Double heavy tri-hadron bound state via delocalized π bond

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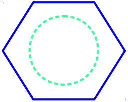
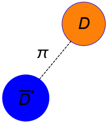
23/05/2018

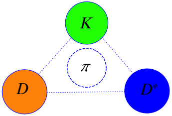
What's the delocalized π bond?



A simplified illustration of a benzene ring as a hexagon with a circle describing the delocalized π *bond* inside.



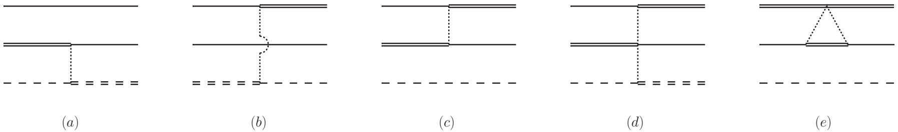
Localized σ bond



Delocalized π bond

$DD^*K \to DDK^* \to$
$D^*DK \to DD^*K$
Two channels: DD^*K and DDK^*.

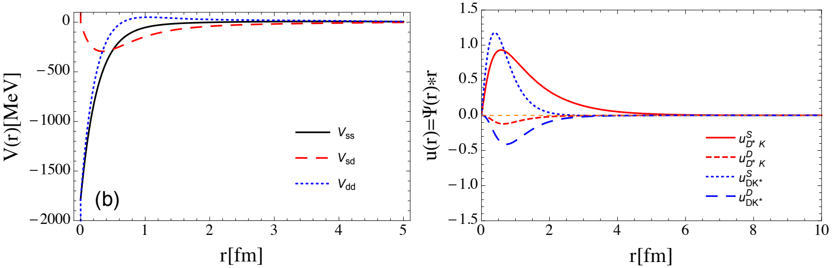
The delocalized π bond in the DD^*K and $D\bar{D}^*K$



- Diagrams (a), (b) and (c) are the leading OPE diagrams for the transitions among the relevant three-body channels, i.e. DD^*K, DDK^* and D^*DK channels. The TPE diagrams, i.e. (d) and (e), are the next-to-leading order contributions.
- The diagrams (a), (b), (c) indicate the transition potentials V_{12}, V_{23} and V_{13}, in order. The (double) solid, dashed lines represent the $D^{(*)}$, $K^{(*)}$ fields. Dotted lines denote pion fields.
- Monopole form factor $\mathcal{F}(q) = \frac{\Lambda^2 - m_\pi^2}{\Lambda^2 - q^2}$, with q the four-momentum of the pion and Λ the cutoff parameter. Λ_{D^*K}, Λ_{DD^*}.

Fix Λ_{D^*K} by reproducing the mass of $D_{s1}(2460)$

- $\mathrm{SU}(2)$ flavor symmetry. To the order $\mathcal{O}(\frac{p_K}{m_K})$. S-D wave mixing, and the coupled channels D^*K and DK^*. With $\Lambda_{D^*K} = 803.2\ \mathrm{MeV}$, we find a D^*K bound state with mass at $D_{s1}(2460)$.
- G-parity rule, i.e., $V_{A\bar{B}} = (-1)^{I_G} V_{AB}$ [1]. $V_{D^*K} = V_{\bar{D}^*K}$.
- $B^*\bar{K}$ with the mass $5772\ \mathrm{MeV}$ [2]. $\Lambda_{B^*\bar{K}} = 1451.0\ \mathrm{MeV}$.



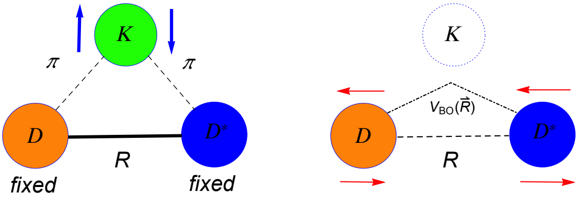
[1] E. Klempt, F. Bradamante, A. Martin and J. M. Richard, Phys. Rept. **368**, 119 (2002).

[2] F. K. Guo, P. N. Shen and H. C. Chiang, Phys. Lett. B **647**, 133 (2007).

Born-Oppenheimer (BO) Approximation

Born-Oppenheimer (BO) approximation with an uncertainty of the order $\mathcal{O}(m_K/m_{D^{(*)}})$.

- Firstly, we keep the two heavy mesons, i.e. D and D^*, at a given fixed location R and study the dynamical behaviour of the light kaon.
- Then, we solve Schrödinger equation of the DD^* system with the effective BO potential created from the interaction with the kaon.

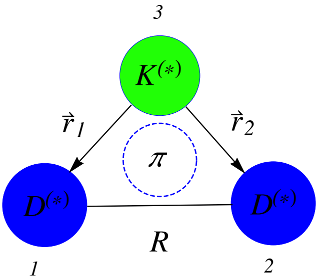


The factorized wave functions

- The BO approximation is based on the factorized wave function

$$|\Psi_T(\vec{R},\vec{r})\rangle = |\Phi(\vec{R})\Psi(\vec{r_1},\vec{r_2})\rangle \ ,$$

with the two charmed mesons and the light kaon located at $\pm\vec{R}/2$ and $\vec{r}$. Here, $\vec{r_1} = \vec{r} + \vec{R}/2$ and $\vec{r_2} = \vec{r} - \vec{R}/2$ are the coordinates of the kaon relative to the first and second interacting D^*.

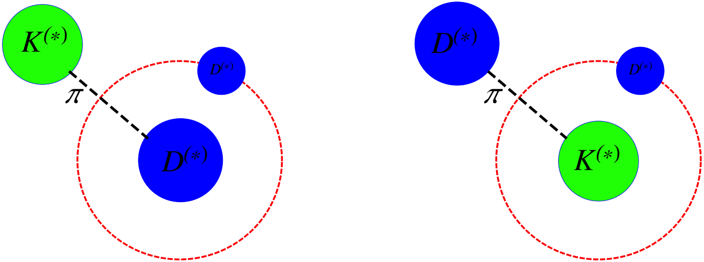


OPE means no hadronic shield effect.

- The wave function of kaon in the DD^*K system is the superposition of the two two-body subsystems

$$\begin{aligned} |\Psi(\vec{r}_1, \vec{r}_2)\rangle &= C_0\{\psi(\vec{r}_2)\,|DD^*K\rangle + \psi(\vec{r}_1)\,|D^*DK\rangle \\ &+ C[\psi'(\vec{r}_1) + \psi'(\vec{r}_2)]|DDK^*\rangle\}\,. \end{aligned} \tag{1}$$

The constant C_0 can be fixed by the normalization constraint of the total wave function.



The separated three-body Schrödinger equation

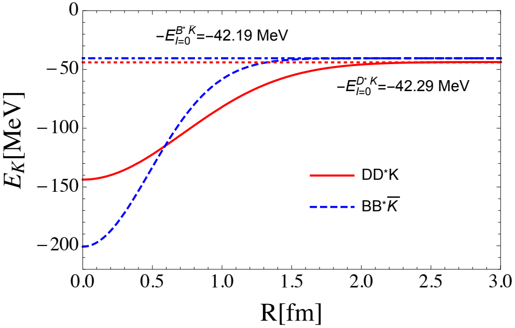
- One is the equation for the kaon

$$H(\vec{r}_1, \vec{r}_2)|\Psi(\vec{r}_1, \vec{r}_2)\rangle = E_K(R)|\Psi(\vec{r}_1, \vec{r}_2)\rangle \tag{2}$$

at any given $\vec{R}$ with

$$H(\vec{r}_1, \vec{r}_2) = \begin{pmatrix} T_{11}(\vec{r}_1, \vec{r}_2) & V_{12}(\vec{r}_2) & 0 \\ V_{21}(\vec{r}_2) & \delta M + T_{22}(\vec{r}_1, \vec{r}_2) & V_{23}(\vec{r}_1) \\ 0 & V_{32}(\vec{r}_1) & T_{33}(\vec{r}_1, \vec{r}_2) \end{pmatrix}.$$

Here, T_{ii} is the relative kinetic energy for the K in the ith channel and $\delta M = M_D + M_{K*} - M_{D*} - M_K$ is the mass gap between the DDK^* and DD^*K (D^*DK) channels. The parameter C in the Eq. (1) can extracted from the variation principle $\partial E_K(R)/\partial C = 0$.



The red dotted and blue dot-dashed horizontal lines are the binding energies of the isosinglet D^*K and $B^*\bar{K}$ systems, respectively.

When the distance R is larger than a certain value, the kaon energy of the three-body system equal to the binding energy of the isosinglet D^*K or $B^*\bar{K}$ two-body system. The two-body binding energies are from the calculations [3].

[3] F. K. Guo, P. N. Shen and H. C. Chiang, Phys. Lett. B **647**, 133 (2007)

- The other one is the Schrödinger equation for the two heavy charmed mesons

$$H'(\vec{R})|\Phi(\vec{R})\rangle = -E_3|\Phi(\vec{R})\rangle \tag{3}$$

with

$$\begin{aligned} H'(\vec{R}) &= T_h(\vec{R}) + V_h(\vec{R}) + V_{\mathrm{BO}}(\vec{R}) \\ &= \begin{pmatrix} T_{DD^*}(\vec{R}) & 0 & V_{13}(\vec{R}) \\ 0 & T_{DD}(\vec{R}) & 0 \\ V_{31}(\vec{R}) & 0 & T_{DD^*}(\vec{R}) \end{pmatrix} + V_{\mathrm{BO}}(\vec{R}) \cdot I, \end{aligned}$$

where $V_{BO}(\vec{R}) = E_K(\vec{R}) + E_B$ is the BO potential provided by the kaon and E_B is the binding energy of the isosinglet D^*K system. The total energy of the three-body system relative to the DD^*K threshold is $E = -(E_3 + E_B)$.

The binding energies of the DD^*K and $BB^*\bar{K}$

- Only the three-body DD^*K system with total isospin $\frac{1}{2}$ and isosinglet D^*K could form a bound state

$$
\begin{aligned}
|D(D^*K)_0\rangle_{\frac{1}{2},\frac{1}{2}} &= \frac{1}{\sqrt{2}}[|D^+(D^{*+}K^0)_0\rangle + |D^+(D^{*0}K^+)_0\rangle], \\
|D(D^*K)_0\rangle_{\frac{1}{2},-\frac{1}{2}} &= \frac{1}{\sqrt{2}}[-|D^0(D^{*+}K^0)_0\rangle - |D^0(D^{*0}K^+)_0\rangle],
\end{aligned}
$$

-

$$
E_{I=1/2}^{DD^*K} = 8.29^{+4.32}_{-3.66}\ \mathrm{MeV}, \quad E_{I=1/2}^{BB^*\bar{K}} = 41.76^{+8.84}_{-8.49}\ \mathrm{MeV},
$$

where the uncertainties are estimated as $m_K/(2\mu_{DD^*})$ and $m_K/(2\mu_{BB^*})$ as discussed above.

-

$$
M_{I=1/2}^{DD^*K} = 4317.92^{+3.66}_{-4.32}\ \mathrm{MeV}, \quad M_{I=1/2}^{BB^*\bar{K}} = 11013.65^{+8.49}_{-8.84}\ \mathrm{MeV},
$$

The binding energies of the $D\bar{D}^*K$ and $B\bar{B}^*\bar{K}$

- For the $I=1/2$ $D\bar{D}^*K$ three-body bound state

$$
\begin{aligned}
|D(\bar{D}^*K)_0\rangle_{\frac{1}{2},\frac{1}{2}} &= \frac{1}{\sqrt{2}}[-|D^+(\bar{D}^{*0}K^0)_0\rangle + |D^+(D^{*-}K^+)_0\rangle], \\
|D(\bar{D}^*K)_0\rangle_{\frac{1}{2},-\frac{1}{2}} &= \frac{1}{\sqrt{2}}[|D^0(\bar{D}^{*0}K^0)_0\rangle - |D^0(D^{*-}K^+)_0\rangle],
\end{aligned}
$$

- For the $D\bar{D}^*K$ and $B\bar{B}^*\bar{K}$ system, the three-body binding energies are

$$
E_{I=1/2}^{D\bar{D}^*K} = 8.29^{+6.55}_{-6.13}\ \mathrm{MeV},\ E_{I=1/2}^{B\bar{B}^*\bar{K}} = 41.76^{+9.02}_{-8.68}\ \mathrm{MeV}
$$

with the additional uncertainty from the missing short-distance interaction.

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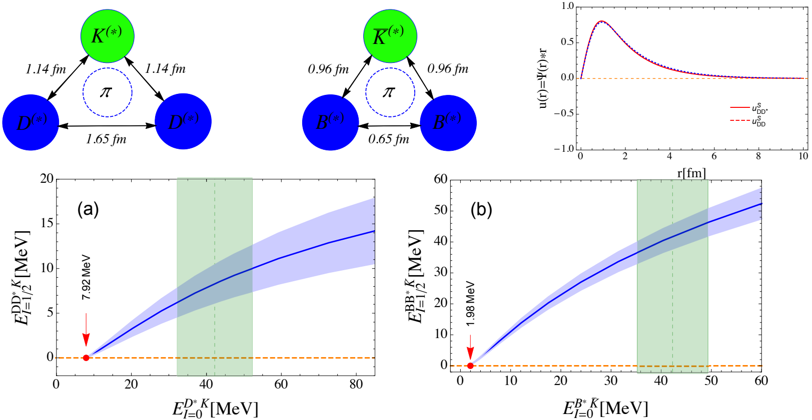
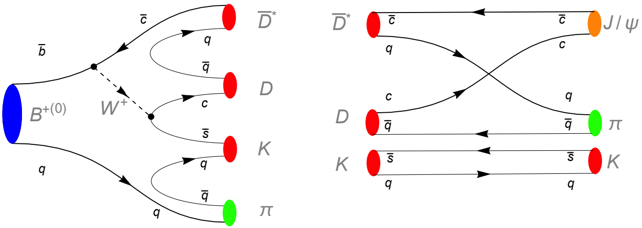
$$
M_{I=1/2}^{D\bar{D}^*K} = 4317.92^{+6.13}_{-6.55}\ \mathrm{MeV},\ M_{I=1/2}^{B\bar{B}^*\bar{K}} = 11013.65^{+8.68}_{-9.02}\ \mathrm{MeV}
$$



The red point is the critical point which indicates the lower limit of the required binding energy of the isosinglet D^*K or $B^*\bar{K}$ to form a three-body bound state. The vertical dashed lines and bands are the central values and uncertainties of the binding energies of the two-body subsystems from the analysis [4].

[4] F. K. Guo, P. N. Shen and H. C. Chiang, Phys. Lett. B **647**, 133 (2007)

Detection from B meson decays in the $J/\psi\pi\pi K$ channel



Aiming at the $X(3872)$, LHCb, Belle and BABAR have collected quite numerous data for B decays in the $J/\psi\pi\pi K$ channel. The existence of the $D\bar{D}^*K$ bound state could be checked from the experimental side by analyzing the current world data on the channels $J/\psi\pi^+K^0$, $J/\psi\pi^0K^+$, $J/\psi\pi^0K^0$, and $\boldsymbol{J/\psi\pi^-K^+}$.

Summary

- Based on the attractive force of the isosinglet D^*K and $B^*\bar{K}$ systems, we predict that there exist two DD^*K and $BB^*\bar{K}$ bound states with $I = 1/2$ and masses $4317.92^{+3.66}_{-4.32}$ MeV and $11013.65^{+8.49}_{-8.84}$ MeV, respectively.
- The $D\bar{D}^*K$ and $B\bar{B}^*\bar{K}$ systems are their analogues with masses $4317.92^{+6.13}_{-6.55}$ MeV and $11013.65^{+8.68}_{-9.02}$ MeV, where the additional uncertainties stemming from the unknown short-distance interaction.
- The existence of the $D\bar{D}^*K$ bound state could be checked from the $B \to J/\psi\pi\pi K$ channel, by focusing on the $J/\psi\pi K$ channel.

Thank you very much!