Status of Charmed Meson Spectroscopy

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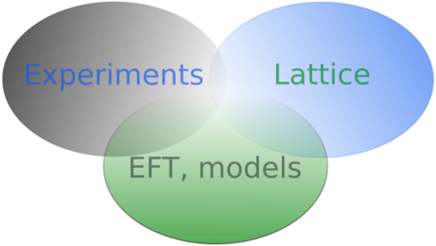
Overview

$D^{\pm}$
D^0
$D^*(2007)^0$
$D^*(2010)^{\pm}$
$D_0^*(2400)^0$
$D_0^*(2400)^{\pm}$
$D_1(2420)^0$
$D_1(2420)^{\pm}$
$D_1(2430)^0$
$D_2^*(2460)^0$
$D_2^*(2460)^{\pm}$
$D(2550)^0$
$D_J^*(2600)$ was $D(2600)$
$D^*(2640)^{\pm}$
$D(2740)^0$
$D(2750)$
$D(3000)^0$

$D_s^{\pm}$
$D_s^{*\pm}$
$D_{s0}^*(2317)^{\pm}$
$D_{s1}(2460)^{\pm}$
$D_{s1}(2536)^{\pm}$
$D_{s2}^*(2573)$
$D_{s1}^*(2700)^{\pm}$
$D_{s1}^*(2860)^{\pm}$
$D_{s3}^*(2860)^{\pm}$
$D_{sJ}(3040)^{\pm}$



Beginning of the interesting story: $D_{s0}^*(2317)$ and $D_{s1}(2460)$

Charm-strange mesons

- $D_{s0}^*(2317)$: 0^+ BaBar (2003)

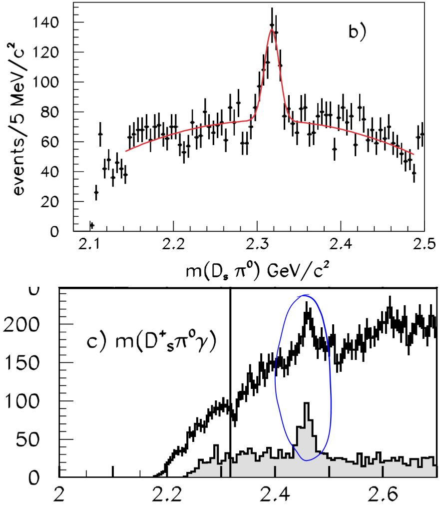
  $M = (2317.7 \pm 0.6)$ MeV,

  $\Gamma < 3.8$ MeV

  The only hadronic decay: $D_s\pi$
- $D_{s1}(2460)$: 1^+ BaBar, CLEO (2003)

  $M = (2459.5 \pm 0.6)$ MeV,

  $\Gamma < 3.5$ MeV
- no isospin partner observed, tiny widths $\Rightarrow$ $I = 0$



BABAR, PRL90(2003)242001

$D_0^*(2400)$ and $D_1(2430)$

- $D_0^*(2400)^0$: $J^P = 0^+, \Gamma = (247 \pm 67)$ MeV — Belle (2004)

  PDG2017:

2318 ± 29	**OUR AVERAGE** Error includes scale factor of 1.7.				
$2297 \pm 8 \pm 20$	3.4k	AUBERT	2009AB	BABR	$B^- \to D^+\pi^-\pi^-$
$2308 \pm 17 \pm 32$		ABE	2004D	BELL	$B^- \to D^+\pi^-\pi^-$
$2407 \pm 21 \pm 35$	9.8k	LINK	2004A	FOCS	γ A

  New measurements by LHCb: (2360 ± 34) MeV — LHCb, PRD92(2015)012012

- $D_1(2430)^0$: $J^P = 1^+, \Gamma = 384^{+130}_{-110}$ MeV

VALUE (MeV)	*DOCUMENT ID*		*TECN*	*COMMENT*
$2427 \pm 26 \pm 25$	ABE	2004D	BELL	$B^- \to D^{*()0+}\pi^-\pi^-$
• • • We do not use the following data for averages, fits, limits, etc. • • •				
2477 ± 28	[1] AUBERT	2006L	BABR	$\overline{B}^0 \to D^{*+}\omega\pi^-$

- Notice: all these experiments used a Breit–Wigner to extract the resonance
  - ☞ $D^{(*)}\pi$-$D^{(*)}\eta$-$D_s^{(*)}\bar{K}$ coupled-channel effects are absent
  - ☞ chiral symmetry constraint on soft pions is absent

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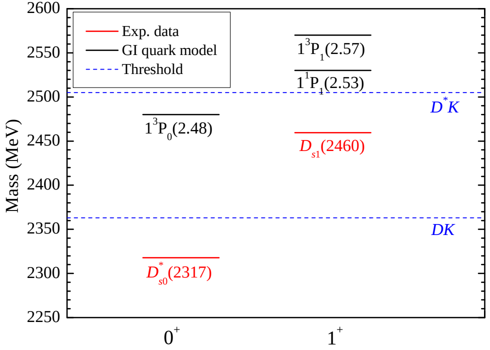
Why are they interesting?



GI quark model: Godfrey, Isgur (1985)

- Why are the masses of $D^*_{s0}(2317)$ and $D_{s1}(2460)$ much lower than quark model predictions for $c\bar{s}$ mesons ?
- Why $\underbrace{M_{D_{s1}(2460)^\pm} - M_{D^*_{s0}(2317)^\pm}}_{=(141.8\pm0.8)\text{ MeV}} \simeq \underbrace{M_{D^{*\pm}} - M_{D^\pm}}_{=(140.67\pm0.08)\text{ MeV}}$ within 2 MeV?
- Why $M_{D^*_0(2400)} \gtrsim M_{D^*_{s0}(2317)}$ and $M_{D_1(2430)} \sim M_{D_{s1}(2460)}$?

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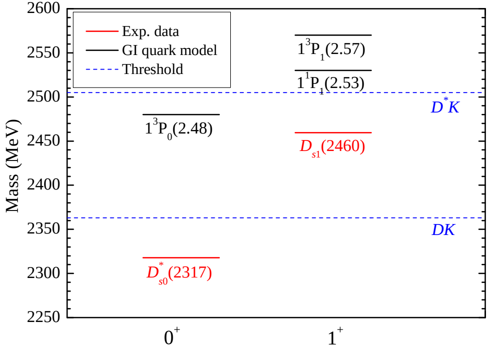

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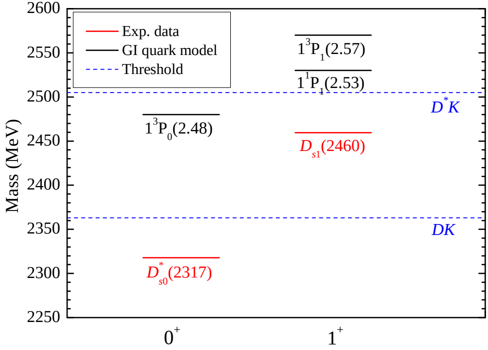

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Lattice studies of the charmed scalar mesons: strange

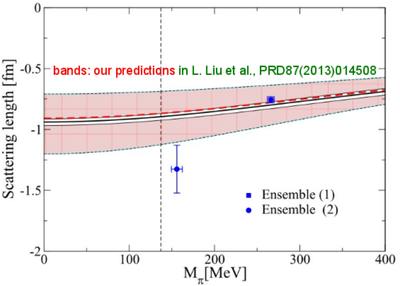
- Early studies using only $c\bar{s}$-type interpolators typically give mass larger than that for $D^*_{s0}(2317)$ Bali (2003); UKQCD (2003); ...

- $c\bar{s} + DK$ interpolators: ∼right mass

  Mohler et al., PRL111(2013)222001

  $M_{D^*_{s0}} - \frac{1}{4}\left(M_{D_s} + 3M_{D^*_s}\right)$:

Mohler et al.	PDG2017
(266 ± 16) MeV	(241.5 ± 0.8) MeV



- New calculation with $M_\pi = 150$ MeV Bali et al. [RQCD Col.], PRD96(2017)074501

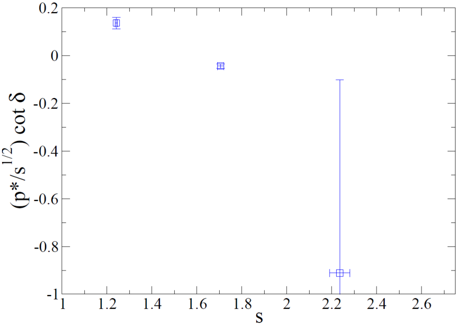
	Energy [MeV]	Expt [MeV]
m_{0-}	1976.9(2)	1966.0(4)
m_{1-}	2094.9(7)	2111.3(6)
m_{0+}	2348(4)(+6)	2317.7(0.6)(2.0)
m_{1+}	2451(4)(+1)	2459.5(0.6)(2.0)

- $(S,I) = (0, \frac{1}{2})$: $c\bar{q} + D\pi$ interpolators:

  Mohler et al., PRD87(2013)034501

  $M_\pi \approx 266$ MeV,
  $M_D \approx 1558$ MeV,
  $M_{D^*} \approx 1690$ MeV

  Lüscher's formula $\Rightarrow$ $D\pi$ phase shifts



$\Rightarrow$ BW parameters of $D_0^*(2400)$ consistent with PDG values

	Mohler et al.	PDG2017
$M_{D_0^*} - \frac{1}{4}(M_D + 3M_{D^*})$	(351 ± 21) MeV	(347 ± 29) MeV
$M_{D_1} - \frac{1}{4}(M_D + 3M_{D^*})$	(380 ± 21) MeV	(456 ± 40) MeV

Lattice studies of the charmed scalar mesons: nonstrange (2)

- $(S, I) = (0, \frac{1}{2})$: first coupled-channel lattice calculation including interpolating fields for $c\bar{q} + D\pi + D\eta + D_s\bar{K}$: Moir et al. [Hadron Spectrum Col.], JHEP1610(2016)011
- $M_\pi = 391$ MeV, $M_D = 1885$ MeV: $D\pi$ threshold (2276.4 ± 0.9) MeV
- for coupled channels:

  parametrizing the T-matrix with the K-matrix formalism

$$T_{ij}^{-1}(s) = K_{ij}^{-1}(s) + I_{ij}(s)$$

  $I_{ij}(s)$: 2-point loop function evaluated with a subtracted dispersion integral

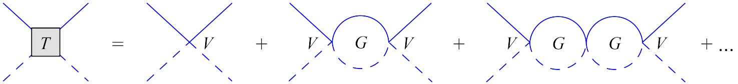
  $K_{ij}(s)$: different forms of the K-matrix were used, summarized as

$$K_{ij}(s) = \left(g_i^{(0)} + g_i^{(1)} s\right) \left(g_j^{(0)} + g_j^{(1)} s\right) \frac{1}{m^2 - s} + \gamma_{ij}^{(0)} + \gamma_{ij}^{(1)} s$$

- $\Rightarrow$ a pole below threshold (2275.9 ± 0.9) MeV. relation to $D_0^*(2400)$?

$D^*_{s0}(2317)$ and $D_{s1}(2460)$ as hadronic molecules

- One possible solution to the 1st puzzle:
  hadronic molecular model: $D^*_{s0}(2317)[DK]$, $D_{s1}(2460)[D^*K]$
  Barnes, Close, Lipkin (2003); van Beveren, Rupp (2003); Kolomeitsev, Lutz (2004); FKG et al. (2006); ...



  $D^{(*)}K$ bound states: poles of the T-matrix

- Solution to the 2nd puzzle as a consequence of heavy quark spin symmetry:
  DK and D^*K interactions almost the same $\Rightarrow$ similar binding energies

$$M_D + M_K - M_{D^*_{s0}(2317)} \simeq M_{D^*} + M_K - M_{D_{s1}(2460)} \pm 4 \text{ MeV}$$

  Uncertainty: binding energy (45 MeV) $\times \frac{\Lambda_{\text{QCD}}}{m_c}\frac{M_K}{\Lambda_\chi}$

$$\Rightarrow \quad M_{D_{s1}(2460)^\pm} - M_{D^*_{s0}(2317)^\pm} \simeq M_{D^{*\pm}} - M_{D^\pm} \text{ is understood}$$

FKG et al., PRL102(2009)242004

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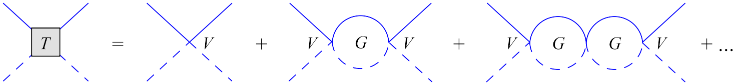

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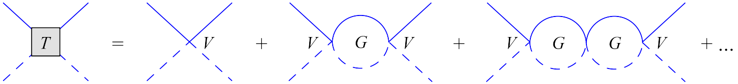
- S-wave charm-meson–light-pseudoscalar-meson interaction



- not far from the thresholds ⇒ chiral EFT for matter field
- D_{s0}^*/D_0^* should appear as poles in scattering amplitudes
  ⇒ needs a nonperturbative treatment: ChPT + unitarization

$$T^{-1}(s) = V^{-1}(s) - G(s)$$

$V(s)$: to be derived from SU(3) chiral Lagrangian, 6 LECs up to NLO

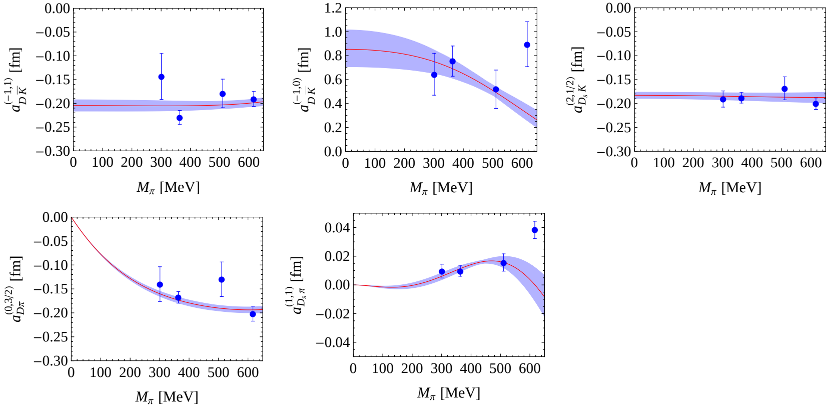
$G(s)$: 2-point scalar loop functions, regularized with a subtraction constant $a(\mu)$

Fit to lattice data

- Fit to lattice data on scattering lengths in 5 simple channels:

  $D\bar{K}(I=1, I=0)$, D_sK, $D\pi(I=3/2)$, $D_s\pi$: no disconnected contribution

  5 parameters: h_2, h_3, h_4, h_5 and $a(\mu)$



Predictions for heavy-strange mesons

- Predictions:

Du et al., arXiv:1712.07957

meson	J^P	prediction	PDG2017	lattice
D_{s0}^*	0^+	2315^{+18}_{-28}	2317.7 ± 0.6	2348^{+7}_{-4}[1]
D_{s1}	1^+	2456^{+15}_{-21}	2459.5 ± 0.6	2451 ± 4[1]
B_{s0}^*	0^+	5720^{+16}_{-23}	–	5711 ± 23[2]
B_{s1}	1^+	5772^{+15}_{-21}	–	5750 ± 25[2]

[1] Bali, Collins, Cox, Schäfer, PRD96(2017)074501

[2] Lang, Mohler, Prelovsek, Woloshyn, PLB750(2015)17

DK component from lattice QCD

- Compositeness $(1-Z)$ related to the S-wave scattering length: Weinberg (1965)

$$a \simeq -2\frac{1-Z}{2-Z}\frac{1}{\sqrt{2\mu E_B}}$$

- From the lattice energy levels in C. Lang et al., PRD90(2014)034510

  $D_{s0}^*(2317)$ contains ∼70% DK Martínez Torres, Oset, Prelovsek, Ramos, JHEP1505,053

- Latest lattice results in G. Bali et al., PRD96(2017)074501

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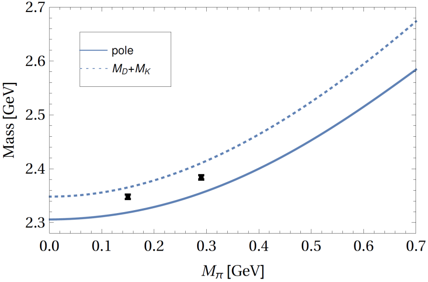


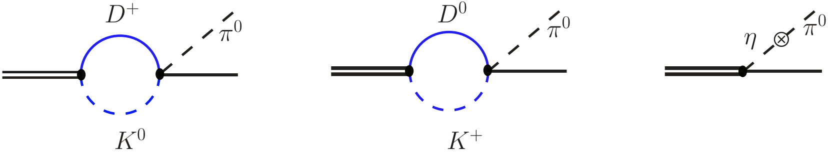
$1-Z = 1.04(0.08)(+0.30)$

M_π [MeV]	150	290
$M_{D_{s0}^*(2317)}$ [MeV]	2348 ± 4	2384 ± 3
M_{D_s} [MeV]	1977 ± 1	1980 ± 1

strong M_π dependence!

curves: prediction in Du et al., EPJC77(2017)728

Decay width of $D_{s0}^*(2317)$



- Large isospin decay width $\Gamma(D_{s0}^*(2317)^+ \to D_s^+ \pi^0) \sim 100$ keV

  Faessler *et al.* (2007); Lutz, Soyeur (2007); FKG *et al.* (2008); Cleven *et al.* (2014)

- $\Gamma(D_{s0}^*(2317)) = (133 \pm 22)$ keV L. Liu *et al.*, PRD86(2013)014508

- Recent result with terms up to $O(p^4)$ in chiral expansion

  X.-Y. Guo, Y. Heo, M. F. M. Lutz, arXiv:1801.10122

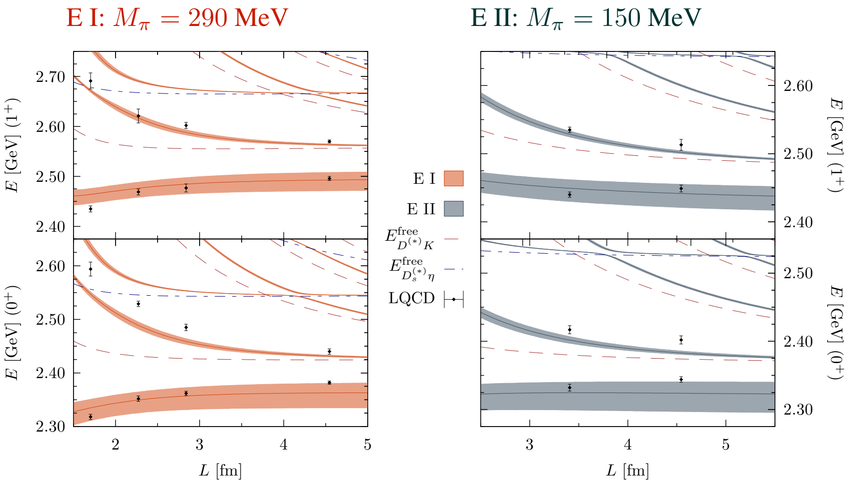
  ☞ LECs from fitting to the lattice results of masses and phase shifts

  ☞ $\Rightarrow \Gamma(D_{s0}^*(2317)) = (110 \pm 6)$ keV

for details $\to$ the talk by X.-Y. Guo on Wednesday, Parallel session-2

Predictions versus recent lattice results: charm-strange

- Postdicted finite volume energy levels for $(S, I) = (1, 0)$, $J^P = 1^+$ & 0^+ versus lattice results by G. Bali, S. Collins, A. Cox, A. Schäfer, PRD96(2017)074501

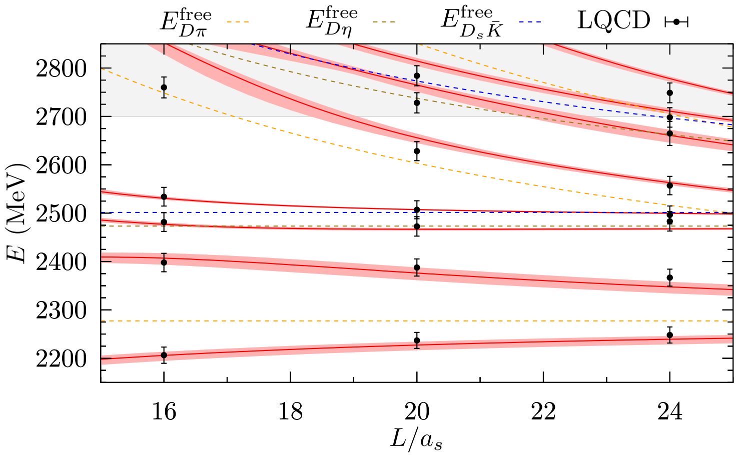


M. Albaladejo, P. Fernandez-Soler, J. Nieves, P. G. Ortega, arXiv:1805.07104

Predictions versus recent lattice results: charm-nonstrange

- Postdicted finite volume energy levels for $I = 1/2$ agree very well with lattice results by G. Moir *et al.* [Hadron Spectrum Collaboration], JHEP1610(2016)011

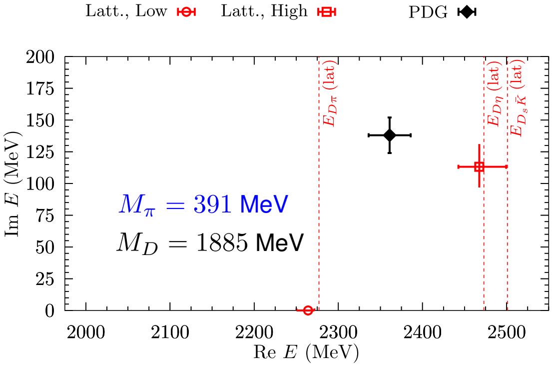
  NOT a fit !



M. Albaladejo, P. Fernandez-Soler, FKG, J. Nieves, PLB767(2017)465

There are two poles (states) !

Masses	M (MeV)	$\Gamma/2$ (MeV)	RS	$\lvert g_{D\pi}\rvert$	$\lvert g_{D\eta}\rvert$	$\lvert g_{D_s\bar{K}}\rvert$
lattice	$2264^{+\ 8}_{-14}$	0	(000)	$7.7^{+1.2}_{-1.1}$	$0.3^{+0.5}_{-0.3}$	$4.2^{+1.1}_{-1.0}$
	2468^{+32}_{-25}	113^{+18}_{-16}	(110)	$5.2^{+0.6}_{-0.4}$	$6.7^{+0.6}_{-0.4}$	$13.2^{+0.6}_{-0.5}$



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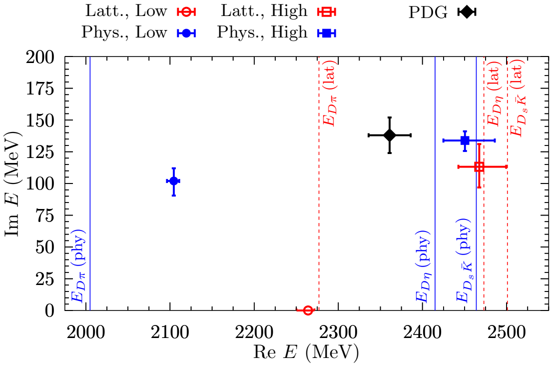
physical	2105^{+6}_{-8}	102^{+10}_{-11}	(100)	$9.4^{+0.2}_{-0.2}$	$1.8^{+0.7}_{-0.7}$	$4.4^{+0.5}_{-0.5}$
	2451^{+36}_{-26}	134^{+7}_{-8}	(110)	$5.0^{+0.7}_{-0.4}$	$6.3^{+0.8}_{-0.5}$	$12.8^{+0.8}_{-0.6}$



Two states in $I = 1/2$ sector

- Two states in $I = 1/2$ sector were found in Kolomeitsev, Lutz (2004); FKG, Shen, Chiang, Ping, Zou (2006); FKG, Hanhart, Meißner (2009); Z.-H. Guo, Meißner, D.-L. Yao (2015)
- remarkable agreement with lattice data ⇒ a strong support
- two states also in other heavy meson sectors $(M, \Gamma/2)$:

	Lower (MeV)	Higher (MeV)	PDG (MeV)
D_0^*	$\left(2105^{+6}_{-8}, 102^{+10}_{-11}\right)$	$\left(2451^{+36}_{-26}, 134^{+7}_{-8}\right)$	$(2318 \pm 29, 134 \pm 20)$
D_1	$\left(2247^{+5}_{-6}, 107^{+11}_{-10}\right)$	$\left(2555^{+47}_{-30}, 203^{+8}_{-9}\right)$	$(2427 \pm 40, 192^{+65}_{-55})$
B_0^*	$\left(5535^{+9}_{-11}, 113^{+15}_{-17}\right)$	$\left(5852^{+16}_{-19}, 36 \pm 5\right)$	–
B_1	$\left(5584^{+9}_{-11}, 119^{+14}_{-17}\right)$	$\left(5912^{+15}_{-18}, 42^{+5}_{-4}\right)$	–

- But is there any experimental support?

  to compare with the most precise measurement of $B^- \to D^+\pi^-\pi^-$ by LHCb PRD94(2016)072001

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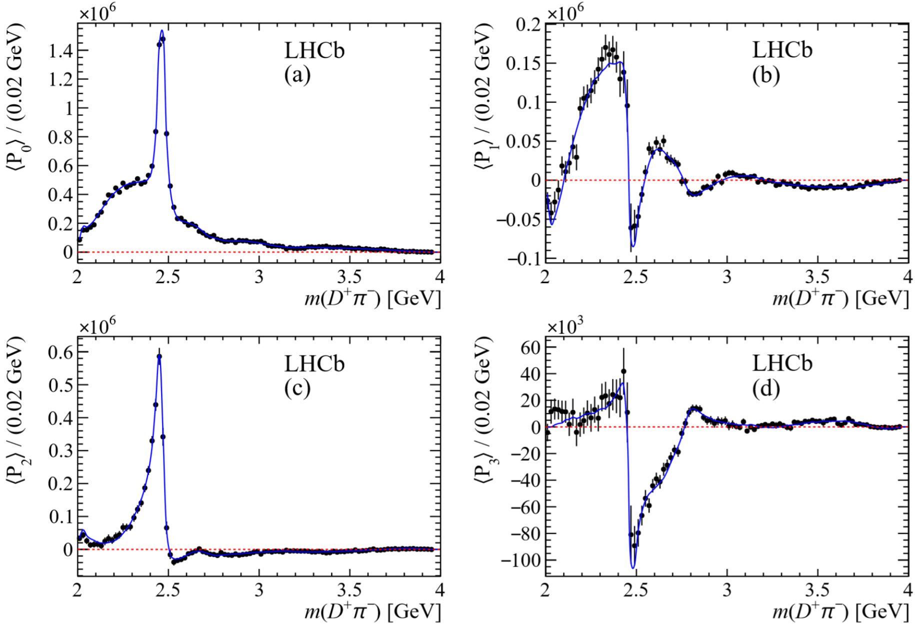
Angular moments of $B^- \to D^+\pi^-\pi^-$

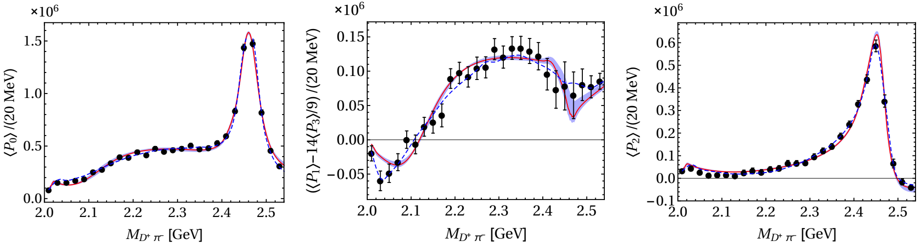
LHCb, PRD94(2016)072001



$$\langle P_0\rangle \propto |\mathcal{A}_0|^2 + |\mathcal{A}_1|^2 + |\mathcal{A}_2|^2 ,$$

$$\langle P_2\rangle \propto \frac{2}{5}|\mathcal{A}_1|^2 + \frac{2}{7}|\mathcal{A}_2|^2 + \frac{2}{\sqrt{5}}|\mathcal{A}_0||\mathcal{A}_2|\cos(\delta_2-\delta_0) ,$$

$$\langle P_{13}\rangle \equiv \langle P_1\rangle - \frac{14}{9}\langle P_3\rangle \propto \frac{2}{\sqrt{3}}|\mathcal{A}_0||\mathcal{A}_1|\cos(\delta_1-\delta_0)$$



- The S-wave $D\pi$ can be well described using our amplitudes with pre-fixed LECs (the same as before)
- Fast variation in [2.4, 2.5] GeV in $\langle P_{13}\rangle$: cusps at $D\eta$ and $D_s\bar{K}$ thresholds

for details $\rightarrow$ the talk by M.-L. Du on Wednesday, Parallel session-2

Summary and outlook (1)

Thanks for the recent experiment, lattice and EFT developments

$\Rightarrow$ likely resolution to all 3 puzzles of positive-parity charm mesons:

- Q: Why are the masses of $D_{s0}^*(2317)$ and $D_{s1}(2460)$ much lower than quark model predictions for $c\bar{s}$ mesons ?

  A: They are dominantly DK and D^*K molecular states, respectively.

- Q: Why $M_{D_{s1}(2460)^\pm} - M_{D_{s0}^*(2317)^\pm} \simeq M_{D^{*\pm}} - M_{D^\pm}$ within 2 MeV ?

  A: Consequence of HQSS as dominantly DK and D^*K molecules.

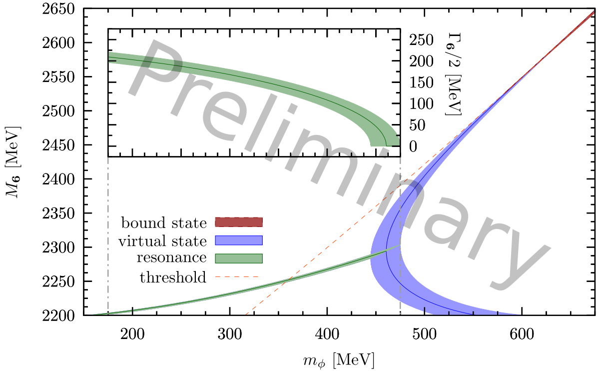
- Why $M_{D_0^*(2400)} \gtrsim M_{D_{s0}^*(2317)}$ and $M_{D_1(2430)} \sim M_{D_{s1}(2460)}$?

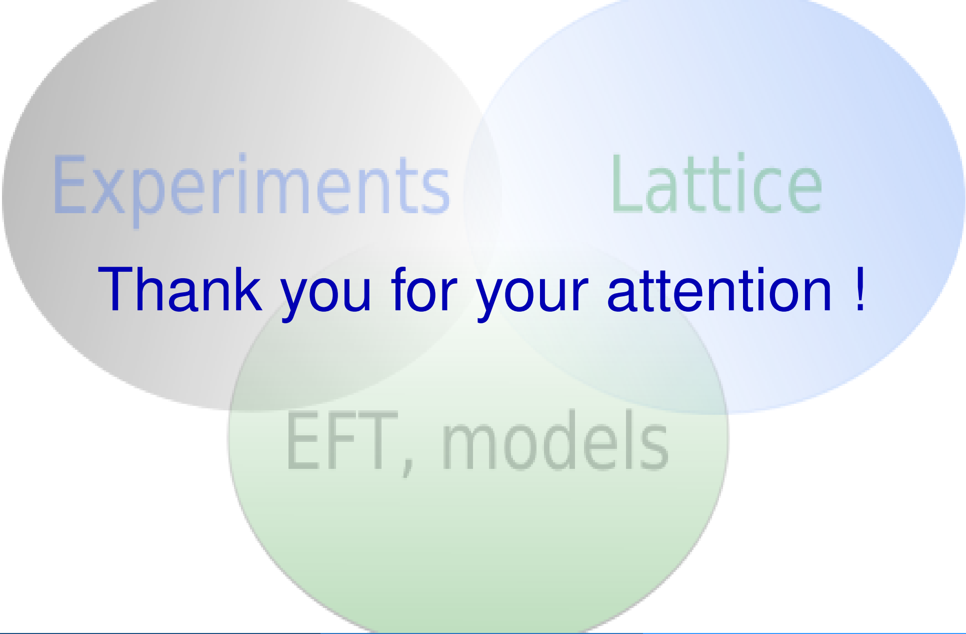
  A: There are two D_0^* and two D_1, and the lower ones have smaller masses.

Summary and outlook (2)

- Suggestions for experimental tests:
  - ☞ $B^- \to D^{*+}\pi^-\pi^-$, strong variations in $\langle P_1\rangle - \frac{14}{9}\langle P_3\rangle$ around the $D^*\eta$ and $D_s^*\bar{K}$ thresholds!
  - The same pattern should be repeated in the bottom sector
- Suggestions for lattice:

  to search for the higher state with a SU(3) symmetric large quark mass





Thank you for your attention !

HQS for $D_{s0}^*(2317)$ and $D_{s1}(2460)$

- Heavy quark flavor symmetry:
  for a singly-heavy hadron, $M_{H_Q} = m_Q + A + \mathcal{O}\left(m_Q^{-1}\right)$
- rough estimates of bottom analogues whatever the D_{sJ} states are

$$M_{B_{s0}^*} = M_{D_{s0}^*(2317)} + \Delta_{b-c} + \mathcal{O}\left(\Lambda_{\text{QCD}}^2\left(\frac{1}{m_c} - \frac{1}{m_b}\right)\right) \simeq (5.65 \pm 0.15)\ \text{GeV}$$

$$M_{B_{s1}} = M_{D_{s1}(2460)} + \Delta_{b-c} + \mathcal{O}\left(\Lambda_{\text{QCD}}^2\left(\frac{1}{m_c} - \frac{1}{m_b}\right)\right) \simeq (5.79 \pm 0.15)\ \text{GeV}$$

  here $\Delta_{b-c} \equiv m_b - m_c \simeq \overline{M}_{B_s} - \overline{M}_{D_s} \simeq 3.33$ GeV, where
  $\overline{M}_{B_s} = 5.403$ GeV, $\overline{M}_{D_s} = 2.076$ GeV: spin-averaged g.s. $Q\bar{s}$ meson masses

  ☞ both to be discovered [1]

- more precise predictions can be made in a given model, e.g. hadronic molecules

[1] The established meson $B_{s1}(5830)$ is probably the bottom partner of $D_{s1}(2536)$.

Easy predictions from HQFS

- Heavy quark flavor symmetry (HQFS) for any hadron containing one heavy quark: velocity remains unchanged in the limit $m_Q \to \infty$: $\Delta v = \frac{\Delta p}{m_Q} = \frac{\Lambda_{\text{QCD}}}{m_Q}$
  $\Rightarrow$ heavy quark is like a static color triplet source, m_Q is irrelevant
- Predicting the bottom-partner masses in 1 minute:

$$M_{B_{s0}^*} \simeq M_B + M_K - 45\,\text{MeV} \simeq 5.730\ \text{GeV}$$
$$M_{B_{s1}} \simeq M_{B^*} + M_K - 45\,\text{MeV} \simeq 5.776\ \text{GeV}$$

nice agreement with lattice results: Lang, Mohler, Prelovsek, Woloshyn, PLB750(2015)17

$$M^{\text{lat.}}_{B_{s0}^*} = (5.711 \pm 0.013 \pm 0.019)\ \text{GeV}$$
$$M^{\text{lat.}}_{B_{s1}} = (5.750 \pm 0.017 \pm 0.019)\ \text{GeV}$$

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Chiral Lagrangian (I)

- The leading order Lagrangian:

$$\mathcal{L}_{\phi P}^{(1)} = D_\mu P D^\mu P^\dagger - m^2 P P^\dagger$$

with $P = (D^0, D^+, D_s^+)$ denoting the D–mesons, and the covariant derivative being

$$\begin{aligned} D_\mu P &= \partial_\mu P + P\Gamma_\mu^\dagger, \quad D_\mu P^\dagger = (\partial_\mu + \Gamma_\mu) P^\dagger, \\ \Gamma_\mu &= \frac{1}{2}\left(u^\dagger \partial_\mu u + u \partial_\mu u^\dagger\right), \end{aligned}$$

where $u_\mu = i\left[u^\dagger(\partial_\mu - i r_\mu)u + u(\partial_\mu - i l_\mu)u^\dagger\right], \quad u = e^{i\lambda_a \phi_a/(2F_0)}$

Burdman, Donoghue (1992); Wise (1992); Yan et al. (1992)

- this gives the Weinberg–Tomozawa term for $P\phi$ scattering

Chiral Lagrangian (II)

- At the next-to-leading order $\mathcal{O}\left(p^2\right)$: FKG, Hanhart, Krewald, Meißner, PLB**666**(2008)251

$$\mathcal{L}_{\phi P}^{(2)} = P\left[-h_0\langle\chi_+\rangle - h_1\chi_+ + h_2\langle u_\mu u^\mu\rangle - h_3 u_\mu u^\mu\right]P^\dagger$$
$$+ D_\mu P\left[h_4\langle u_\mu u^\nu\rangle - h_5\{u^\mu, u^\nu\}\right]D_\nu P^\dagger \,,$$

$\chi_\pm = u^\dagger\chi u^\dagger \pm u\chi^\dagger u, \quad \chi = 2B_0\,\mathrm{diag}(m_u, m_d, m_s)$

- LECs: $h_{1,3,5} = \mathcal{O}\left(N_c^0\right), h_{2,4,6} = \mathcal{O}\left(N_c^{-1}\right)$

  $M_{D_s} - M_D \Rightarrow h_1 = 0.42$

  h_0: can be fixed from lattice results of charmed meson masses

  $h_{2,3,4,5}$: to be fixed from lattice results on scattering lengths

- Extensions to $\mathcal{O}\left(p^3\right)$, see Y.-R. Liu, X. Liu, S.-L. Zhu, PRD**79**(2009)094026; L.-S. Geng et al., PRD**82**(2010)054022; D.-L. Yao, M.-L. Du, FKG, U.-G. Meißner, JHEP**1511**(2015)058;

  M.-L. Du, FKG, U.-G. Meißner, D.-L. Yao, EPJC**77**(2017)728

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Energy levels in a finite volume

- Goal: predict finite volume (FV) energy levels for $I = 1/2$, and compare with recent lattice data by the `Hadron Spectrum Col.` in JHEP1610(2016)011
  $\Rightarrow$ insights into $D_0^*(2400)$
- In a FV, momentum gets quantized: $\vec{q} = \frac{2\pi}{L}\vec{n}$, $\vec{n} \in \mathbb{Z}^3$
- Loop integral $G(s)$ gets modified: $\int d^3\vec{q} \to \frac{1}{L^3}\sum_{\vec{q}}$, and one gets

M. Döring, U.-G. Meißner, E. Oset, A. Rusetsky, EPJA47(2011)139

$$\widetilde{G}(s,L) = G(s) + \lim_{\Lambda\to+\infty} \underbrace{\left[\frac{1}{L^3} \sum_{\vec{n}}^{|\vec{q}|<\Lambda} I(\vec{q}) - \int_0^{\Lambda} \frac{q^2 \mathrm{d}q}{2\pi^2}\, I(\vec{q}) \right]}_{\text{finite volume effect}}$$

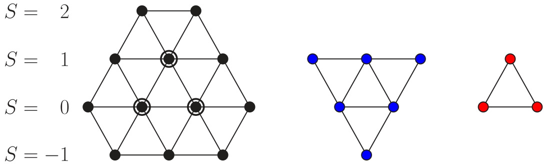
$I(\vec{q})$: loop integrand

- FV energy levels obtained by as poles of $\widetilde{T}(s,L)$:

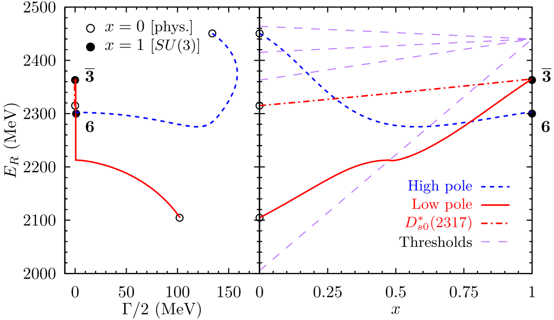
$$\widetilde{T}^{-1}(s,L) = V^{-1}(s) - \widetilde{G}(s,L)$$

SU(3) analysis

- In the SU(3) limit, irreps: $\overline{\mathbf{3}} \otimes \mathbf{8} = \overline{\mathbf{15}} \oplus \mathbf{6} \oplus \overline{\mathbf{3}}$



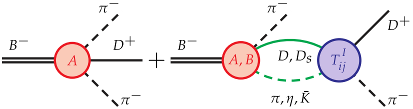
- Evolution of the two poles (LO) from the physical to the SU(3) symmetric case



Fit to LHCb data

Du et al., arXiv:1712.07957 [hep-ph]

- $B^- \to D^+\pi^-\pi^-$ contains coupled-channel $D\pi$ FSI
- consider S, P, D waves: $\mathcal{A}(B^- \to D^+\pi^-\pi^-) = \mathcal{A}_0(s) + \mathcal{A}_1(s) + \mathcal{A}_2(s)$
  - ☞ P-wave: D^*, $D^*(2680)$; D-wave: $D_2(2460)$ as in the LHCb paper
  - ☞ S-wave: use the coupled-channel (1: $D\pi$; $2: D\eta$; $3: D_s\bar{K}$) amplitudes with all parameters fixed before



  - ☞ only 2 parameters in S-wave: C and a subtraction constant in $G_i(s)$

SU(3)+chiral ⇒ $\mathcal{A}_0(s) \propto E_\pi \left[2 + G_{D\pi}(s)\left(\frac{5}{3}T^{1/2}_{11}(s) + \frac{1}{3}T^{3/2}(s)\right)\right]$

$$+\frac{1}{3}E_\eta G_{D\eta}(s)T^{1/2}_{21}(s) + \sqrt{\frac{2}{3}}E_{\bar{K}}G_{D_s\bar{K}}(s)T^{1/2}_{31}(s)$$

$$+C\, E_\eta G_{D\eta}(s)T^{1/2}_{21},$$

$\text{Im}\, G_i(s) = -\rho_i(s) \Rightarrow$ Unitarity: $\text{Im}\mathcal{A}_{0,i}(s) = -\sum_j T^*_{ij}(s)\rho_j(s)\mathcal{A}_{0,j}(s)$

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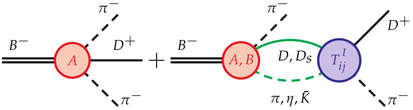

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