A new parametrization for the scalar isoscalar form factor

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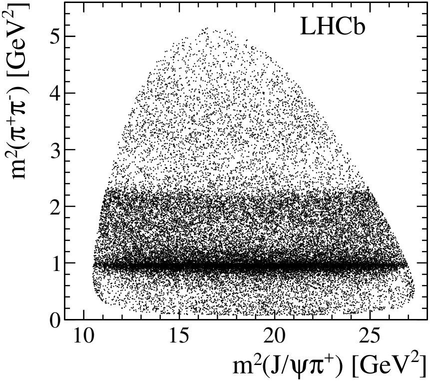
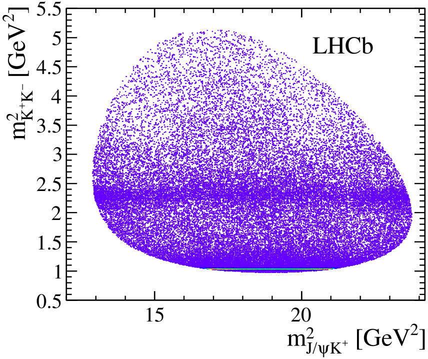
The decay $\bar{B}_s^0 \to J/\psi\, \pi^+\pi^-(K^+K^-)$

$\bar{B}_s^0 \to J/\psi\, \pi^+\pi^-$ by LHCb [2014]

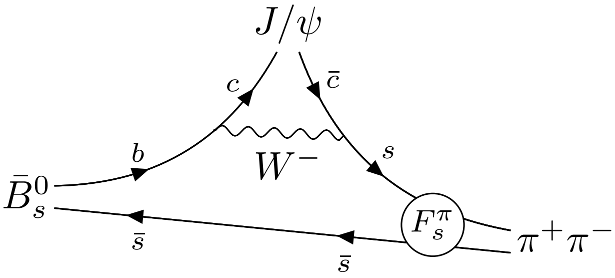


$\bar{B}_s^0 \to J/\psi\, K^+K^-$ by LHCb [2017]



- S-wave resonances: $f_0(500)$, $f_0(980)$, $f_0(1500)$ and $f_0(1790)$ (LHCb [2014])
- P-wave resonances: $\phi(1020)$ and $\phi(1680)$ (LHCb [2017])
- D-wave resonances: $f_2(1270)$, $f_2'(1525)$, $f_2(1750)$ and $f_2(1950)$ (LHCb [2017])

Previous study $\bar{B}_s^0 \to J/\psi\,\pi^+\pi^-$



- Dispersive study Daub et al. [2016] using $\pi\pi$ and $K\bar{K}$ coupled channels
- S-wave of $\pi\pi$ system modeled by the scalar isoscalar form factor F_s^π
- D-wave of $\pi\pi$ system modeled by Breit-Wigner functions
- Works well up to $m_{\pi\pi} = 1.05\,\mathrm{GeV} \Rightarrow$ covers $f_0(500)$ and $f_0(980)$

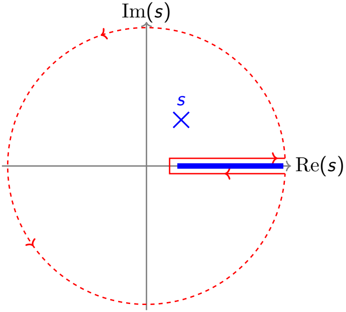
Extension to higher energies:

- Include additional resonances $f_0(1500)$ and $f_0(1790)$
- Include further inelasticities such as 4π modeled as $\rho\rho/\sigma\sigma$

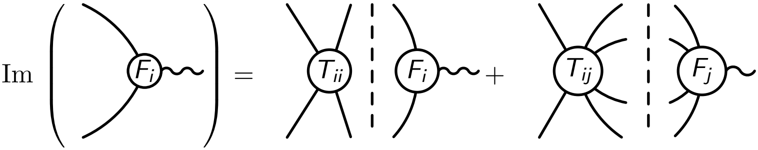
Introduction into dispersion theory

- Analyticity and Cauchy's theorem

$$F_i(s) = \frac{1}{\pi} \int_{s_{th}}^{\infty} \mathrm{d}z \, \frac{\mathrm{Im}\, F_i(z)}{z - s - i\epsilon}$$



- $\mathrm{Im}\, F_i(z)$ obtainable by the unitary equation



$$\mathrm{Im}\, F_i(s) = T_{ij}^*(s)\sigma_j(s)F_j(s) = T_{ij}^*(s)\frac{2p_j}{\sqrt{s}}F_j(s)$$

Problems of dispersion theory

- Obtain the coupled channel integral equation

$$F_i(s) = \frac{1}{\pi} \int_{s_{th}}^{\infty} \frac{\mathrm{d}z}{z - s - i\epsilon} \, T_{ij}^*(z)\sigma_j(z)F_j(z)$$

- Need $T_{ij}(s)$ for all relevant channels to arbitrary large energies (not possible/realizable)
- Know $T_{ij}(s)$ up to $s_{\max} \Rightarrow F_i(s)$ is well described for $s < s_{\max}$
  scattering phases known up to $1\,\mathrm{GeV} \Rightarrow$ analysis works up to $1\,\mathrm{GeV}$
- Need a parametrization that
  1. fulfills analyticity and unitarity
  2. is consistent with high precision analysis at low energies (Roy-Steiner equations) Garcia-Martin et al. [2011], Büttiker et al. [2004]
  3. includes further inelastic channels
  4. describes the high energy data
- Such a parametrization was provided by Hanhart [2012]

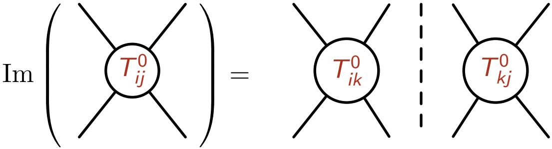
A new parametrization for the form factor

- Full parametrization for the scattering matrix T and the form factor F

$$T(s) = T^0(s) + \Gamma(s)\,\left[\mathbb{1} - V^R(s)\,\Sigma(s)\right]^{-1}\,V^R(s)\,\Gamma^t(s)$$

$$F(s) = \Gamma(s)\left[\mathbb{1} - V^R(s)\,\Sigma(s)\right]^{-1}\,M(s)$$

- Unitary scattering amplitude $T^0(s)$



- Defined by scattering phases and inelasticies

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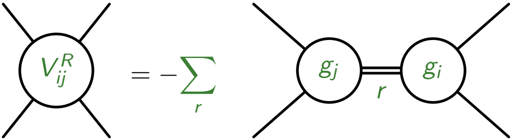
$$F(s) = \Gamma(s)\left[\mathbb{1} - V^R(s)\,\Sigma(s)\right]^{-1} M(s)$$

- Resonance exchange potential $V^R(s)$



$$V_{ij}^R(s) = -\sum_r g_i^r \frac{s}{m_r^2(s - m_r^2)} g_j^r$$

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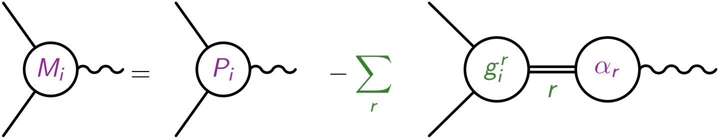
$$F(s) = \Gamma(s)\,\left[\mathbb{1} - V^R(s)\,\Sigma(s)\right]^{-1}\,M(s)$$

- Source term $M(s)$



$$M_i(s) \quad = \quad c_i + d_i\,s + \ldots - \sum_r \qquad g_i^r\,\frac{s}{s - m_r^2}\,\alpha_r$$

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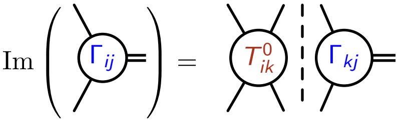
$$T(s) = T^0(s) + \Gamma(s)\,\left[\mathbb{1} - V^R(s)\,\Sigma(s)\right]^{-1}\,V^R(s)\,\Gamma^t(s)$$

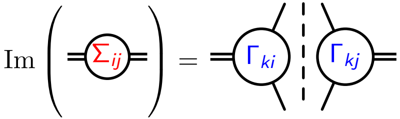
$$F(s) = \Gamma(s)\,\left[\mathbb{1} - V^R(s)\,\Sigma(s)\right]^{-1}\,M(s)$$

- Vertex factor $\Gamma(s)$



$$\Rightarrow \Gamma_{ij}(s) = \frac{1}{\pi}\int_{s_{th}}^{\infty} \mathrm{d}z\, \frac{T^0_{ik}(z)^*\,\sigma_k(z)\,\Gamma_{kj}(z)}{z - s - i\epsilon}$$

- Self energy $\Sigma(s)$



$$\Rightarrow \Sigma_{ij}(s) = \frac{s}{\pi}\int_{s_{th}}^{\infty} \frac{\mathrm{d}z}{z}\, \frac{\Gamma^*_{ki}(z)\sigma_k(z)\Gamma_{kj}(z)}{z - s - i\epsilon}$$

Angular moments averages $\langle Y_L^0 \rangle$

- Observables are angular moments averages

$$\left\langle Y_L^0 \right\rangle (\sqrt{s}) = \int_{-1}^{1} \mathrm{d}\cos\Theta \, \frac{\mathrm{d}^2\Gamma}{\mathrm{d}\sqrt{s}\,\mathrm{d}\cos\Theta} \, Y_L^0(\cos\Theta)$$

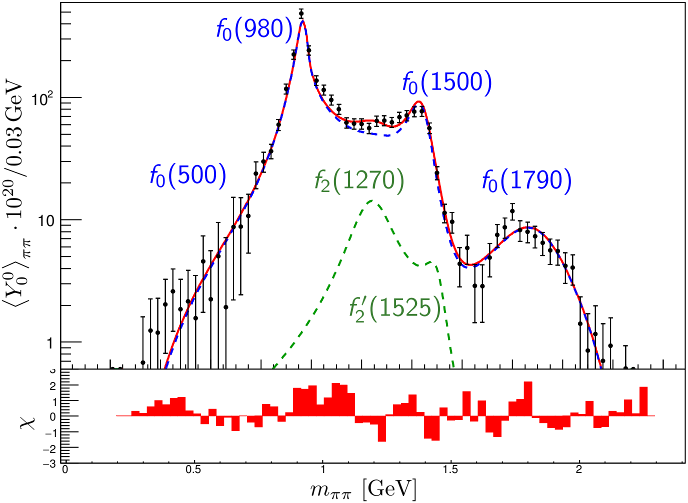
- Expressing them by a normalization $\mathcal{N}$, the scalar form factor F_S and the P-wave amplitude F_P^τ as well as the D-wave amplitude F_D^τ

$$\sqrt{4\pi}\left\langle Y_0^0 \right\rangle = X\sigma_\pi\sqrt{s}\left\{ X^2\mathcal{N}^2|F_S|^2 + \sum_{\tau=0,\|,\perp} |F_P^\tau|^2 + \sum_{\tau=0,\|,\perp} |F_D^\tau|^2 \right\}$$

$$\sqrt{4\pi}\left\langle Y_2^0 \right\rangle = X\sigma_\pi\sqrt{s}\left\{ 2X\mathcal{N}\mathrm{Re}\left(F_S\left(F_D^0\right)^* \right) + \sum_{\tau=0,\perp,\|} \left(c_\tau\,|F_P^\tau|^2 + d_\tau\,|F_D^\tau|^2 \right) \right\}$$

- P- and D-wave amplitudes are modeled for simplicity with Breit-Wigner functions.

Fit $\langle Y_0^0 \rangle$ for the decay $\bar{B}_s^0 \to J/\psi\, \pi^+\pi^-$



Fit ($\rho\rho$) with two additional resonances. *S*- and *D*-waves.

$$\frac{\chi^2}{\mathrm{ndf}} = \frac{413.22}{384 - 30} \approx 1.17$$

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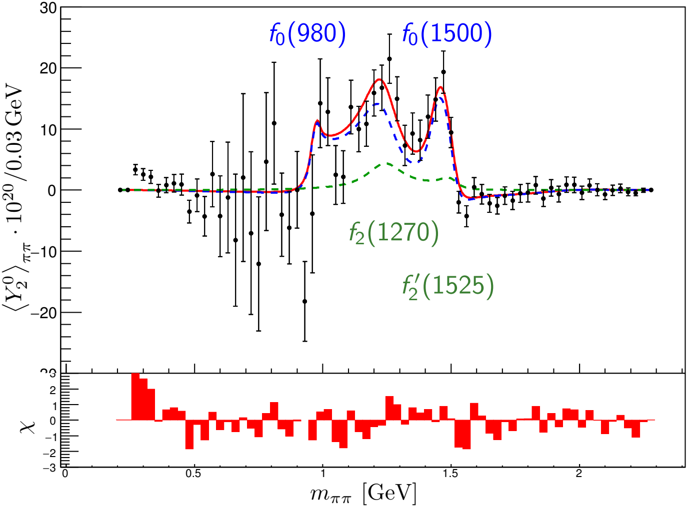

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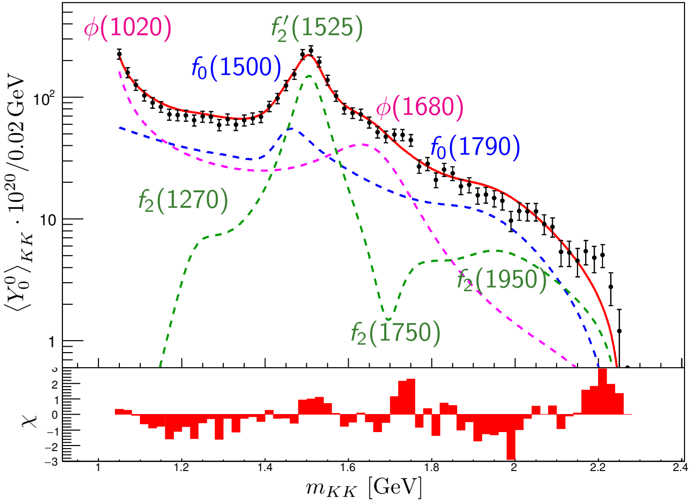

Fit ($\rho\rho$) with two additional resonances. S-, P- and D-waves.

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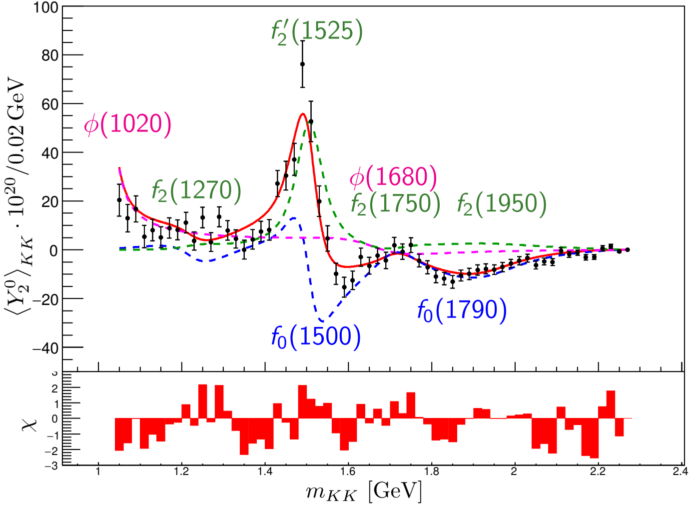
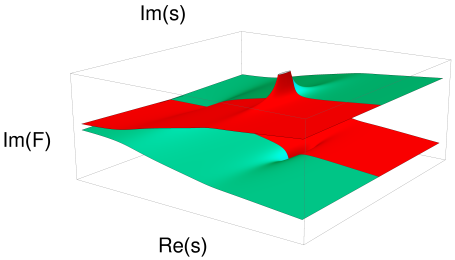


Fit ($\rho\rho$) with two additional resonances. SD-, P- and D-waves.

$$\frac{\chi^2}{\text{ndf}} = \frac{413.22}{384 - 30} \approx 1.17$$

Padé approximants

- Consider the form factor $F(s)$ as a meromorphic function on several Riemann-Sheets
- Sheets are smoothly connected across the cut
- Resonance poles on unphysical sheets

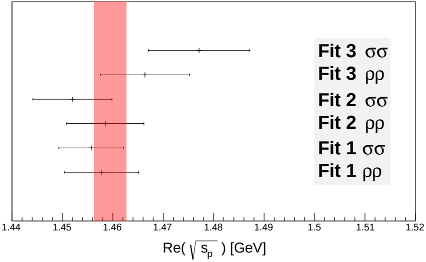


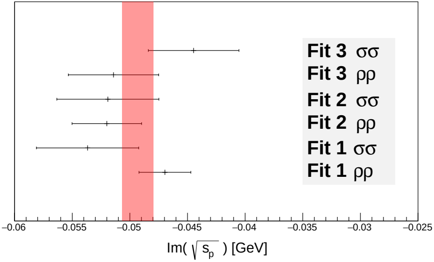
Padé expansion

$$F(s) \approx P_M^N(s, s_0) = \frac{\sum_{n=0}^{N} a_n \, (s - s_0)^n}{\sum_{m=0}^{M} b_m \, (s - s_0)^m}$$

Expansion breaks down when it hits a threshold.

Pole position: $f_0(1500)$

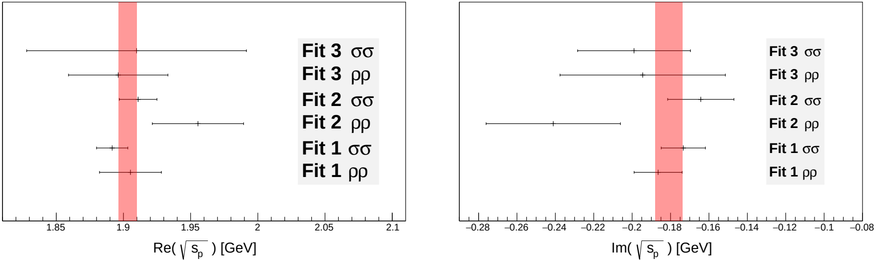




$$\sqrt{s_{f_0(1500)}}_{\mathrm{Pade}} = [(1.4595 \pm 0.0032) - i\ (0.0493 \pm 0.0014)]\ \mathrm{GeV}$$
$$\sqrt{s_{f_0(1500)}}_{\mathrm{LHCb}} = [(1.4659 \pm 0.0031) - i\ (0.0575 \pm 0.0035)]\ \mathrm{GeV}$$

$\frac{\chi^2}{\mathrm{ndf}}$	$\rho\rho$	$\sigma\sigma$
Fit 1=2 resonances, constant polynomial in $M(s)$	1.17	1.23
Fit 2=2 resonances, linear polynomial in $M(s)$	1.12	1.19
Fit 3=3 resonances, constant polynomial in $M(s)$	0.97	1.06

Pole position: $f_0(1790)$



$$\sqrt{s_{f_0(1790)}}_{\mathrm{Pade}} = [(1.903 \pm 0.008) - i\ (0.1809 \pm 0.007\ )]\ \mathrm{GeV}$$
$$\sqrt{s_{f_0(1790)}}_{\mathrm{LHCb}} = [(1.809 \pm 0.022) - i\ (0.1315 \pm 0.015\ )]\ \mathrm{GeV}$$

$\frac{\chi^2}{\mathrm{ndf}}$	$\rho\rho$	$\sigma\sigma$
Fit 1=2 resonances, constant polynomial in $M(s)$	1.17	1.23
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Fit 3=3 resonances, constant polynomial in $M(s)$	0.97	1.06

Summary and outlook

Summary

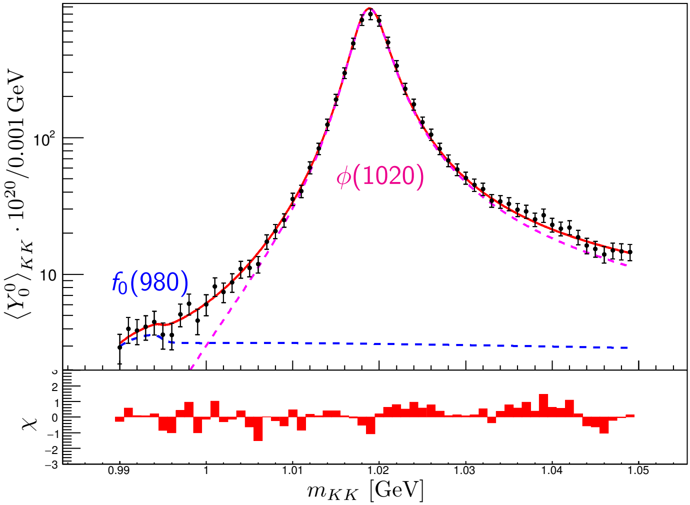
- New parametrization with improved analyticity and unitarity properties
- Applied successfully to $\bar{B}_s^0 \to J/\psi\, \pi^+\pi^-(K^+K^-)$
- Extraction of resonance poles on next unphysical sheets via Padé approximants

Outlook

- Include scattering data to reduce ambiguity between source term and resonance potential
- Improve the D-wave model
- Include additional inelastic channels to increase precision
- ...

Thank you for your attention!

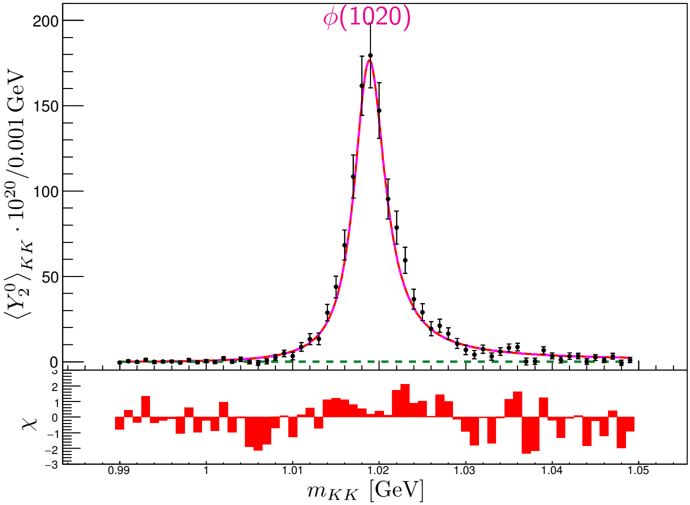
Fit $\langle Y_0^0 \rangle$ for the decay $\bar{B}_s^0 \to J/\psi\, K^+ K^-$



Fit ($\rho\rho$) with two additional resonances. S-, P- and D-waves.

$$\frac{\chi^2}{\mathrm{ndf}} = \frac{413.22}{384 - 30 - 1} \approx 1.17$$

Fit $\langle Y_2^0 \rangle$ for the decay $\bar{B}_s^0 \to J/\psi K^+ K^-$



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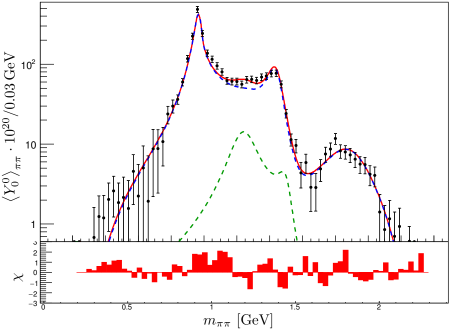
Comparison $\langle Y_0^0 \rangle$ for the decay $\bar{B}_s^0 \to J/\psi\, \pi^+\pi^-$



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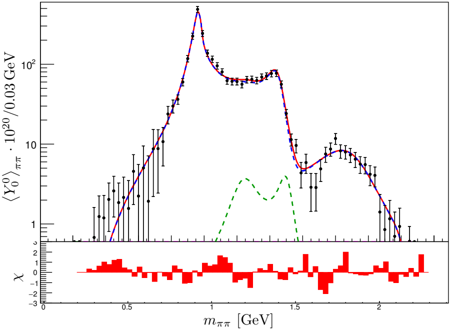

Fit $(\rho\rho)$ three additional resonances

$$\frac{\chi^2}{\mathrm{ndf}} = \frac{338.13}{384 - 35 - 1} \approx 0.97$$

Fit, S-, P- and D-waves.

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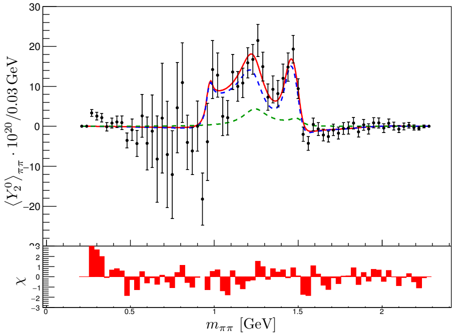

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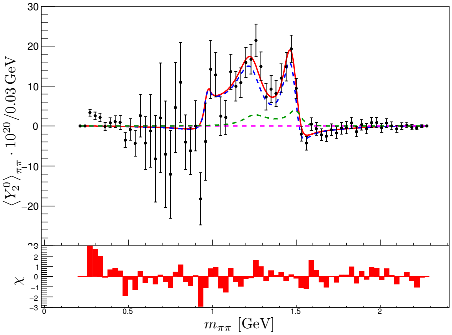

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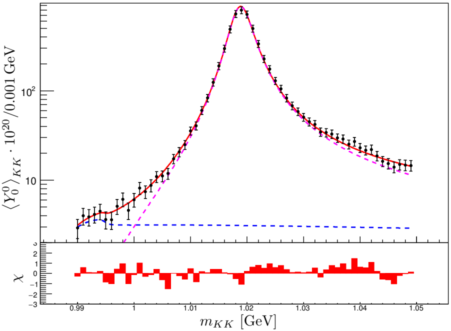

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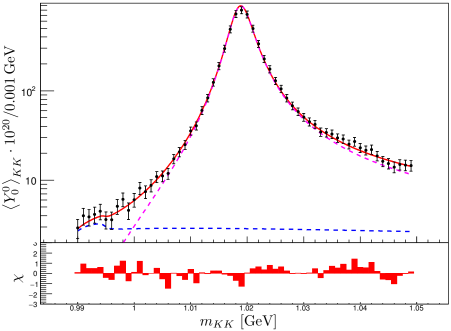

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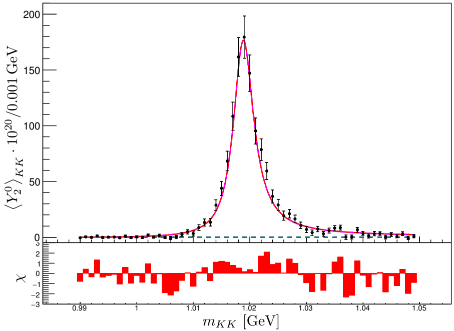

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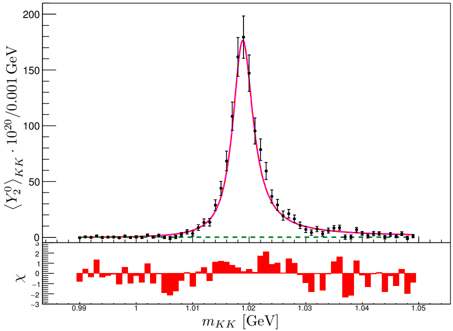

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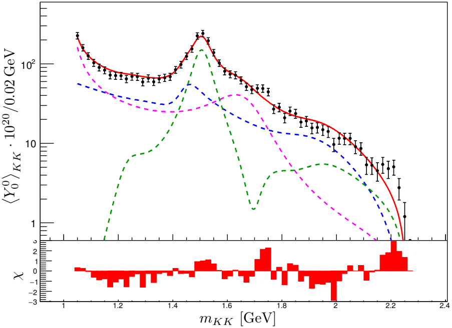

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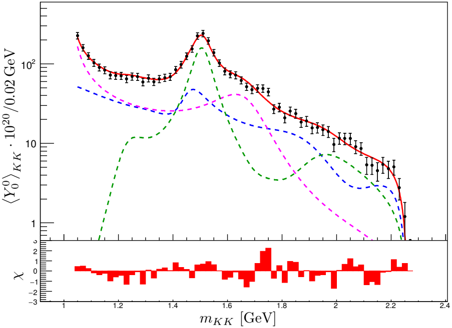

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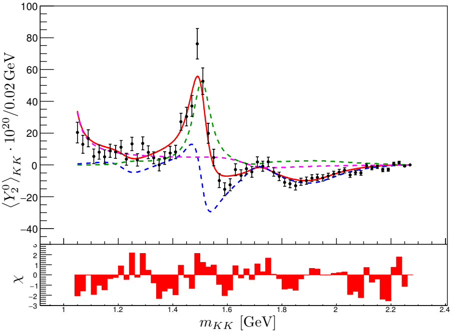

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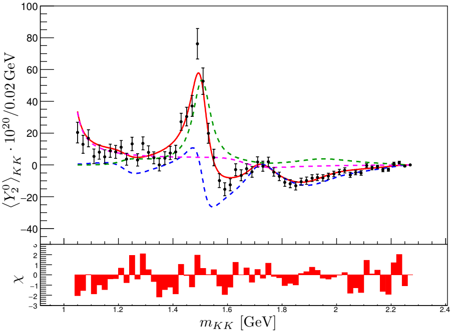

Fit ($\rho\rho$) three additional resonances

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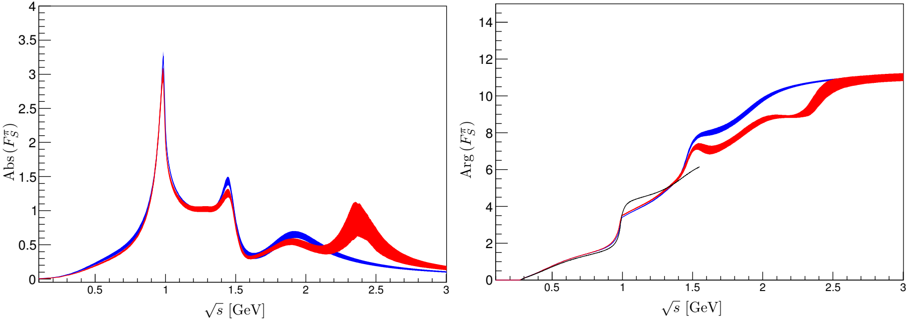
Fit, SD-, P- and D-waves.

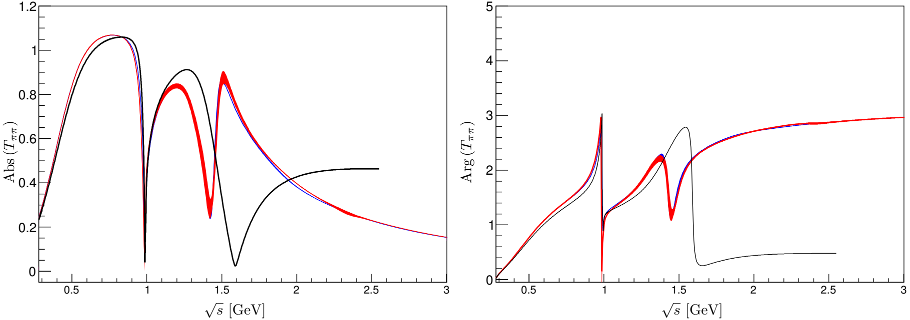
Comparison of the scalar pion form factor



Absolute value (left) and argument (right) of the scalar pion form factor F_S^π. Shown are the fits with two resonances, three resonances and the $\pi\pi$ scattering phaseshift δ_π.

- Absolute value is strongly fixed by the angular moments $\left\langle Y_0^0 \right\rangle$.
- A certain ambiguity in the phase, since we also fit the S-D-wave interference.
- Ambiguity of the resonance potential and source term by only fitting the form factor.
- Low energies are slightly shifted by inclusion of the S-channel resonances.
- Position of the third resonance not determined through the data.

Comparison of $\pi\pi$ scattering matrix element



Absolute value (left) and argument (right) of the pion pion scattering amplitude $T_{\pi\pi}$. Shown are the fits with two resonances, three resonances and the input.

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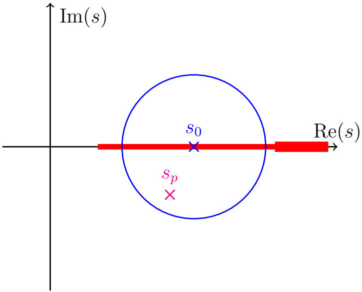
Method: Padé pole extraction

- Vary the expansion point s_0.
- Choose fitting range, such that the expected pole is included in a circle. Avoid threshold points.
- Fit the Padé approximant for varying N

$$P_1^N(s, s_0) = \frac{\sum_{n=1}^{N} a_n (s - s_0)^n}{1 + b_0 (s - s_0)}$$

and extract the Padé pole.



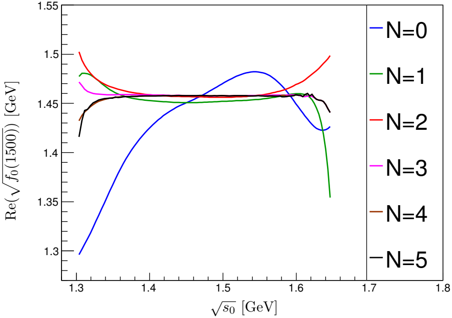
Extraction of the best pole

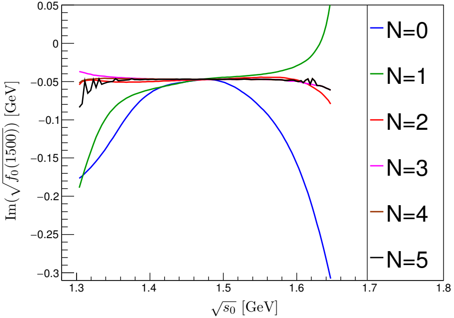
Search for s_0 such that

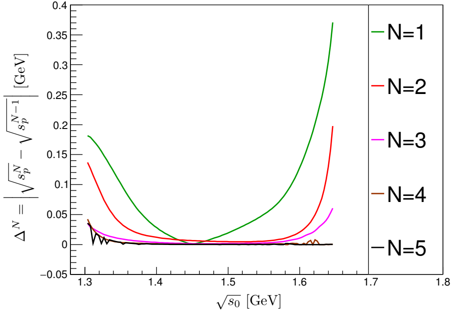
1) the extracted pole position stabilizes
2) the systematics get minimized

$$\Delta^N = \left| \sqrt{s_p^N} - \sqrt{s_p^{N-1}} \right|$$

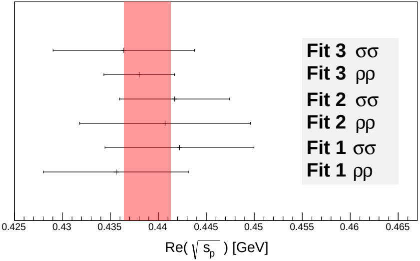
Method: Padé pole extraction

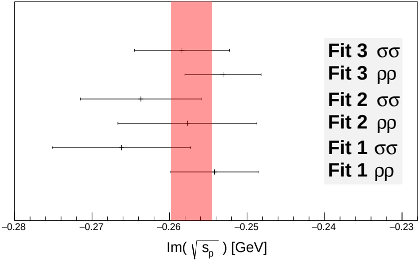






Pole position: $f_0(500)$



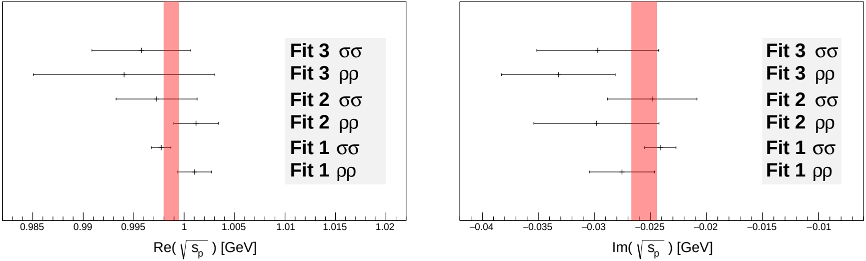


$$\sqrt{s_{f_0(500)}}_{\mathrm{Pade}} = [(0.439 \pm 0.002) - i\ (0.257 \pm 0.003)]\ \mathrm{GeV}$$

$$\sqrt{s_{f_0(500)}}_{\mathrm{Dai/Pennington}} = [\ 0.441 \qquad - i\ \ 0.272 \qquad]\ \mathrm{GeV}$$

$\frac{\chi^2}{\mathrm{ndf}}$	$\rho\rho$	$\sigma\sigma$
Fit 1=2 resonances, constant polynomial in $M(s)$	1.17	1.23
Fit 2=2 resonances, linear polynomial in $M(s)$	1.12	1.19
Fit 3=3 resonances, constant polynomial in $M(s)$	0.97	1.06

Pole position: $f_0(980)$



$$\sqrt{s_{f_0(980)}}_{\mathrm{Pade}} = [(0.9967 \pm 0.0008) - i\ (0.0256 \pm 0.0011)]\ \mathrm{GeV}$$

$$\sqrt{s_{f_0(980)}}_{\mathrm{Dai/Pennington}} = [\ 0.998 \quad - i\ \ 0.021 \quad]\ \mathrm{GeV}$$

$\frac{\chi^2}{\mathrm{ndf}}$	$\rho\rho$	$\sigma\sigma$
Fit 1=2 resonances, constant polynomial in $M(s)$	1.17	1.23
Fit 2=2 resonances, linear polynomial in $M(s)$	1.12	1.19
Fit 3=3 resonances, constant polynomial in $M(s)$	0.97	1.06

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