Hidden model dependence in amplitude analysis

[Eur.Phys.J.C78 (2018) no.3], [arXiv:1805.0211]

Misha Mikhasenko

Joint Physics Analysis Center,
COMPASS @ CERN,
Universität Bonn, HISKP, Bonn, Germany

CHARM 2018

25/05/2018

Bundesministerium
für Bildung
und Forschung

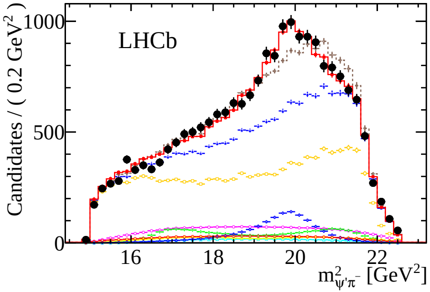
To the memory of Mike Pennington

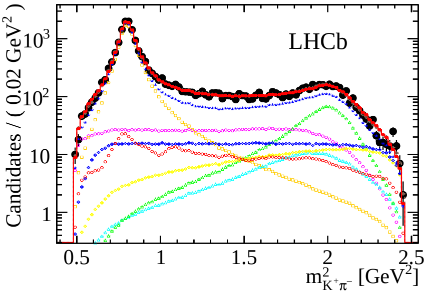
He was a great scientist and very kind person.

Overview

$Z_c(4430)$ was confirmed by LHCb [Phys.Rev.Lett. 112 (2014)]

LHCb analyses of $Z_c(4430)$ in the decay $B \to \psi' \pi K$.





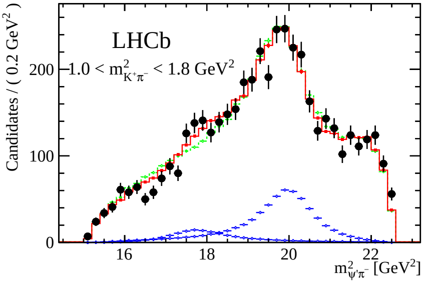
The Dalitz plot analysis

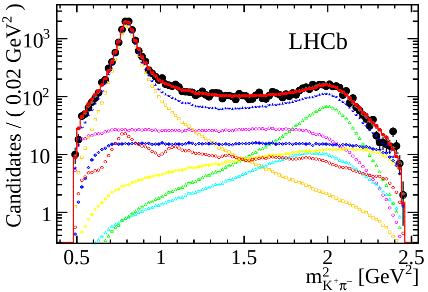
- The main dynamics is in πK
- The Z_1^+ have very high significance ($13 - 19\sigma$).

Resonance	Mass (MeV/c^2)	Γ (MeV/c^2)	J^P
$K^*(800)^0$	682±29	547±24	0^+
$K^*(892)^0$	895.81±0.19	47.4±0.6	1^-
$K^*(1410)^0$	1414±15	232±21	1^-
$K_0^*(1430)^0$	1425±50	270±80	0^+
$K_2^*(1430)^0$	1432.4±1.3	109±5	2^+
$K^*(1680)^0$	1717±27	322±110	1^-
$K_3^*(1780)^0$	1776±7	159±21	3^-

$Z_c(4430)$ was confirmed by LHCb [Phys.Rev.Lett. 112 (2014)]

LHCb analyses of $Z_c(4430)$ in the decay $B \to \psi' \pi K$.





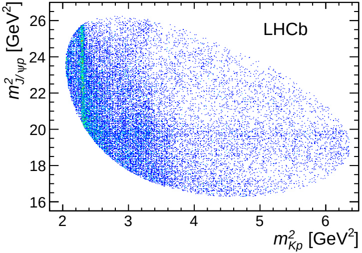
The Dalitz plot analysis

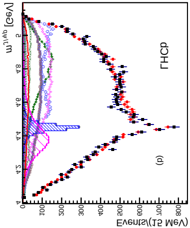
- The main dynamics is in πK
- The Z_1^+ have very high significance ($13 - 19\sigma$).

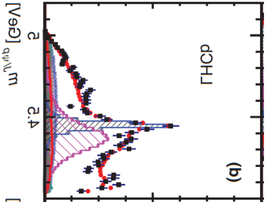
Resonance	Mass (MeV/c^2)	Γ (MeV/c^2)	J^P
$K^*(800)^0$	682±29	547±24	0^+
$K^*(892)^0$	895.81±0.19	47.4±0.6	1^-
$K^*(1410)^0$	1414±15	232±21	1^-
$K_0^*(1430)^0$	1425±50	270±80	0^+
$K_2^*(1430)^0$	1432.4±1.3	109±5	2^+
$K^*(1680)^0$	1717±27	322±110	1^-
$K_3^*(1780)^0$	1776±7	159±21	3^-

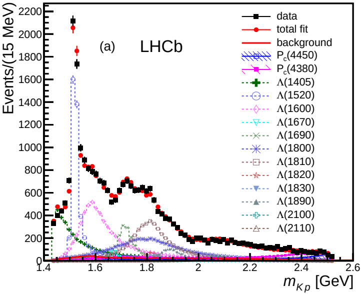
$P_c(4450)$ in the decay $\Lambda_b^0 \to J/\psi\, p\, K$

[Phys.Rev.Lett. 115]





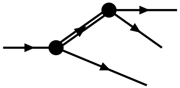




- High significance of pentaquark signals
- The extracted quantum number $(3/2^-, 5/2^+)$ are ambiguous.
- The parameters rely on description of the pK channel.

Questions to the amplitude construction

Spin formalism

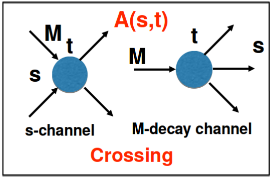


How to write the amplitude for interaction of particles with spins

- What do we know about kinematics? $\Leftarrow$ Reps. of Rotation Group
- How can we constrain the dynamics? $\Leftarrow$ Analyticity, Unitarity

Formalisms on the market:

- Helicity, Spin-Orbit, Zemach tensors, Chung's tensor corrections, Covariant projection formalism (Fillipini, Bonn-Gatchina group (EPJ, 2005)), . . .



- Models have different energy dependence,
- Some introduce spurious singularities
- Some do not satisfy crossing
- Some invoke relativistic corrections

Discussion on the relativistic corrections

How can I not be confused?

"Relativity Is Not an Essential Complication." Let us imagine a reaction amplitude M expressed, first of all, in covariant form; it will be a Lorentz invariant function of the four-momenta of the reaction and the covariant spin tensors. **We are then at liberty to refer the momenta to the CMF (center-of-mass frame) of the reaction and to refer the tensor specifying the spin of each particle to that particle's RF (rest frame).** The spin tensors for massless particles can be broken up into multipole contributions referred to the CMF. . Ch. Zemach, 1965

The extended use of noncovariant formalisms is rather surprising, ... The reasons for this puzzling situation are not very clear to us. . . . **Zemach** made the statement "relativity is not an essential complication", which probably has been **misunderstood by many analysts**.
. Filippini *et al.* Phys.Rev. D51 (1995) 2247-2261

We have devised a method of constructing our covariant decay amplitudes, such that they lead to the **Zemach** amplitudes when the **Lorentz factors** are set one S.U.Chung, PRD 78, 2008

This is the **nonrelativistic L-S coupling scheme** as introduced by Jacob and Wick in Ref. [26], which is equivalent to the nonrelativistic Zemach tensors [38,39]. Relativistic corrections as worked out in Ref. [40] are not applied. PRD 95 (2017) 032004

As mentioned in Sec. III B, we **do not apply relativistic corrections** to the decay amplitudes in the partial-wave analysis. First studies show that the effect on the shapes of the selected 18 waves is small. PRD 95 (2017) 032004

Discussion on the relativistic corrections

How can I not be confused?

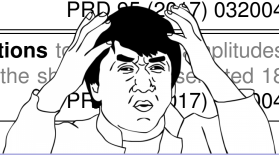
"Relativity Is Not an Essential Complication." Let us imagine a reaction amplitude M expressed, first of all, in covariant form; it will be a Lorentz invariant function of the four-momenta of the reaction and the covariant spin tensors. **We are then at liberty to refer the momenta to the CMF (center-of-mass frame) of the reaction and to refer the tensor specifying the spin of each particle to that particle's RF (rest frame).** The spin tensors for massless particles can be broken up into multipole contributions referred to the CMF. . Ch. Zemach, 1965

The extended use of noncovariant formalisms is rather surprising, ... The reasons for this puzzling situation are not very clear to us. . . . **Zemach** made the statement "relativity is not an essential complication", which probably has been **misunderstood by many analysts**.
. Filippini *et al.* Phys.Rev. D51 (1995) 2247-2261

We have devised a method of constructing our covariant decay amplitudes, such that they lead to the **Zemach** amplitudes when the **Lorentz factors** are set one S.U.Chung, PRD 78, 2008

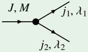
This is the **nonrelativistic L-S coupling scheme** as introduced by Jacob and Wick in Ref. [26], which is equivalent to the nonrelativistic Zemach tensors [38,39]. Relativistic corrections as worked out in Ref. [40] are not applied. PRD 95 (20 7) 032004

As mentioned in Sec. III B, we **do not apply relativistic corrections** t plitudes in the partial-wave analysis. First studies show that the effect on the sh se ted 18 waves is small. PR 17) 04



Helicity amplitude [Jacob-Wick; Collins; Chung]

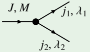
Gives exact and only angular dependence

- Use Wigner-D matrix for every decay
  $\mathcal{M}_{M;\lambda_1,\lambda_2} =$ 

  $\sim D^{J*}_{M\lambda}(\Omega)$, taking Ω from the rest frame
- Sum intermediate helicities.

Helicity amplitude [Jacob-Wick; Collins; Chung]

Gives exact and only angular dependence

- Use Wigner-D matrix for every decay
  $\mathcal{M}_{M;\lambda_1,\lambda_2} =$ 

  $\sim D^{J*}_{M\lambda}(\Omega)$, taking Ω from the rest frame
- Sum intermediate helicities.

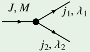
Helicities are frame-dependent

- Match helicities for final state particles with spin

$$|\mathcal{M}|^2 = \sum_{\lambda_{\Lambda_b^0}} \sum_{\lambda_p} \sum_{\Delta\lambda_\mu} \left| \mathcal{M}^{\Lambda^*}_{\lambda_{\Lambda_b^0},\lambda_p,\Delta\lambda_\mu} + e^{i\,\Delta\lambda_\mu \alpha_\mu} \sum_{\lambda_p^{P_c}} d^{\frac{1}{2}}_{\lambda_p^{P_c},\lambda_p}(\theta_p)\, \mathcal{M}^{P_c}_{\lambda_{\Lambda_b^0},\lambda_p^{P_c},\Delta\lambda_\mu} \right|^2,$$

Helicity amplitude [Jacob-Wick; Collins; Chung]

Gives exact and only angular dependence

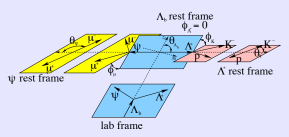
- Use Wigner-D matrix for every decay
  $\mathcal{M}_{M;\lambda_1,\lambda_2} =$ 

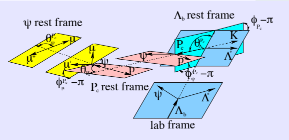
  $\sim D^{J*}_{M\lambda}(\Omega)$, taking Ω from the rest frame
- Sum intermediate helicities.

Helicities are frame-dependent

- Match helicities for final state particles with spin

$$|\mathcal{M}|^2 = \sum_{\lambda_{\Lambda_b^0}} \sum_{\lambda_p} \sum_{\Delta\lambda_\mu} \left| \mathcal{M}^{\Lambda^*}_{\lambda_{\Lambda_b^0},\lambda_p,\Delta\lambda_\mu} + e^{i\,\Delta\lambda_\mu \alpha_\mu} \sum_{\lambda_p^{P_c}} d^{\frac{1}{2}}_{\lambda_p^{P_c},\lambda_p}(\theta_p)\, \mathcal{M}^{P_c}_{\lambda_{\Lambda_b^0},\lambda_p^{P_c},\Delta\lambda_\mu} \right|^2,$$

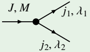




Helicity amplitude [Jacob-Wick; Collins; Chung]

Gives exact and only angular dependence

- Use Wigner-D matrix for every decay

$\mathcal{M}_{M;\lambda_1,\lambda_2} =$ 

$\sim D^{J*}_{M\lambda}(\Omega)$, taking Ω from the rest frame

- Sum intermediate helicities.

Helicities are frame-dependent

- Match helicities for final state particles with spin

$$|\mathcal{M}|^2 = \sum_{\lambda_{\Lambda_b^0}} \sum_{\lambda_p} \sum_{\Delta\lambda_\mu} \left| \mathcal{M}^{\Lambda^*}_{\lambda_{\Lambda_b^0}, \lambda_p, \Delta\lambda_\mu} + e^{i\,\Delta\lambda_\mu \alpha_\mu} \sum_{\lambda_p^{P_c}} d^{\frac{1}{2}}_{\lambda_p^{P_c}, \lambda_p}(\theta_p)\, \mathcal{M}^{P_c}_{\lambda_{\Lambda_b^0}, \lambda_p^{P_c}, \Delta\lambda_\mu} \right|^2,$$

Helicities are not independent [Collins]

- Parity relates $\mathcal{M}_{\pm\lambda,\ldots}$
- **Conspiracy relations** (at threshold only S-wave survive).

$\mathcal{M}_\lambda(s) = \sum_L \langle L0S\lambda|J\lambda\rangle\, \mathcal{M}_L(s) \quad \Rightarrow \quad \mathcal{M}_\lambda(s_{\mathrm{th}}) = \langle 00S\lambda|J\lambda\rangle\, \mathcal{M}_0(s_{\mathrm{th}})$

Tensor formalism

[Anisovich et al., EPJ, 2005]

How it works

- explicitely Lorentz-invariant $\mathcal{M} = \varepsilon_\mu(p_\psi, \lambda_\psi) A^\mu_\lambda$ can be evaluated in any frame
- No partial waves $A^\mu_\lambda(s,t) = (p_3 - p_4)^\mu F(s,t) + (p_3 + p_4)^\mu G(s,t).$

Tensor formalism

[Anisovich et al., EPJ, 2005]

How it works

- explicitely Lorentz-invariant $\mathcal{M} = \varepsilon_\mu(p_\psi, \lambda_\psi) A^\mu_\lambda$ can be evaluated in any frame
- No partial waves $A^\mu_\lambda(s,t) = (p_3 - p_4)^\mu F(s,t) + (p_3 + p_4)^\mu G(s,t).$

Covariant projection method (CPM)

- Particle with spin j is described by $\epsilon_{\mu_1 \dots \mu_j}$
- Recipe to construct partial wave amplitude $a \to bc$
  - Combine tensors $\epsilon^b_{\mu\dots}$ and $\epsilon^c_{\mu\dots}$ to a tensor $S_{\mu_1\dots\mu_S}$ with S-indices
  - Build spherical tensors $L_{\mu_1\dots\mu_L}$ for the orbital angular momentum.
  - Combine the tensors $S_{\mu\dots}$ and $L_{\mu\dots}$ to the total tensor $J_{\mu_1\dots\mu_j}$

Tensor formalism

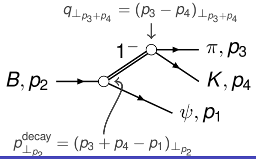
[Anisovich et al., EPJ, 2005]

How it works

- explicitely Lorentz-invariant $\mathcal{M} = \varepsilon_\mu(p_\psi, \lambda_\psi) A_\lambda^\mu$ can be evaluated in any frame
- No partial waves $A_\lambda^\mu(s,t) = (p_3 - p_4)^\mu F(s,t) + (p_3 + p_4)^\mu\, G(s,t).$

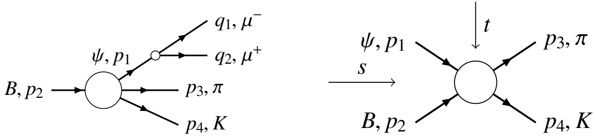
Covariant projection method (CPM)

- Particle with spin j is described by $\epsilon_{\mu_1 \dots \mu_j}$
- Recipe to construct partial wave amplitude $a \to bc$
  - Combine tensors $\epsilon^b_{\mu\dots}$ and $\epsilon^c_{\mu\dots}$ to a tensor $S_{\mu_1\dots\mu_S}$ with S-indices
  - Build spherical tensors $L_{\mu_1\dots\mu_L}$ for the orbital angular momentum.
  - Combine the tensors $S_{\mu\dots}$ and $L_{\mu\dots}$ to the total tensor $J_{\mu_1\dots\mu_j}$



$$= \epsilon_\rho(\lambda, p_1) \underbrace{\left[g_S(s) g_{\rho\mu} + g_D(s) L_{\rho\mu}(p)\right]}_{\text{S,D-waves}} \left(-g^{\mu\nu} + \frac{P^\mu P^\nu}{s}\right) \underbrace{L_\nu(q)}_{\text{P-wave}},$$

Formalism for $B \to \psi\pi K$ [Eur.Phys.J. C78 (2018) no.3, 229]



Weak decay

- Parity is conserved for two cases separately (pseudoscalar B, i.e. 0^-, scalar B, i.e. 0^+).
- Parity-conserving reaction & parity-violating reaction
- Interaction between "stable" particles: B, ψ, π, K.

Isobar model

$$\mathcal{A}_\lambda(s,t,u) = A_\lambda^{(s)}(s,t,u) + A_\lambda^{(t)}(s,t,u) + A_\lambda^{(u)}(s,t,u),$$

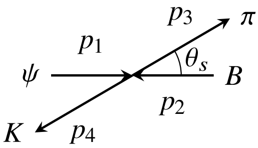
Partial wave expansion

Parity violating process *s*-channels isobars

Crossing

- To incorporate πK-resonances,
  we consider crossed-symmetric process $B\psi \to \pi K$.

$$A_\lambda = \frac{1}{4\pi}\sum_{j=|\lambda|}^{\infty}(2j+1)A_\lambda^j(s)\, d_{\lambda 0}^j(\cos\theta_s),$$



$$p(s) = |\vec{p}_1| = |\vec{p}_2| = \frac{\sqrt{s-(m_1+m_2)^2}\sqrt{s-(m_1-m_2)^2}}{2\sqrt{s}} \equiv \frac{\lambda_{12}^{1/2}}{2\sqrt{s}}$$

Note on $A_\lambda^j(s)$

- it includes resonances,
- it has several special point: $s_\pm = (m_1 \pm m_2)^2$, $s_{34}^\pm = (m_3 \pm m_4)^2$.

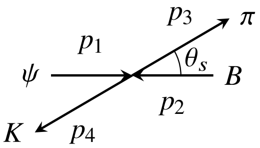
Partial wave expansion

Parity violating process *s*-channels isobars

Crossing

- To incorporate πK-resonances,
  we consider crossed-symmetric process $B\psi \to \pi K$.

$$A_\lambda = \frac{1}{4\pi}\sum_{j=|\lambda|}^{\infty}(2j+1)A_\lambda^j(s)\, d_{\lambda 0}^j(\cos\theta_s),$$



$$q(s) = |\vec{p}_3| = |\vec{p}_4| = \frac{\sqrt{s-(m_3+m_4)^2}\sqrt{s-(m_3-m_4)^2}}{2\sqrt{s}} \equiv \frac{\lambda_{34}^{1/2}}{2\sqrt{s}}$$

Note on $A_\lambda^j(s)$

- it includes resonances,
- it has several special point: $s_\pm = (m_1 \pm m_2)^2$, $s_{34}^\pm = (m_3 \pm m_4)^2$.

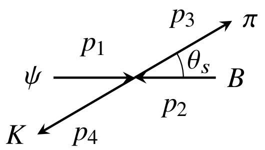
Partial wave expansion

Parity violating process *s*-channels isobars

Crossing

- To incorporate πK-resonances, we consider crossed-symmetric process $B\psi \to \pi K$.

$$A_\lambda = \frac{1}{4\pi}\sum_{j=|\lambda|}^{\infty}(2j+1)A_\lambda^j(s)\, d_{\lambda 0}^j(\cos\theta_s),$$



$$\cos\theta_s = \frac{s(t-u)+(m_3^2-m_4^2)(m_1^2-m_2^2)}{\lambda_{12}^{1/2}\lambda_{34}^{1/2}}, \quad A_\lambda^j \sim p^{L_1}q^{L_2} \longrightarrow \lambda_{12}^{L_1/2}\lambda_{34}^{L_2/2}$$

Note on $A_\lambda^j(s)$

- it includes resonances,
- it has several special point: $s_\pm = (m_1 \pm m_2)^2$, $s_{34}^\pm = (m_3 \pm m_4)^2$.

Singularities of $A^j_\lambda(s)$

Kinematic singularity: K-functions [Collins, Martin-Spearman]

- To cancel $d^j_{\lambda 0}(z_s)$ singularities. $\lambda_{ij} = \left(s - (m_i + m_j)^2\right)\left(s - (m_i - m_j)^2\right)$.
  - $d^j_{\lambda 0}(z_s) \sim \lambda_{12}^{(j-|\lambda|)/2} \lambda_{34}^{(j-|\lambda|)/2}$,
  - $d^j_{\lambda 0}(z_s) \sim \sin^{|\lambda|}(\theta)$, $\quad d^j_{\lambda 0}(z_s) = \xi_{\lambda 0}(z_s)\hat{d}^j_{\lambda 0}(z_s)$, $\quad z_s = \cos\theta_s$
- extra p or q for consistency with $p^{L_1} q^{L_2}$ at threshold.
- residues factorization gives extra $\sqrt{s}$ $\quad \Rightarrow \quad \boxed{A^j_\lambda = K_{\lambda 0}\hat{A}^j_\lambda}$.

Singularities of $A^j_\lambda(s)$

Kinematic singularity: K-functions [Collins, Martin-Spearman]

- To cancel $d^j_{\lambda 0}(z_s)$ singularities. $\lambda_{ij} = \left(s-(m_i+m_j)^2\right)\left(s-(m_i-m_j)^2\right)$.
  - $d^j_{\lambda 0}(z_s) \sim \lambda_{12}^{(j-|\lambda|)/2}\lambda_{34}^{(j-|\lambda|)/2}$,
  - $d^j_{\lambda 0}(z_s) \sim \sin^{|\lambda|}(\theta)$, $\quad d^j_{\lambda 0}(z_s) = \xi_{\lambda 0}(z_s)\hat{d}^j_{\lambda 0}(z_s)$, $\quad z_s = \cos\theta_s$
- extra p or q for consistency with $p^{L_1}q^{L_2}$ at threshold.
- residues factorization gives extra $\sqrt{s}$ $\quad \Rightarrow \quad \boxed{A^j_\lambda = K_{\lambda 0}\hat{A}^j_\lambda}$.

Dynamical singularities of $\hat{A}^j_\lambda$

- Resonances $\sim \left[m^2 - s - im\Gamma(s)\right]^{-1}$ (right singularities).
- Barrier factors (left singularities)
  - Blatt-Weisskopf factors
    $$B_1(s) = \frac{1+z_0}{1+z}, \quad B_2(s) = \frac{9+3z_0+z_0^2}{9+3z+z^2}, \quad z = Rp.$$
  - Production factors, e.g. $1/s$

Helicity amplitudes are not independent

Covariant amplitude

General covariant amplitude

$$A_\lambda(s,t) = \epsilon_\mu(\lambda, p_1)\left[(p_3 - p_4)^\mu - \frac{m_3^2 - m_4^2}{s}(p_3 + p_4)^\mu\right] C(s,t) + \epsilon_\mu(\lambda, p_1)(p_3 + p_4)^\mu\, B(s,t).$$

Helicity amplitudes are not independent

Covariant amplitude

General covariant amplitude

$$A_\lambda(s,t) = \epsilon_\mu(\lambda, p_1)\left[(p_3 - p_4)^\mu - \frac{m_3^2 - m_4^2}{s}(p_3 + p_4)^\mu\right] C(s,t) + \epsilon_\mu(\lambda, p_1)(p_3 + p_4)^\mu\, B(s,t).$$

Matching covariant amplitude to helicity amplitude:

$$-C(s,t)\frac{\boxed{\text{xxx}}}{4m_1^2 s} + B(s,t)\frac{\lambda_{12}}{4m_1^2} = \frac{A_0}{K_{00}\,\xi_{00}(z_s)} = \frac{1}{4\pi}\left(\sum_{j>0}(2j+1)(pq)^j \hat{A}_0^j(s)\hat{d}_{00}^j(z_s) + \frac{\lambda_{12}}{4m_1^2}\hat{A}_0^0\right),$$

$$\pm\sqrt{2}\; C(s,t) = \frac{A_\pm}{K_{\pm 0}\,\xi_{10}(z_s)} = \pm\frac{1}{4\pi}\sum_{j>0}(2j+1)(pq)^{j-1}\hat{A}_\pm^j(s)\,\hat{d}_{10}^j(z_s).$$

Helicity amplitudes are not independent

Covariant amplitude

General covariant amplitude

$$A_\lambda(s,t) = \epsilon_\mu(\lambda, p_1)\left[(p_3 - p_4)^\mu - \frac{m_3^2 - m_4^2}{s}(p_3+p_4)^\mu\right] C(s,t) + \epsilon_\mu(\lambda,p_1)(p_3+p_4)^\mu\, B(s,t).$$

Matching covariant amplitude to helicity amplitude:

$$-C(s,t)\frac{\boxed{\text{xxx}}}{4m_1^2 s} + B(s,t)\frac{\lambda_{12}}{4m_1^2} = \frac{A_0}{K_{00}\,\xi_{00}(z_s)} = \frac{1}{4\pi}\left(\sum_{j>0}(2j+1)(pq)^j \hat{A}_0^j(s)\hat{d}_{00}^j(z_s) + \frac{\lambda_{12}}{4m_1^2}\hat{A}_0^0\right),$$

$$\pm\sqrt{2}\;C(s,t) = \frac{A_\pm}{K_{\pm 0}\,\xi_{10}(z_s)} = \pm\frac{1}{4\pi}\sum_{j>0}(2j+1)(pq)^{j-1}\hat{A}_\pm^j(s)\,\hat{d}_{10}^j(z_s).$$

Condition for $B(s,t)$:

$$4\pi B(s,t) = \hat{A}_0^0 + \frac{4m_1^2}{\lambda_{12}}\sum_{j>0}(2j+1)(pq)^j\left[\hat{A}_0^j(s)\hat{d}_{00}^j(z_s) + \frac{s+m_1^2-m_2^2}{\sqrt{2}m_1^2}\hat{A}_+^j(s)\,z_s\hat{d}_{10}^j(z_s)\right].$$

Helicity amplitudes are not independent

Covariant amplitude

General covariant amplitude

$$A_\lambda(s,t) = \epsilon_\mu(\lambda, p_1)\left[(p_3 - p_4)^\mu - \frac{m_3^2 - m_4^2}{s}(p_3+p_4)^\mu\right] C(s,t) + \epsilon_\mu(\lambda, p_1)(p_3+p_4)^\mu\, B(s,t).$$

Matching covariant amplitude to helicity amplitude:

$$-C(s,t)\frac{\boxed{\text{xxx}}}{4m_1^2 s} + B(s,t)\frac{\lambda_{12}}{4m_1^2} = \frac{A_0}{K_{00}\,\xi_{00}(z_s)} = \frac{1}{4\pi}\left(\sum_{j>0}(2j+1)(pq)^j \hat{A}_0^j(s)\hat{d}_{00}^j(z_s) + \frac{\lambda_{12}}{4m_1^2}\hat{A}_0^0\right),$$

$$\pm\sqrt{2}\;C(s,t) = \frac{A_\pm}{K_{\pm 0}\,\xi_{10}(z_s)} = \pm\frac{1}{4\pi}\sum_{j>0}(2j+1)(pq)^{j-1}\hat{A}_\pm^j(s)\,\hat{d}_{10}^j(z_s).$$

Condition for $B(s,t)$:

$$4\pi B(s,t) = \hat{A}_0^0 + \frac{4m_1^2}{\lambda_{12}}\sum_{j>0}(2j+1)(pq)^j\left[\hat{A}_0^j(s)\hat{d}_{00}^j(z_s) + \frac{s+m_1^2-m_2^2}{\sqrt{2}m_1^2}\hat{A}_+^j(s)\, z_s\hat{d}_{10}^j(z_s)\right].$$

Helicity amplitudes are not independent

Covariant amplitude

General covariant amplitude

$$A_\lambda(s,t) = \epsilon_\mu(\lambda,p_1)\left[(p_3-p_4)^\mu - \frac{m_3^2-m_4^2}{s}(p_3+p_4)^\mu\right] C(s,t) + \epsilon_\mu(\lambda,p_1)(p_3+p_4)^\mu\, B(s,t).$$

Matching covariant amplitude to helicity amplitude:

$$-\,C(s,t)\frac{\boxed{\text{xxx}}}{4m_1^2 s} + B(s,t)\frac{\lambda_{12}}{4m_1^2} = \boxed{\frac{A_0}{K_{00}\,\xi_{00}(z_s)} = \frac{1}{4\pi}\left(\sum_{j>0}(2j+1)(pq)^j \hat{A}_0^j(s)\hat{d}_{00}^j(z_s) + \frac{\lambda_{12}}{4m_1^2}\hat{A}_0^0\right)},$$

$$\pm\sqrt{2}\;C(s,t) = \boxed{\frac{A_\pm}{K_{\pm 0}\,\xi_{10}(z_s)} = \pm\frac{1}{4\pi}\sum_{j>0}(2j+1)(pq)^{j-1}\hat{A}_\pm^j(s)\,\hat{d}_{10}^j(z_s).}$$

Condition for $B(s,t)$:

$$4\pi B(s,t) = \hat{A}_0^0 + \frac{4m_1^2}{\lambda_{12}}\sum_{j>0}(2j+1)(pq)^j\left[\hat{A}_0^j(s)\hat{d}_{00}^j(z_s) + \frac{s+m_1^2-m_2^2}{\sqrt{2}m_1^2}\hat{A}_+^j(s)\,z_s\hat{d}_{10}^j(z_s)\right].$$

Helicity amplitudes are not independent

Covariant amplitude

General covariant amplitude

$$A_\lambda(s,t) = \epsilon_\mu(\lambda, p_1)\left[(p_3-p_4)^\mu - \frac{m_3^2-m_4^2}{s}(p_3+p_4)^\mu\right] C(s,t) + \epsilon_\mu(\lambda,p_1)(p_3+p_4)^\mu\, B(s,t).$$

Matching covariant amplitude to helicity amplitude:

$$-C(s,t)\frac{\boxed{\text{xxx}}}{4m_1^2 s} + B(s,t)\frac{\lambda_{12}}{4m_1^2} = \frac{A_0}{K_{00}\,\xi_{00}(z_s)} = \frac{1}{4\pi}\left(\sum_{j>0}(2j+1)(pq)^j \hat{A}_0^j(s)\hat{d}_{00}^j(z_s) + \frac{\lambda_{12}}{4m_1^2}\hat{A}_0^0\right),$$

$$\pm\sqrt{2}\;C(s,t) = \frac{A_\pm}{K_{\pm 0}\,\xi_{10}(z_s)} = \pm\frac{1}{4\pi}\sum_{j>0}(2j+1)(pq)^{j-1}\hat{A}_\pm^j(s)\,\hat{d}_{10}^j(z_s).$$

Condition for $B(s,t)$:

$$4\pi B(s,t) = \hat{A}_0^0 + \frac{4m_1^2}{\lambda_{12}}\sum_{j>0}(2j+1)(pq)^j \boxed{\left[\hat{A}_0^j(s)\hat{d}_{00}^j(z_s) + \frac{s+m_1^2-m_2^2}{\sqrt{2}m_1^2}\hat{A}_+^j(s)\,z_s\hat{d}_{10}^j(z_s)\right]}.$$

Conspiracy between helicity amplitudes

Jackson-Hite, Collins, Martin Spearman

$$0 \xleftarrow[s\to s_\pm]{} [\dots] = \hat{A}_0^j(s)\frac{(z_s)^j}{\langle j-1,0;1,0|j,0\rangle} - \frac{s+m_1^2-m_2^2}{\sqrt{2}m_1^2}\hat{A}_+^j(s)\,\frac{(z_s)^j}{\sqrt{2}\langle j-1,0;1,1|j,1\rangle}.$$

Conspiracy between helicity amplitudes

Jackson-Hite, Collins, Martin Spearman

$$0 \xleftarrow[s \to s_\pm]{} [\dots] = \hat{A}_0^j(s) \frac{(z_s)^j}{\langle j-1,0;1,0|j,0\rangle} - \frac{s+m_1^2-m_2^2}{\sqrt{2}m_1^2} \hat{A}_+^j(s) \frac{(z_s)^j}{\sqrt{2}\langle j-1,0;1,1|j,1\rangle}.$$

General form

$$A_+^j = p^{j-1} q^j \left[\langle j-1,0;1,1|j,1\rangle g_j(s) + f_j(s)\right],$$

$$A_0^j = p^{j-1} q^j \left[\langle j-1,0;1,0|j,0\rangle \frac{s+m_1^2-m_2^2}{2m_1\sqrt{s}} g_j'(s) + \frac{m_1}{\sqrt{s}} f_j'(s)\right].$$

with $g_j(s_\pm) = g_j'(s_\pm)$ and $f_j(s_\pm) = f_j'(s_\pm) = 0$.

Conspiracy between helicity amplitudes

Jackson-Hite, Collins, Martin Spearman

$$0 \xleftarrow[s\to s_\pm]{} [\dots] = \hat{A}_0^j(s) \frac{(z_s)^j}{\langle j-1,0;1,0|j,0\rangle} - \frac{s+m_1^2-m_2^2}{\sqrt{2}m_1^2} \hat{A}_+^j(s) \frac{(z_s)^j}{\sqrt{2}\langle j-1,0;1,1|j,1\rangle}.$$

General form

$$A_+^j = p^{j-1}q^j\left[\langle j-1,0;1,1|j,1\rangle g_j(s) + f_j(s)\right],$$

$$A_0^j = p^{j-1}q^j\left[\langle j-1,0;1,0|j,0\rangle \frac{s+m_1^2-m_2^2}{2m_1\sqrt{s}} g_j'(s) + \frac{m_1}{\sqrt{s}} f_j'(s)\right].$$

with $g_j(s_\pm) = g_j'(s_\pm)$ and $f_j(s_\pm) = f_j'(s_\pm) = 0$.

What is interesting:

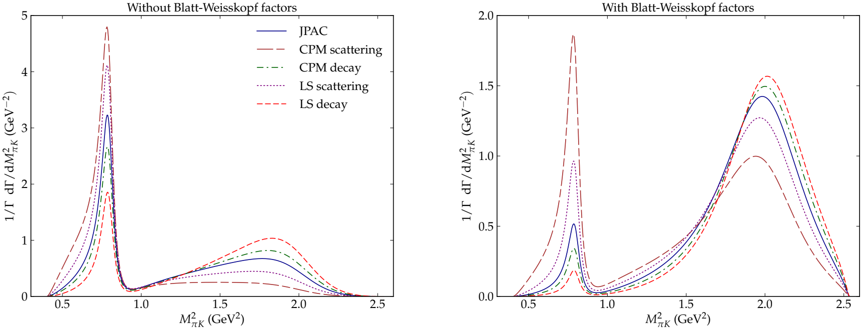
- We recognize LS-scheme $A_\lambda^j(s) = \sum_l \left(\frac{2l+1}{2j+1}\right)^{1/2} \langle l,0;1,\lambda|j,\lambda\rangle G_l^j(s)$, but with extra factor.

Visual difference

of the various formalisms

As an example:

- $K^*(782)$ and $K^*(1410)$ in the πK-channel, $\Gamma_{K^*} = 232$ MeV.



Conclusion

Conclusion

Choice of amplitude model:

- There is no “God-given” formalism.
- Non-covariant != Non-relativistic.
- Customary approaches incorporate spurious (unmotivated) energy dependence

  Violates crossing symmetry: the recipes give different amplitude for scattering and decay
- Analyticity helps to clarify differences between formalisms.
- We suggest a method to construct an amplitude for the reaction with the minimal model-dependence.

Conclusion

Choice of amplitude model:

- There is no “God-given” formalism.
- Non-covariant != Non-relativistic.
- Customary approaches incorporate spurious (unmotivated) energy dependence

  Violates crossing symmetry: the recipes give different amplitude for scattering and decay
- Analyticity helps to clarify differences between formalisms.
- We suggest a method to construct an amplitude for the reaction with the minimal model-dependence.

- The systematics due to the model dependence is often not estimated.
- Data analyses with the developed models is ongoing

  [T.Skwarnicki, JPAC-LHCb]

Thank you for the attention

Thanks to my collaborators:
JPAC Collaboration (A. Pilloni , J. Nys, M. Albaladejo,
C. Fernandez-Ramirez, A. Jackura, V. Mathieu, N. Sherrill,
T. Skwarnicki, A.P. Szczepaniak)

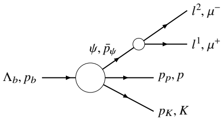
The $\Lambda_0 \to J/\psi[\to \mu^+\mu^-]\, p\, K^-$ decay [arXiv:1805.02113]

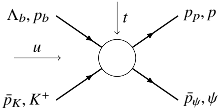
What is the right formalism to search for resonances?
II. The pentaquark chain

A. Pilloni,[1,*] J. Nys,[1,2,3,4,†] M. Mikhasenko,[5,‡] M. Albaladejo,[6] C. Fernández-Ramírez,[7] A. Jackura,[3,4] V. Mathieu,[1] N. Sherrill,[3,4] T. Skwarnicki,[8] and A. P. Szczepaniak[1,3,4]
(Joint Physics Analysis Center)
[1] *Theory Center, Thomas Jefferson National Accelerator Facility, Newport News, VA 23606, USA*
[2] *Department of Physics and Astronomy, Ghent University, Belgium*
[3] *Center for Exploration of Energy and Matter, Indiana University, Bloomington, IN 47403, USA*
[4] *Physics Department, Indiana University, Bloomington, IN 47405, USA*
[5] *Universität Bonn, Helmholtz-Institut für Strahlen- und Kernphysik, 53115 Bonn, Germany*
[6] *Departamento de Física, Universidad de Murcia, E-30071 Murcia, Spain*
[7] *Instituto de Ciencias Nucleares, Universidad Nacional Autónoma de México, Ciudad de México 04510, Mexico*
[8] *Syracuse University, Syracuse, NY 13244, USA*

We discuss the differences between several partial-wave analysis formalisms used in the construction of three-body decay amplitudes involving fermions. Specifically, we consider the decay $\Lambda_b \to \psi\, pK^-$, where the hidden charm pentaquark signal has been reported. We analyze the analytical properties of the amplitudes and separate kinematical and dynamical singularities. The result is an amplitude with the minimal energy dependence compatible with the S-matrix principles.

PACS numbers: 11.55.Bq, 11.80.Cr, 11.80.Et





The pentaquark chain

Same game, but much more complicated

- Three particles with spin ⇒ three helicity indices.
- The kinematic singularity free amplitudes are obtained from the **P**arity **C**onserving **H**elicity **A**mplitudes.

The pentaquark chain

Same game, but much more complicated

- Three particles with spin $\Rightarrow$ three helicity indices.
- The kinematic singularity free amplitudes are obtained from the **P**arity **C**onserving **H**elicity **A**mplitudes.
- The basis of scalar functions is more complicated

$$A_{\lambda_p,\lambda_b\lambda_\psi}(s,t) = \epsilon_\mu(p_\psi,\lambda_\psi)\,\bar{u}(p_{\bar{p}},\lambda_{\bar{p}})\left(\sum_{i=1}^{6} C_i(s,t)\,M_i^\mu\right)u(p_b,\lambda_b),$$

$$M_1^\mu = \gamma^5\,p_b^\mu, \qquad M_2^\mu = \gamma^5\,p_{\bar{p}}^\mu, \qquad M_3^\mu = \gamma^5\,\not{p}_\psi\,p_b^\mu,$$
$$M_4^\mu = \gamma^5\,\not{p}_\psi\,p_{\bar{p}}^\mu, \qquad M_5^\mu = \gamma^5\gamma^\mu, \qquad M_6^\mu = \gamma^5\,\not{p}_\psi\,\gamma^\mu.$$

The pentaquark chain

Same game, but much more complicated

- Three particles with spin $\Rightarrow$ three helicity indices.
- The kinematic singularity free amplitudes are obtained from the **P**arity **C**onserving **H**elicity **A**mplitudes.
- The basis of scalar functions is more complicated

$$A_{\lambda_p,\lambda_b\lambda_\psi}(s,t) = \epsilon_\mu(p_\psi,\lambda_\psi)\,\bar{u}(p_{\bar{p}},\lambda_{\bar{p}})\left(\sum_{i=1}^{6} C_i(s,t)\,M_i^\mu\right)u(p_b,\lambda_b),$$

$$M_1^\mu = \gamma^5\,p_b^\mu, \qquad M_2^\mu = \gamma^5\,p_{\bar{p}}^\mu, \qquad M_3^\mu = \gamma^5\,\not{p}_\psi\,p_b^\mu,$$
$$M_4^\mu = \gamma^5\,\not{p}_\psi\,p_{\bar{p}}^\mu, \qquad M_5^\mu = \gamma^5\gamma^\mu, \qquad M_6^\mu = \gamma^5\,\not{p}_\psi\,\gamma^\mu.$$

- Fermion-boson scattering has extra singularities.
- Matching is required at several orders of z_s.

Visual difference

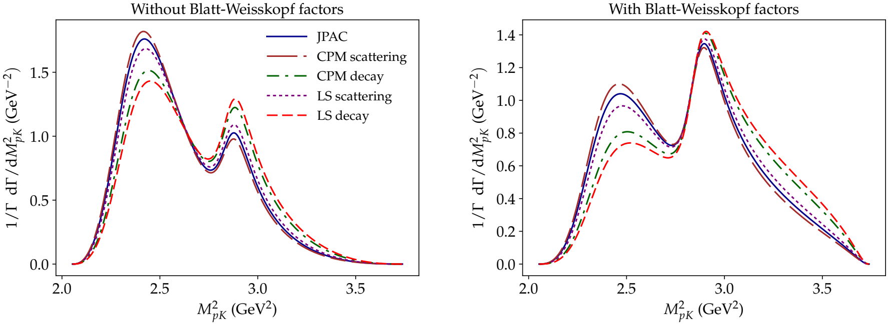
[arXiv:1805.02113]

Model for the dynamical function

- $\Lambda^*(1520)$ and $\Lambda^*(1690)$ in the pK-channel.

$$T_{\Lambda^*}(s) \equiv \frac{10}{M^2_{\Lambda(1520)} - s - iM_{\Lambda(1520)}\Gamma_{\Lambda(1520)}} + \frac{1}{M^2_{\Lambda(1690)} - s - iM_{\Lambda(1690)}\Gamma_{\Lambda(1690)}}.$$

The width is enlarged for the illustration purpose.



The work can be extended

Beyond four legs

- Photoproduction of two pions.
- Diffractive production of three particle system at COMPASS:
  - quasi-2 $\rightarrow$ 3 scattering
  - partial wave expansion in 3π-system
  - partial wave expansion in subchannels, 2π-system

Blatt-Weisskopf

- Phenomenological function

$$B_1(q) = \left(\frac{1}{1+q^2R^2}\right)^{1/2}, \quad B_2(p) = \left(\frac{1}{9+3p^2R^2+p^4R^4}\right)^{1/2}.$$

coming from I

$$U_l(r) = \begin{cases} -V_0 & \text{for } r \le R \\ \frac{l(l+1)}{2mr^2} & \text{for } r > R \end{cases},$$

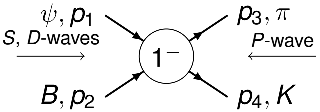
$$v_l(k) \to \int \frac{\mathrm{d}z}{2} \frac{P_l(z)}{M^2-t} = \frac{1}{4k^2} Q_l\left(1+\frac{M^2}{2k^2}\right).$$

Spin-orbit partial waves

LS-scheme: Jacob-Wick

$$A_\lambda^j = p^{j-1} q^j \left[\left(\frac{2j-1}{2j+1}\right)^{1/2} \langle j-1, 0; 1, \lambda | j, \lambda \rangle \hat{G}^j_{j-1}(s) + \left(\frac{2j+3}{2j+1}\right)^{1/2} \langle j+1, 0; 1, \lambda | j, \lambda \rangle p^2 \hat{G}^j_{j+1}(s) \right]$$

$$A_\lambda^1 = q \left[\frac{1}{\sqrt{3}} \hat{G}^1_0(s) + \sqrt{\frac{5}{3}} \langle 2, 0; 1, \lambda | j, \lambda \rangle p^2 \hat{G}^1_2(s) \right]$$



Matching:

$$g_j(s) = g_j'(s) \frac{s + m_1^2 - m_2^2}{2m_1\sqrt{s}} = \left(\frac{2j-1}{2j+1}\right)^{1/2} \hat{G}^j_{j-1}(s),$$

$$f_j(s) = \frac{m_1}{\sqrt{s}} f_j'(s) = p^2 \left(\frac{2j+3}{2j+1}\right)^{1/2} \hat{G}^j_{j+1}(s).$$

Specific model:

- $g_j'(s)$ has a pole at $s_p = m_2^2 - m_1^2$,
- $f_j'(s)$ has extra-$\sqrt{s}$ singularity.

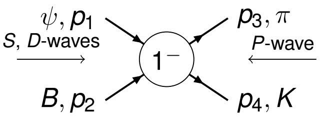
Covariant projection tensor formalism for scattering

Filippini(PRD 51,1995), Bonn-Gatchina group (EPJ, 2005), Chung-Friedrich(PRD78,2008)

$$A_\lambda = \epsilon_\rho(\lambda, p_1)\,\underbrace{[g_S(s) g_{\rho\mu} + g_D(s) X_{\rho\mu}(p)]}_{\text{S,D-waves}} \left(-g^{\mu\nu} + \frac{P^\mu P^\nu}{s}\right) \underbrace{X_\nu(q)}_{\text{P-wave}},$$

$$A_0 = q\frac{E_1}{m_1}\cos\theta_s\left[g_S(s) - p^2 g_D(s)\right],$$

$$A_+ = -q\frac{\sin\theta_s}{\sqrt{2}}\left[g_S(s) + \frac{p^2}{2} g_D(s)\right].$$



Matching:

$$g_1 = \frac{4\pi}{3} g_S, \quad g_1' = \frac{4\pi}{3} g_S,$$

$$f_1 = \frac{2\pi\lambda_{12}}{3s} g_D, \quad f_1' = -\frac{4\pi\lambda_{12}}{3s}\frac{s + m_1^2 - m_2^2}{m_1^2} g_D.$$

Specific model:

- just fine.
- except extra $1/s$ singularity.

g_S and g_D are not S- and $D-$waves.

Covariant projection tensor formalism for the decay

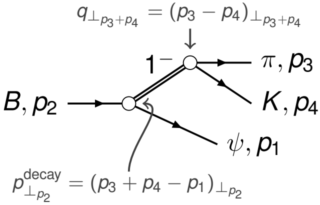
Filipini-Rotondi, Bonn-Gatchina, Friedrich-Chung

$$A_\lambda = \epsilon_\rho(\lambda, p_1) \underbrace{\left[g_S(s) g_{\rho\mu} + g_D(s) X_{\rho\mu}(p^{\text{decay}})\right]}_{\text{S,D-waves}} \left(-g^{\mu\nu} + \frac{P^\mu P^\nu}{s}\right) \underbrace{X_\nu(q)}_{\text{P-wave}},$$

$$X^{\rho\mu} = \frac{3}{2}\left[p_\perp^\rho p_\perp^\mu - \frac{1}{3} g_\perp^{\rho\mu} p_\perp^2\right].$$

$$A_0 = q\cos\theta_s \left[\frac{E_1}{m_1} g_S(s) - \gamma p^2 g_D(s) \frac{s}{m_2^2} \frac{s - m_1^2 - m_2^2}{2 m_1 m_2}\right],$$

$$A_+ = -q\frac{\sin\theta_s}{\sqrt{2}}\left[g_S(s) + \frac{p^2}{2} g_D(s) \frac{s}{m_2^2}\right].$$



- g_S and g_D are not S- and $D-$waves.
- High polynomial in s is an artifact of the model.
- Trace back of the formalism is contribution to dynamic functions!

Minimal model dependence

Covariant formalism

$$A_\lambda(s,t) = \epsilon_\mu(\lambda, p_1)\left[(p_3 - p_4)^\mu - \frac{m_3^2 - m_4^2}{s}(p_3+p_4)^\mu\right] C(s,t) + \epsilon_\mu(\lambda, p_1)(p_3+p_4)^\mu\, B(s,t).$$

Minimal model dependence

Covariant formalism

$$A_\lambda(s,t) = \epsilon_\mu(\lambda, p_1)\left[(p_3 - p_4)^\mu - \frac{m_3^2 - m_4^2}{s}(p_3+p_4)^\mu\right] C(s,t) + \epsilon_\mu(\lambda, p_1)(p_3+p_4)^\mu\, B(s,t).$$

$$4\pi C(s,t) = \frac{1}{\sqrt{2}} \sum_{j>0} (2j+1)(pq)^{j-1} \hat{A}^j_{\pm}(s)\, \hat{d}^j_{10}(z_s).$$

$$4\pi B(s,t) = \hat{A}^0_0 + \frac{4m_1^2}{\lambda_{12}} \sum_{j>0} (2j+1)(pq)^j \left[\hat{A}^j_0(s) \hat{d}^j_{00}(z_s) + \frac{s + m_1^2 - m_2^2}{\sqrt{2} m_1^2} \hat{A}^j_{+}(s)\, z_s \hat{d}^j_{10}(z_s)\right].$$

Minimal model dependence

Covariant formalism

$$A_\lambda(s,t) = \epsilon_\mu(\lambda,p_1)\left[(p_3-p_4)^\mu - \frac{m_3^2-m_4^2}{s}(p_3+p_4)^\mu\right] C(s,t) + \epsilon_\mu(\lambda,p_1)(p_3+p_4)^\mu\, B(s,t).$$

$$4\pi C(s,t) = \frac{1}{\sqrt{2}}\sum_{j>0}(2j+1)(pq)^{j-1}\hat{A}^j_{\pm}(s)\,\hat{d}^j_{10}(z_s).$$

$$4\pi B(s,t) = \hat{A}^0_0 + \frac{4m_1^2}{\lambda_{12}}\sum_{j>0}(2j+1)(pq)^j\left[\hat{A}^j_0(s)\hat{d}^j_{00}(z_s) + \frac{s+m_1^2-m_2^2}{\sqrt{2}m_1^2}\hat{A}^j_+(s)\,z_s\hat{d}^j_{10}(z_s)\right].$$

General form of the helicity amplitudes

$$A^j_+ = p^{j-1}q^j\left[\langle j-1,0;1,1|j,1\rangle g_j(s) + f_j(s)\right],$$

$$A^j_0 = p^{j-1}q^j\left[\langle j-1,0;1,0|j,0\rangle\frac{s+m_1^2-m_2^2}{2m_1\sqrt{s}}g'_j(s) + \frac{m_1}{\sqrt{s}}f'_j(s)\right].$$

with $g_j(s_\pm) = g'_j(s_\pm)$ and $f_j(s_\pm) = f'_j(s_\pm) = 0$.