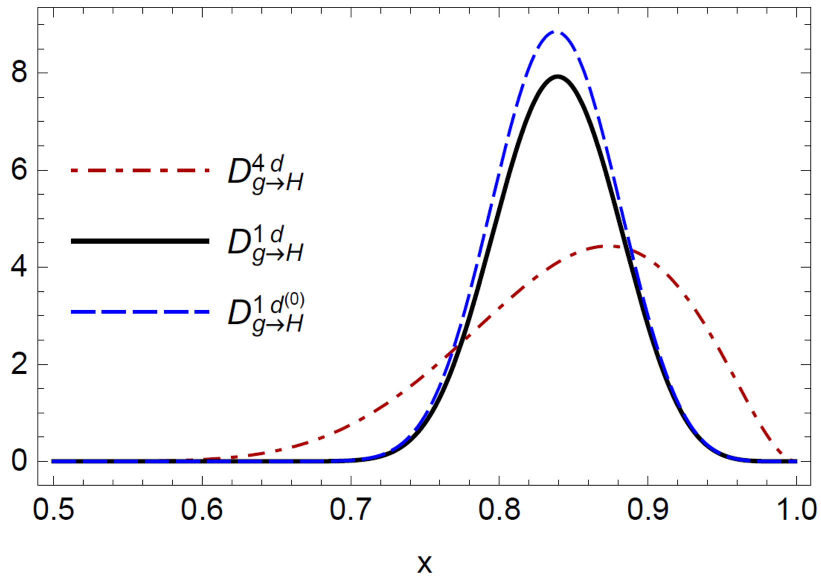
Gluon fragmenting to quarkonium

- **Numerical comparison between SGF & NRQCD**
- **Model input**

$$F^{H}_{^3S_1^{[8]}}(P, P_H) = a\, k^2 \exp(-\frac{k_0^2 + k^2}{\Lambda^2})$$

Beneke, Schuler, Wolf, 0001062

- $\Lambda \sim m_Q v^2$, choose $500MeV$
- Conclusion independent of the model



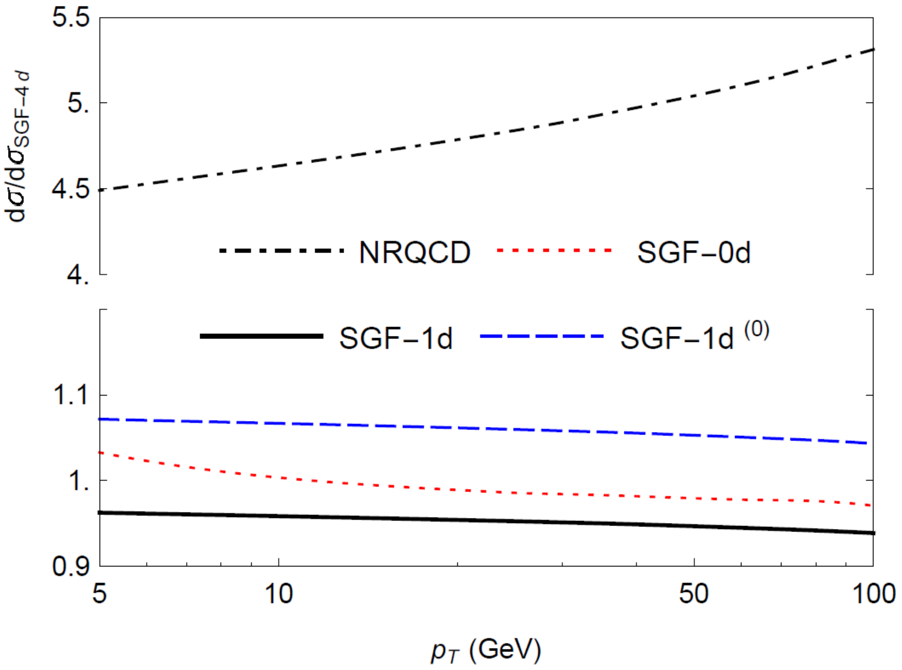
- SGF-4d and -1d have different shape, but have the same accumulated value

$$\int_0^1 dx\, D^{4d}_{g\to H}(x) = \int_0^1 dx\, D^{1d}_{g\to H}(x)$$

- -1$\mathrm{d}^{(0)}$: leading term of expansion m_Q around $M/2$, very close to -1d.

Cross section ratio

- **Assume SGF-4d is exact**



- 1d very close to 4d, deviation less than 6%
- Expanding m_Q results in about 10% uncertainty
- 0d with $z_0 = 0.86$ well reproduce 4d
- NRQCD overshoots 4d by a factor of 4 —not reliable

- **Rough explanation**

$$0.86^9 \approx 1/4 \sim (1 - v^2/2)^9$$

Similar effects exist in many EFTs

Summary

- **NRQCD factorization: polarization puzzle, hierarchy problem, universality problem**
  - Possible reason: convergence of v^2 expansion is too bad because of soft gluon emission
- **Soft gluon factorization (SGF)**
  - Soft gluons effects are considered, should have much better convergence of v^2 expansion
- **Two important expansions**
  - From 4d to 1d, with small v^2 correction
  - Expansion m_Q around $M/2$, good convergence

Outlook

➢ Proof of SGF to all order in perturbation theory

- Similar to the proof of NRQCD to all order in α_s and v^2
- One-loop proof is available; two-loop should not be hard

➢ Phenomenological study

- Complexity is similar to NRQCD, thanks to the two expansions
- Most established codes reuse directly (FDC, Helac-Onia,…)
- Many NRQCD results should be redone, a lot of works

➢ May resolve difficulties in NRQCD

- Universality problem: importance of v^2 correction depends on process
- Hierarchy problem: relative importance of different channels changed in SGF
- Polarization puzzle: may also have large v^2 correction

Thank you!

What is new?

- **Factorization in full QCD but not NRQCD effective field theory**
  - More convenient to deal with power corrections in full QCD than EFT
- **Momentum difference between $Q\bar{Q}$ and H considered**
  - No further large power corrections
- **Difference between shape function models**
  - External $Q\bar{Q}$ in hard part are on mass shell, gauge invariant for hard part
  - Operator definition of SGDs with gauge links to ensure gauge invariance
  - Soft factor in SGDs

Simplification: 0d form

➢ SGF-0d

- If $F_n^H(z)$ peaks around $z = z_n \sim 1 - O(\lambda/m_H)$
- Approximate $F_n^H(z) \approx \delta(z - z_n)\langle O_n^H \rangle$

$$(2\pi)^3 2P_H^0 \frac{d\sigma_H}{d^3P_H} \approx \sum_n d\hat{\sigma}_n(P_H/z_n)\langle \mathcal{O}_n^H \rangle_l \, S_l(0)$$

➢ Roughly recover NRQCD if choosing $z_n = 1$

- May result in large corrections

Simplification: expansion of m_Q

➢ At least two hard scales in short distance

- **Invariant mass of $Q\bar{Q}$ pair M and quark mass m_Q**
- **Relation:** $M = 2m_Q + O(\lambda)$

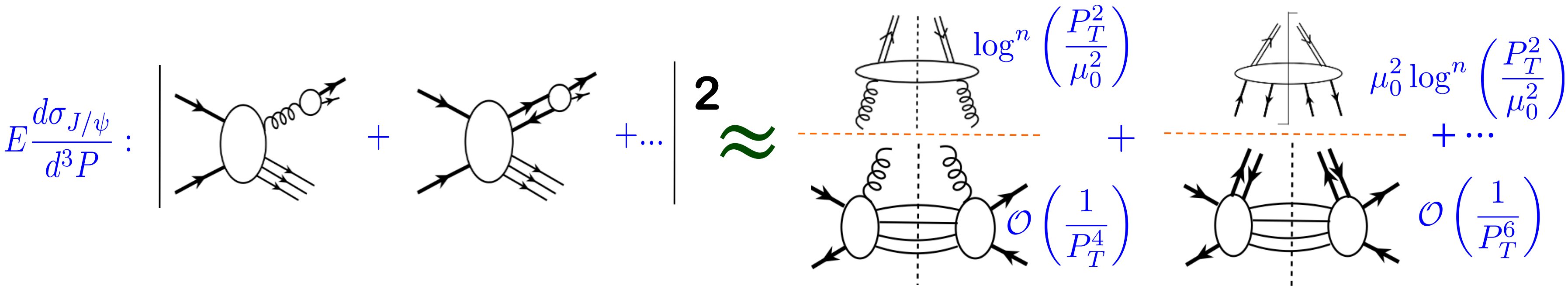
➢ Expansion

- $d\hat{\sigma}(m_Q, M) = d\hat{\sigma}\left(\frac{M}{2}, M\right) + \left(m_Q - \frac{M}{2}\right) d\hat{\sigma}'\left(\frac{M}{2}, M\right) + \cdots$
- **Good convergence**

➢ Comparing with NRQCD expansion

- $d\hat{\sigma}(m_Q, M) = d\hat{\sigma}(m_Q, 2m_Q) + (M - 2m_Q) d\hat{\sigma}'(m_Q, 2m_Q) + \cdots$
- **Bad convergence:** $d\hat{\sigma}(m_Q, M)$ **may** $\propto M^{-5}$

J/ψ production via gluon fragmentation



- **Easy to calculate**

$$d\sigma_H(p_T) = \int dx\, d\hat{\sigma}_g(p_T/x) D_{g\to H}(x)$$

- $d\hat{\sigma}_g$: **well known**
- $D_{g\to H}$: **calculated by using NRQCD or SGF**