Charmonium and exotics from lattice QCD

Francesco Knechtli

- Charmonium and exotics
- Hadro-charmonium
- Decoupling of the charm quark
- Conclusions

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Part I

Charmonium and exotics

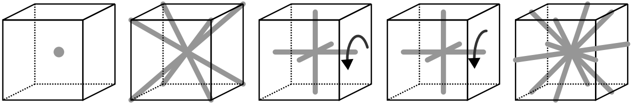
Lattice QCD

Charmonium $c\bar{c}$ resonances on the lattice:

- masses are well understood if the states are treated as stable
- states above the open charm tresholds $D\bar{D}$ etc. decay strongly and multi-hadron channels need to be included
- On finite Euclidean lattice: no dynamical real-time, no asymptotic states
- Workaround: scattering data can be inferred from the spectrum of QCD in a finite volume below the inelastic threshold [Lüscher, Comm. Math. Phys. 105 (1986); Lüscher, Nucl. Phys. B354 (1991)]. For a recent review [Briceno, Dudek and Young, 1706.06223]

Lattice techniques 1

Quantum numbers on the lattice: J^{PC}



[FK, Peardon and Günther, Lattice Quantum Chromodynamics: Practical Essentials, Springer Briefs in Physics]

- parity P and charge conjugation C ✓
- for states at rest, $SO(3)$ of infinite volume continuum is reduced to the 24 proper rotations of the cube O
- five irreducible representations (irreps) Λ of O contain each an infinite number of continuum spins $J \geq 0$
- components M of a spin J are distributed across lattice irreps (subduction)

Start from continuum operators $\mathcal{O}^{J,M}$ to construct operators in lattice irrep Λ

$$\mathcal{O}^{[J]}_{\Lambda,\lambda} = \sum_M \mathcal{S}^{\Lambda,\lambda}_{J,M} \mathcal{O}^{J,M}$$

[Dudek et al, 0909.0200, 1004.4930]

Lattice techniques 2

$\mathcal{O}^\dagger(0)$ ($\mathcal{O}(t)$) are creation (annihilation) operators for hadrons at rest. Masses are extracted from two-point function at separation t in Euclidean time

$$C_{ij}(t) = \langle 0|\mathcal{O}_i(t)\,\mathcal{O}_j^\dagger(0)|0\rangle$$

i, j label states in a basis for a given irrep Λ.
Simplest meson operators are the bilinears $\sum_{\underline{x}} \bar{\psi}(\underline{x}, t)\Gamma\psi(\underline{x}, t)$. For operators with spins $J \geq 2$ and exotic quantum numbers like $(0^{+-}, 1^{-+}, 2^{+-})$ need [Dudek et al, 1004.4930]

$$\mathcal{O} = \sum_{\underline{x}} \bar{\psi}(\underline{x}, t)\Gamma \overleftrightarrow{D}_i \overleftrightarrow{D}_j \cdots \psi(\underline{x}, t)\ , \quad \overleftrightarrow{D} \equiv \overleftarrow{D} - \overrightarrow{D}$$

In order to study resonances one needs to include operators for multi-hadron states. Basis can be enlarged with tetraquark operators.
$C_{ij}(t)$ has the spectral decomposition

$$C_{ij}(t) = \sum_{\mathfrak{n}} \frac{Z_i^{\mathfrak{n}*} Z_j^{\mathfrak{n}}}{2M_{\mathfrak{n}}} e^{-M_{\mathfrak{n}} t}\ , \quad \text{with overlaps} \quad Z_i^{\mathfrak{n}} \equiv \langle \mathfrak{n}|\mathcal{O}_i^\dagger|0\rangle$$

Lattice techniques 3

Quark field smearing

To improve the overlaps $Z_i^{\mathfrak{n}} \equiv \langle \mathfrak{n} | \mathcal{O}_i^\dagger | 0 \rangle$ expose long-range degrees of freedom that best create physical states

$$\psi(\underline{x}, t) \longrightarrow \tilde{\psi}(\underline{x}, t) = \sum_{\underline{y}} \Box[U](\underline{x}, \underline{y}) \psi(\underline{y}, t)$$

Distillation: $\Box = VV^\dagger$ projector onto the space of the N_D lowest eigenstates of the 3d gauge-covariant Laplacian [Hadron Spectrum Collaboration, Peardon et al, 0905.2160], enables computation of all-to-all quark propagators

Variational method

Masses of states $M_{\mathfrak{n}}$ are extracted from the generalized eigenvalue problem

$$C_{ij}(t) v_j^{\mathfrak{n}} = \lambda^{\mathfrak{n}}(t, t_0) C_{ij}(t_0) v_j^{\mathfrak{n}}$$

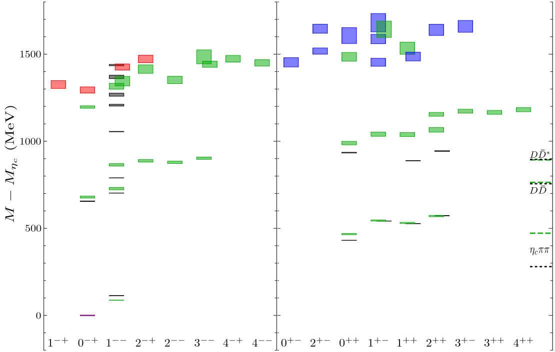
where $\lambda^{\mathfrak{n}}(t, t_0) \simeq \mathrm{e}^{-M_{\mathfrak{n}}(t-t_0)}$ [Michael, Nucl. Phys. B259 (1985); Lüscher and Wolff, Nucl. Phys. B339 (1990)]. Overlaps can be extracted $Z_i^{\mathfrak{n}} = \sqrt{2M_{\mathfrak{n}}} \mathrm{e}^{M_{\mathfrak{n}} t_0/2} v_j^{\mathfrak{n}*} C_{ji}(t_0)$.

Charmonium and its excited states

Calculation by the Hadron Spectrum Collaboration:

- 128×24^3 and 256×32^3 ensembles generated with $N_\mathrm{f} = 2 + 1$ dynamical quarks; $m_\mathrm{strange} \approx m_\mathrm{strange}^\mathrm{phys}$, $m_\mathrm{up} = m_\mathrm{down} = m_\mathrm{light}$ corresponds to $M_\pi = 391\,\mathrm{MeV}$ and $M_\pi = 236\,\mathrm{MeV}$
- Quenched (not dynamical) charm quark, its mass is tuned to reproduce the physical η_c mass
- Anisotropic lattices $a_s \approx 0.12\,\mathrm{fm}$, $a_t \approx 0.034\,\mathrm{fm}$; charmonium two-point function $\sim \mathrm{e}^{-(a_t M_\mathrm{n})(t/a_t)}$
- Only bilinears, no multi-hadron operators included; states above threshold are treated as stable $\Rightarrow$ mass is accurate up to hadronic width
- Improved techniques (operator construction, distillation, variational method) described above are used
- Charm-annihilation (disconnected) diagrams are not included (OZI suppressed)

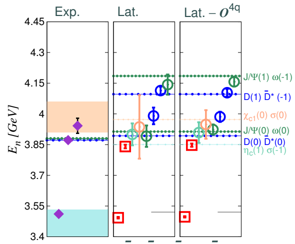
Charmonium and its excited states contd



[Hadron Spectrum Collaboration: Cheung, O'Hara, Moir, Peardon, Ryan, Thomas and Tims, 1610.01073]

- Results on the $M_\pi = 236\,\mathrm{MeV}$ ensemble compared to PDG (black)
- Many states follow the $n^{2S+1}L_J$ pattern of quark potential models
- Excess states are hybrid mesons $\bar{q}qg$: lightest and first excited hybrid supermultiplet; four hybrids are **exotic**
- Every state has a preference to overlap only on operators subduced from a single continuum spin

The $X(3872)$



(a) $I=0$: $\bar{c}c$ ($\bar{u}u + \bar{d}d$) & $\bar{c}c$

[Padmanath, Lang and Prelovsek, 1503.03257]

$J^{PC} = 1^{++}$, $M = 3871.69 \pm 0.17\,\mathrm{MeV}$,
$m_{D^0} + m_{D^{*0}} = 3871.69 \pm 0.09\,\mathrm{MeV}$ [Olsen, 1511.01589]

- Additional energy eigenstate appears compared to non-interacting spectrum
- Two levels with dominant overlap with $\bar{c}c$ and $D\bar{D}^*$ operators
- Interpretation as pure molecule or pure tetraquark is unlikely
- Mass extracted from $D\bar{D}^*$ scattering analysis with two energy levels: $m_X - (m_{D^0} + m_{D^{*0}}) = -8(15)\,\mathrm{MeV}$
- agreement with previous results [Prelovsek and Leskovec, 1307.5172; Lee, DeTar, Mohler and Na, 1411.1389]
- systematic finite size $L \simeq 2\,\mathrm{fm}$ and discretization $a = 0.12\,\mathrm{fm}$ effects (pion mass is $m_\pi = 266\,\mathrm{MeV}$)

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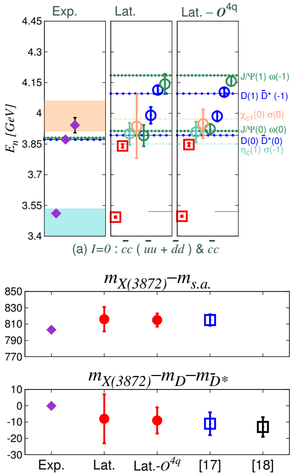

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Part II

Hadro-charmonium

Motivation

- LHCb found pentaquark candidates P_c^+ of exotic quark content $uudc\bar{c}$ in the decay $\Lambda_b \to (J/\psi\ p)\ K$ [LHCb: Aaij et al, 1507.03414, 1604.05708]
- 5 quark (4 q, 1 $\bar{q}$) systems are very difficult to study directly on the lattice, in particular if many decay channels are possible
- $20\,\mathrm{MeV}$ binding energy reported for charmonium-nucleon system for a rather large light quark mass ($m_\pi \approx 800\,\mathrm{MeV}$) and coarse lattice spacing $a \approx 0.145\,\mathrm{fm}$ [NPLQCD Collaboration: Beane et al, 1410.7069]

Hadro-quarkonia

- Hadro-quarkonium model: quarkonium core embedded in a light hadron cloud [Dubynskiy and Voloshin, 0803.2224]
- Based on attractive color dipole-dipole van der Waals interaction between point-like quarkonium and hadron
- Could explain the LHCb pentaquark, examples of close-by charmonium-baryon systems:

$J^P = \frac{3}{2}^-$: $m(\Delta) + m(J/\psi) \approx 4329\,\mathrm{MeV}$ vs. $P_c^+(4380)$ (width $200\,\mathrm{MeV}$)

$J^P = \frac{5}{2}^+$: $m(N) + m(\chi_{c2}) \approx 4496\,\mathrm{MeV}$ vs. $P_c^+(4450)$ (width $40\,\mathrm{MeV}$)

Static quarks

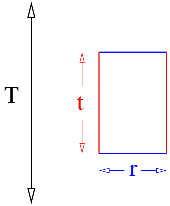
We consider quarkonium $\bar{Q}Q$ in the static limit $m_Q \to \infty$. To leading order in (p)NRQCD, quarkonia can be approximated by the non-relativistic Schrödinger equation with a static potential $V_0(r)$.

Static potential V_0 in the vacuum

$\mathcal{Q}_{\mathbf{r}}^\dagger(\mathbf{z})$ is a lattice operator which creates a static quark at $\mathbf{z}+\mathbf{r}/2$ and an anti-quark at $\mathbf{z}-\mathbf{r}/2$

$$V_0(r) = -\lim_{t\to\infty} \frac{\mathrm{d}}{\mathrm{d}t} \ln\langle 0|\mathcal{Q}_{\mathbf{r}} \mathcal{T}^{t/a} \mathcal{Q}_{\mathbf{r}}^\dagger|0\rangle$$

assuming the theory has a transfer matrix $\mathcal{T} = e^{-a\mathbb{H}}$



T is the temporal size of the lattice and $W(r,t)$ a rectangular Wilson loop

$$\langle W(r,t)\rangle \overset{T\to\infty}{\sim} \langle 0|\mathcal{Q}_{\mathbf{r}} \mathcal{T}^{t/a} \mathcal{Q}_{\mathbf{r}}^\dagger|0\rangle = \sum_n c_n c_n^* \mathrm{e}^{-V_n(r)t}$$

Extrap. $t\to\infty$ to extract the ground state potential V_0

Hadro-quarkonium in the static limit

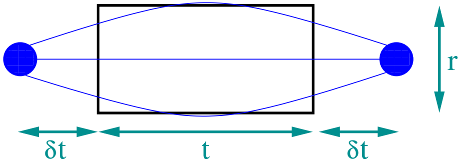
Test of the hadro-quarkonium model in the static limit: Does the static potential become more or less attractive, when a light hadron H is "added"?

Static potential V_H in the presence of a hadron

$\overline{\mathcal{H}}$ is a lattice operator which creates a hadron $|H\rangle = \overline{\mathcal{H}}|0\rangle$

$$V_H(r) = -\lim_{t\to\infty} \frac{\mathrm{d}}{\mathrm{d}t} \ln\langle H|\mathcal{Q}_{\mathbf{r}}\mathcal{T}^{t/a}\mathcal{Q}_{\mathbf{r}}^{\dagger}|H\rangle$$

Hadron is at zero-momentum $\mathcal{H} \equiv \sum_{\mathbf{x}} \mathcal{H}(\mathbf{x})$ (momentum can also be injected)



Put a hadronic source at time 0, (almost) reach ground state $|H\rangle$ at the time δt
Create a static $\bar{Q}Q$ pair and propagate over time t
Annihilate $\bar{Q}Q$ pair at $t + \delta t$ and the light hadron at $t + 2\delta t$

Hadro-quarkonium correlator

Potential shift ΔV_H

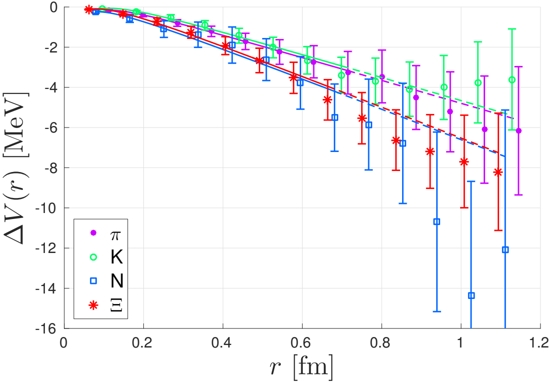
$$\begin{aligned}\Delta V_H(r,\delta t) &= V_H(r,\delta t) - V_0(r) \\ &= -\lim_{t\to\infty}\frac{\mathrm{d}}{\mathrm{d}t}\ln\frac{\langle W(r,t)C_{H,\mathrm{2pt}}(t+2\delta t)\rangle}{\langle W(r,t)\rangle\langle C_{H,\mathrm{2pt}}(t+2\delta t)\rangle}\end{aligned}$$

and extrapolating $\delta t \to \infty$.
Wilson loops $W(r,t)$ are averaged over spatial position and hadronic two-point functions $\langle C_{H,\mathrm{2pt}}(t+2\delta t)\rangle = \langle 0|\mathcal{H}\mathcal{T}^{(t+2\delta t)/a}\overline{\mathcal{H}}|0\rangle$ are averaged over the sink position (zero momentum).

$N_\mathrm{f} = 2+1$ ensemble C101 (96×48^3 sites) generated by the **C**oordinated **L**attice **S**imulations consortium [Bruno et al, 1411.3982]:

- $m_\pi = 220\,\mathrm{MeV}$, $m_\mathrm{K} = 470\,\mathrm{MeV}$, $L \approx 4.1\,\mathrm{fm}$, $a = 0.0854(15)\,\mathrm{fm}$
- High statistics: 1552 configs, separated by 4 MDUs, times 12 hadron sources (1 forward, 1 backward, 11 forward and backward propagating ⇒ 24 2-point functions). Wilson loops at all positions and in all directions.

Potential shifts



[Alberti, Bali, Collins, FK, Moir and Söldner, 1608.06537]

Shown are data for $\Delta V_H(r, \delta t = 5a)$. Curves represent the parametrization

$$\Delta V_H(r) = \Delta\mu_H - \frac{\Delta c_H}{r} + \Delta\sigma_H r$$

Parameters $\Delta\mu_H$, Δc_H, $\Delta\sigma_H$ describe the modifications to the Cornell potential $V_0 = \mu - c/r + \sigma r$

Volume check

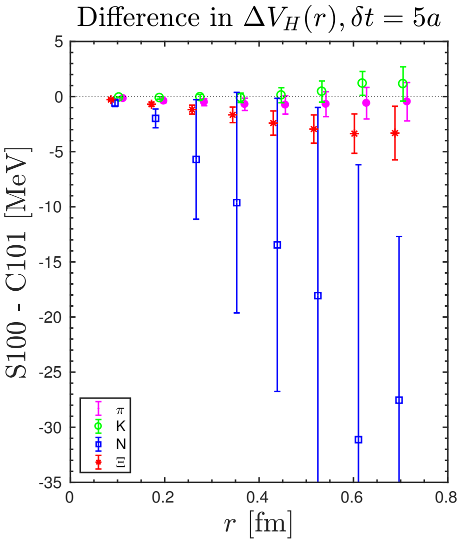
CLS ensemble S100

- same a and quark masses as C101 but
- smaller $L \approx 2.7\,\mathrm{fm}$
- 940 configurations times 10 hadron sources (forward and backward propagating $\Rightarrow$ 20 2-point functions)

Check for finite volume effects:

$$\Delta V_H(r)^{\mathrm{S100}} - \Delta V_H(r)^{\mathrm{C101}}$$

- no significant finite volume effects visible for distances $r > 0.3\,\mathrm{fm}$
- only statistical errors shown



Phenomenological implications

Non-relativistic approach (potential NRQCD) to describe quarkonia $\overline{Q}Q$

$$H\psi_{nL} = M_{nL}^{(0)}\psi_{nL}\,, \quad H = 2(m_Q - \delta m_Q) + \frac{p^2}{m_Q} + V_0(r) + \ldots$$

Quarkonium levels $M_{nL}^{(0)}$ (n radial, L angular momentum quantum numbers)

Mass/Mass difference	$1S\,[\mathrm{MeV}]$	$1P\,[\mathrm{MeV}]$	$2S\,[\mathrm{MeV}]$
M_{nL} (experiment)	3068.6	3525.3	3674.4
$M_{nL}^{(0)}$ (Schrödinger)	3068.6	3483.3	3674.4
$\Delta M^{(\pi)}$	-1.7	-3.1	-4.0
$\Delta M^{(K)}$	-1.5	-2.9	-3.8
$\Delta M^{(\rho)}$	-2.5	-4.9	-6.5
$\Delta M^{(K^*)}$	-1.6	-3.2	-4.2
$\Delta M^{(\phi)}$	-1.6	-3.2	-4.3
$\Delta M^{(N)}$	-2.4	-4.3	-5.5
$\Delta M^{(\Xi)}$	-2.0	-3.9	-5.1
$\Delta M^{(\Delta)}$	-0.9	-1.0	-1.0
$\Delta M^{(\Xi^*)}$	-2.6	-4.8	-6.3

Application to charmonium:

- adjust m_c and δm_c to reproduce experimental $1S$ and $2S$ charmonia
- replace $V_0 \longrightarrow V_H = V_0 + \Delta V_H$ and compute $M_{nL}^{(H)}$
- compute mass differences $\Delta M_{nL}^{(H)} = M_{nL}^{(H)} - M_{nL}^{(0)}$
- caveats: relativistic corrections are not small for charmonium and $m_H \approx m_c$ for baryons

$\Delta M^{(H)} < 0 \Rightarrow$ charmonium “within” a hadron H is energetically favorable

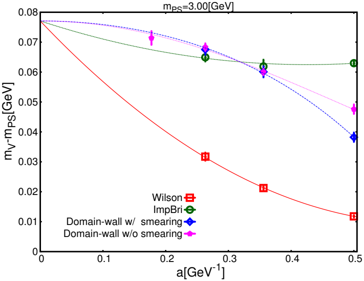
Part III

Decoupling of the charm quark

Charm sea effects

At present most simulations of lattice QCD are done with $N_f = 2 + 1$ dynamical quarks (up, down and strange). The inclusion of dynamical heavy quarks (charm) requires

- high precision in low energy observables to resolve tiny charm sea effects
- small lattice spacings to control cut-off effects proportional to the heavy quark mass: $m_c = 1.28\,\mathrm{GeV}$ with $a > 0.05\,\mathrm{fm} \equiv 0.1\,\mathrm{GeV}^{-1} \Rightarrow am_c > 0.3$



quenched study, constrained continuum limit (not physical)

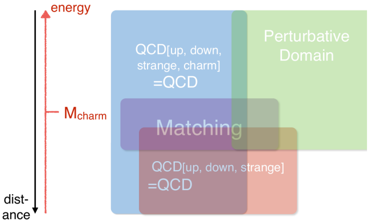
[Cho, Hashimoto, Jüttner, Kaneko, Marinkovic, Noaki and Tsang, `1504.01630`]

Effective theory of decoupling

Effective theory for energies $E \ll M$ (M is the mass of the heavy quark)

[Weinberg, Phys. Lett. B91 (1980)]

$$\begin{aligned}\mathscr{L}^{(N_\mathrm{f})}_{\mathrm{QCD}} &= \mathscr{L}^{(N_\mathrm{f}-1)}_{\mathrm{QCD}}(\psi_{\mathrm{light}}, \bar{\psi}_{\mathrm{light}}, A_\mu; g_-(M), m_-(M)) \\ &\quad + \frac{1}{M^2}\mathscr{L}_6 \\ \mathscr{L}_6 &= \bar{\psi}_{\mathrm{light}}\sigma_{\mu\nu}F_{\mu\nu}m_{\mathrm{light}}\psi_{\mathrm{light}} + \ldots\end{aligned}$$



Effective theory couplings $g_- = g_{\mathrm{light}}$, $m_- = m_{\mathrm{light}}$ determined by matching $\leftrightarrow$ decouping of the heavy quark

Model study

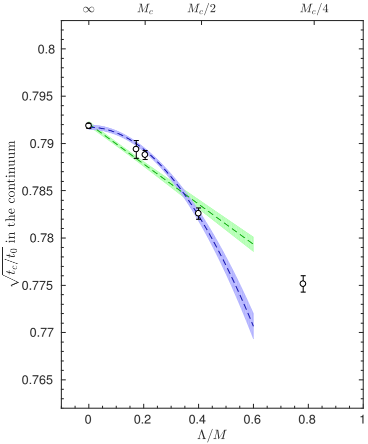
Perturbative matching

- By itself perturbation theory for decoupling of the charm quark seems to work well. This is used in the determination of $\alpha_{\overline{\mathrm{MS}}}(M_Z) = 0.1185(8)(3)$ from QCD_3 simulations by the ALPHA Collaboration [Bruno et al., 1706.03821] (the non-lattice value quoted by the PDG is $0.1174(16)$)
- Non-perturbative check?
- In perturbation theory power corrections in $1/M$ are neglected. What is their size?

Non-perturbative model study on the lattice

- To avoid a multi-scale problem in comparing QCD_4 and QCD_3, we study a model, QCD_2 with $N_\mathrm{f} = 2$ degenerate quarks of mass $1.2\,M_\mathrm{c} \gtrsim M \gtrsim M_\mathrm{c}/8$
- Effective theory for $E \ll M$ is a Yang–Mills (YM) theory ($N_\mathrm{f} = 0$, $M = \infty$) at leading order
- We can afford very small lattice spacings down to $a = 0.023\,\mathrm{fm}$ and control the continuum limit

Validity of the effective theory for charm



[FK, Korzec, Leder and Moir, 1706.04982]

For ratios of low energy hadronic scales the effective theory predicts

$$\left.\frac{m^{\text{had},1}(M)}{m^{\text{had},2}(M)}\right|_{N_f=2} = \left.\frac{m^{\text{had},1}}{m^{\text{had},2}}\right|_{N_f=0} + k/M^2$$

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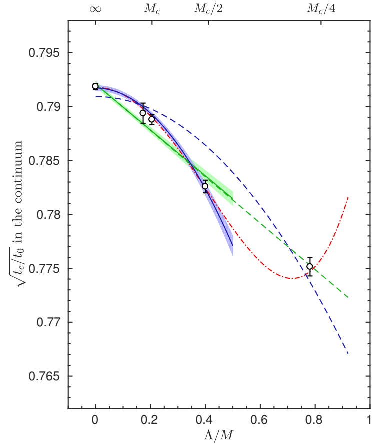

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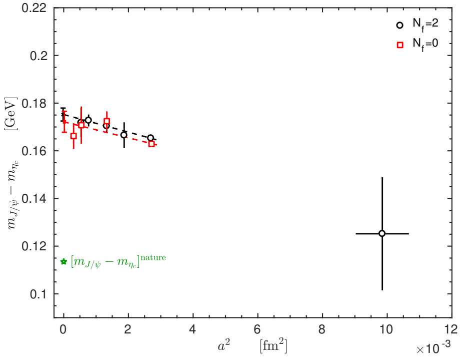
Impact of sea charm quarks

Continuum extrapolation of hyperfine splitting at $m_{\eta_c} = 3.25\,\mathrm{GeV}$



[Korzec, FK, Cali, Leder and Moir, `1612.07634`; Cali, FK, Korzec, work in progress]

- Decoupling of charm quark works for binding energies of charmonium
- Thanks to lattice spacings $a \lesssim 0.05\,\mathrm{fm}$ continuum extrapolations linear in a^2 are under control
- Light sea quarks, disconnected contributions and electromagnetism are presumably responsible for the deviation to physical number

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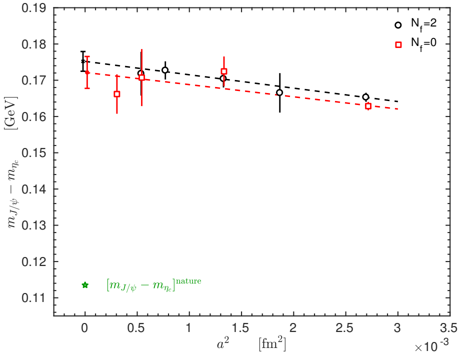

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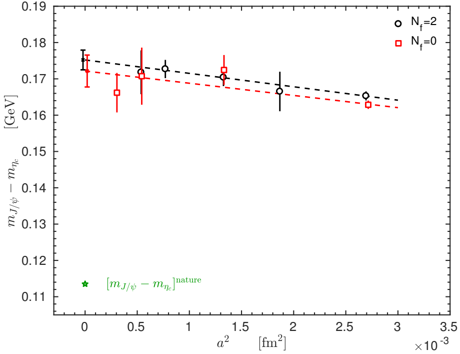

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Part IV

Conclusions

Conclusions

Charmonium and exotics

- Lattice QCD provides the techniques to study charmonium resonances
- Lattice simulations can identify candidates for exotic states and elucidate their nature

Hadro-charmonium

- Hadro-charmonium in the static limit yields stronger binding of charmonium but only by few MeV's like deuterium
- Modification of the static potential in a hadron is interesting for charmonium in medium

Decoupling of the charm quark

- Decoupling of charm at low energies is consistent with the effective theory beyond leading order
- Decoupling applies to binding energies of charmonium