Lattice Predictions for Bound Heavy Tetraquarks

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The observed heavy hadron spectrum suggests a phenomenological binding mechanism from "good" diquark configurations for tetraquarks containing heavy quarks, e.g. $qq'\bar{b}\bar{b}$ and $qq'\bar{c}\bar{b}$.

Assumptions and observations:

- HQS $\rightarrow$ heavy quark spin decouples and $(\bar{h}\bar{h})_3 \leftrightarrow h$
- Good approx. in $(B^* - B)/(\Xi^*_{bb} - \Xi_{bb})$ and $(B^*_s - B_s)/(\Omega^*_{bb} - \Omega_{bb})$

Naive binding using the spin average $B_{sp} = \frac{1}{4}[3(\text{spin0}) + (\text{spin1})]$:

- $\Sigma_b - \Lambda_b \approx 194\text{MeV}$ vs. $B_{sp} - \Sigma_b \sim -145\text{MeV}$
- $\Xi'_b - \Xi_b \approx 162\text{MeV}$ vs. $B_{sp} - \Xi'_b \sim -106\text{MeV}$

Predictions:

- Deeper binding with heavier quarks, $\sim 1/m_Q$
- Binding set by the reduced mass of $\bar{Q}$ and $\bar{Q}'$ in the $\bar{Q}'\bar{Q}$ diquark
- Deeper binding for lighter quarks in the qq' diquark

$\Rightarrow$ Great opportunity for ab initio theory prediction.

Direct calculation `PhysRevLett.118.142001`

Diquark-Diquark operator:

$$D(x) = \Big((q_a^{\alpha}(x))^T (C\gamma_5)^{\alpha\beta} q'^{\beta}_b(x) \Big) \times \\ \Big[(\bar{Q}_a^{\kappa}(x)(C\gamma_i)^{\kappa\rho}(\bar{Q}'^{\rho}_b(x))^T - (\bar{Q}_b^{\kappa}(x)(C\gamma_i)^{\kappa\rho}(\bar{Q}'^{\rho}_a(x))^T \Big] .$$

Dimeson-Dimeson operator:

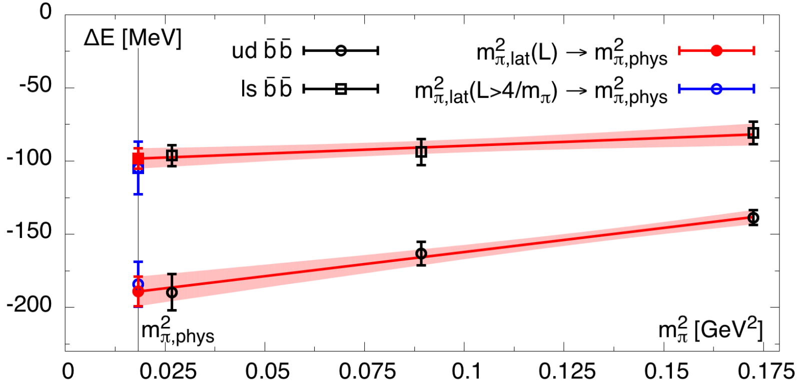
$$M(x) = \bar{b}_a^{\alpha}(x)\gamma_5^{\alpha\beta} u_a^{\beta}(x)\ \bar{b}_b^{\kappa}(x)\gamma_i^{\kappa\rho} d_b^{\rho}(x) - \bar{b}_a^{\alpha}(x)\gamma_5^{\alpha\beta} d_a^{\beta}(x)\ \bar{b}_b^{\kappa}(x)\gamma_i^{\kappa\rho} u_b^{\rho}(x) .$$

Compute the energies from the 2×2 GEVP

$$F(t) = \begin{pmatrix} G_{DD}(t) & G_{DM}(t) \\ G_{MD}(t) & G_{MM}(t) \end{pmatrix}, \quad F(t)\nu = \lambda(t) F(t_0)\nu ,$$

$$G_{\mathcal{O}_1\mathcal{O}_2} = \frac{C_{\mathcal{O}_1\mathcal{O}_2}(t)}{C_{PP}(t) C_{VV}(t)} ,\ \lambda(t) = A e^{-\Delta E (t-t_0)} .$$

Tetraquarks with $\bar{Q}'\bar{Q}$: 3×3 GEVP via second two-meson threshold.



Physical point: $\Delta E_{ud\bar{b}\bar{b}} = 189(10)(3)$ MeV and $\Delta E_{ls\bar{b}\bar{b}} = 98(7)(3)$ MeV

PACS-CS,'09	$32^3 \times 64$	$a^{-1} = 2.194$[GeV]	$m_{s,lat} = m_{s,phys}$
Label	E_H	E_M	E_L
m_π [MeV]	415	299	163
$m_\pi L$	6.1	4.4	2.4

New results: Heavy quark mass dependence and $ud\bar{c}\bar{b}$

Predictions:

- Deeper binding with heavier quarks, $\sim 1/m_Q$
- Binding set by the reduced mass of $\bar{Q}$ and $\bar{Q}'$ in the $\bar{Q}'\bar{Q}$ diquark
- Deeper binding for lighter quarks in the qq' diquark ✓

The direct calculation of $ud\bar{b}\bar{b}$ and $\ell s\bar{b}\bar{b}$ validates the prediction of deeper binding with lighter qq' diquarks.

Can further insight into the binding mechanism be gained?

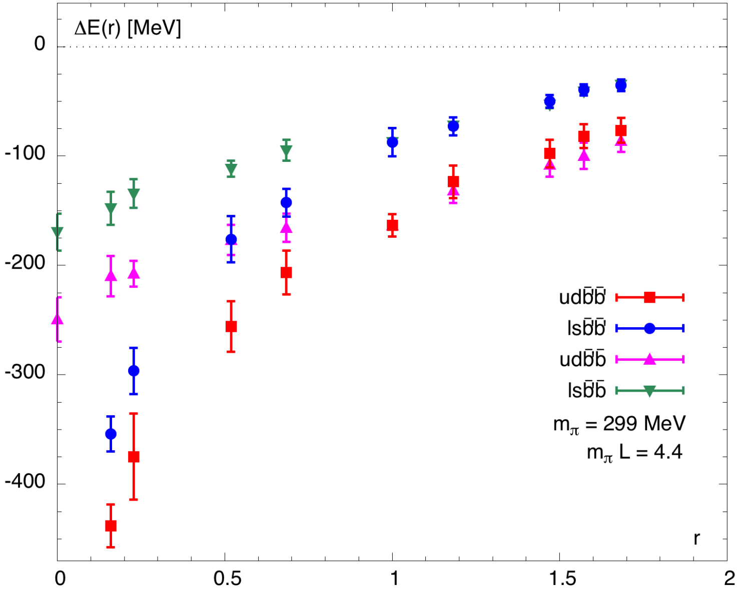
- To test the other two predictions we vary the $\bar{Q}'$ mass
- Channels investigated are: $ud\bar{Q}'\bar{Q}'$, $ls\bar{Q}'\bar{Q}'$, $ud\bar{Q}'\bar{b}$ and $ls\bar{Q}'\bar{b}$

Heavy quark mass dependence

- $m_{\text{bare}}(Q')$ is varied at $m_\pi = 299$MeV and $m_\pi L = 4.4$ to ensure **small volume effects** but **significant chance of binding**. *("lighter is better")*
- For the ratio $m^{b'}/m^b$ we compare the spin averages $\frac{1}{4}[3\eta_b + \Upsilon](Q')$

$m_{\text{bare}}(Q')$	$m^{b'}/m^b := 1/r$
0.9	0.594(3)
1.0	0.636(2)
1.2	0.680(5)
1.6	0.846(7)
1.93	1
3.0	1.463(12)
4.0	1.928(17)
8.0	4.395(35)
10.0	6.287(48)
static	∞

- Static quarks are only used in $\bar{Q}'\bar{Q}$-type channels.


ΔE(r) [MeV]
0
-100
-200
-300
-400
0
0.5
1
1.5
2
r
udb̄′b̄′
lsb̄′b̄′
udb̄′b̄
lsb̄′b̄
m_π = 299 MeV
m_π L = 4.4

Heavy quark mass dependence

Predictions:

- Deeper binding with heavier quarks, $\sim 1/m_Q$ ✓
- Binding set by the reduced mass of $\bar{Q}$ and $\bar{Q}'$ in the $\bar{Q}'\bar{Q}$ diquark ✓
- Deeper binding for lighter quarks in the qq' diquark ✓

The calculation of $ud\bar{Q}'\bar{Q}'$, $ls\bar{Q}'\bar{Q}'$, $ud\bar{Q}'\bar{b}$ and $ls\bar{Q}'\bar{b}$ at unphysical heavy quark mass validates the remaining two predictions of the simple binding mechanism in mind.

- Is it possible to further quantify these findings in a model?
- Can we gain insight away from the HQS validity regime, e.g. in the charm quark region?

Towards a phenomenological model description

Expected terms contributing to the tetraquark binding:

1. Coulomb attraction between two heavy antiquarks $\rightarrow \mathcal{O}(\mu)$

2. Mass independent term from the good light diquark $\rightarrow \mathcal{O}(const.)$

3. Residual heavy-light interaction (tetraquark state) $\rightarrow \mathcal{O}(1/m_{Q_1} + 1/m_{Q_2})$

4. Residual heavy-light interaction (two-meson threshold states) $\rightarrow \mathcal{O}(1/m_{Q_1} + 1/m_{Q_2})$

Towards a phenomenological model description

4. Residual heavy-light interaction (two-meson threshold states)
   - Caveat: Correct two-meson threshold has to be chosen depending on $m_{b'} < m_b$ or $m_{b'} > m_b$, i.e. $r < 1$ or $r > 1$
   - In $ud\bar{Q}'\bar{b}$ one has B^*P' for $r < 1$ and BV' for $r > 1$
   - Can be determined from the observed $B^* - B$, $B_s^* - B_s$, $D^* - D$ and $D_s^* - D_s$ splittings and an additional dependence on m_{Q_1} and m_{Q_2}.

Towards a phenomenological model description

$\bar{Q}'\bar{Q}'$:

$$\Delta E_{ud\bar{Q}'\bar{Q}'} = \frac{C_0}{2r} + C_1^{ud} + C_2^{ud}\,(2r) + (23\text{ MeV})\,r,$$

$$\Delta E_{\ell s\bar{Q}'\bar{Q}'} = \frac{C_0}{2r} + C_1^{\ell s} + C_2^{\ell s}\,(2r) + (24\text{ MeV})\,r,$$

$\bar{Q}'\bar{b}$, $r < 1$:

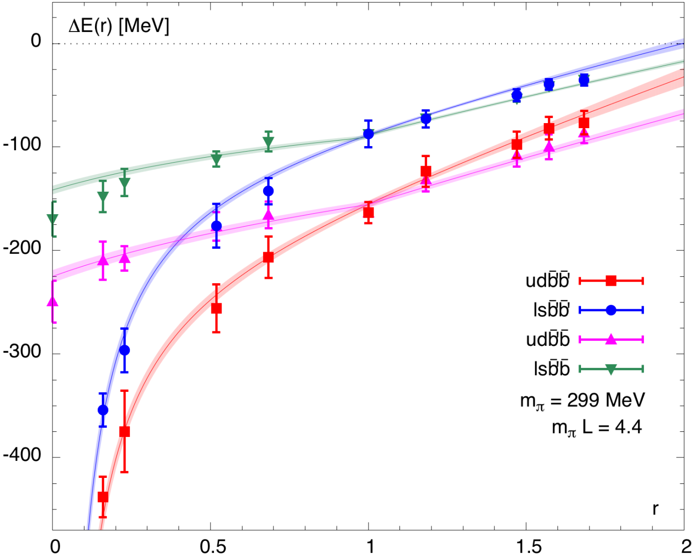
$$\Delta E_{ud\bar{Q}'\bar{b}} = \frac{C_0}{1+r} + C_1^{ud} + C_2^{ud}\,(1+r) + (34\text{ MeV} - 11\text{ MeV}\,r)\,,$$

$$\Delta E_{\ell s\bar{Q}'\bar{b}} = \frac{C_0}{1+r} + C_1^{\ell s} + C_2^{\ell s}\,(1+r) + (34\text{ MeV} - 12\text{ MeV}r)\,,$$

$\bar{Q}'\bar{b}$, $r > 1$:

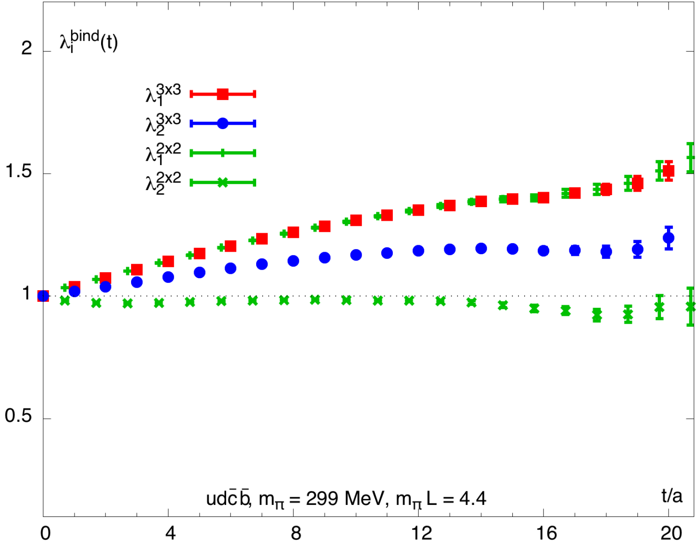
$$\Delta E_{ud\bar{Q}'\bar{b}} = \frac{C_0}{1+r} + C_1^{ud} + C_2^{ud}\,(1+r) + (34\text{ MeV}\,r - 11\text{ MeV})\,,$$

$$\Delta E_{\ell s\bar{Q}'\bar{b}} = \frac{C_0}{1+r} + C_1^{\ell s} + C_2^{\ell s}\,(1+r) + (36\text{ MeV}\,r - 11\text{ MeV})\,.$$



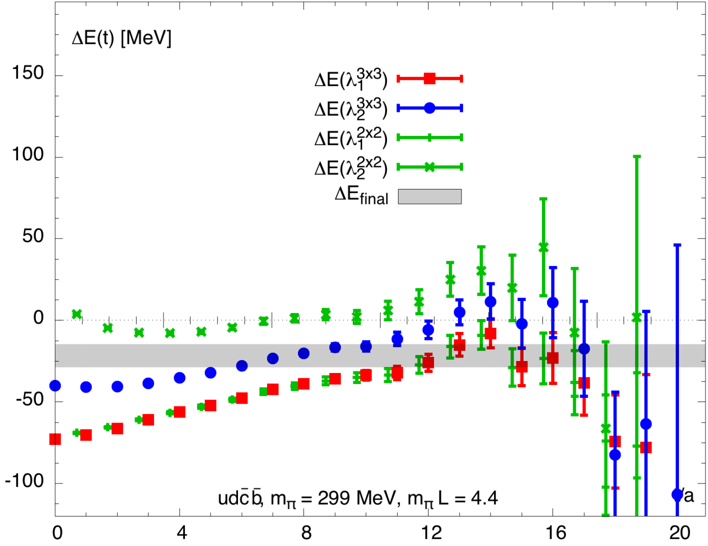
- Most likely additional bound tetraquark in charm quark region: $ud\bar{c}\bar{b}$
- Excellent candidate for direct calculation at physical quark masses!

Direct calculation of $ud\bar{c}\bar{b}$ tetraquarks - at $m_\pi = 299$MeV



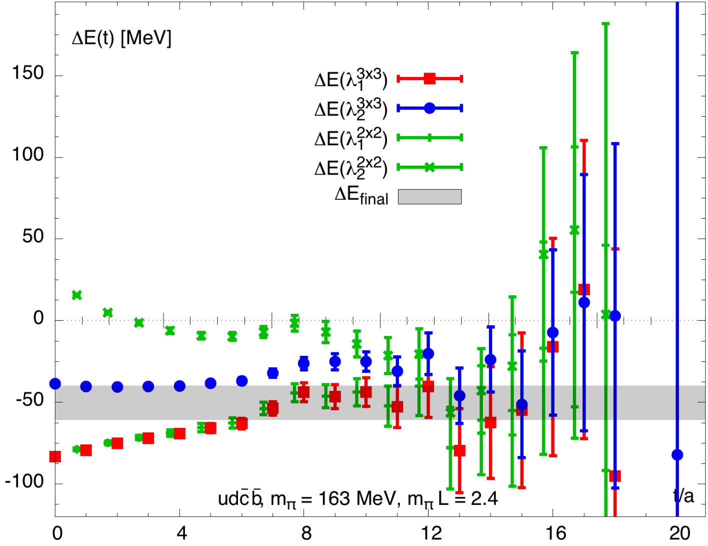
- Rising exponential hints at state below two-meson threshold

Energies at $m_\pi = 299\text{MeV}$



- Effective energy reveals (bound?) state below two-meson threshold

Energies at $m_\pi = 163\text{MeV}$



- Effective energy also reveals state below two-meson threshold

Future work

At $m_\pi = 299, 163$ MeV we can identify a state below the two-meson threshold.

We find evidence of binding in the $ud\bar{c}\bar{b}$ channel at the level of $15 - 65$ MeV, close to the electro-stable threshold.

Extrapolate to physical point?

- Expect naive volume effects for our lightest ensemble ($m_\pi L = 2.4$) around $\Delta E_L^{V,\mathrm{exp}} \approx (0.1)\Delta E^{binding}$
- Possibe binding induced by FV scattering at the level of $\Delta E_L^{V,\mathrm{scatt}} \approx (0.3 - 0.5)\Delta E^{binding}$

$\Rightarrow$ Need proper study of volume effects!

🤔

Ongoing effort:

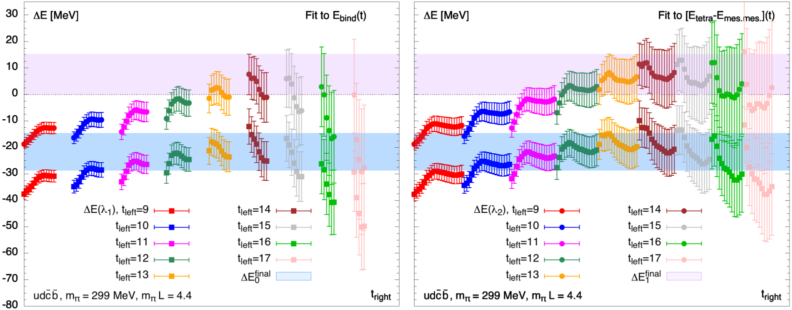
- Increase number of points in the extrapolation to m_π^{phys}
- Add more lattice volumes, in particular at $m_\pi = 163$MeV

Conclusions

- Studying $ud\bar{Q}'\bar{Q}'$, $ls\bar{Q}'\bar{Q}'$, $ud\bar{Q}'\bar{b}$ and $ls\bar{Q}'\bar{b}$ for unphysically heavy quarks we find excellent agreement with phenomenological considerations based on a binding mechanism induced from the observed spectrum.
- The study suggests the $ud\bar{c}\bar{b}$ is the most likely bound tetraquark of this kind in the charm quark region
- In a direct lattice calculation in the charm quark region we indeed find evidence of a $ud\bar{c}\bar{b}$ tetraquark state, close to the electro-stable threshold.
- With future, running, calculations the binding energy dependence on the lattice volume will be pinned down and the extrapolation to the physical point performed

Backup

Backup: Energy of $ud\bar{Q}'\bar{b}$ on E_M



Backup: Ensembles in preparation

κ_l	L	T	m_π[MeV]	$m_\pi L$	status
0.13781	32	64	163	2.4	avail.
	48	64		3.6	config.
	64	64		4.8	therm.
0.13779	32	64	$\sim$185	2.7	config.
0.13777	32	64	$\sim$205	3.0	config.
	48	64		4.5	config.
	64	64		6.0	therm.
0.13770	32	64	299	4.4	avail.
0.13754	32	64	415	6.1	avail.

Table: Throughout $a^{-1} = 2.194\text{GeV}^{-1}$ and $\kappa_{s,sea} = 0.13640$