Study of radiative decays $\Upsilon(1S)\to\gamma\pi^+\pi^-$ and $\Upsilon(1S)\to\gamma K^-K^+$

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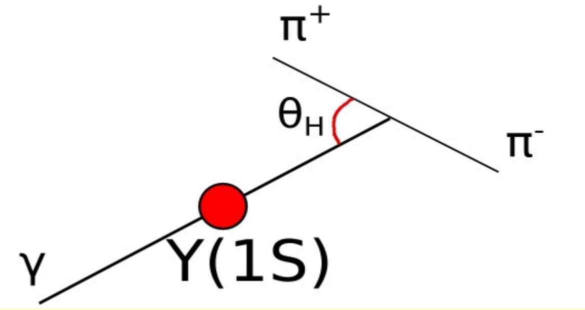
Outline

- Motivation/Introduction
- Event Reconstruction
- Study of two-pion and two-kaon invariant mass spectra
- Spin analyses
- Summary

Comments before start

- The paper has been accepted for publication in PRD (arXiv:1804.04044)
- The system with two pseudoscalars (h^+h^-) produced via the decay $\Upsilon(1S) \to \gamma h^+h^-$ has quantum numbers J^{PC} (I) = even^{++} (0)
- The helicity angle θ_H as the angle formed by the h^+, in the h^+h^- rest frame, and the γ in the $h^+h^-\gamma$ rest frame.



- There is a "soup" of J^{PC} (I) = even^{++} (0) mesons in nature:

- Despite the long history of the study of the f-like states, located close to each other and with broad widths, we lack precise knowledge of their properties, mixing angles, nature and etc

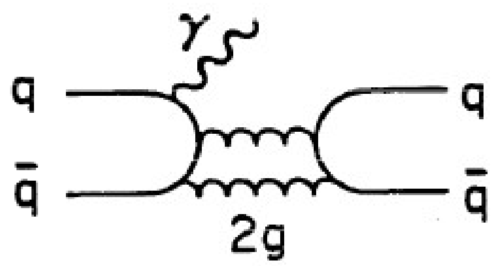
Physics Motivations

- The search for gluonium states is still a hot topic for QCD.

- Lattice QCD calculations predict the lightest gluonium states to have quantum numbers $J^{PC} = 0^{++}$ and 2^{++} and to be in the mass region below 2.5 GeV/c^2 [PRD73 014516].

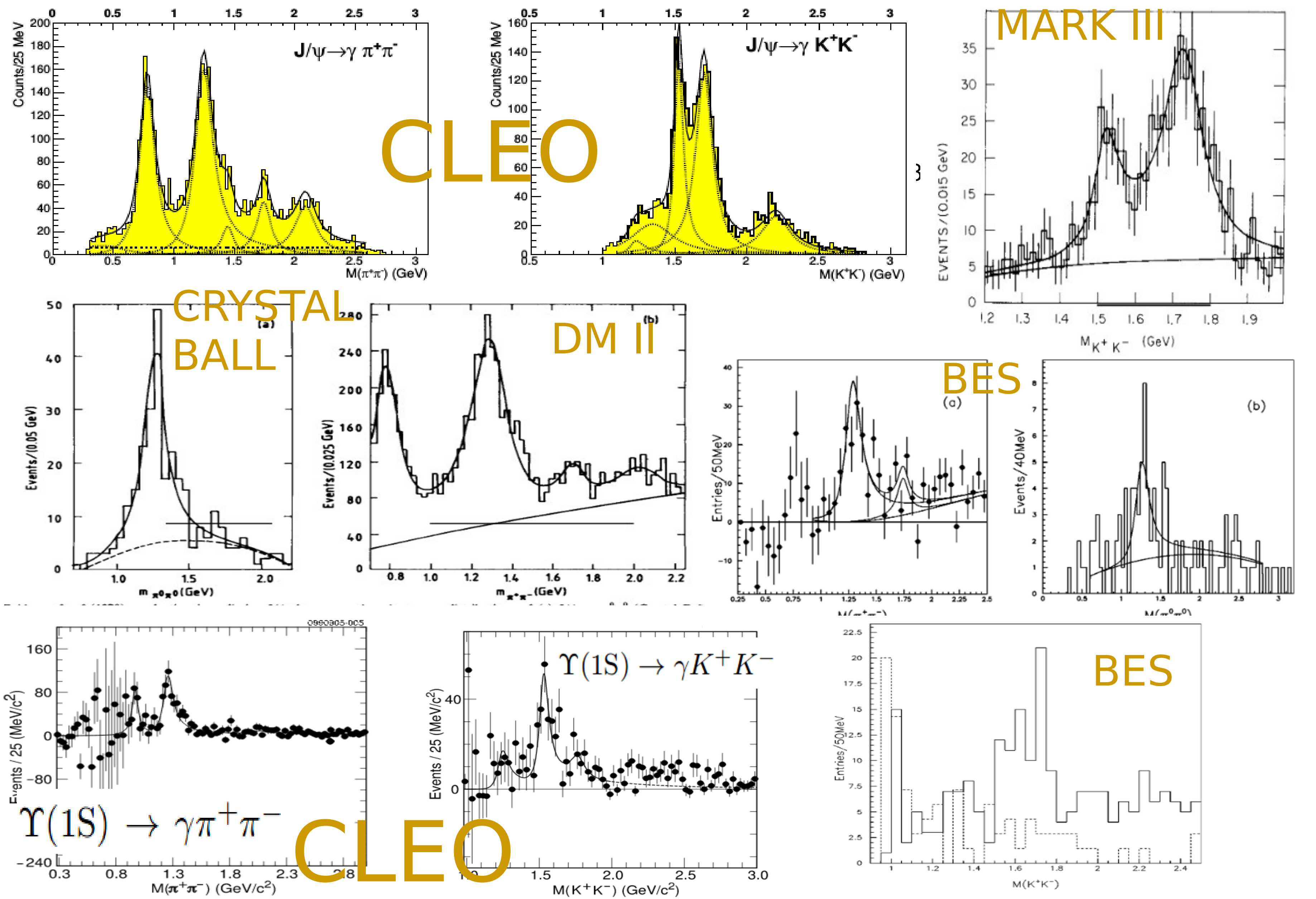
- Possible candidate for the $J^{PC} = 0^{++}$ glueball is the $f_0(1710)$. For this resonance early analyses assigned $J^{PC} = 2^{++}$.

There are a lot of sources for the production of f-like states. Among them – radiative decay of J/ψ, $\psi(2S)$ or $\Upsilon(1S)$:



- So, it is important to improve the precision of the parameters of f-like mesons and to check complementarity of beauty and charm hadron physics in the radiative decays.

Physics Motivations: Other experiments



Analysis strategy: EVENTS RECONSTRUCTION

- Used integrated luminosities of 13.6 fb^{-1} and 28.0 fb^{-1} at the Υ(2S) and Υ (3S) resonances

- We use the following full reconstructed decay chains:

$$\Upsilon(2S)/\Upsilon(3S) \rightarrow \pi_s^+\pi_s^- \Upsilon(1S)$$
$$\rightarrow \gamma h^+h^-$$

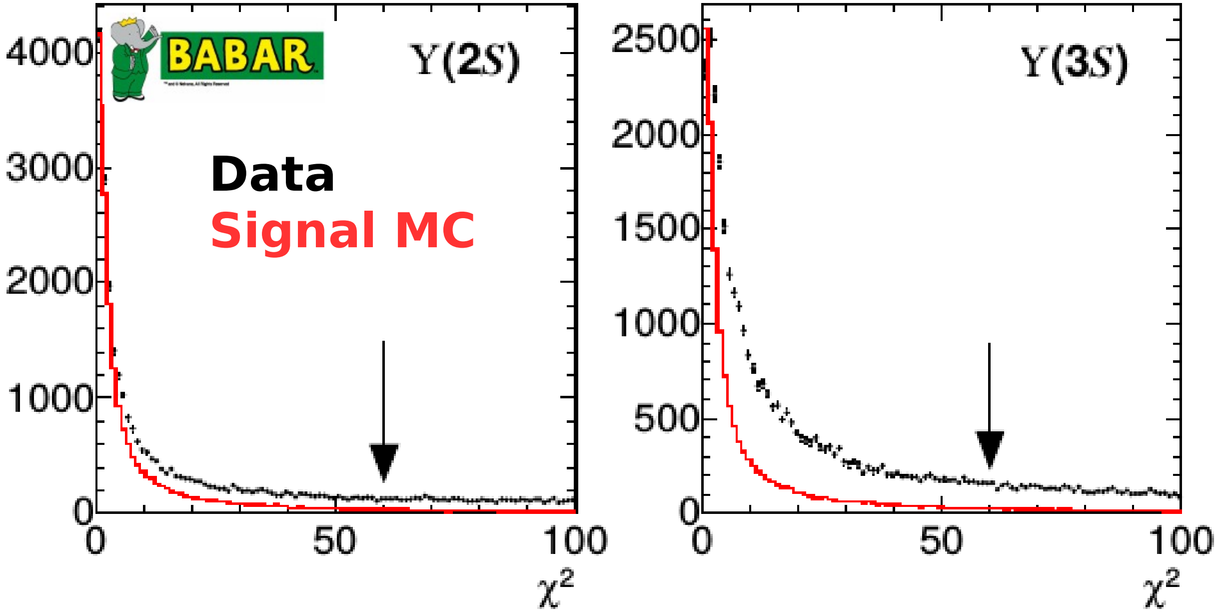
where h = π, K.

- The chain of the "reference" decay

$$\Upsilon(2S)/\Upsilon(3S) \rightarrow \pi_s^+\pi_s^- \Upsilon(1S)$$
$$\rightarrow \mu^+\mu^-$$

- We consider only events containing exactly four well-measured tracks with transverse momentum greater than 0.1 GeV/c

- We also require exactly one well-reconstructed γ in the calorimeter having an energy greater than 2.5 GeV



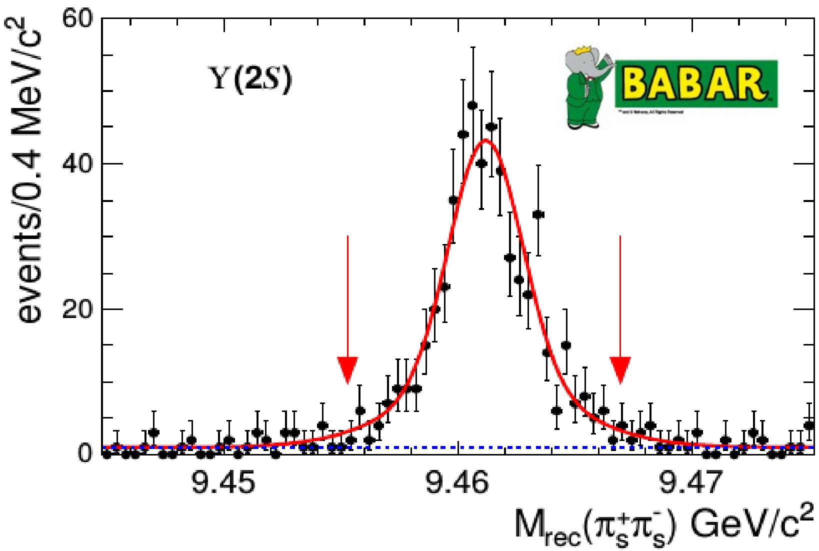
χ^2 distribution used for defining the momentum balance

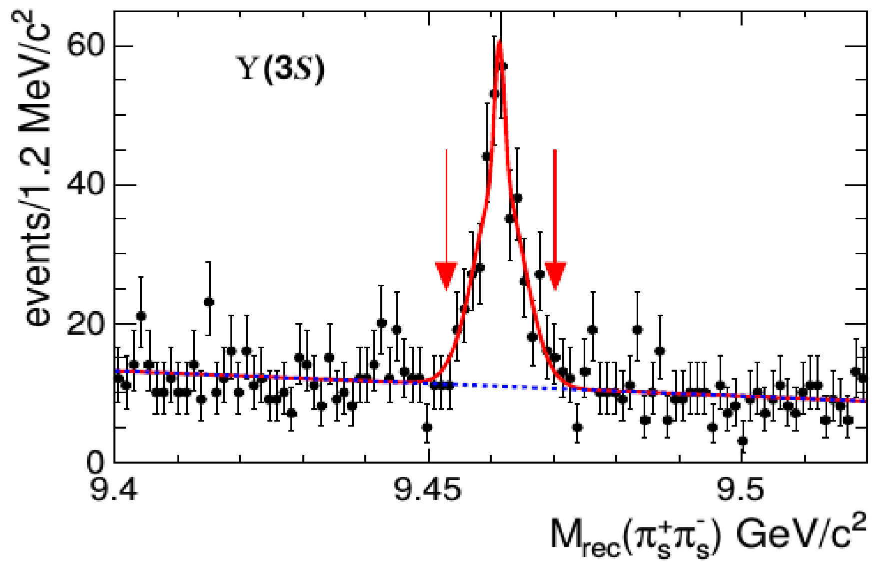
- $$\chi^2 = \sum_{i=1}^{3} \frac{(\Delta \mathbf{p}_i - \langle \Delta \mathbf{p}_i \rangle)^2}{\sigma_i^2}$$

- $\Delta \mathbf{p}_i = \mathbf{p}_i^{e^+} + \mathbf{p}_i^{e^-} - \sum_{j=1}^{5} \mathbf{p}_i^j$ - the missing laboratory three-momenta components

Analysis strategy: RECOILING MASS

$$M^2_{\rm rec}(\pi_s^+\pi_s^-) = |p_{e^+} + p_{e^-} - p_{\pi_s^+} - p_{\pi_s^-}|^2$$

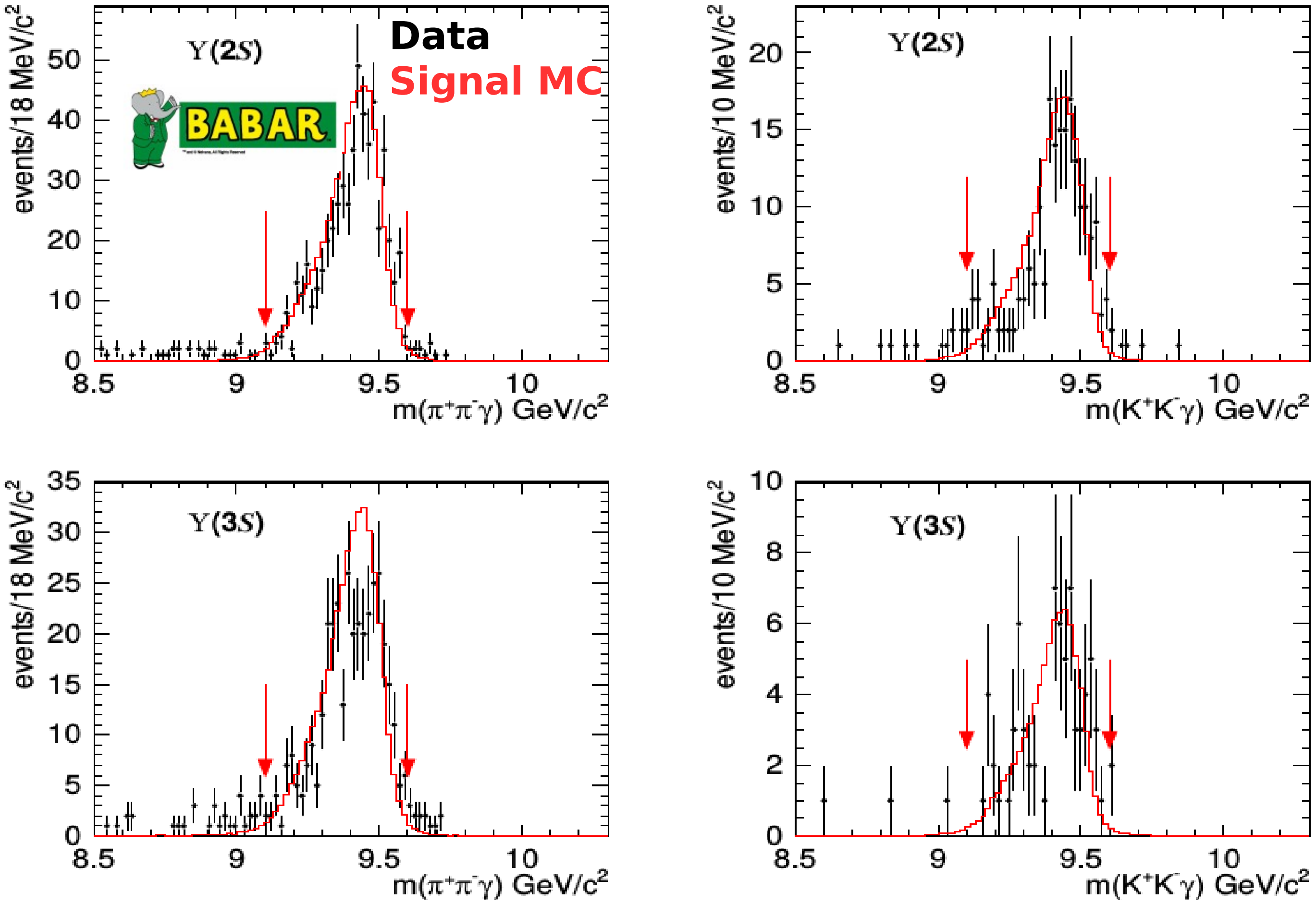




Combinatorial recoiling mass M_{rec} *to* $\pi_s^+\pi_s^-$ *candidates*

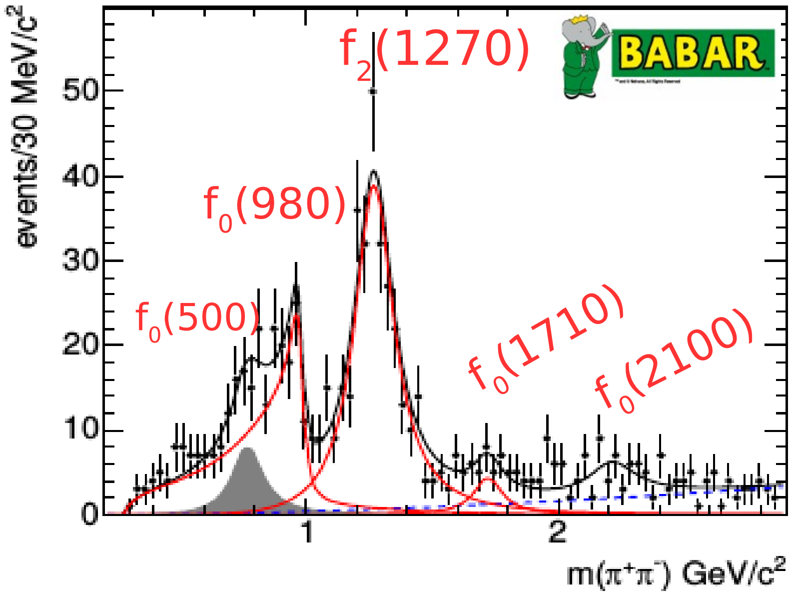
- Sidebands in the recoiling mass spectra are used for the study of background in further analysis.

Analysis strategy: THE ISOLATION OF $\Upsilon(1S)$

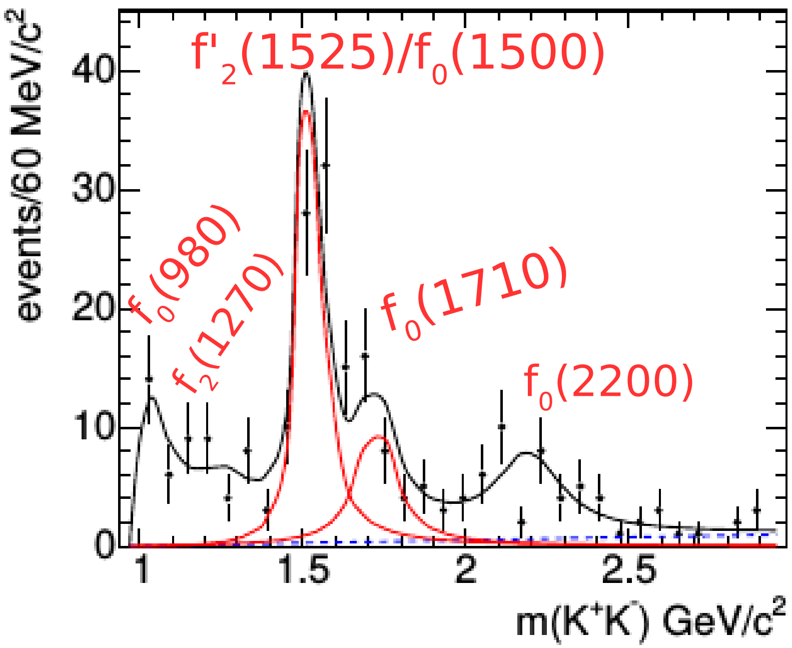


$M(\gamma h^+ h^-)$ mass distributions after the M_{rec} $(\pi_s^+\pi_s^-)$ selection

- We require $9.1\ \mathrm{GeV}/c^2 < M(\gamma h^+ h^-) < 9.6\ \mathrm{GeV}/c^2$



- 16 free parameters
- $\chi^2/ndf = 182/152$, $P(\chi^2) = 5\%$
- For the $\Upsilon(3S)$ data we also include $\rho(770)^0$ background.
- S-wave = $|BW[f_0(500)(m)] + c\cdot BW[f_0(980)(m)e^{i\varphi}]|^2$
- The fraction of S-wave events associated with the $f_0(500)$ is $(27.7 \pm 3.1)\%$
- $m(f_0(500)) = 0.856 \pm 0.086$ GeV/c^2 $\Gamma(f_0(500)) = 1.279 \pm 0.324$ GeV
- $m(f_0(2100)) = 2.208 \pm 0.068$ GeV/c^2.



- 6 free parameters
- $\chi^2/ndf = 35/29$, $P(\chi^2) = 20\%$
- Fits with only $f_2'(1525)$ and $f_0(1500)$ are performed. We label this contribution as $f_J(1500)$.

Two pions and two kaons invariant mass spectra

STUDY OF THE $\pi^+\pi^-$ AND K^+K^- MASS SPECTRA

Resonances ($\pi^+\pi^-$)	Yield $\Upsilon(2S)$	Yield $\Upsilon(3S)$	Significance
S-wave	$133 \pm 16 \pm 13$	87 ± 13	12.8σ
$f_2(1270)$	$255 \pm 19 \pm 8$	$77 \pm 7 \pm 4$	15.9σ
$f_0(1710)$	$24 \pm 8 \pm 6$	$6 \pm 8 \pm 3$	2.5σ
$f_0(2100)$	33 ± 9	8 ± 15	
$\rho(770)^0$		54 ± 23	

Resonances (K^+K^-)	Yield $\Upsilon(2S) + \Upsilon(3S)$	Significance
$f_0(980)$	47 ± 9	5.6σ
$f_J(1500)$	$77 \pm 10 \pm 10$	8.9σ
$f_0(1710)$	$36 \pm 9 \pm 6$	4.7σ
$f_2(1270)$	15 ± 8	
$f_0(2200)$	38 ± 8	

Resonances yields and statistical significances from the fits.

- The observation of a significant S-wave was not possible in the study of J/ψ radiative decay to $\pi^+\pi^-$ because of the presence of a irreducible background from $J/\psi \rightarrow \pi^+\pi^-\pi^0$ [PRD 35, 2077 (1987)].
- Systematic uncertainties are dominated by the uncertainties on resonances parameters

Branching fractions

- We compute branching fraction *B(R)* for resonance R using the expression

$$\mathcal{B}(R) = \frac{N(\Upsilon(nS)\to\pi^+\pi^-\Upsilon(1S)(\to R\gamma))}{N(\Upsilon(nS)\to\pi^+\pi^-\Upsilon(1S)(\to\mu^+\mu^-))} \cdot \mathcal{B}(\Upsilon(1S)(\to\mu^+\mu^-))$$

where N indicates the efficiency corrected yield for the given resonance.

Resonance	$\mathcal{B}(10^{-5})$
$\pi\pi$ S-wave	$4.63 \pm 0.56 \pm 0.48$
$f_2(1270)$	$10.15 \pm 0.59 \ {}^{+0.54}_{-0.43}$
$f_0(1710)\to\pi\pi$	$0.79 \pm 0.26 \pm 0.17$
$f_J(1500)\to K\bar{K}$	$3.97 \pm 0.52 \pm 0.55$
$f_2'(1525)$	$2.13 \pm 0.28 \pm 0.72$
$f_0(1500)\to K\bar{K}$	$2.08 \pm 0.27 \pm 0.65$
$f_0(1710)\to K\bar{K}$	$2.02 \pm 0.51 \pm 0.35$

BABAR

Γ_{34}	$\gamma f_0(980)$	< 3	$\times 10^{-5}$	90%
Γ_{35}	$\gamma f_2'(1525)$	$(3.7 \ {}^{+1.2}_{-1.1})$	$\times 10^{-5}$	
Γ_{36}	$\gamma f_2(1270)$	(1.01 ± 0.09)	$\times 10^{-4}$	
Γ_{38}	$\gamma f_0(1500)$	< 1.5	$\times 10^{-5}$	90%
Γ_{39}	$\gamma f_0(1710)$	< 2.6	$\times 10^{-4}$	90%
Γ_{40}	$\gamma f_0(1710) \to \gamma K^+ K^-$	< 7	$\times 10^{-6}$	90%

PDG

- We correct the efficiency corrected yields for isospin and for PDG measured branching fractions.

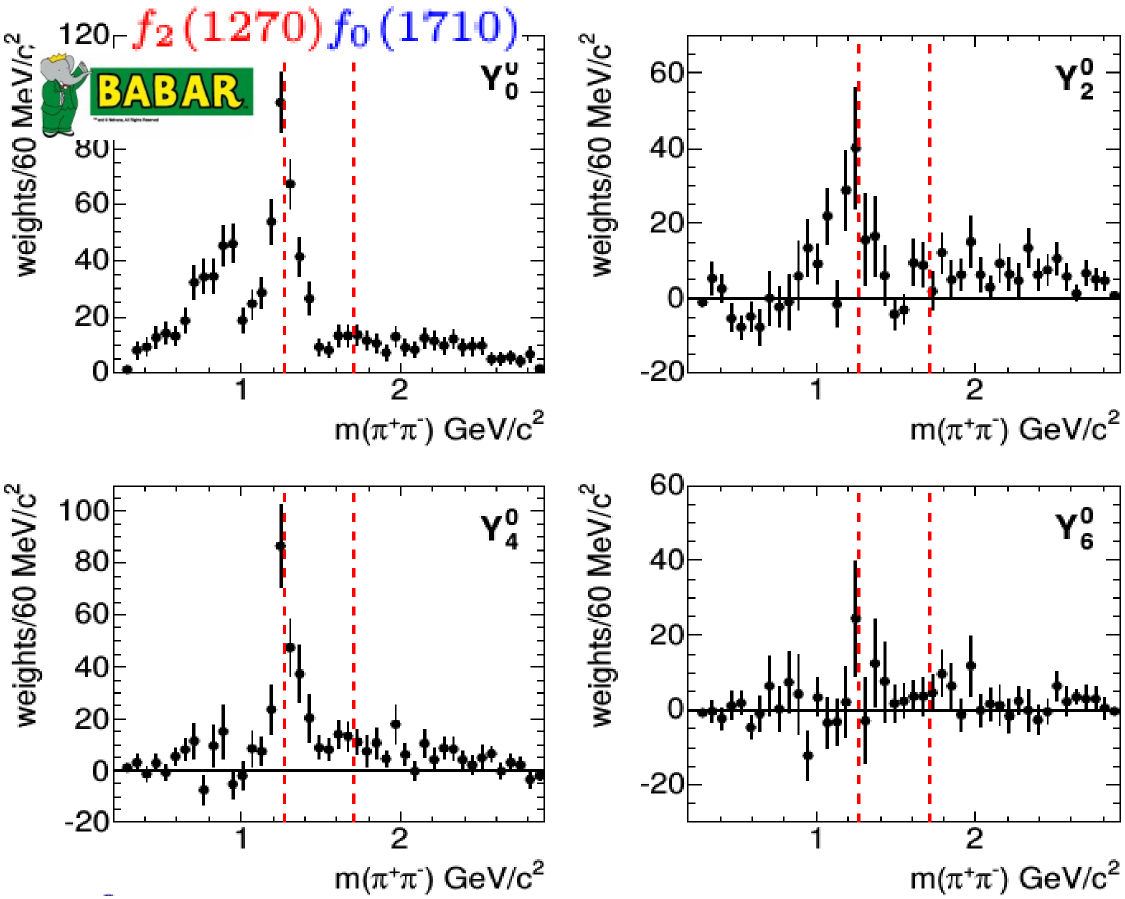
- $\dfrac{\mathcal{B}(f_0(1710)\to\pi\pi)}{\mathcal{B}(f_0(1710)\to K\bar{K})} = 0.64 \pm 0.27_{\text{stat}} \pm 0.18_{\text{sys}}$

$0.41^{+0.11}_{-0.17}$

BES2 $e^+e^- \to J/\psi \to \gamma\pi^+\pi^-$

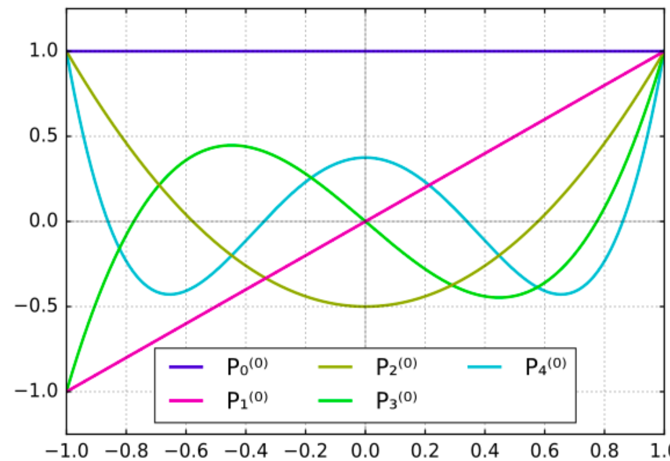
Legendre polynomial moments, $\pi^+\pi^-$.

- Efficiency corrected $\pi^+\pi^-$ mass spectrum weighted by Legendre polynomial moments:



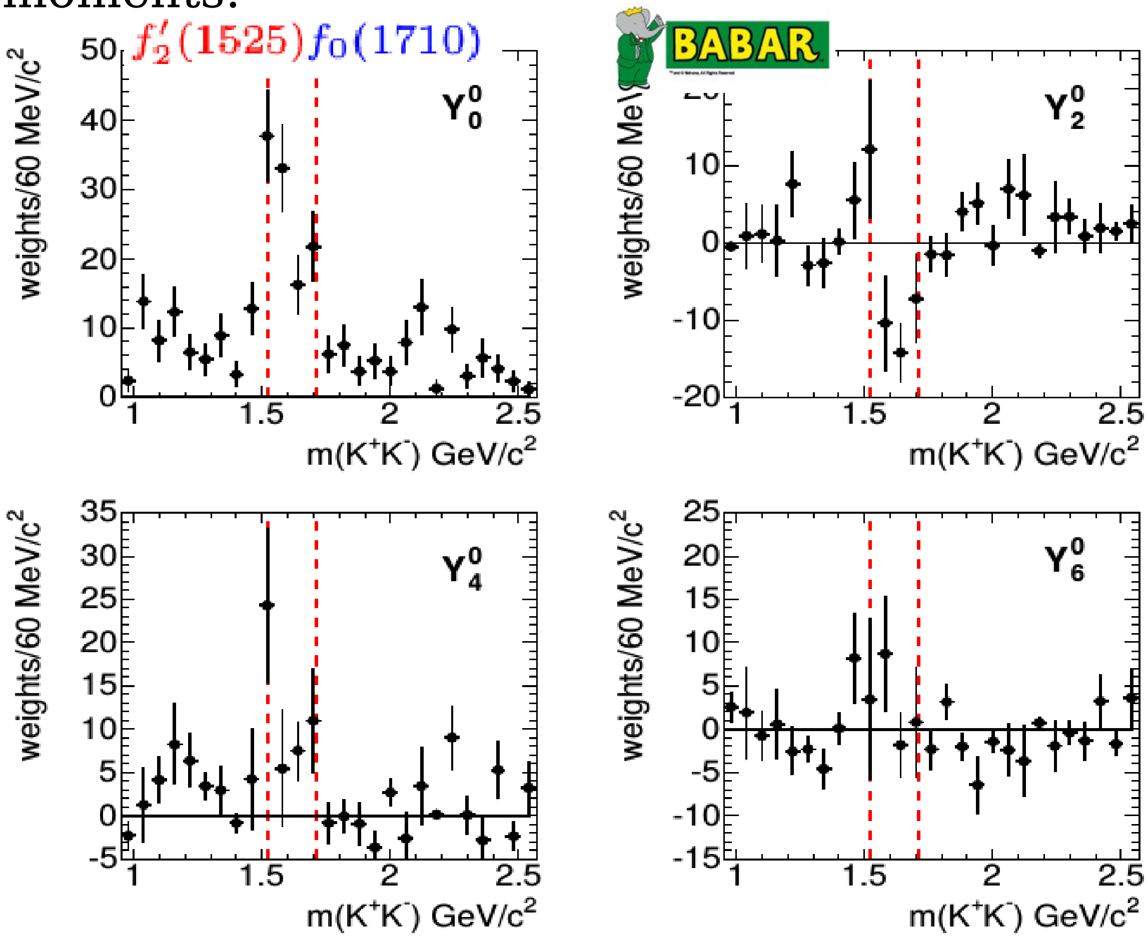
$$\sqrt{4\pi} < Y^0_0 >= S^2 + D^2$$
$$\sqrt{4\pi} < Y^0_2 >= 2SD\cos\phi_{SD} + 0.639D^2$$
$$\sqrt{4\pi} < Y^0_4 >= 0.857D^2$$

$$Y_{\ell,m}(\theta,\phi) = \sqrt{\frac{(2\ell+1)(\ell-m)!}{4\pi(\ell+m)!}}\, P^m_\ell(\cos\theta)\, e^{im\phi}$$



- Y_2^0 is related to the S-D interference, clearly visible at the $f_2(1270)$ mass.
- Y_4^0 is related to D-wave, clearly visible at the $f_2(1270)$ mass.

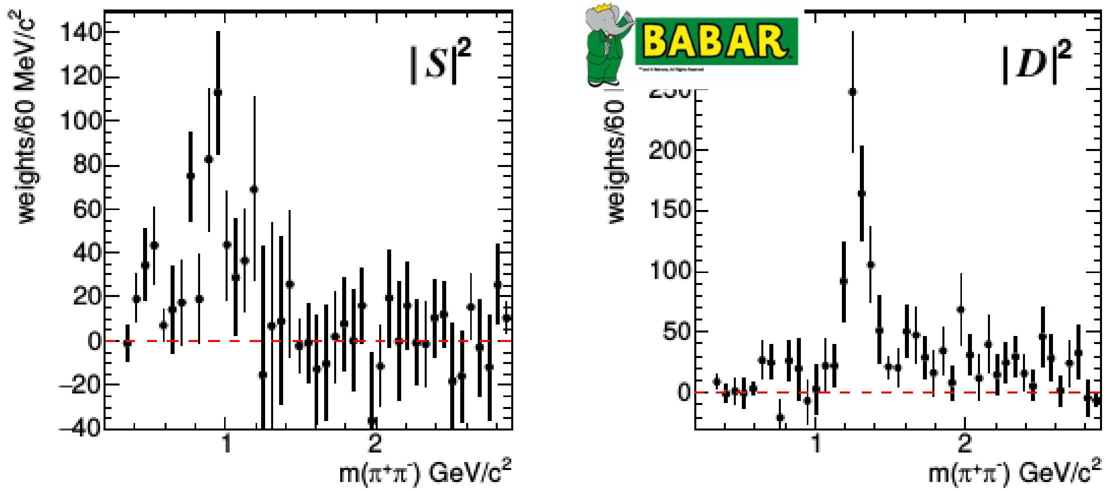
- Efficiency corrected K^+K^- mass spectrum weighted by Legendre polynomial moments:



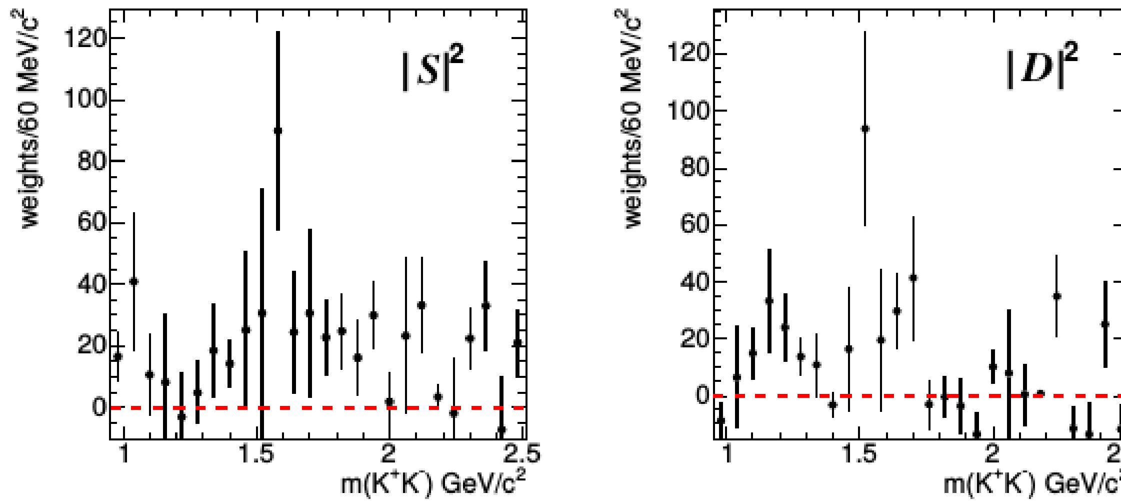
- Y_2^0 is related to the S-D interference, clearly visible at the $f_2'(1525)$ mass.
- Y_4^0 is related to D-wave, clearly visible at the $f_2'(1525)$ mass.
- Activity in Y_2^0 and Y_4^0 in the $f_0(1710)$ region.

$$\sqrt{4\pi} < Y_0^0 >= S^2 + D^2$$
$$\sqrt{4\pi} < Y_2^0 >= 2SD\cos\phi_{SD} + 0.639D^2$$
$$\sqrt{4\pi} < Y_4^0 >= 0.857D^2$$

By direct solving



(a) S and (b) D-wave contributions to the production of $\pi^+\pi^-$



(a) S and (b) D-wave contributions to the production of K^+K^-

- **$\pi^+\pi^-$:** The accumulation of events at threshold in fact belongs to S-wave

- **K^+K^-:** The structure around 1.5 GeV can be explained as the sum of contributions of $f_0(1500)$ and $f_2'(1525)$

- We define θ_γ as the angle formed by the radiative photon in the $h^+h^-\gamma$ rest frame with respect to the Υ(1S) direction in the Υ(2S)/Υ (3S) rest frame.
- We perform a 2-D unbinned maximum likelihood fit to ($\cos\theta_\gamma$ vs. $\cos\theta_H$) spectrum in the regions around resonances.
- The Likelihood function L is written as:

$$\mathcal{L} = \prod_{n=1}^{N} [f_{sig} \frac{\epsilon(\cos\theta_H, \cos\theta_\gamma) A_s(\cos\theta_H, \cos\theta_\gamma)}{\int A_s(\cos\theta_H, \cos\theta_\gamma)\epsilon(\cos\theta_H, \cos\theta_\gamma) \mathrm{d}\cos\theta_H \mathrm{d}\cos\theta_\gamma} +$$
$$(1 - f_{sig}) \frac{\epsilon(\cos\theta_H, \cos\theta_\gamma) A_b(\cos\theta_H, \cos\theta_\gamma)}{\int A_b(\cos\theta_H, \cos\theta_\gamma)\epsilon(\cos\theta_H, \cos\theta_\gamma) \mathrm{d}\cos\theta_H \mathrm{d}\cos\theta_\gamma}]$$

- f_{sig} is the fraction of signal, $\varepsilon(\cos\theta_H, \cos\theta_\gamma)$ is the fitted efficiency.

- A_s and A_b are the probability densities for signal and background, respectively. The form of A_s depends strongly on the spin of the resonance.

- We consider as background only the contamination due to the tails of nearby adjacent resonances.

- 104 events

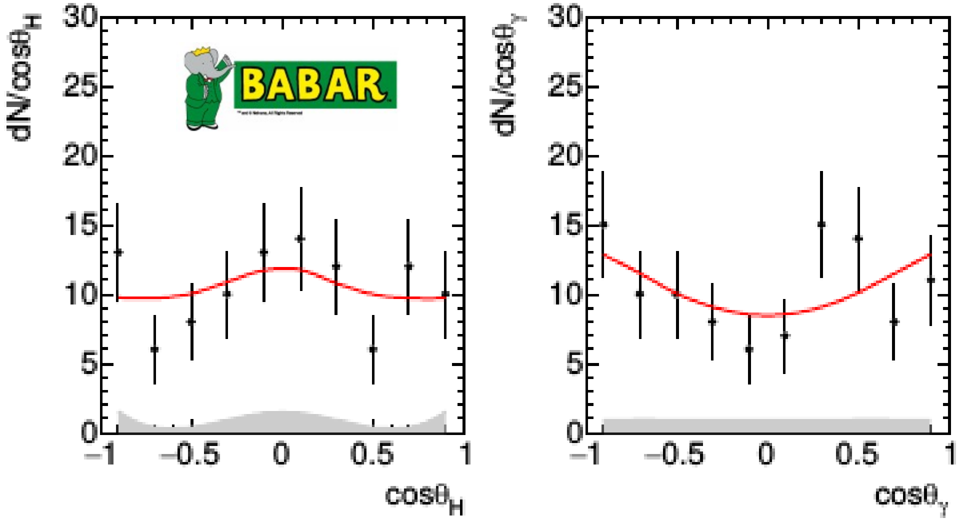
- Background (in gray) from $f_2(1270)$ is 9%

- Only one free parameter: The ratio of the amplitudes corresponding to helicities 0 and 1 of Υ(1S)

- Figure of merit:

$f = (\chi^2(\cos\theta_H)+\chi^2(\cos\theta_\gamma))/ndf$

$ndf = Nbins - Nparam$



Uncorrected (a)cos θ_H and (b)cos θ_γ distributions

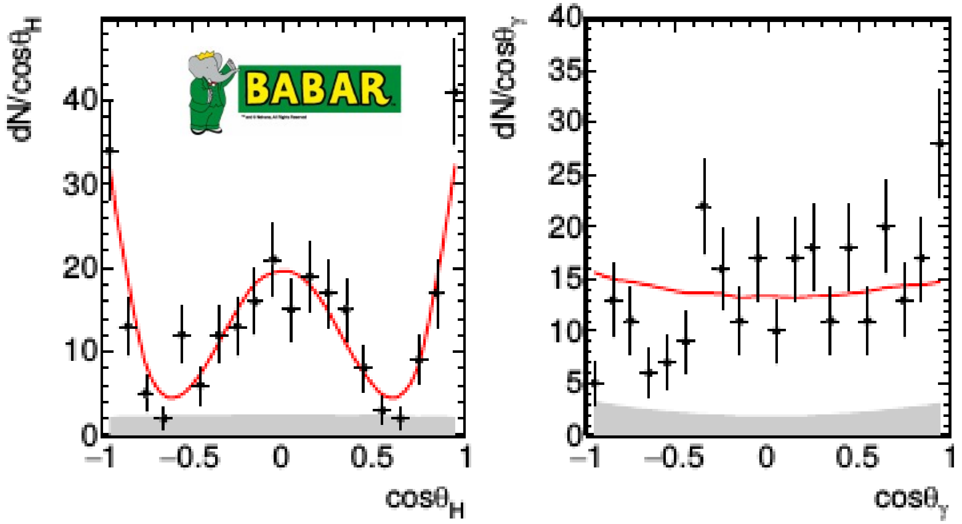
We obtain:

- f = 14.3/19

- a good description of the data consistent with the spin 0 hypothesis

The $f_2(1270) \to \pi^+\pi^-$: $1.092 < m_{\pi^+\pi^-} < 1.460$

- 280 events
- One free parameter is the ratio of the amplitudes corresponding to helicities 0 and 1 of $\Upsilon(1S)$
- Two parameters are the ratios of the amplitudes corresponding to helicities 0, 1 and 2 of $f_2(1270)$.



Uncorrected (a)cos θ_H and (b)cos θ_γ distributions

- Background from S-wave is 16%.

- f = 70/37 for spin 2

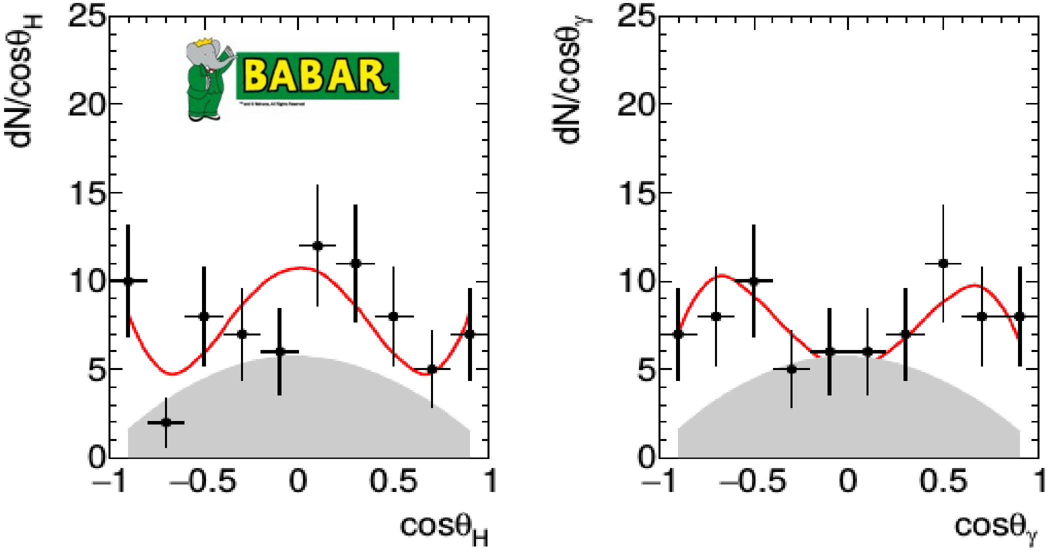
- a good description of the cos θ_H projection

- a poor description of the cos θ_γ projection. This may be due to the possible unaccounted presence of additional scalar components

The $f_J(1500) \to K^+K^-$: $1.424 < m_{\pi+\pi-} < 1.620$

- 76 events

Fit #1
- **superposition of S and D waves** (we assign S to $f_0(1500)$, D to $f_2'(1525)$)
- Three helicity contributions as free parameters, and one free S-wave contribution
- **f = 8.5/16 = 1.22**
- The shaded area represents the spin-0 contribution
- adequate description of the data



Uncorrected (a)cos θ_H and (b)cos θ_γ distributions

Fit #2
- the presence of **the spin-2 $f_2'(1525)$ only**

$\Delta(-2\log \boldsymbol{L}) = 1.3$

- Due the low statistics we cannot statistically distinguish between the two hypotheses.

Summary

- Spin-parity analyses and branching fraction measurements are reported for the resonances observed in the $\pi^+\pi^-$ and K^+K^- mass spectra.

- We observed of broad S-wave, $f_0(980)$, and $f_2(1270)$ resonances in the $\pi^+\pi^-$ mass spectrum.

- We observed a signal in the 1500 MeV/c^2 mass region of the K^+K^- mass spectrum. The spin analysis indicates contributions from $f_2'(1525)$ and $f_0(1500)$ resonances.

- We report observation of $f_0(1710)$ in both $\pi^+\pi^-$ and K^+K^- mass spectra with combined significance of 5.7σ.

Thank You!

BACK UP

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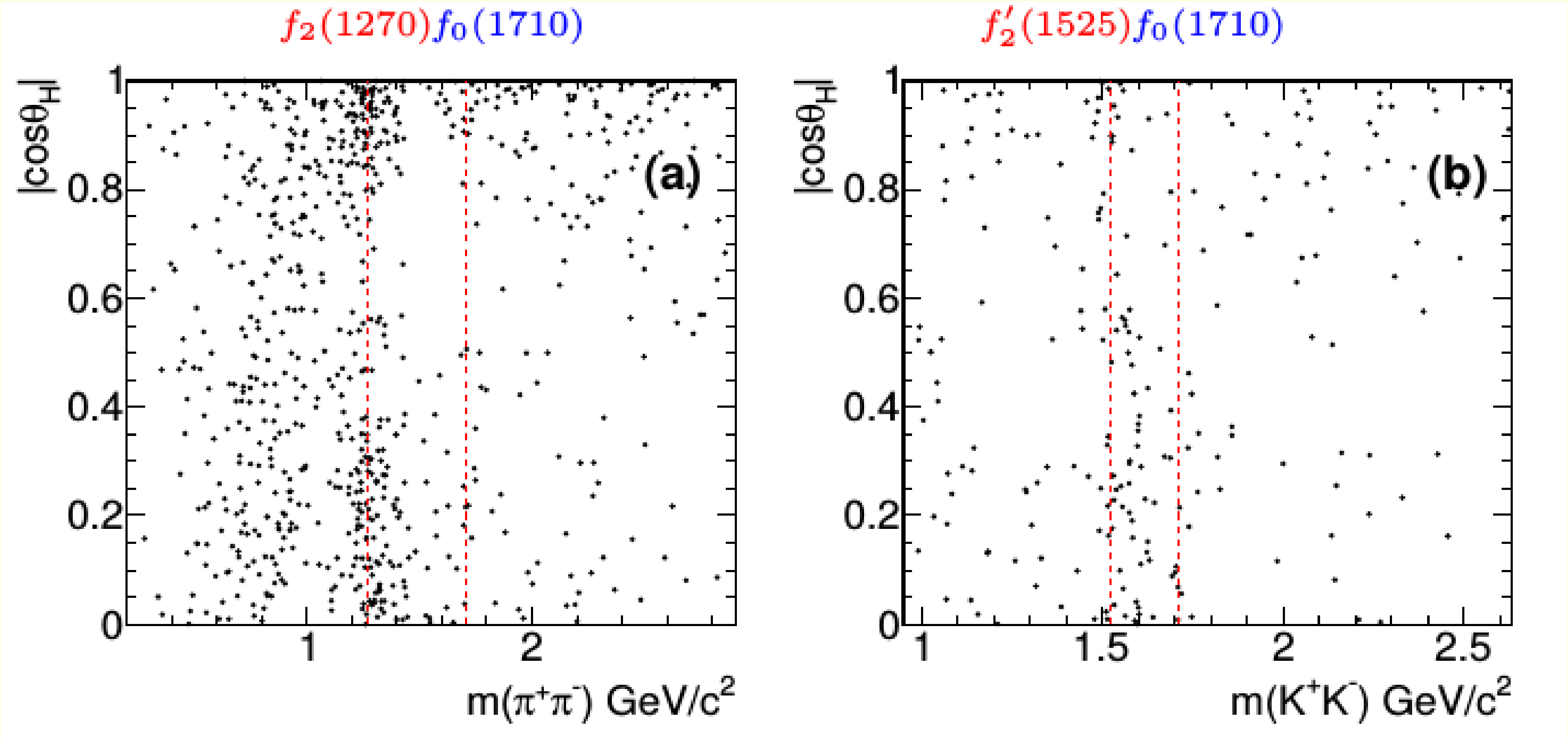
Angular analysis



Scatter diagram $\cos\theta_H$ vs. $m(\pi^+\pi^-)$ and $\cos\theta_H$ vs. $m(K^+K^-)$.

- We observe clearly the spin-2 structure of the $f_2(1270)$.

$$
\begin{aligned}
\frac{dU}{d\cos\theta_\gamma\, d\cos\theta_{\pi^+}} \;=\;& \frac{15}{1024}|E_{00}|^2 \left[6|A_{01}|^2 \left(22|C_{10}|^2 + 8|C_{11}|^2 + 9|C_{12}|^2\right) +\right.\\
& 2|A_{00}|^2 \left(22|C_{10}|^2 + 24|C_{11}|^2 + 9|C_{12}|^2\right) +\\
& 24 \left(|A_{00}|^2 + 3|A_{01}|^2\right) \left(2|C_{10}|^2 - |C_{12}|^2\right) \cos 2\theta_{\pi^+} +\\
& 6 \left(|A_{00}|^2 \left(6|C_{10}|^2 - 8|C_{11}|^2 + |C_{12}|^2\right) +\right.\\
& \left.|A_{01}|^2 \left(18|C_{10}|^2 - 8|C_{11}|^2 + 3|C_{12}|^2\right)\right) \cos 4\theta_{\pi^+} -\\
& 2 \left(|A_{00}|^2 - |A_{01}|^2\right) \cos 2\theta_\gamma \left(22|C_{10}|^2 - 24|C_{11}|^2 + 9|C_{12}|^2 +\right.\\
& 12 \left(2|C_{10}|^2 - |C_{12}|^2\right) \cos 2\theta_{\pi^+} +\\
& \left.\left.3 \left(6|C_{10}|^2 + 8|C_{11}|^2 + |C_{12}|^2\right) \cos 4\theta_{\pi^+}\right)\right]
\end{aligned}
$$

$$
\frac{dU_{\theta_\gamma}}{d\cos\theta_\gamma} = \frac{3}{8}|C_{10}|^2|E_{00}|^2 \left(|A_{00}|^2 + 3|A_{01}|^2 - \left(|A_{00}|^2 - |A_{01}|^2\right) \cos 2\theta_\gamma\right)
$$

TABLE III: Results from the helicity amplitude fits to resonances decaying to $\pi^+\pi^-$ and K^+K^-.

Resonance	mass range (GeV/c^2)	events	spin	χ_H, χ_γ, χ^2_t/ndf	$\|A_{00}\|^2/\|A_{01}\|^2$		
$\pi\pi$ S-wave	0.6-1.0	104	0	5.8, 8.4, 14.3/19	0.09 ± 0.33		
					$\|A_{01}\|^2/\|A_{00}\|^2$	$\|C_{11}\|^2/\|C_{10}\|^2$	$\|C_{12}\|^2/\|C_{10}\|^2$
$f_2(1270)\to\pi^+\pi^-$	1.092-1.460	280	2	24.0, 46.0, 70/37	1.07 ± 0.31	0.00 ± 0.03	0.29 ± 0.08
$f_2'(1525)\to K^+K^-$	1.424-1.620	36	2	6.7, 1.8, 8.5/16	47.9 ± 10.8	0.42 ± 0.36	1.43 ± 0.35
$f_0(1500)\to K^+K^-$		40	0		0.04 ± 0.07		

The moments $\sigma\langle Y_l^m\rangle$ in terms of the amplitudes L_0 and $L_\pm$

$$\sqrt{4\pi}\,\sigma\langle Y_0^0\rangle = S_0^2 + P_0^2 + P_-^2 + P_+^2 + D_0^2 + D_-^2 + D_+^2 + F_0^2 + F_-^2 + F_+^2 + G_0^2 + G_-^2 + G_+^2 + H_0^2 + H_-^2 + H_+^2$$

$$\sqrt{4\pi}\,\sigma\langle Y_1^0\rangle = 2.S_0P_0 + 1.789P_0D_0 + 1.549(P_-D_- + P_+D_+) + 1.757D_0F_0 + 1.656(D_-F_- + D_+F_+) + 1.746F_0G_0 + 1.690(F_-G_- + F_+G_+) + 1.741G_0H_0 + 1.706(G_-H_- + G_+H_+)$$

$$\sqrt{4\pi}\,\sigma\langle Y_1^1\rangle = 1.414S_0P_- + 1.095P_0D_- - 0.632P_-D_0 - 1.014D_0F_- - 0.717D_-F_0 + 0.976F_0G_- - 0.756F_-G_0 + 0.953G_0H_- - 0.778G_-H_0$$

$$\sqrt{4\pi}\,\sigma\langle Y_2^0\rangle = 2.S_0D_0 + 0.894P_0^2 + 1.757P_0F_0 - 0.447(P_-^2 + P_+^2) + 1.434(P_-F_- + P_+F_+) + 0.639D_0^2 + 1.714D_0G_0 + 0.319(D_-^2 + D_+^2) + 1.565(D_-G_- + D_+G_+) + 0.596F_0^2 + 1.699F_0H_0 + 0.447(F_-^2 + F_+^2) + 1.612(F_-H_- + F_+H_+) + 0.581G_0^2 + 0.494(G_-^2 + G_+^2) + 0.573H_0^2 + 0.516(H_-^2 + H_+^2)$$

$$\sqrt{4\pi}\,\sigma\langle Y_2^1\rangle = 1.414S_0D_- + 1.095P_0P_- + 1.171P_0F_- - 0.717P_-F_0 + 0.452D_0D_- + 1.107D_0G_- - 0.808D_-G_0 + 0.298F_0F_- + 1.074F_0H_- - 0.849F_-H_0 + 0.225G_0G_- + 0.181H_0H_-$$

$$\sqrt{4\pi}\,\sigma\langle Y_2^2\rangle = 0.548(P_-^2 - P_+^2) - 0.293(P_-F_- - P_+F_+) + 0.391(D_-^2 - D_+^2) - 0.319(D_-G_- - D_+G_+) + 0.365(F_-^2 - F_+^2) - 0.329(F_-H_- - F_+H_+) + 0.356(G_-^2 - G_+^2) + 0.351(H_-^2 - H_+^2)$$

$$\sqrt{4\pi}\,\sigma\langle Y_3^0\rangle = 2.S_0F_0 + 1.757P_0D_0 + 1.746P_0G_0 - 1.014(P_-D_- + P_+D_+) + 1.380(P_-G_- + P_+G_+) + 1.193D_0F_0 + 1.699D_0H_0 + 0.422(D_-F_- + D_+F_+) + 1.519(D_-H_- + D_+H_+) + 1.091F_0G_0 + 0.704(F_-G_- + F_+G_+) + 1.052G_0H_0 + 0.816(G_-H_- + G_+H_+)$$

$$\sqrt{4\pi}\,\sigma\langle Y_3^1\rangle = 1.414S_0F_- + 1.171P_0D_- + 1.195P_0G_- + 1.014P_-D_0 - 0.756P_-G_0 + 0.632D_0F_- + 1.140D_0H_- + 0.298D_-F_0 - 0.849D_-H_0 + 0.498F_0G_- + 0.129F_-G_0 + 0.431G_0H_- + 0.048G_-H_0$$

$$\sqrt{4\pi}\,\sigma\langle Y_3^2\rangle = 0.926(P_-D_- - P_+D_+) - 0.378(P_-G_- - P_+G_+) + 0.577(D_-F_- - D_+F_+) - 0.416(D_-H_- - D_+H_+) + 0.514(F_-G_- - F_+G_+) + 0.490(G_-H_- - G_+H_+)$$

$$\sqrt{4\pi}\,\sigma\langle Y_4^0\rangle = 2.S_0G_0 + 1.746P_0F_0 + 1.741P_0H_0 - 1.069(P_-F_- + P_+F_+) + 1.348(P_-H_- + P_+H_+) + 0.857D_0^2 + 1.162D_0G_0 - 0.571(D_-^2 + D_+^2) + 0.318(D_-G_- + D_+G_+) + 0.545F_0^2 + 1.052F_0H_0 + 0.091(F_-^2 + F_+^2) + 0.610(F_-H_- + F_+H_+) + 0.485G_0^2 + 0.243(G_-^2 + G_+^2) + 0.462H_0^2 + 0.308(H_-^2 + H_+^2)$$

$$\sqrt{4\pi}\,\sigma\langle Y_4^1\rangle = 1.414S_0G_- + 1.195P_0F_- + 1.206P_0H_- + 0.976P_-F_0 - 0.778P_-H_0 + 1.107D_0D_- + 0.698D_0G_- + 0.225D_-G_0 + 0.498F_0F_- + 0.577F_0H_- + 0.048F_-H_0 + 0.343G_0G_- + 0.266H_0H_-$$

$$\sqrt{4\pi}\,\sigma\langle Y_4^2\rangle = 0.845(P_-F_- - P_+F_+) - 0.426(P_-H_- - P_+H_+) + 0.452(D_-^2 - D_+^2) + 0.453(D_-G_- - D_+G_+) + 0.287(F_-^2 - F_+^2) + 0.377(F_-H_- - F_+F_+) + 0.256(G_-^2 - G_+^2) + 0.243(H_-^2 - H_+^2)$$

$$\sqrt{4\pi}\,\sigma\langle Y_5^0\rangle = 2.S_0H_0 + 1.741P_0G_0 - 1.101(P_-G_- + P_+G_+) + 1.699D_0F_0 + 1.147D_0H_0 - 1.201(D_-F_- + D_+F_+) + 0.256(D_-H_- + D_+H_+) + 1.052F_0G_0 + 0.068(F_-G_- + F_+G_+) + 0.923G_0H_0 + 0.377(G_-H_- + G_+H_+)$$

$$\sqrt{4\pi}\,\sigma\langle Y_5^1\rangle = 1.414S_0H_- + 1.206P_0G_- + 0.953P_-G_0 + 1.140D_0F_- + 0.730D_0H_- + 1.074D_-F_0 + 0.181D_-H_0 + 0.577F_0G_- + 0.431F_-G_0 + 0.435G_0H_- + 0.266G_-H_0$$

$$\sqrt{4\pi}\,\sigma\langle Y_5^2\rangle = 0.798(P_-G_- - P_+G_+) + 0.870(D_-F_- - D_+F_+) + 0.372(D_-H_- - D_+H_+) + 0.525(F_-G_- - F_+G_+) + 0.455(G_-H_- - G_+H_+)$$

$$\sqrt{4\pi}\,\sigma\langle Y_6^0\rangle = 1.738P_0H_0 - 1.122(P_-H_- + P_+H_+) + 1.691D_0G_0 - 1.235(D_-G_- + D_+G_+) + 0.840F_0^2 + 1.032F_0H_0 - 0.630(F_-^2 + F_+^2) + 0.504G_0^2 - 0.025(G_-^2 + G_+^2) + 0.435H_0^2 + 0.131(H_-^2 + H_+^2)$$

$$\sqrt{4\pi}\,\sigma\langle Y_6^1\rangle = 1.212P_0H_- + 0.939P_-H_0 + 1.155D_0G_- + 1.055D_-G_0 + 1.112F_0F_- + 0.617F_0H_- + 0.390F_-H_0 + 0.517G_0G_- + 0.364H_0H_-$$

$$\sqrt{4\pi}\,\sigma\langle Y_6^2\rangle = 0.766(P_-H_- - P_+H_+) + 0.844(D_-G_- - D_+G_+) + 431(F_-^2 - F_+^2) + 0.478(F_-H_- - F_+H_+) + 0.258(G_-^2 - G_+^2) + 0.223(H_-^2 - H_+^2)$$

$$\sqrt{4\pi}\,\sigma\langle Y_7^0\rangle = 1.687D_0H_0 - 1.258(D_-H_- + D_+H_+) + 1.672F_0G_0 - 1.295(F_-G_- + F_+G_+) + 0.986G_0H_0 - 0.121(G_-H_- + G_+H_+)$$

$$\sqrt{4\pi}\,\sigma\langle Y_7^1\rangle = 1.164D_0H_- + 1.041D_-H_0 + 1.130F_0G_- + 1.095F_-G_0 + 0.562G_0H_- + 0.479G_-H_0$$

$$\sqrt{4\pi}\,\sigma\langle Y_7^2\rangle = 0.823(D_-H_- - D_+H_+) + 0.848(F_-G_- - F_+G_+) + 0.494(G_-H_- - G_+H_+)$$

$$\sqrt{4\pi}\,\sigma\langle Y_8^0\rangle = 1.667F_0H_0 - 1.318(F_-H_- + F_+H_+) + 0.831G_0^2 - 0.665(G_-^2 + G_+^2) + 0.481H_0^2 - 0.096(H_-^2 + H_+^2)$$

$$\sqrt{4\pi}\,\sigma\langle Y_8^1\rangle = 1.141F_0H_- + 1.083F_-H_0 + 1.115G_0G_- + 0.527H_0H_-$$

$$\sqrt{4\pi}\,\sigma\langle Y_8^2\rangle = 0.835(F_-H_- - F_+H_+) + 0.421(G_-^2 - G_+^2) + 0.244(H_-^2 - H_+^2)$$

$$\sqrt{4\pi}\,\sigma\langle Y_9^0\rangle = 1.656G_0H_0 - 1.352(G_-H_- + G_+H_+)$$

$$\sqrt{4\pi}\,\sigma\langle Y_9^1\rangle = 1.127G_0H_- + 1.104G_-H_0$$

$$\sqrt{4\pi}\,\sigma\langle Y_9^2\rangle = 0.836(G_-H_- - G_+H_+)$$

$$\sqrt{4\pi}\,\sigma\langle Y_{10}^0\rangle = 0.825H_0^2 - 0.688(H_-^2 + H_+^2)$$

$$\sqrt{4\pi}\,\sigma\langle Y_{10}^1\rangle = 1.117H_0H_-$$

$$\sqrt{4\pi}\,\sigma\langle Y_{10}^2\rangle = 0.416(H_-^2 - H_+^2)$$

We use the abbreviated notation $LL' = \mathrm{Re}(LL'^*)$. A summation over helicity flip and non-flip indices at the nucleon vertex is implicit, that is $\mathrm{Re}(LL'^*) = \mathrm{Re}(L_{\text{flip}}L'^*_{\text{flip}}) + \mathrm{Re}(L_{\text{non-flip}}L'^*_{\text{non-flip}})$, see sect. 2.