

Properties of $Z_b(10610)$ and $Z_b(10650)$ from an analysis of experimental line shapes

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Charm 2018, Novosibirsk

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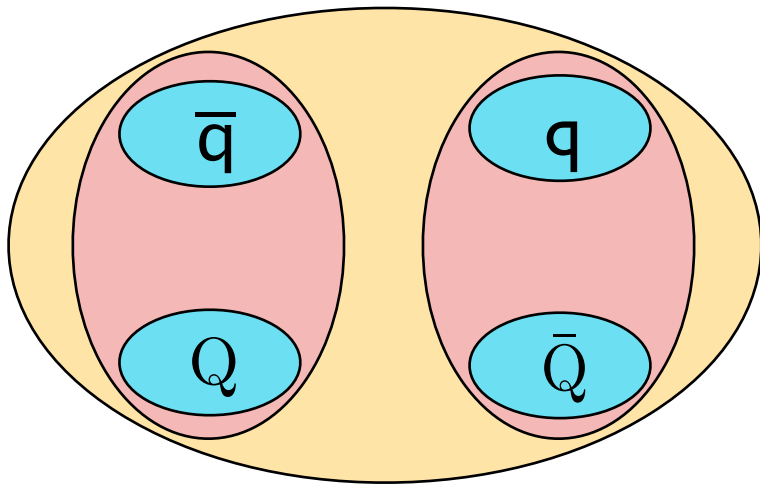
arXiv: 1805.07453

Intro to hadronic molecules

more in a review talk by Alexey Nefediev on Monday

- Plenty of observed XYZ states inconsistent with a quark model picture

Hadronic molecules — a special class of exotic states



- ➡ reside very close to hadronic thresholds and couple to them in S-wave
- ➡ decay predominantly to open-flavour channels
- ➡ specific analytic properties: unitarity cut

observable coupling of a state to a hadronic channel

$$\bar{g}_{\text{eff}} = \frac{\sqrt{2\mu\epsilon}}{\mu} \lambda^2 + O\left(\frac{\sqrt{2\mu\epsilon}}{\beta}\right) \leq \frac{\sqrt{2\mu\epsilon}}{\mu}$$

probability of finding a molecular component in the full w.f.

— non-analytic in binding energy ϵ

Weinberg 1963-65

our work 2004

...

Natural candidates:

$X(3872)$ and $Z_b(10610)/Z_b(10650)$

Belle (2010-2016)

Heavy-quark spin symmetry (HQSS)

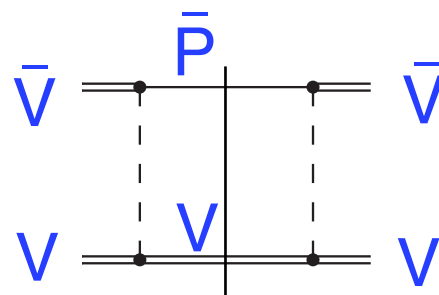
- In the limit $\Lambda_{\text{QCD}}/m_Q \rightarrow 0$ strong interactions are independent of HQ spin
- Spin decouples $\Rightarrow$ spin partner states $\Rightarrow$ test of the nature! $\mathcal{H} \propto \frac{\sigma \cdot B}{m_Q}$
Cleven et al. (2015)

$\Rightarrow$ plenty of predictions for the partners using contact interactions

Bondar et al. (2011), Voloshin (2011), Mehen and Powell (2011), Guo et al. (2015)

- Role of one-pion exchange (OPE) for the molecular partners can be non-trivial
Our works: PLB 763, 20 (2016), JHEP 1706, 158 (2017)

— HQSS violation from the mass splitting δ is enhanced by strong S-D transitions from OPE



The diagram shows a heavy quark V and an anti-heavy quark $\bar{V}$ interacting via one-pion exchange. The quark lines are horizontal, with the anti-quark at the top and the quark at the bottom. A vertical line represents the pion exchange, with dashed lines indicating the quark-to-pion and anti-quark-to-pion vertices. The pion is labeled P at the top and V at the bottom.

$$p_{\text{typ}} \simeq \sqrt{2\mu\delta} \approx 500 \text{ MeV} \geq m_\pi$$

$$\delta = m_V - m_P$$

$$V = D^* \text{ and } B^*$$

$$P = D \text{ and } B$$

- Assuming molecules are bound states $\Rightarrow$ visible shifts of spin partners from thresholds, especially in the c-quark sector

But accurate line shape analyses with the OPE included are needed!

Molecular states from line shape analyses: Goals

- Analyse line shapes in a chiral EFT approach
 - obeying chiral and HQ symmetries of QCD
 - manifestly analytic and unitary

The goals:

- infer the role of $SU(3)$ GB octet interactions (π and η exchanges) and HQSS violation
- extract the molecular state poles directly from line shapes
- predict HQSS partner states parameter free
- investigate chiral extrapolations of molecular states $\longleftrightarrow$ lattice

Can be applied to various molecule candidates: $X(3872)$, $Z_c(3900)$, $Z_c(4020)$, ...
 $Z_b(10610)$, $Z_b(10650)$...

This Talk — application to $J^{PC} = 1^{+-}$ $Z_b(10610)$ and $Z_b(10650)$

Formalism for line shapes $\Upsilon(10860) \rightarrow \pi Z_b^{(')} \rightarrow \pi\alpha$

- Input: experimental distributions for

$$\Upsilon(10860) \rightarrow \pi Z_b^{(')} \rightarrow \pi\alpha \quad \alpha = BB^*, \quad B^*B^*, \quad h_b(1P)\pi, \quad h_b(2P)\pi$$

and branching fractions for $\alpha = B\bar{B}^*, B^*\bar{B}^*, h_b(1P)\pi, h_b(2P)\pi, \Upsilon(1S)\pi, \Upsilon(2S)\pi, \Upsilon(3S)\pi$
 Belle: Bondar et al. (2012), Garmash et al. (2016)

- $\Upsilon(mS)\pi\pi$ distributions not included: involve sizeable $\pi\pi$ FSI

— Recent calculations for $\Upsilon(3S) \rightarrow \Upsilon(1S)\pi\pi$, $\Upsilon(4S) \rightarrow \Upsilon(1S, 2S)\pi\pi$ but not yet for $\Upsilon(5S) \rightarrow \Upsilon(mS)\pi\pi$
 Chen et al. (2016-2017)

Production amplitudes for the events dominated by the Z_b 's poles:

$$U_{\text{el}} = \begin{array}{c} \Upsilon(5S) \\ \text{---} \pi \\ \text{---} B^{(*)} \\ \text{---} \bar{B}^* \end{array} + \begin{array}{c} \text{---} \pi \\ \text{---} B \\ \text{---} \bar{B}^* \end{array} B^{(*)} + \begin{array}{c} \text{---} \pi \\ \text{---} \bar{B}^* \\ \text{---} \bar{B}^* \end{array} B^{(*)}$$

$$U_{\text{inel}} = \begin{array}{c} \Upsilon(5S) \\ \text{---} \pi \\ \text{---} B \\ \text{---} \bar{B}^* \end{array} \pi + \begin{array}{c} \text{---} \pi \\ \text{---} \bar{B}^* \\ \text{---} \bar{B}^* \end{array} \pi$$

$h_b(mP)$ $h_b(mP)$

👉 Inelastic source $\Upsilon(5S) \rightarrow h_b(nP)\pi\pi$ requires flip in the HQ spin $\Rightarrow$ suppressed by HQSS

Formalism for line shapes: effective potential

- Production amplitudes U : from coupled-channel Lippmann-Schwinger Eqs. (LSE)
 2 elastic channels x 2: $\alpha = B\bar{B}^*[^3S_1], B\bar{B}^*[^3D_1], B^*\bar{B}^*[^3S_1], B^*\bar{B}^*[^3D_1]$
 5 inelastic channels: $\alpha = h_b(1P)\pi, h_b(2P)\pi, \Upsilon(1S)\pi, \Upsilon(2S)\pi, \Upsilon(3S)\pi$
- Assumption: interaction of light $q\bar{q}$ mesons with heavy quarkonia is suppressed
- Effective elastic potentials: $V_{\alpha\beta}^{\text{eff}}(M, p, p') = V_{\alpha\beta}^{\text{CT}}(M, p, p') + V_{\alpha\beta}^{\pi}(M, p, p') + V_{\alpha\beta}^{\eta}(M, p, p')$,

Contact terms:
$$V_{\alpha\beta}^{\text{CT}} = v_{\alpha\beta} - \underbrace{\frac{i}{2\pi\sqrt{s}} \sum_j m_{H_i} m_{h_i} g_{j\alpha} g_{j\beta} k_j^{2l_j+1}}_{\text{unitarity contribution from inelastic channels}}$$

elastic CT's:
$$v(p, p') = \begin{pmatrix} \mathcal{C}_d(1 + \epsilon) + \mathcal{D}_d(p^2 + p'^2) & \mathcal{D}_{SD}p'^2 & \mathcal{C}_l + \mathcal{D}_l(p^2 + p'^2) & \mathcal{D}_{SD}p'^2 \\ \mathcal{D}_{SD}p^2 & 0 & \mathcal{D}_{SD}p^2 & 0 \\ \mathcal{C}_l + \mathcal{D}_l(p^2 + p'^2) & \mathcal{D}_{SD}p'^2 & \mathcal{C}_d(1 - \epsilon) + \mathcal{D}_d(p^2 + p'^2) & \mathcal{D}_{SD}p'^2 \\ \mathcal{D}_{SD}p^2 & 0 & \mathcal{D}_{SD}p^2 & 0 \end{pmatrix}.$$

$\mathcal{C}_d, \mathcal{C}_l - \mathcal{O}(1)$

$\mathcal{D}_d, \mathcal{D}_l, \mathcal{D}_{SD} - \mathcal{O}(p^2)$

ϵ — HQSS violation

Formalism for line shapes: Fitting procedure

- Input:

- experimental distributions for

$$\Upsilon(10860) \rightarrow \pi Z_b^{(')} \rightarrow \pi \alpha$$

$$\alpha = B\bar{B}^*, \quad B^*\bar{B}^*, \quad h_b(1P)\pi, \quad h_b(2P)\pi$$

- branching fractions for

$$\alpha = BB^*, \quad B^*B^*, \quad h_b(1P)\pi, \quad h_b(2P)\pi, \quad \Upsilon(1S)\pi, \quad \Upsilon(2S)\pi, \quad \Upsilon(3S)\pi$$

- Parameters of the fits:

2 elastic S-S waves contact terms at LO

3 elastic contact terms at NLO: 1 S-D waves , 2 S-S waves

5 inelastic-elastic constants

7 HQSS violation parameters if included : 1 elastic + 6 inelastic

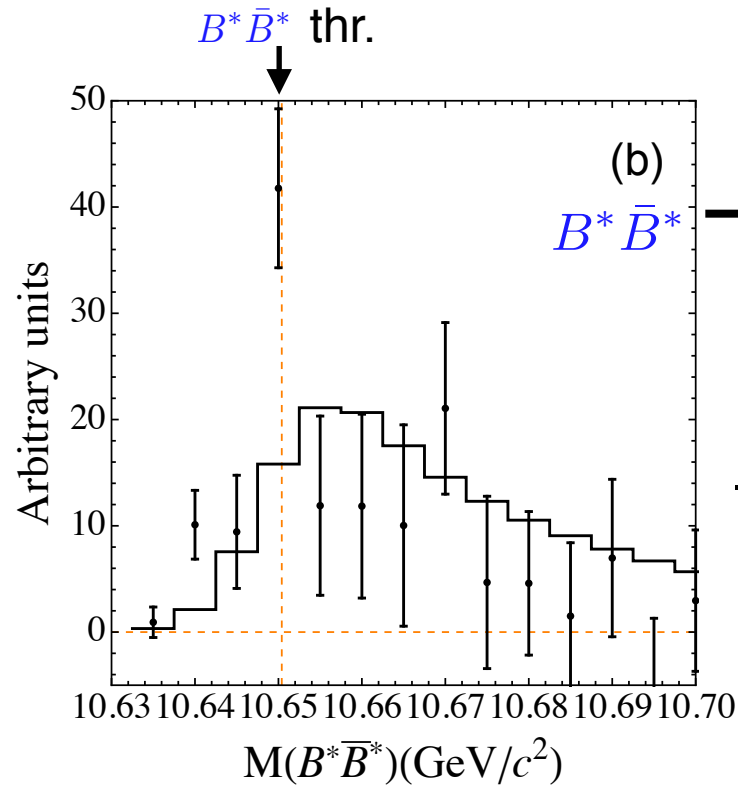
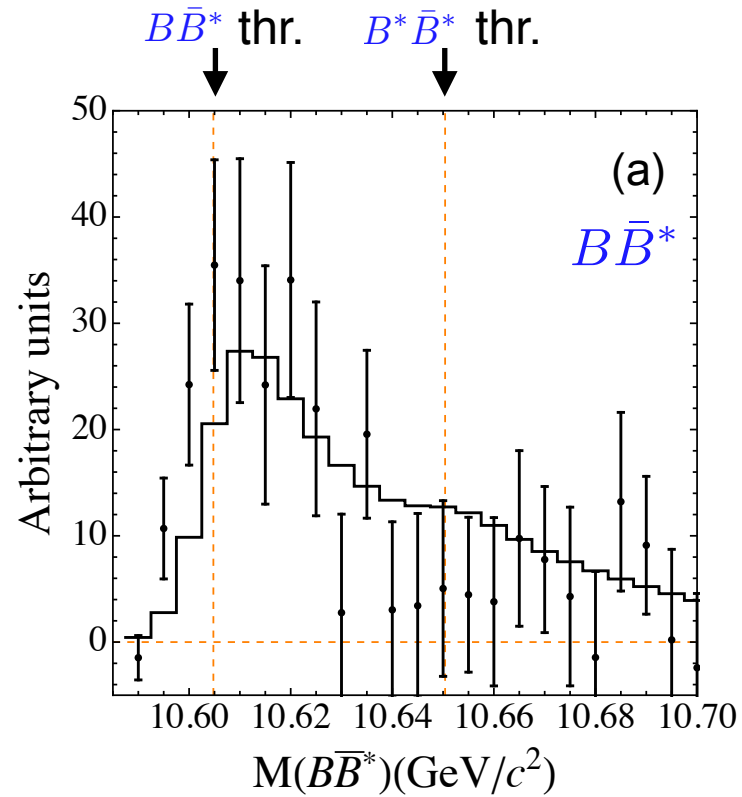
One π and one η -exchange potentials are parameter free!

Results: contact theory (CT) at LO

arXiv: 1805.07453

$$\chi^2 \equiv \frac{\chi^2}{\text{dof.}}$$

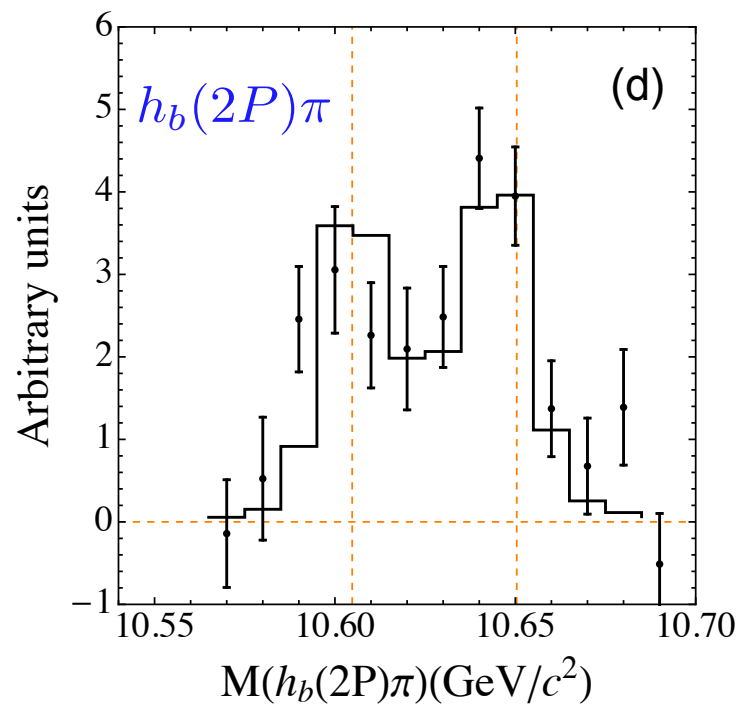
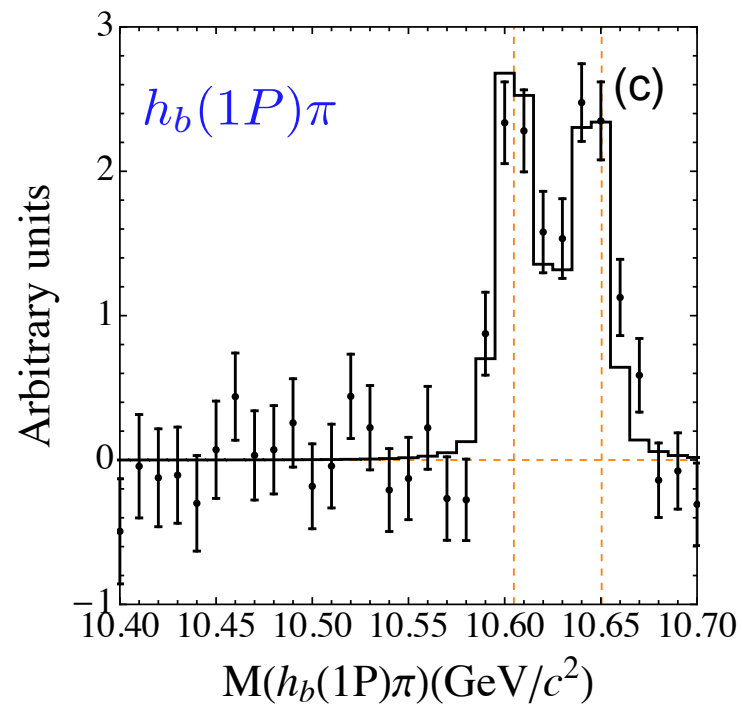
$$\chi^2=1.25$$



— fit A: LO CT's

— HQSS is preserved in the potentials

— consistent with the parameterisation by Guo et al. (2016)



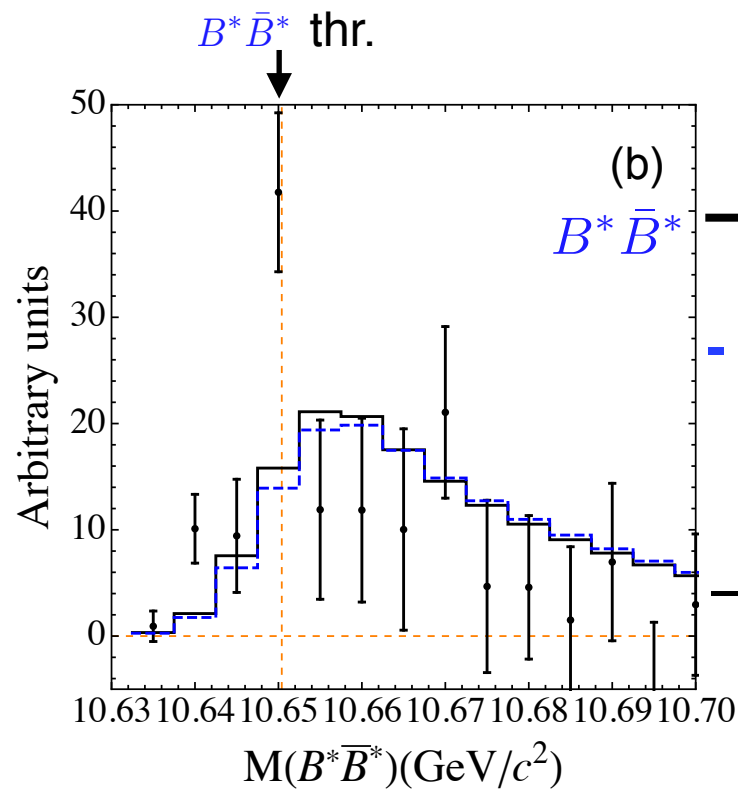
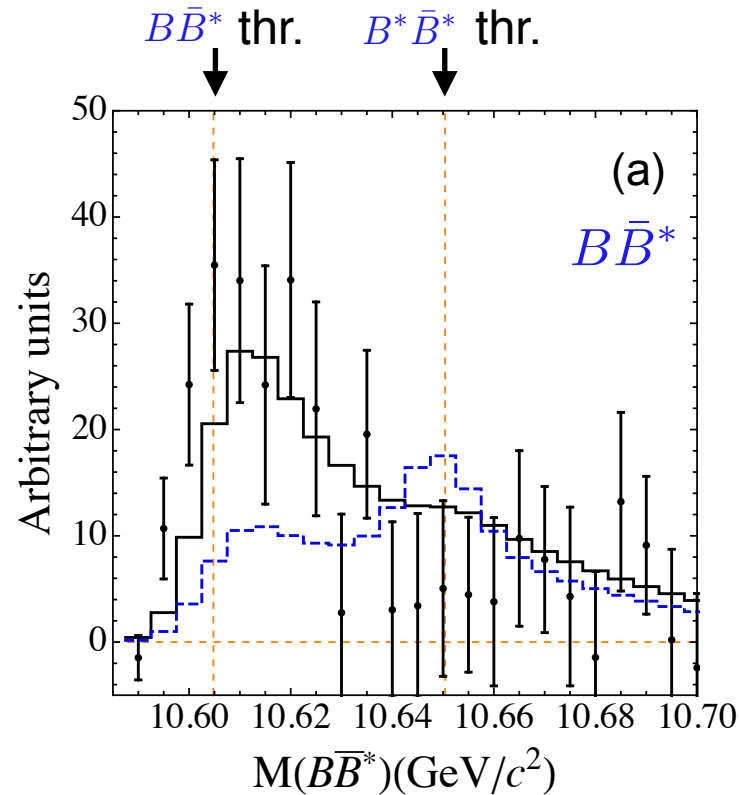
Results: LO CT's + OPE

arXiv: 1805.07453

$$\chi^2 \equiv \frac{\chi^2}{\text{dof.}}$$

$$\chi^2=1.25$$

$$\chi^2=1.72$$

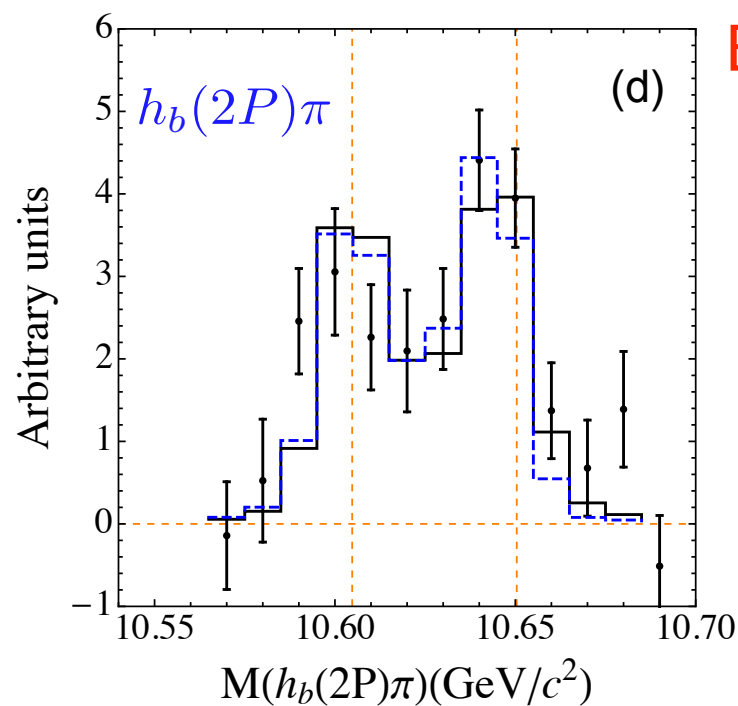
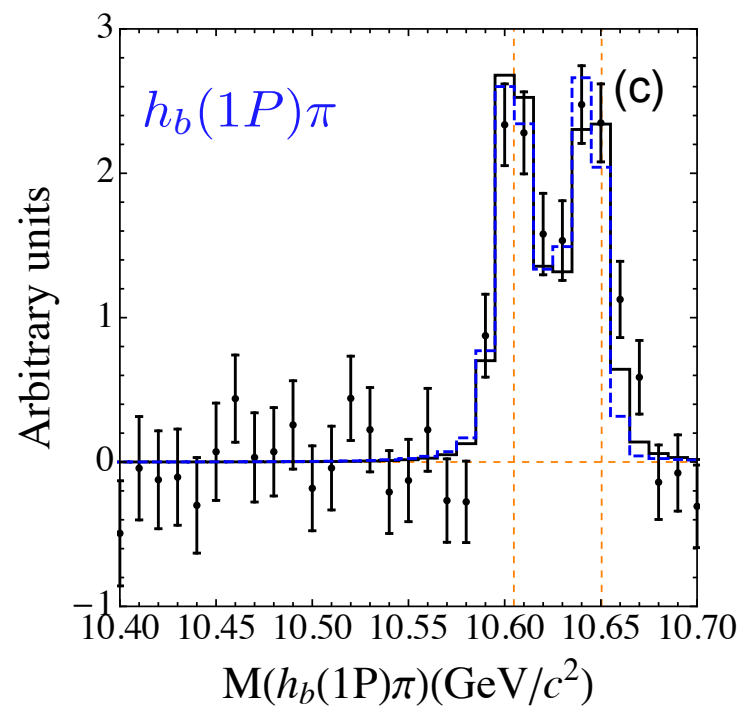


— fit A: LO CT's

- - - fit B: fit A + OPE

— dynamical HQSS violation due to OPE

— OPE: clear bump structure at $B^*\bar{B}^*$ thr. due to coupled-channels



— Data by Belle: no structure

“Light-quark spin symmetry”
Voloshin (2016)

- Strong coupled-channel dynamics from OPE is inconsistent with the data

Results : Including HQSS violation in the CT's

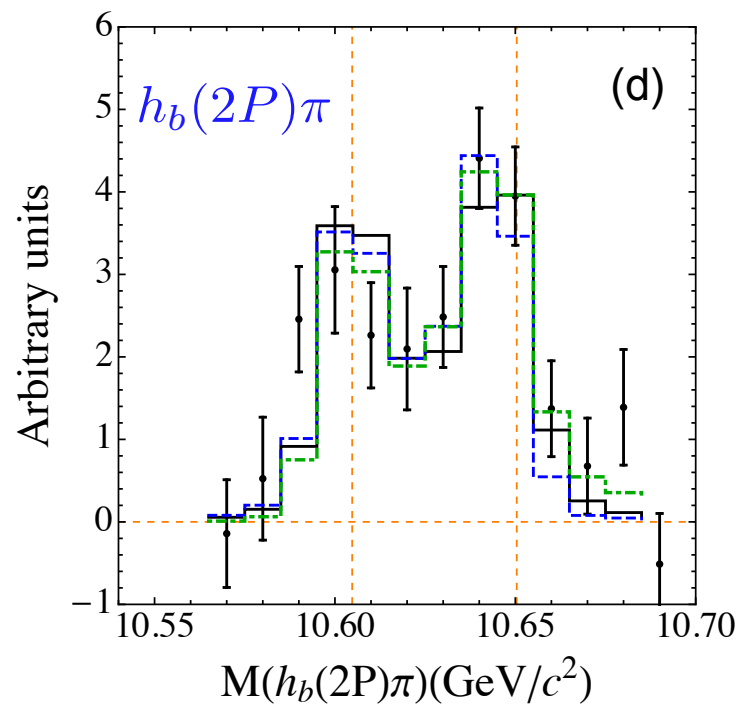
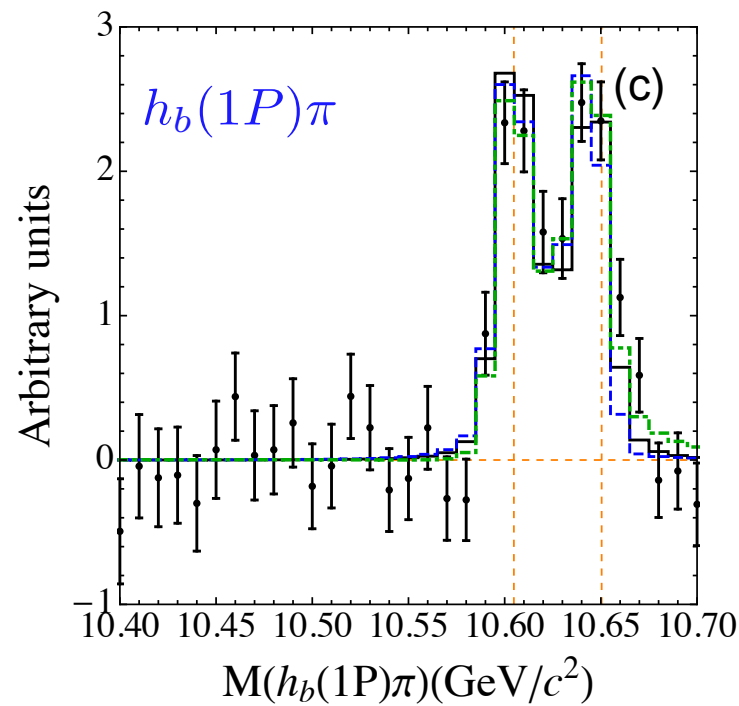
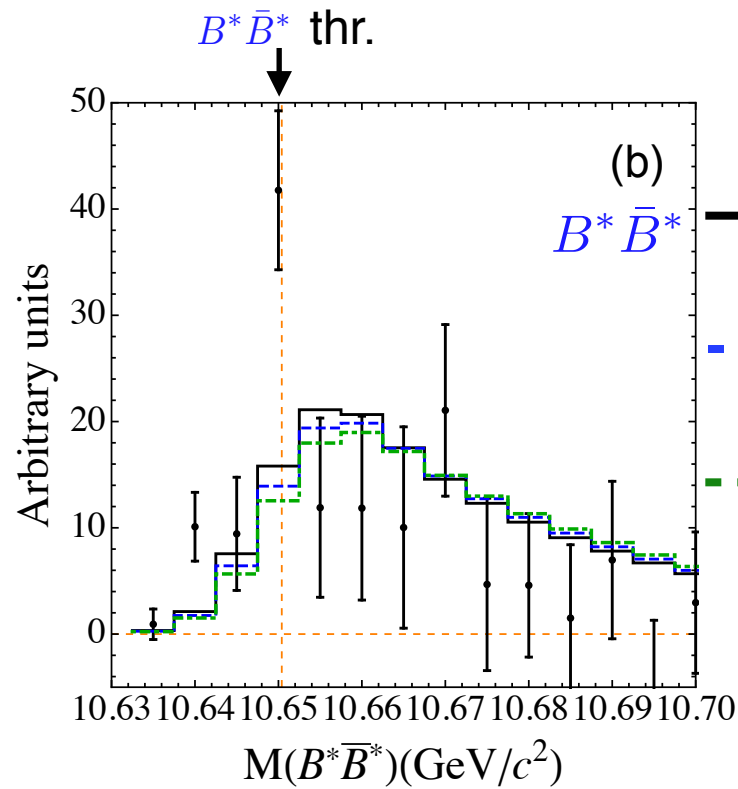
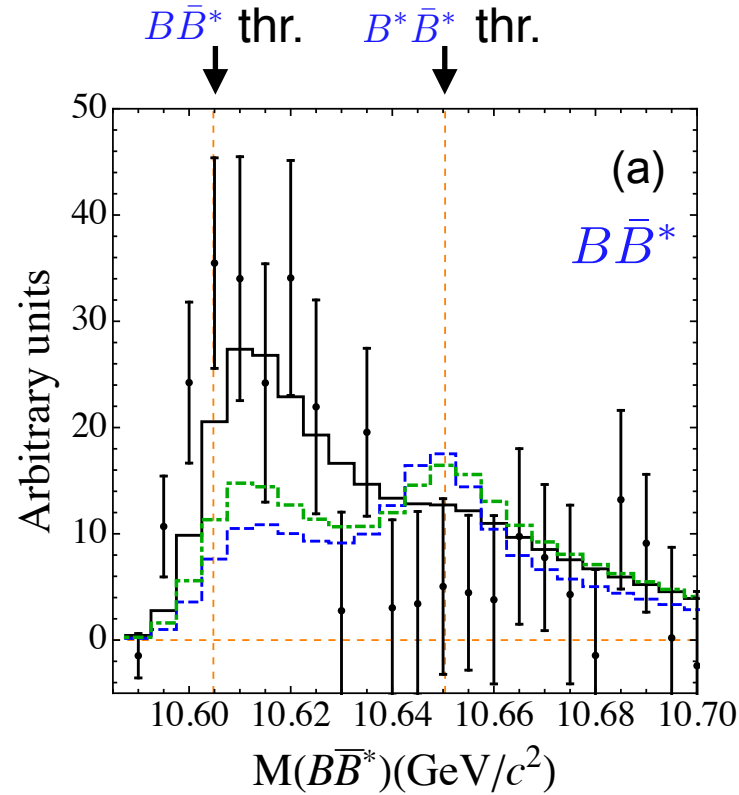
arXiv: 1805.07453

$$\chi^2 \equiv \frac{\chi^2}{\text{dof.}}$$

$$\chi^2=1.25$$

$$\chi^2=1.72$$

$$\chi^2=1.57$$



7 new parameters are added due to HQSS violation

● Violation of HQSS in the contact potentials is not supported by the data!

Results: LO CT's + OPE + NLO CT's

arXiv: 1805.07453

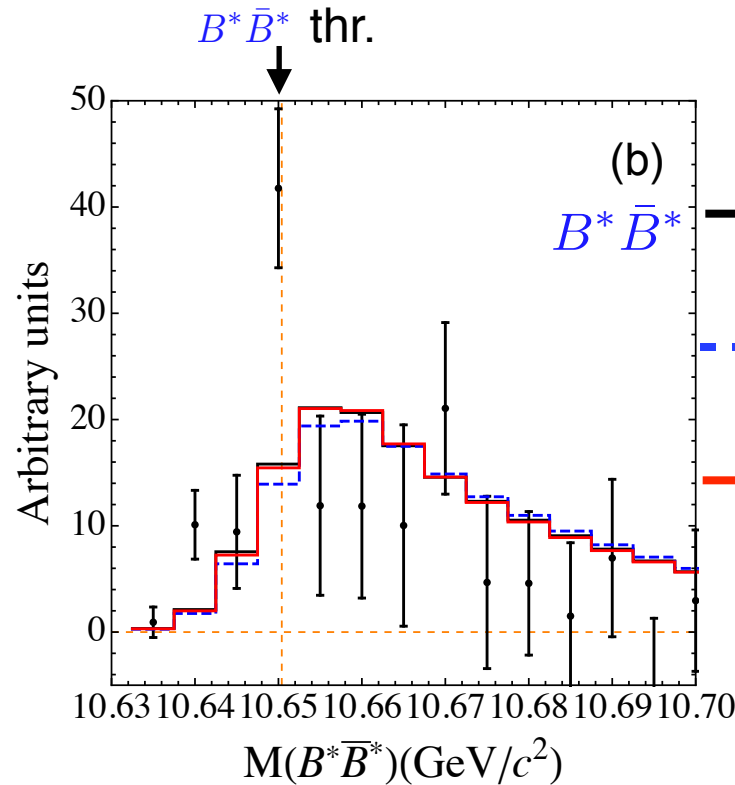
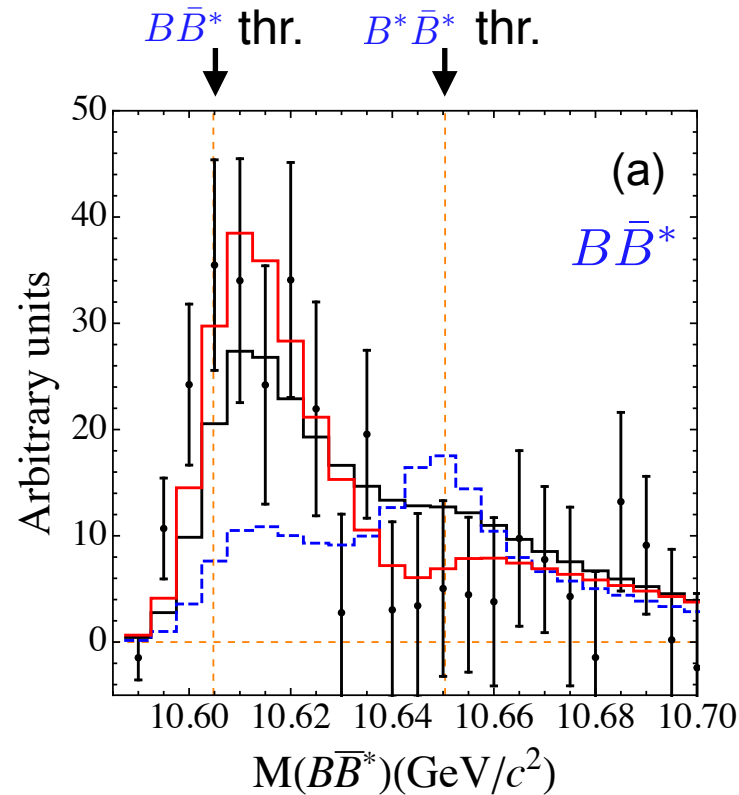
$$\chi^2 \equiv \frac{\chi^2}{\text{dof.}}$$

$$\chi^2=1.25$$

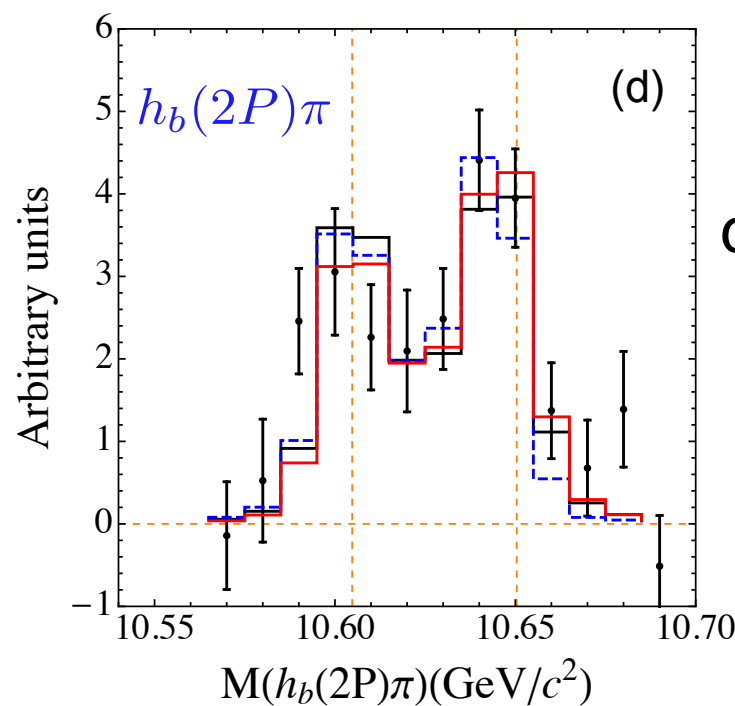
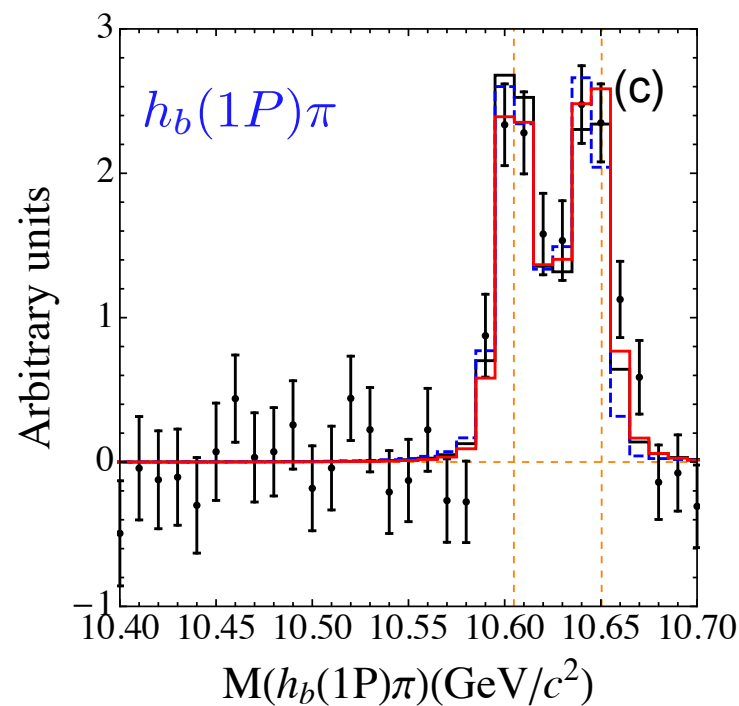
$$\chi^2=1.72$$

$$\chi^2=1.10$$

$$\chi^2=1.11$$



—back to HQSS preserving potentials!



—The effect from two S-S NLO contact terms $\mathcal{D}_d$, $\mathcal{D}_l$ is small

fit E: fit D + 1η $\chi^2=1.11$
 $\Rightarrow$ Indistinguishable from fit D

- A single contact term $\mathcal{D}_{SD}$ at NLO cancels largely the S-D OPE and improves the fit
- 1π and 1η can be largely absorbed into redefinition of the contact terms at NLO

Extracting the poles

- W/O inelastic channels two-channel problem: BB^* and B^*B^*

Conformal mapping of 4 RS surface to a single sheet surface in omega-plane:

$$k_1 = \sqrt{\frac{\mu_1 \delta}{2}} \left(\omega + \frac{1}{\omega} \right), \quad k_2 = \sqrt{\frac{\mu_2 \delta}{2}} \left(\omega - \frac{1}{\omega} \right)$$

$$\delta = m_{B^*} - m_B$$

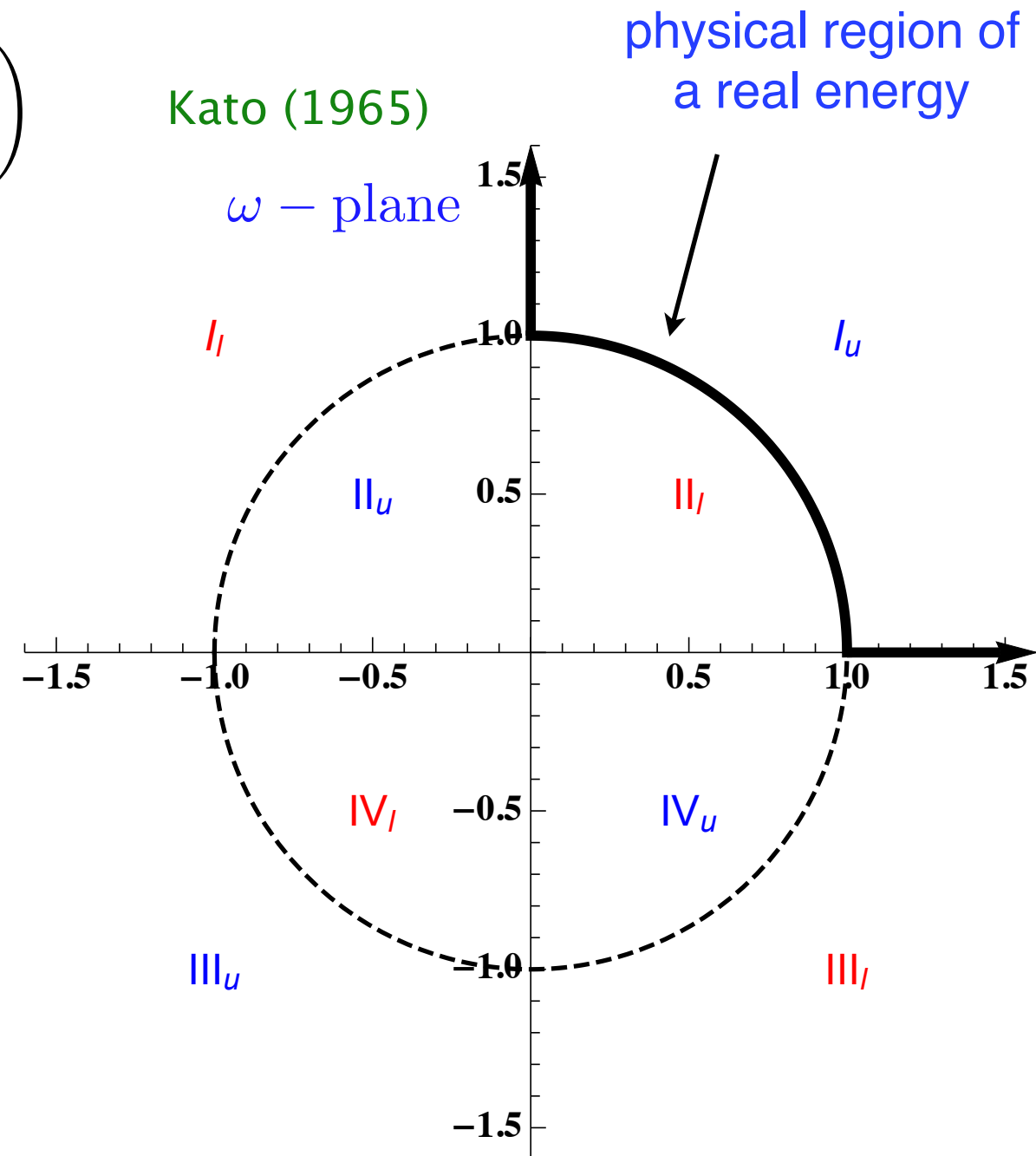
Definition of Riemann Sheets (RS):

$$\begin{aligned} \text{RS-I : } & \text{Im } k_1 > 0, \quad \text{Im } k_2 > 0, \\ \text{RS-II : } & \text{Im } k_1 < 0, \quad \text{Im } k_2 > 0, \\ \text{RS-III : } & \text{Im } k_1 < 0, \quad \text{Im } k_2 < 0, \\ \text{RS-IV : } & \text{Im } k_1 > 0, \quad \text{Im } k_2 < 0. \end{aligned}$$

Energy relative to the BB^* threshold:

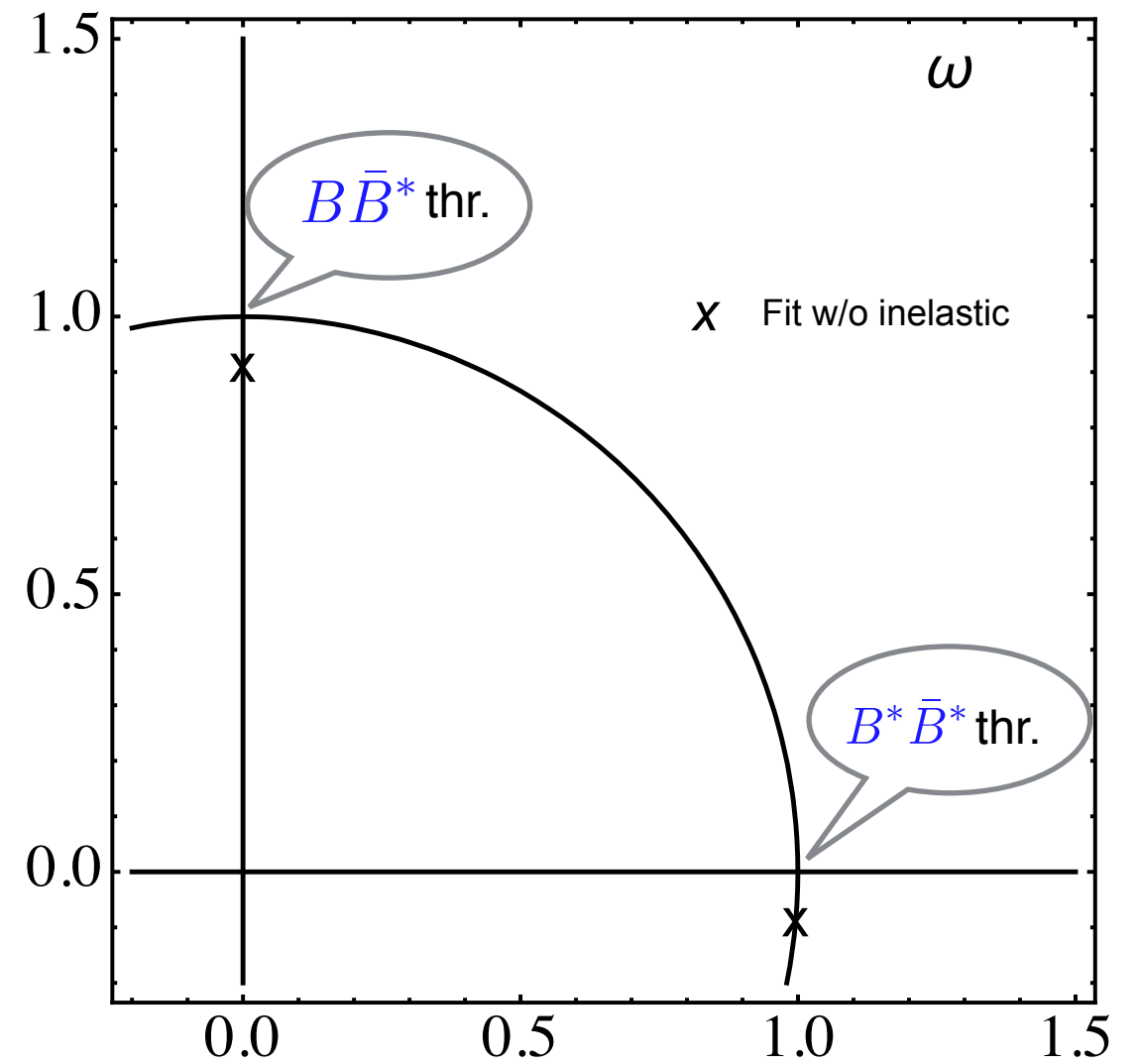
$$E = \frac{k_1^2}{2\mu_1} = \frac{k_2^2}{2\mu_2} + \delta = \frac{\delta}{4} \left(\omega^2 + \frac{1}{\omega^2} + 2 \right)$$

- Only the poles close to the physical region are relevant



Poles of Zb(10610) and Zb(10650)

- NO inelastic channels: both states are virtual with energies just below their thresholds



Poles of Zb(10610) and Zb(10650)

- Include inelastic ch. and treat them all as a third distant channel (k_{in}): 4 RS $\Rightarrow$ 8 RS

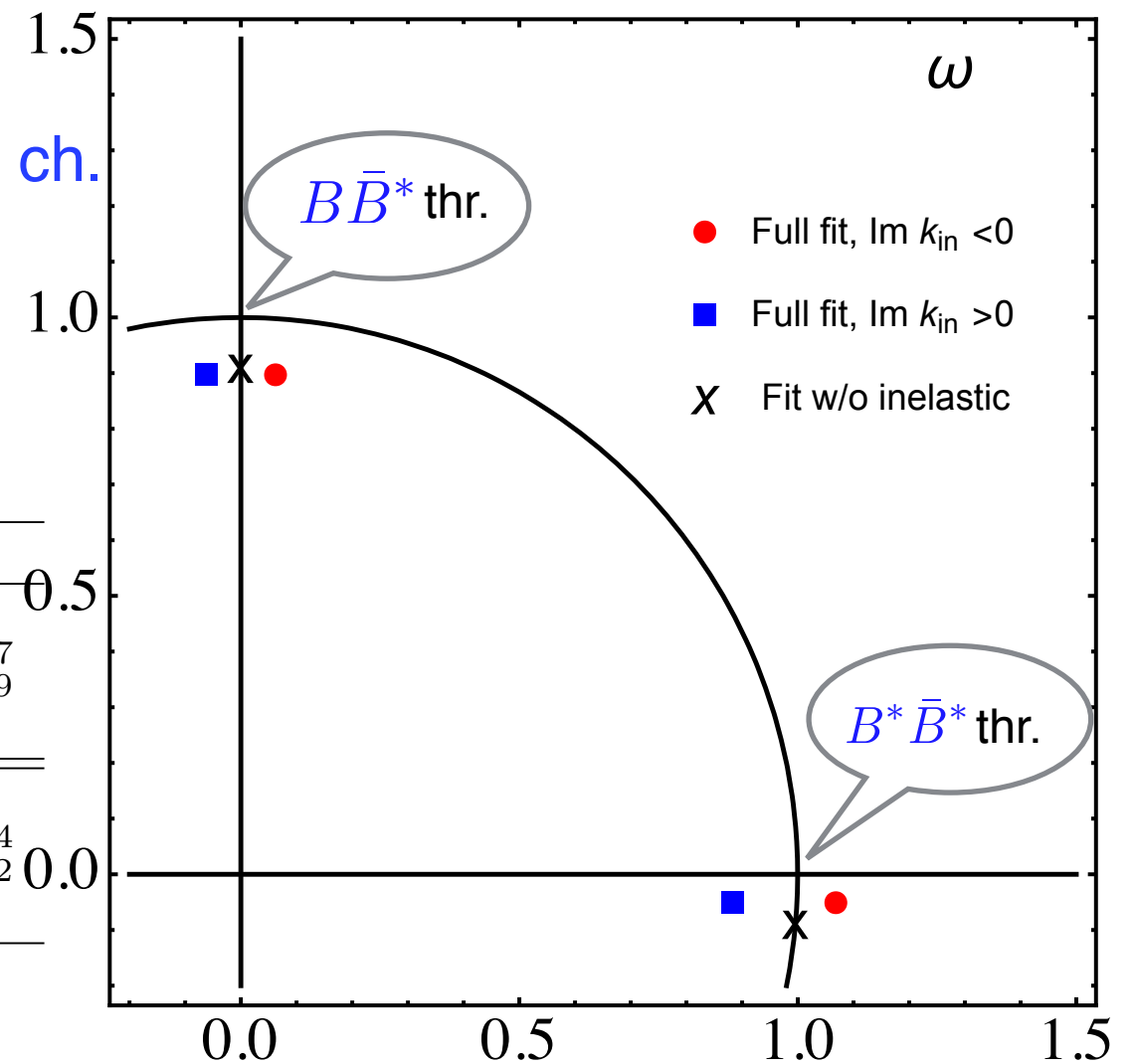
$\Rightarrow$ number of poles doubles

Poles are close to the one pole scenario w/o inelastic ch.

$\Rightarrow$ Role of inelastic channels is subleading

The poles

scheme	$E_{Z_b}(\text{MeV})$	$E_{Z'_b}(\text{MeV})$
full fit in HQSS limit ($\text{Im } k_{in} < 0$)	$-0.28_{+0.30}^{-0.21} - i0.67_{+0.51}^{-0.41}$	$+0.11_{+1.02}^{-0.28} - i0.29_{+0.39}^{-0.97}$
full fit in HQSS limit ($\text{Im } k_{in} > 0$)	$-0.28_{+0.30}^{-0.21} + i0.67_{+0.41}^{-0.51}$	$+0.54_{+0.46}^{-0.37} + i0.63_{+0.52}^{-0.74}$



- Zb(10610) is a shallow virtual state just below $B \bar{B}^*$ threshold
- Zb(10650) resides just above $B^* \bar{B}^*$ threshold

Conclusions

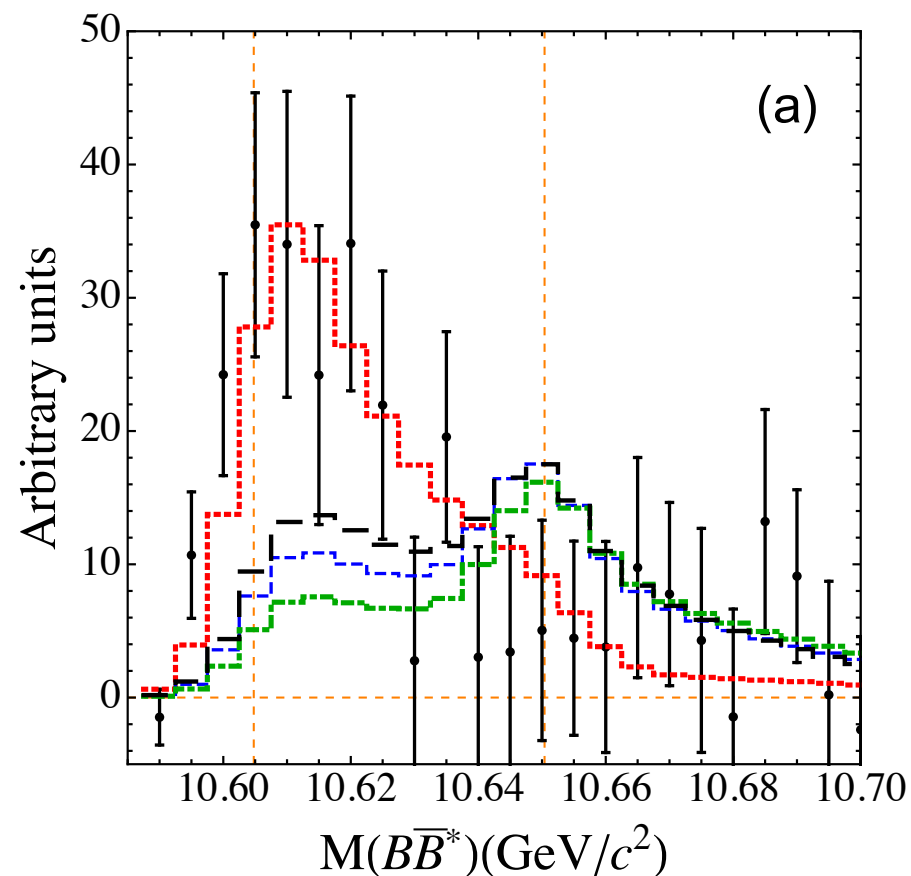
A systematic EFT approach consistent with chiral and heavy quark symmetries to probe various molecular candidates in c and b -sectors is proposed

- Analysis of the line shapes $\Upsilon(10860) \rightarrow \pi Z_b^{(')} \rightarrow \pi \alpha$ and partial BR's with $\frac{\chi^2}{\text{dof.}} \simeq 1$
- Data are consistent with HQSS
- S-D tensor **OPE force can strongly affect the line shapes** due to coupled-channels
- Non-trivial cancellation of the OPE by a single S-D contact term at NLO
 - Origin: not yet understood — related to “light-quark spin symmetry”
- The poles of **Zb(10610)/Zb(10650)** are extracted:
both are shallow states located in the vicinity of the nearby hadronic thresholds

Outlook: — **Parameter free predictions for spin partners** (In progress)
— Reanalyse the role of OPE for spin partners (In progress)
👉 Reanalyse the line shapes in the charm sector

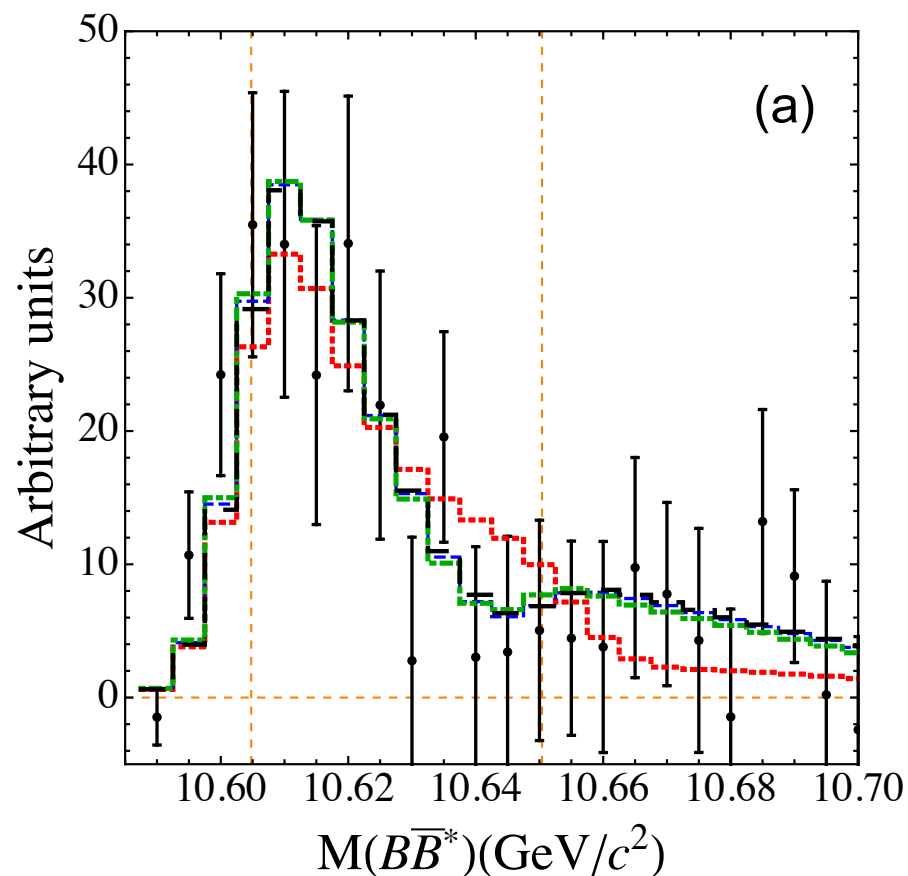
Dependence on the regulator

LO CT's+OPE

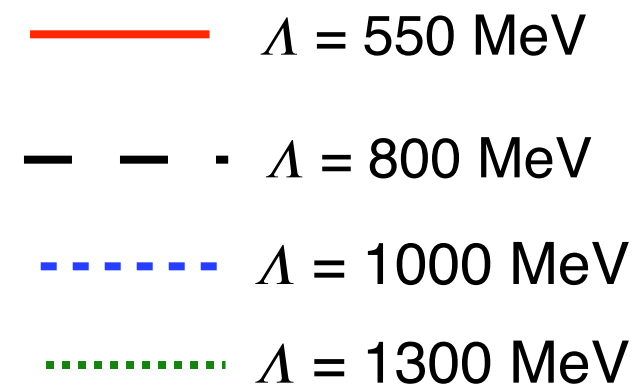


Visible cutoff dependence

LO+NLO CT's+OPE



Negligible cutoff dependence



Necessity for the NLO CT's is indicated by renormalisability

Formalism for line shapes: long-range potentials

- Goldstone-Boson Lagrangian: $\mathcal{L}_\Phi = -\frac{g_Q}{4f_\pi} \text{Tr} (\boldsymbol{\sigma} \cdot \boldsymbol{\nabla} \Phi_{ab} \bar{H}_b \bar{H}_a^\dagger) + \text{h.c.},$

Heavy fields — $H_a = B_a + \mathbf{B}_a^* \cdot \boldsymbol{\sigma}$, $\Phi = \begin{pmatrix} \pi^0 + \sqrt{\frac{1}{3}}\eta & \sqrt{2}\pi^+ \\ \sqrt{2}\pi^- & -\pi^0 + \sqrt{\frac{1}{3}}\eta \end{pmatrix}$ —relevant part of SU(3) GB matrix

with $g_b = g_c \approx 0.57$ from $D^* \rightarrow D\pi$ width and HQSS

- Exemplary GB potential for OPE:

$$V_{B\bar{B}^* \rightarrow B^*\bar{B}}(\mathbf{p}, \mathbf{p}') = -\frac{2g_b^2}{(4\pi f_\pi)^2} \tau_1 \cdot \tau_2^c (\epsilon_1 \cdot \mathbf{q})(\epsilon_2'^* \cdot \mathbf{q}) \left(\frac{1}{D_{BB\pi}(\mathbf{p}, \mathbf{p}')} + \frac{1}{D_{B^*B^*\pi}(\mathbf{p}, \mathbf{p}')} \right)$$

👉 TOPT propagators with NR heavy mesons and relativistic pions

$$D_{BB\pi}(\mathbf{p}, \mathbf{p}') = 2E_\pi(\mathbf{q}) \left(m + m + \frac{\mathbf{p}^2}{2m} + \frac{\mathbf{p}'^2}{2m} + E_\pi(\mathbf{q}) - \sqrt{s} \right) \quad E_\pi(\mathbf{q}) = \sqrt{\mathbf{q}^2 + m_\pi^2}$$

$$D_{B^*B^*\pi}(\mathbf{p}, \mathbf{p}') = 2E_\pi(\mathbf{q}) \left(m_* + m_* + \frac{\mathbf{p}^2}{2m_*} + \frac{\mathbf{p}'^2}{2m_*} + E_\pi(\mathbf{q}) - \sqrt{s} \right)$$

One π and one η -exchange potentials are parameter free!