D mixing parameter y in the factorisation-assisted topological-amplitude approach

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arXiv: 1705.07335

The 9th International Workshop on Charm Physics
Budker INP, 25.05.2018

Outline

- Motivation
  - Difficulties and efforts in understanding D-Dbar mixing in theory
- D-Dbar Mixing in the FAT approach
  - Handle flavor SU(3) breaking effects
- Summary and Outlook

Description of meson Mixing

- The time evolution of two flavor eigenstates

$$i\frac{\partial}{\partial t}\begin{pmatrix} M^0(t) \\ \overline{M}^0(t) \end{pmatrix} = \left(\mathbf{M} - \frac{i}{2}\mathbf{\Gamma}\right)\begin{pmatrix} M^0(t) \\ \overline{M}^0(t) \end{pmatrix}$$

$$\boxed{\begin{aligned} M^0 &= (q_1\bar{q}_2) \\ \overline{M}^0 &= (\bar{q}_1 q_2) \end{aligned}}$$

- Mass eigenstates

$$|M_{L,H}\rangle = p|M^0\rangle \pm q|\overline{M}^0\rangle$$

- Diagonalise the Hamiltonian matrix

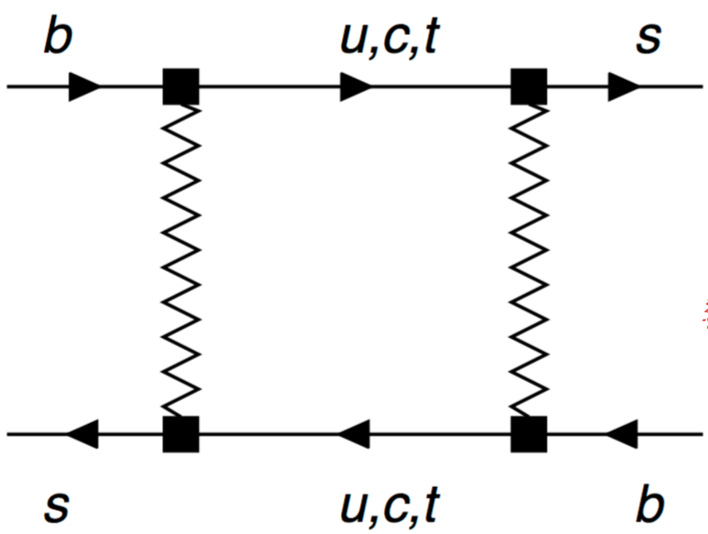
$$\Delta m - \frac{i}{2}\Delta\Gamma = 2\sqrt{\left(\mathbf{M}_{12} - \frac{i}{2}\mathbf{\Gamma}_{12}\right)\left(\mathbf{M}^*_{12} - \frac{i}{2}\mathbf{\Gamma}^*_{12}\right)}$$

$$\left(\frac{q}{p}\right)^2 = \frac{\mathbf{M}^*_{12} - \frac{i}{2}\mathbf{\Gamma}^*_{12}}{\mathbf{M}_{12} - \frac{i}{2}\mathbf{\Gamma}_{12}}$$

physical observables

$$\boxed{y_D \equiv \frac{\Delta\Gamma_D}{2\Gamma_D}}$$

Theoretical study of B Mixing



heavy quark expansion

- Wilson coefficients
- matrix elements of high-dimension operators

- Wilson coefficients are calculated perturbatively up to N…NLO-QCD
- Hadronic matrix elements of operators are calculated by nonperturbative approaches like Lattice or Sum Rule

$$
\begin{aligned}
Q &= \bar{s}_\alpha \gamma_\mu (1-\gamma_5) b_\alpha \, \bar{s}_\beta \gamma^\mu (1-\gamma_5) b_\beta \\
\langle B_s | Q | \overline{B}_s \rangle &= \frac{8}{3} M_{B_s}^2 \, f_{B_s}^2 B
\end{aligned}
$$

$\boxed{\Rightarrow \text{ expanded by } 1/m_b}$

[Lenz, Nierste, 06'; Lenz Rauh, 17']

Theoretical study of B Mixing

$$\Delta M_s^{\text{th}} = (19.30 \pm 6.68)\text{ps}^{-1}$$

$$\Delta \Gamma_s^{\text{th}} = (0.096 \pm 0.039)\text{ps}^{-1}$$

$$\Delta M_d^{\text{th}} = (0.53 \pm 0.02)\text{ps}^{-1}$$

[Lenz, Nierste, 06']

$$\Delta M_s^{\text{exp}} = (17.757 \pm 0.021)\text{ps}^{-1}$$

$$\Delta \Gamma_s^{\text{exp}} = (0.082 \pm 0.007)\text{ps}^{-1}$$

$$\Delta M_d^{\text{exp}} = (0.5096 \pm 0.0034)\text{ps}^{-1}$$

[PDG, 2016]

Match perfectly!

Apply it to D mixing?

D mixing: Theory vs Exp.

- Including corrections up to $1/m_c$ and NLO-QCD, y_D can only reach order of 10^{-6}. [Bobrowski, Lenz, Riedl, Rohrwild, 10']

$$y_D \equiv \frac{\Delta\Gamma_D}{2\Gamma_D}$$

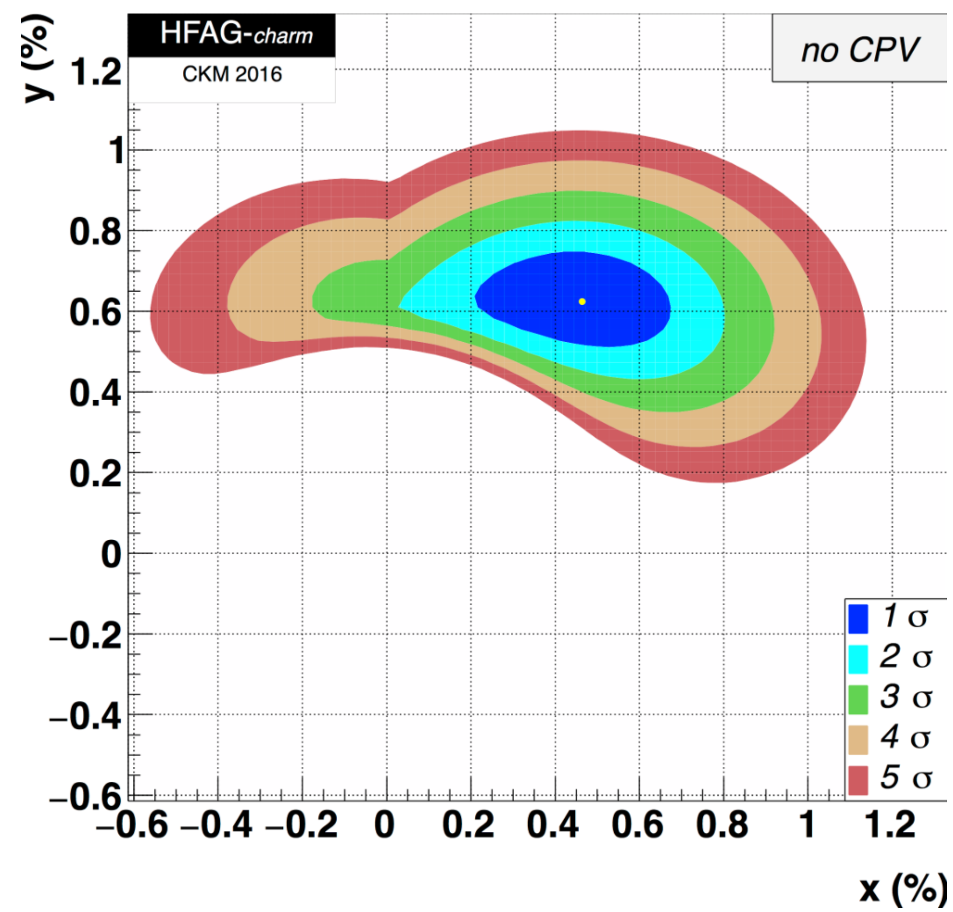
- Current world average results [HFLAV, 16']
  - ✳ If CP is conserved

$$y = (0.62 \pm 0.08)\%$$

  - ✳ If CP violation is allowed

$$y = (0.69^{+0.06}_{-0.07})\%$$

3-order difference!



[See Prof. Alan Schwartz's talk for the update]

What is the problem?

Problem on dynamics

- $m_c \sim 1.3$ GeV
  - neither heavy enough for heavy quark expansion, $1/m_c$

$$y_D = y_D^{(0)} + \frac{\Lambda}{m_c} y_D^{(1)} + \boxed{\mathcal{O}\left(\frac{\Lambda^2}{m_c^2}\right)}$$

might be large

  - nor light enough for chiral perturbation theory

Exclusive Approaches

Assuming no CPV

phase space

mass eigenstates

$$y = \frac{1}{2\Gamma}\sum_n \rho_n \left(|\mathcal{A}(D_+ \to n)|^2 - |\mathcal{A}(D_- \to n)|^2\right)$$

$$= \frac{1}{\Gamma}\sum_n \eta_{\rm CP}(n)\rho_n \mathcal{R}e\left[\mathcal{A}(D^0 \to n)\mathcal{A}^*(D^0 \to \bar{n})\right]$$

$$= \sum_n \eta_{\rm CKM}(n)\eta_{\rm CP}(n)\cos\delta_n \sqrt{\mathcal{B}(D^0 \to n)\mathcal{B}(D^0 \to \bar{n})}$$

$(-1)^{n_s}$

strong phase difference

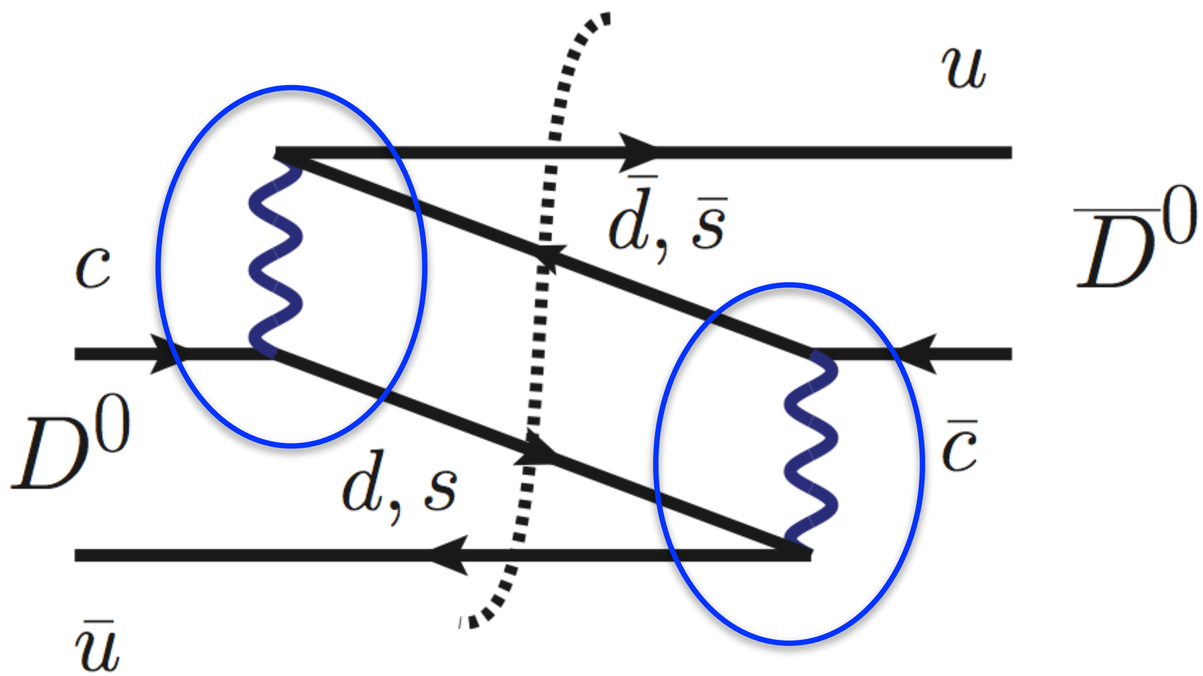
Sum up all the intermediate (final) states

⇨ **To measure or predict branching fractions**

Exclusive Approaches

Effort 1. SU(3) breaking effects from phase space

[Falk,Grossman,Ligeti,Petrov, 01']



$$V_{ud}^* V_{cd} + V_{us}^* V_{cs} \approx 0$$

y vanishes in the SU(3) limit

$$\boxed{y \sim \sin^2\theta_C \times [SU(3)\ \text{breaking}]^2}$$

Promising SU(3) breaking effects from PS, e.g., $\pi\pi\pi\pi$ *v.s.* $KKKK$

⇨ **This is qualitative. What about quantitative?**

Exclusive Approaches

Effort 2. Try PP & PV (best measured)

[Cheng,Chiang, 10']

$$\boxed{\mathcal{B}(\pi^+\pi^-) + \mathcal{B}(K^+K^-) - 2\cos\delta_{K^+\pi^-}\sqrt{\mathcal{B}(K^-\pi^+)\mathcal{B}(K^+\pi^-)}}$$

$$+\mathcal{B}(\pi^0\pi^0) + \mathcal{B}(K^0\bar{K}^0) - 2\cos\delta_{K^0\pi^0}\sqrt{\mathcal{B}(\bar{K}^0\pi^0)\mathcal{B}(K^0\pi^0)}$$

$$+\mathcal{B}(\pi^0\eta) + \mathcal{B}(\pi^0\eta') + \mathcal{B}(\eta\eta) + \mathcal{B}(\eta\eta')$$

$$-2\cos\delta_{K^0\eta}\sqrt{\mathcal{B}(\bar{K}^0\eta)\mathcal{B}(K^0\eta)} - 2\cos\delta_{K^0\eta'}\sqrt{\mathcal{B}(\bar{K}^0\eta')\mathcal{B}(K^0\eta')}$$

vanish in the SU(3) symmetry limit

$$\boxed{y_{PP}}$$

Exclusive Approaches

Effort 2. Try PP & PV (best measured)

[Cheng,Chiang, 10']

$$
\begin{aligned}
&Br(\pi^0\rho^0)+Br(\pi^0\omega)+Br(\pi^0\phi)+Br(\eta\omega)+Br(\eta'\omega)+Br(\eta\phi)+Br(\eta\rho^0)+Br(\eta'\rho^0)\\
&-2\cos\delta_{K^{*-}\pi^+}\sqrt{Br(K^{*-}\pi^+)Br(K^{*+}\pi^-)}-2\cos\delta_{K^{*0}\pi^0}\sqrt{Br(K^{*0}\pi^0)Br(\bar{K}^{*0}\pi^0)}\\
&-2\cos\delta_{K^-\rho^+}\sqrt{Br(K^-\rho^+)Br(K^+\rho^-)}-2\cos\delta_{K^0\rho^0}\sqrt{Br(K^0\rho^0)Br(\bar{K}^0\rho^0)}\\
&-2\cos\delta_{K^{*0}\eta}\sqrt{Br(K^{*0}\eta)Br(\bar{K}^{*0}\eta)}-2\cos\delta_{K^{*0}\eta'}\sqrt{Br(K^{*0}\eta')Br(\bar{K}^{*0}\eta')}\\
&-2\cos\delta_{K^0\omega}\sqrt{Br(K^0\omega)Br(\bar{K}^0\omega)}-2\cos\delta_{K^0\phi}\sqrt{Br(K^0\phi)Br(\bar{K}^0\phi)}\\
&+2\cos\delta_{K^+K^{*-}}\sqrt{Br(K^+K^{*-})Br(K^-K^{*+})}+2\cos\delta_{K^0\bar{K}^{*0}}\sqrt{Br(K^0\bar{K}^{*0})Br(\bar{K}^0K^{*0})}\\
&+2\cos\delta_{\pi^+\rho^-}\sqrt{Br(\pi^+\rho^-)Br(\pi^-\rho^+)}
\end{aligned}
$$

$$\boxed{y_{PV}}$$

Exclusive Approaches

Effort 2. Try PP & PV (best measured)

- Take branching ratios from exp (fit for unmeasured channels) and assume no strong phase difference

[Cheng,Chiang, 10']

$$\boxed{\begin{aligned} y_{PP} &= (0.86 \pm 0.41) \times 10^{-3} \\ y_{PV} &= (2.69 \pm 2.53) \times 10^{-3} \;\; (A, A1) \\ y_{PV} &= (1.52 \pm 2.20) \times 10^{-3} \;\; (S, S1) \end{aligned}}$$

VS

$$\boxed{y_{\rm exp} = (0.62 \pm 0.08)\%}$$

Same order, but large uncertainties! Why?

- Contributions to y may cancel between different modes, but uncertainties always add together (assuming independent, but not true).
- The uncertainties of y are determined by the worst channels (with largest uncertainties).

Exclusive Approaches

Effort 2. Try PP & PV (best measured)

$$
\begin{aligned}
y_{PP} &= (0.86 \pm 0.41) \times 10^{-3} \\
y_{PV} &= (2.69 \pm 2.53) \times 10^{-3} \;\; (A, A1) \\
y_{PV} &= (1.52 \pm 2.20) \times 10^{-3} \;\; (S, S1)
\end{aligned}
$$

[Cheng,Chiang, 10']

$$
\mathcal{B}(\eta\eta)_{\rm exp} = (1.70 \pm 0.20) \times 10^{-3}
$$

$$
\mathcal{B}(\eta\eta')_{\rm exp} = (1.07 \pm 0.26) \times 10^{-3}
$$

$$
\sigma[B(\eta\eta) + B(\eta\eta')] = 0.33 \times 10^{-3}
$$

one term in yPP

$$
\mathcal{B}(\pi^+ K^{*-})_{\rm exp} = (54.3 \pm 4.4) \times 10^{-3}
$$

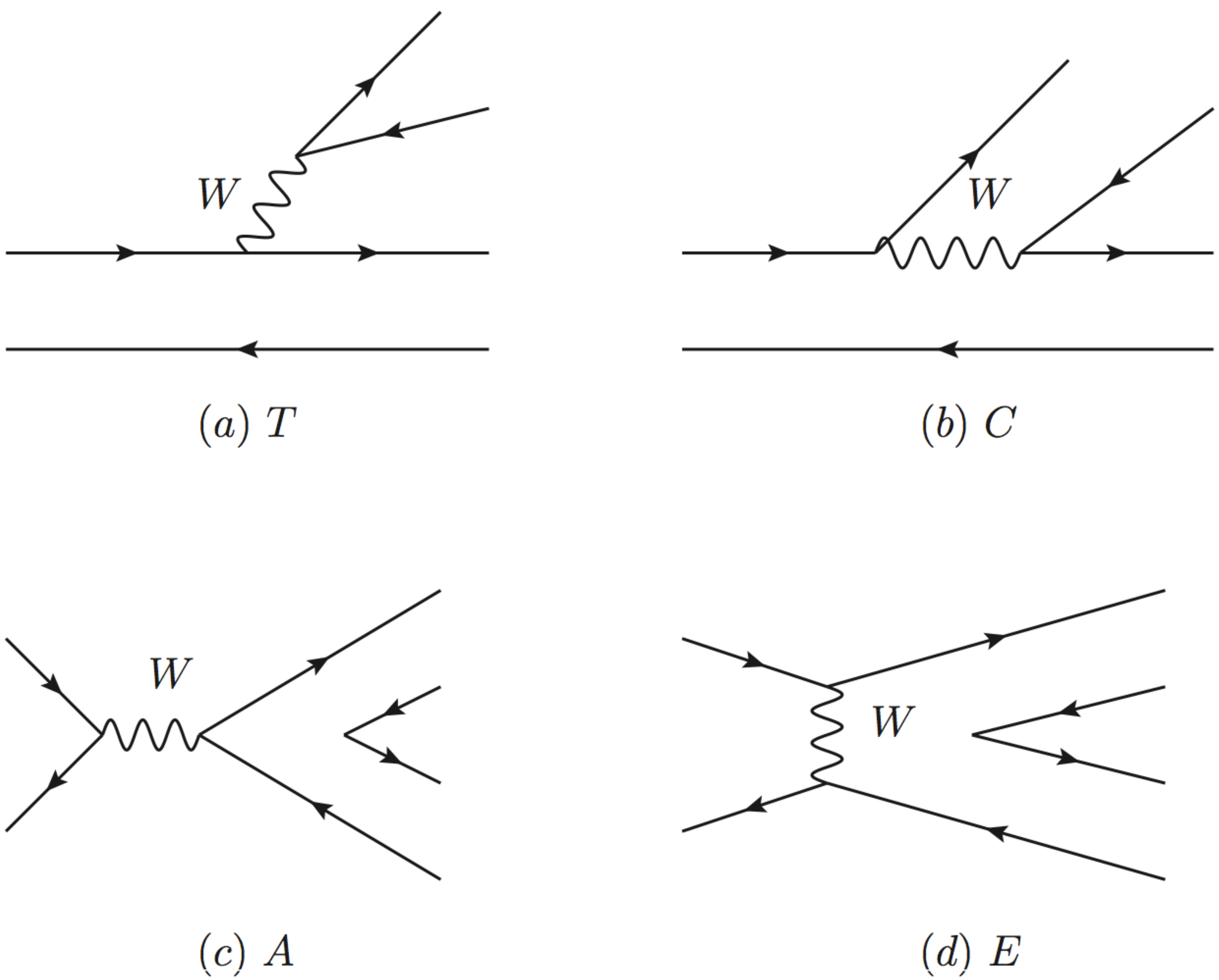
$$
\mathcal{B}(\pi^- K^{*+})_{\rm exp} = (0.345^{+0.180}_{-0.102}) \times 10^{-3}
$$

$$
\sigma[-2\sqrt{\mathcal{B}(\pi^+ K^{*-})\mathcal{B}(\pi^- K^{*+})}] = 1.8 \times 10^{-3}
$$

one term in yPV

- To reduce uncertainties, we need to find correlations between the modes.

Factorization-Assisted Topological-Amplitude Approach



[Bhattacharya, Rosner, 08',10']

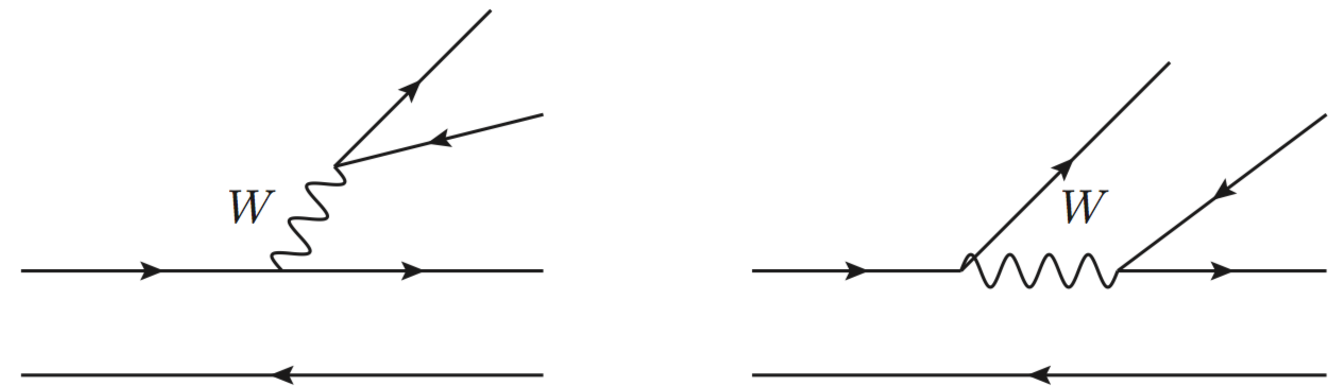
[Cheng, Chiang, 10']

- Factorize each topological amplitude:
- Short-distance dynamics: Wilson coefficients
- Long-distance dynamics: hadronic matrix elements

[Li, Lu, Yu, 1203.3120]
[QQ, Li, Lu, Yu, 1305.7021]

Emission Amplitudes (PP)

- Color-favored Tree (T)
- Color-suppressed (C)



$(a)\ T$ $\qquad$ $(b)\ C$

$$\boxed{\langle P_1 P_2 | \mathcal{H}_{\text{eff}} | D \rangle_{T,C} = \frac{G_{\text{F}}}{\sqrt{2}} V_{\text{CKM}} a_{1,2}(\mu) f_{P_2} (m_D^2 - m_{P_1}^2) F_0^{DP_1}(m_{P_2}^2)}$$

$$a_1(\mu) = C_2(\mu) + \frac{C_1(\mu)}{N_c}$$

Non-factorizable

$$a_2(\mu) = C_1(\mu) + C_2(\mu) \left[\frac{1}{N_c} + \chi_{nf} e^{i\phi} \right]$$

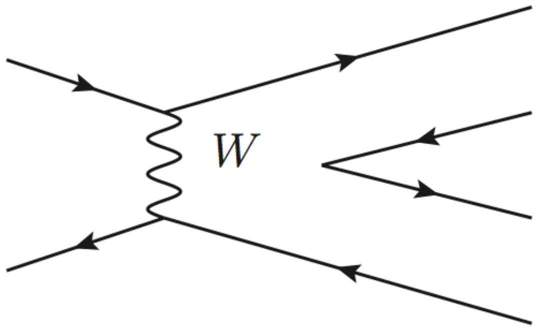
$$\mu = \sqrt{\Lambda m_D (1 - r_2^2)}, \qquad r_2 = m_{P_2}^2 / m_D^2$$

SU(3) breaking effects in amplitudes

[Li, Lu, Yu, 1203.3120]

Annihilation Amplitude (PP)

- W-exchange (E)



(d) E

$$\boxed{\langle P_1 P_2 | \mathcal{H}_{\rm eff} | D \rangle_E = \frac{G_{\rm F}}{\sqrt{2}} V_{\rm CKM} b^E_{q,s}(\mu) f_D m_D^2 \left(\frac{f_{P_1} f_{P_2}}{f_\pi^2} \right)}$$

Dominated by **non-factorizable** contribution

$$b^E_{q,s}(\mu) = C_2(\mu) \chi^E_{q,s} e^{i\phi^E_{q,s}} + S_\pi$$

SU(3) breaking effects in amplitudes

$$\boxed{\chi_q \neq \chi_s}$$

Glauber strong phase for pions: [Li, Mishima, 11']

pion= 1. Nambu-Goldstone boson
2. qqbar bound state

[Li, Lu, Yu, 1203.3120]

Amplitudes (PV)

- Color-favored Tree (T)
- Color-suppressed (C)

$$\langle PV|\mathcal{H}_{\text{eff}}|D\rangle_{T,C} = \frac{G_{\text{F}}}{\sqrt{2}} V_{\text{CKM}} a_{1,2}^{P}(\mu) f_V m_V F_1^{DP}(m_V^2) 2(\varepsilon_V \cdot p_D)$$

$$\langle VP|\mathcal{H}_{\text{eff}}|D\rangle_{T,C} = \frac{G_{\text{F}}}{\sqrt{2}} V_{\text{CKM}} a_{1,2}^{V}(\mu) f_P m_V A_0^{DV}(m_P^2) 2(\varepsilon_V \cdot p_D)$$

- W-exchange (E)

$$\langle PV|\mathcal{H}_{\text{eff}}|D\rangle_{E} = \frac{G_{\text{F}}}{\sqrt{2}} V_{\text{CKM}} C_2(\mu) \chi_{q,s}^{E} e^{i\phi_{q,s}^{E}} f_D m_D \frac{f_P}{f_\pi} \frac{f_V}{f_\rho} (\varepsilon_V \cdot p_D)$$

[QQ, Li, Lu, Yu, 1305.7021]

To extract nonperturbative parameters from abundant and precise data

- PP

$$\chi^C_{\rm nf} = -0.81 \pm 0.01, \qquad \phi^C_{\rm nf} = 0.22 \pm 0.14, \qquad S_\pi = -0.92 \pm 0.07,$$
$$\chi^E_q = 0.056 \pm 0.002, \quad \phi^E_q = 5.03 \pm 0.06, \quad \chi^E_s = 0.130 \pm 0.008, \quad \phi^E_s = 4.37 \pm 0.10$$

- PV

$$S_\pi = -1.88 \pm 0.12, \qquad \chi^C_P = 0.63 \pm 0.03, \qquad \phi^C_P = 1.57 \pm 0.11,$$
$$\chi^C_V = 0.71 \pm 0.03, \qquad \phi^C_V = 2.77 \pm 0.10, \qquad \chi^E_q = 0.49 \pm 0.03,$$
$$\phi^E_q = 1.61 \pm 0.07, \qquad \chi^E_s = 0.54 \pm 0.03, \qquad \phi^E_s = 2.23 \pm 0.08$$

PP: 7 parameters for 13 data $\rightarrow$ χ^2 per d.o.f. = 1.1
PV: 9 parameters for 19 data $\rightarrow$ χ^2 per d.o.f. = 1.8

[Jiang, Yu, QQ, Li, Lu, 1705.07335]

D^0 decay rates of PP and PV channels
agree well with experiments

($\times 10^{-3}$)

Modes	$\mathcal{B}$(exp)	$\mathcal{B}$(FAT)	Modes	$\mathcal{B}$(exp)	$\mathcal{B}$(FAT)	Modes	$\mathcal{B}$(exp)	$\mathcal{B}$(FAT)
$\pi^0\overline{K}^0$	24.0 ± 0.8	24.2 ± 0.8	$\pi^0\overline{K}^{*0}$	37.5 ± 2.9	35.9 ± 2.2	$\overline{K}^0\rho^0$	$12.8^{+1.4}_{-1.6}$	13.5 ± 1.4
π^+K^-	39.3 ± 0.4	39.2 ± 0.4	π^+K^{*-}	54.3 ± 4.4	62.5 ± 2.7	$K^-\rho^+$	111.0 ± 9.0	105.0 ± 5.2
$\eta\overline{K}^0$	9.70 ± 0.6	9.6 ± 0.6	$\eta\overline{K}^{*0}$	9.6 ± 3.0	6.1 ± 1.0	$\overline{K}^0\omega$	22.2 ± 1.2	22.3 ± 1.1
$\eta'\overline{K}^0$	19.0 ± 1.0	19.5 ± 1.0	$\eta'\overline{K}^{*0}$	< 1.10	0.19 ± 0.01	$\overline{K}^0\phi$	$8.47^{+0.66}_{-0.34}$	8.2 ± 0.6
$\pi^+\pi^-$	1.421 ± 0.025	1.44 ± 0.02	$\pi^+\rho^-$	5.09 ± 0.34	4.5 ± 0.2	$\pi^-\rho^+$	10.0 ± 0.6	9.2 ± 0.3
K^+K^-	4.01 ± 0.07	4.05 ± 0.07	K^+K^{*-}	1.62 ± 0.15	1.8 ± 0.1	K^-K^{*+}	4.50 ± 0.30	4.3 ± 0.2
$K^0\overline{K}^0$	0.36 ± 0.08	0.29 ± 0.07	$K^0\overline{K}^{*0}$	0.18 ± 0.04	0.19 ± 0.03	$\overline{K}^0K^{*0}$	0.21 ± 0.04	0.19 ± 0.03
$\pi^0\eta$	0.69 ± 0.07	0.74 ± 0.03	$\eta\rho^0$		1.4 ± 0.2	$\pi^0\omega$	0.117 ± 0.035	0.10 ± 0.03
$\pi^0\eta'$	0.91 ± 0.14	1.08 ± 0.05	$\eta'\rho^0$		0.25 ± 0.01	$\pi^0\phi$	1.35 ± 0.10	1.4 ± 0.1
$\eta\eta$	1.70 ± 0.20	1.86 ± 0.06	$\eta\omega$	2.21 ± 0.23	2.0 ± 0.1	$\eta\phi$	0.14 ± 0.05	0.18 ± 0.04
$\eta\eta'$	1.07 ± 0.26	1.05 ± 0.08	$\eta'\omega$		0.044 ± 0.004			
$\pi^0\pi^0$	0.826 ± 0.035	0.78 ± 0.03	$\pi^0\rho^0$	3.82 ± 0.29	4.1 ± 0.2			
π^0K^0		0.069 ± 0.002	π^0K^{*0}		0.103 ± 0.006	$K^0\rho^0$		0.039 ± 0.004
π^-K^+	0.133 ± 0.009	0.133 ± 0.001	π^-K^{*+}	$0.345^{+0.180}_{-0.102}$	0.40 ± 0.02	$K^+\rho^-$		0.144 ± 0.009
ηK^0		0.027 ± 0.002	ηK^{*0}		0.017 ± 0.003	$K^0\omega$		0.064 ± 0.003
$\eta'K^0$		0.056 ± 0.003	$\eta'K^{*0}$		0.00055 ± 0.00004	$K^0\phi$		0.024 ± 0.002

Back to our topic in the end

D-Dbar Mixing
in the FAT approach

$$\boxed{\begin{aligned} y &= \frac{1}{2\Gamma}\sum_n \rho_n \eta_{\rm CP}(n)\left(\langle D^0|H_w|n\rangle\langle\bar{n}|H_w|D^0\rangle + \langle D^0|H_w|\bar{n}\rangle\langle n|H_w|D^0\rangle\right) \\ &= \sum_n \eta_{\rm CKM}(n)\eta_{\rm CP}(n)\cos\delta_n\sqrt{Br(D^0\to n)Br(D^0\to\bar{n})}\,, \end{aligned}}$$

$$CP|M_1M_2\rangle = \eta_{\rm CP}(M_1)\eta_{\rm CP}(M_2)(-1)^L|M_1M_2\rangle = \eta_{\rm CP}(M_1M_2)|M_1M_2\rangle$$

Our results

$$y_{PP} = (1.00 \pm 0.19) \times 10^{-3}$$
$$y_{PV} = (1.12 \pm 0.72) \times 10^{-3}$$

$$y_{PP+PV} = (0.21 \pm 0.07)\%$$

[Jiang, Yu, QQ, Li, Lu, 1705.07335]

$$y_{PP} = (0.86 \pm 0.41) \times 10^{-3}$$
$$y_{PV} = (2.69 \pm 2.53) \times 10^{-3} \quad (A, A1)$$
$$y_{PV} = (1.52 \pm 2.20) \times 10^{-3} \quad (S, S1)$$

[Cheng,Chiang, 10']

$$\boxed{y_{\mathrm{exp}} = (0.62 \pm 0.08)\%}$$

[HFLAV, 16']

- **In Cheng and Chiang's work, uncertainties are summed up channel by channel.**
- **In our approach, uncertainties are controlled by most precisely measured channels.**

PP	PV	VV	PA	PS	PT	semilep
10%	30 %	10 %	10 %	5 %	0.2%	15%

Two-body D^0 decay branching fractions [PDG]

Summary and Outlook

- The inclusive approach based on heavy quark expansion can not explain the yD data. [Bobrowski, Lenz, Riedl, Rohrwild, 10']
- In the direction of exclusive approaches, considering the SU(3) breaking only from phase spaces, the order of a SM y is natural. [Falk,Grossman,Ligeti,Petrov, 01']
- Adding up the measured PP and PV branching ratios, the central value for y is consistent with data, but with very large uncertainties (from the worst measured channels). [Cheng,Chiang, 10']
- In our approach (FAT), SU(3) breaking is well described for PP and PV, and thus predict well the yD with uncertainties under control (determined by the most precisely measured channels).
- The PP and PV channels contribute about 1/3 of the D-Dbar mixing parameter **y**.
- We are looking forward to more data of branching ratios and angular distribution of the VV, PA, PS and multi-body modes.

[Li, Lu, Yu, 1203.3120]

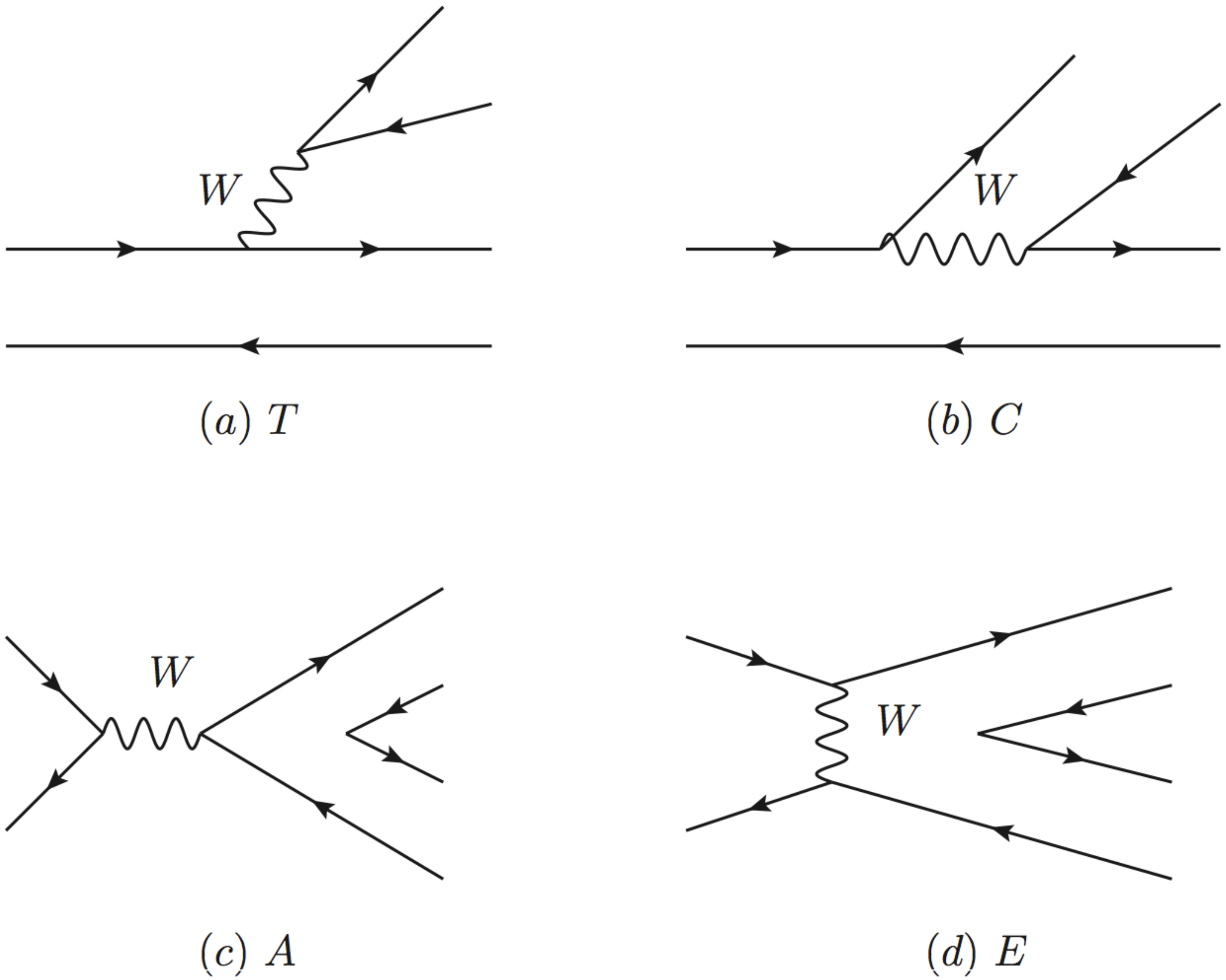
[QQ, Li, Lu, Yu, 1305.7021]

[Jiang, Yu, QQ, Li, Lu, 1705.07335]

Thank you for your attention!

Backup

Topological-Amplitude Approach



[Bhattacharya, Rosner, 08’,10’]

[Cheng, Chiang, 10']

Estimation of VV contribution

- ***No enough data for fit***
  - only 4 channels with measured total decay rates (18 parameters)
  - only 1 channel with measured longitudinal decay rate (6 parameters)
- ***Apply PV parameters to longitudinal VV channels***

Neutral D^0 decay rates of VV channels

to see whether the estimation is acceptable

$(\times 10^{-3})$

Modes	$\mathcal{B}_{\rm tot}(\rm exp)$	$\mathcal{B}_{\rm long}(\rm exp)$	$\mathcal{B}_{\rm long}(\rm FAT)$
$\rho^0 \overline{K}^{*0}$	15.9 ± 3.5		14.3 ± 1.6
$\rho^+ K^{*-}$	65.0 ± 25.0		41.8 ± 2.4
$\overline{K}^{*0}\omega$	11.0 ± 5.0		37.7 ± 2.7
$\rho^+\rho^-$			4.1 ± 0.3
$K^{*+}K^{*-}$			1.18 ± 0.06
$K^{*0}\overline{K}^{*0}$			0.043 ± 0.006
$\rho^0\rho^0$	1.83 ± 0.13	1.25 ± 0.13	1.4 ± 0.2
$\rho^0\omega$			1.37 ± 0.08
$\rho^0\phi$			0.65 ± 0.04
$\omega\omega$			0.53 ± 0.08
$\omega\phi$			1.4 ± 0.1
$\rho^0 K^{*0}$			0.041 ± 0.005
$\rho^- K^{*+}$			0.143 ± 0.008
$K^{*0}\omega$			0.108 ± 0.008

Estimation: $y_{VV} = (0.28 \pm 0.47) \times 10^{-3}$ (Longitudinal)

$$y_{PP+PV+VV} = 0.24\%$$

Exp: $y_D = (0.62 \pm 0.08)\%$

Two-body D^0 decay branching fractions

PP	PV	VV	PA	PS	PT	semilep
10%	30 %	10 %	10 %	5 %	0.2%	15%

[PDG]

η_{CP}:	+	+	+, S,D	+	-
			-, P wave		

More efforts expected from experimentalists

to measure VV, PA, PS and multi-body modes

x from dispersion relation

$$Ref(s) = \frac{1}{\pi} P \int_0^\infty \frac{ds'}{s'-s} Imf(s')$$

In the heavy quark limit

$$\Delta m = -\frac{1}{2\pi} \mathrm{P} \int_{2m_\pi}^{\infty} \mathrm{d}E \left[\frac{\Delta\Gamma(E)}{E-m_D} + \mathcal{O}\left(\frac{\Lambda_{\mathrm{QCD}}}{E} \right) \right]$$

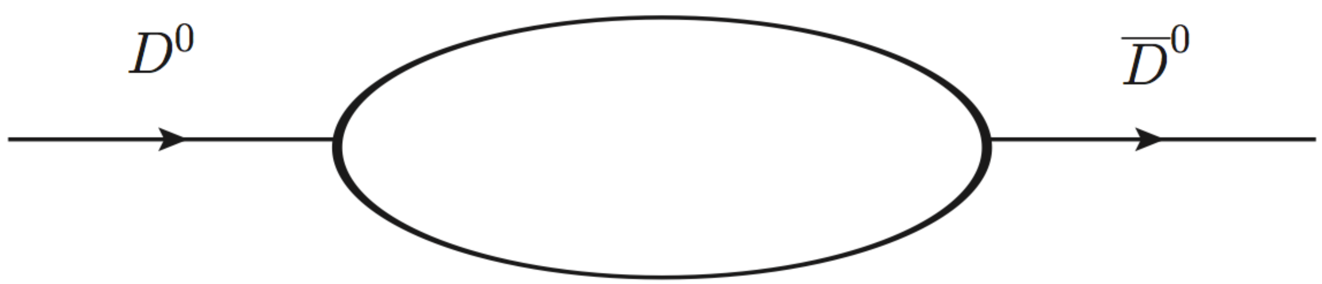
[Falk,Grossman,Ligeti,Nir,Petrov,2004]

x from dispersion relation

$$Ref(s) = \frac{1}{\pi} P \int_0^\infty \frac{ds'}{s' - s} Imf(s')$$

$$\Delta m = 2M_{12} = 2Re[\Pi(m_D^2)]$$

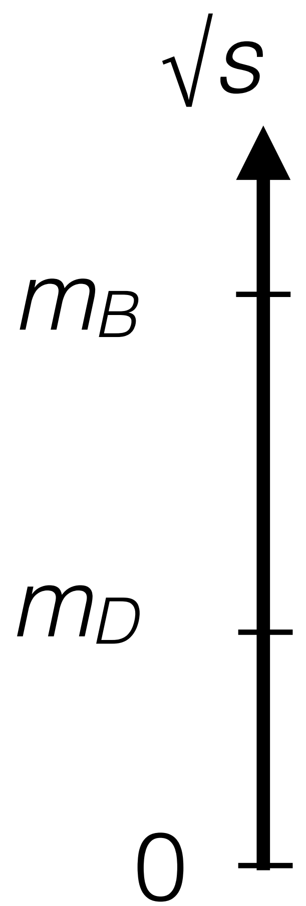
$$\Delta\Gamma = 2\Gamma_{12} = 2Im[\Pi(m_D^2)]$$



$$\Delta m(m_D^2) = \frac{2}{\pi} P \int_{(2m_\pi^2)^2}^\infty \frac{\Delta\Gamma(s)}{s - m_D^2} ds$$

x from dispersion relation

$$\Delta m(m_D^2) = \frac{2}{\pi} P \int_{(2m_\pi^2)^2}^{\infty} \frac{\Delta\Gamma(s)}{s - m_D^2} ds$$



vanishes due to SU(3) symmetry

$y(m_D^2)$

Simulate nearby m_D

dynamics required

$y(4m_\pi^2) = 0$

$$D^0 \to \pi^+\pi^- \quad v.s. \quad D^0 \to K^+K^-$$

$$\mathcal{A}(D^0 \to \pi^+\pi^-) = \frac{G_F}{\sqrt{2}}\lambda_d \left(T^{\pi\pi} + E^{\pi\pi}\right)$$

Glauber phase

$$= \frac{G_F}{\sqrt{2}} V_{cd}^* V_{ud} \left[a_1(\mu)(m_D^2 - m_\pi^2) f_\pi F_0^{D\pi}(m_\pi^2) + C_2(\mu) e^{i(\phi_q^E + 2S_\pi)} \chi_q^E f_D m_D^2\right]$$

Main difference

$$\mathcal{A}(D^0 \to K^+K^-) = \frac{G_F}{\sqrt{2}}\lambda_s \left(T^{KK} + E^{KK}\right)$$

$$= \frac{G_F}{\sqrt{2}} V_{cs}^* V_{us} \left[a_1(\mu)(m_D^2 - m_K^2) f_K F_0^{DK}(m_K^2) + C_2(\mu) e^{i\phi_q^E} \chi_q^E f_D m_D^2 \frac{f_K^2}{f_\pi^2}\right]$$

Modes	Br(FSI)	Br(diagram)	Br(pole)	Br(exp)	Br(this work)
$D^0 \to \pi^+\pi^-$	1.59	2.24 ± 0.10	2.2 ± 0.5	1.45 ± 0.05	1.43
$D^0 \to K^+K^-$	4.56	1.92 ± 0.08	3.0 ± 0.8	4.07 ± 0.10	4.19

$$T^{\pi\pi} = 2.73, \qquad E^{\pi\pi} = 0.82 e^{-i142^\circ},$$

$$T^{KK} = 3.65, \qquad E^{KK} = 1.2 e^{-i85^\circ},$$

Result of CP asymmetries

- Difference of CPV in D->KK and D-> pipi

$$\Delta A_{CP}^{\text{dir}} = A_{CP}(D^0 \to K^- K^+) - A_{CP}(D^0 \to \pi^- \pi^+)$$

- Our prediction:

$$\boxed{\Delta A_{CP} = (-0.57 \sim -1.87) \times 10^{-3}}$$ [Li, Lu, Yu, 12']

- After our prediction, the world average value is lowered down by the LHCb results

Exp: $\Delta A_{CP} = (-2.53 \pm 1.04) \times 10^{-3}$ [HFAG2014]

Measurements of ΔA_{CP}

$$\Delta A_{CP}^{\mathrm{dir}} = A_{CP}(D^0 \to K^- K^+) - A_{CP}(D^0 \to \pi^- \pi^+)$$

Measurements	ΔA_{CP}	Publication	World Average
2011LHCb (D*)	(-0.82±0.24)%	PRL108,111602	ICHEP2012: (-0.74±0.15)% HFAG2012: (-0.68±0.15)%
2012 CDF	(-0.62±0.23)%	PRL109,111801	
2012 Belle	(-0.87±0.41)%	1212,1975	
2013LHCb (D*)	(-0.34±0.18)%	LHCb-CONF-2013-03	HFAG2013: (-0.33±0.12)%
2013LHCb (B)	(+0.49±0.33)%	PLB723(2013)33	
2014LHCb (B)	(+0.14±0.18)%	JHEP07(2014)041	HFAG2014: (-0.25±0.10)%
2016LHCb (D*)	(-0.10±0.09)%	PRL116,191601	(-0.14±0.07)%

Measurements of ΔA_{CP}

$$\Delta A_{CP}^{\text{dir}} = A_{CP}(D^0 \to K^- K^+) - A_{CP}(D^0 \to \pi^- \pi^+)$$

Measurements	ΔA_{CP}	Publication	World Average
2011LHCb (D*)	(-0.82±0.24)%		
2012 CDF	(-0.62±0.23)%		
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2013LHCb (D*)	(-0.34±0.18)%	LHCb-CONF-2013-03	HFAG2013: (-0.33±0.12)%
2013LHCb (B)	(+0.49±0.33)%	PLB723(2013)33	
2014LHCb (B)	(+0.14±0.18)%	JHEP07(2014)041	HFAG2014: (-0.25±0.10)%
2016LHCb (D*)	(-0.10±0.09)%	PRL116,191601	$\boxed{\textbf{(-0.14±0.07)\%}}$

Prediction in 2012
[Li,Lu,**FSY**,PRD86,036012]
ΔA_{CP}=(-0.06 ~ -0.19)%