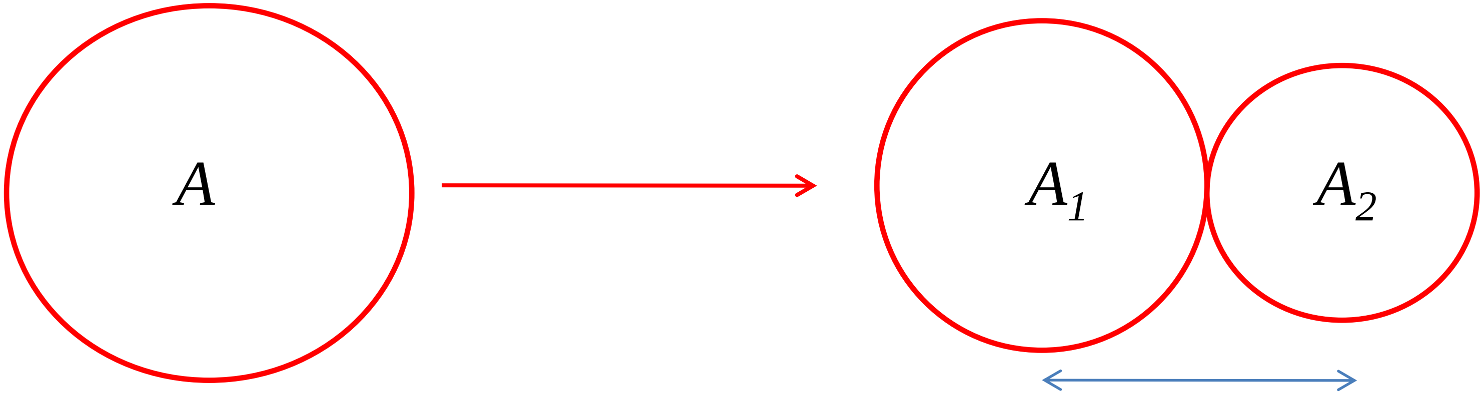
Study of clustering nuclei

Clusters in nuclei



$A = A_1 + A_2$ -- *nuclear mass*

R -- *relative distance*

$\xi = 2A_2/A$ – *mass asymmetry*

- Each cluster configuration is described by the wave function:

$$\psi_{ijk}\left(\vec{r}_1,\ldots,\vec{r}_{A_0},\vec{R}\right) = \hat{A}\left[\phi_i\left(A_1\right)\phi_j\left(A_2\right)\chi_k\left(\vec{R}\right)\right]$$

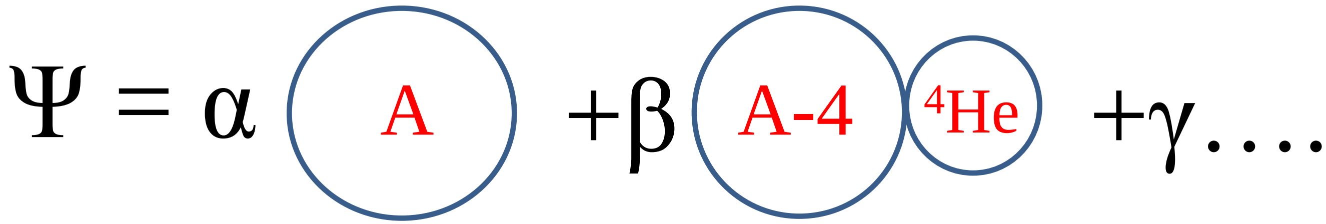
-Total wave function of a nuclear state can generally be presented as a superposition of various cluster configurations:

$$\Psi\left(\vec{r}_1,\ldots,\vec{r}_{A_0}\right) = \sum_h \sum_{ijk} a^h_{ijk}\psi^h_{ijk}\left(\vec{r}_1,\ldots,\vec{r}_{A_0},\vec{R}_{touch}\right)$$

Cluster effects in nuclei

There are many indications of importance of cluster degrees of freedom in nuclei in different mass regions, different deformations and excitation energies.

- Reflection-Asymmetric Deformations in actinides and rare-earth nuclei

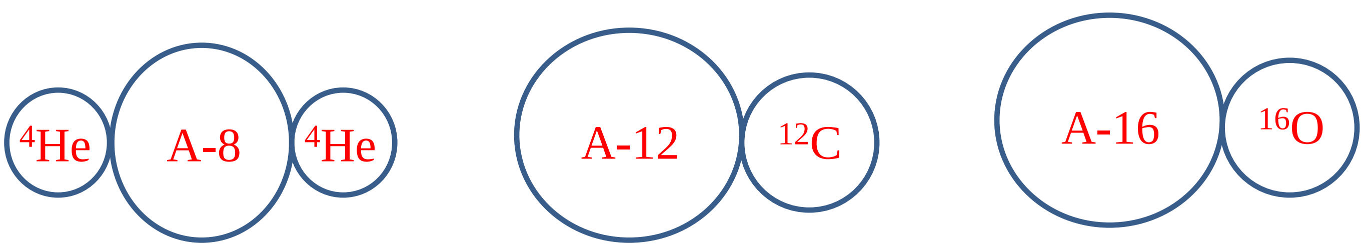


The potential energy of an α-cluster systems are close to or even lower than the binding energy of the rare-earth nuclei and actinides.

Thus, an α-cluster systems play an important role in the structure of these nuclei.

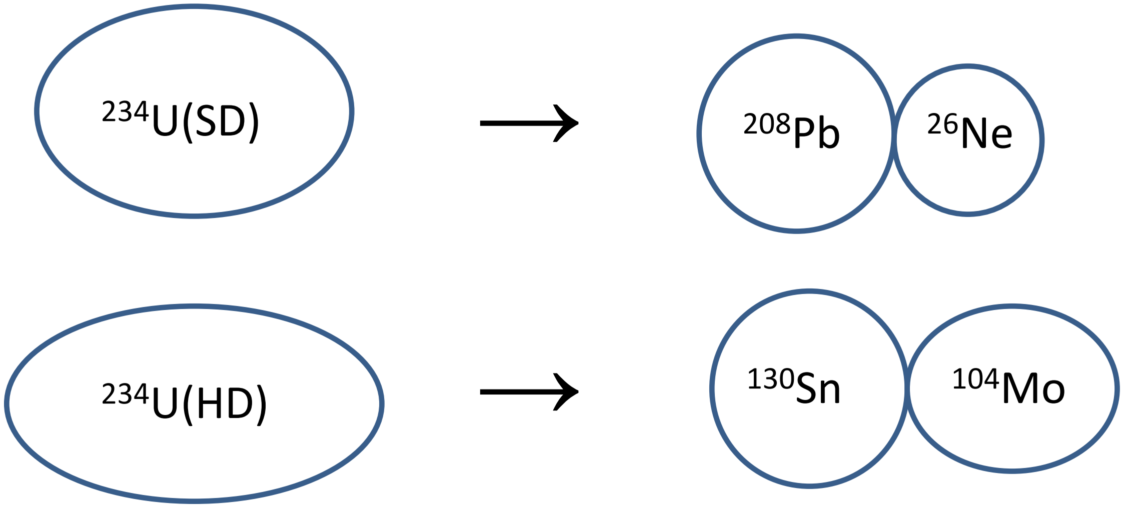
- Other cluster states

With excitation energy increase, due to the additional binding of certain clusters, other cluster systems can become important.



Cluster effects in nuclei

- Extremely elongated nuclear states (Super- and Hyperdeformation)



- α-decay, cluster radioactivity, spontaneous fission

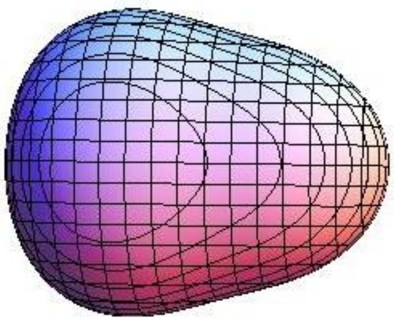
Reflection Asymmetric Deformation in Heavy Nuclei

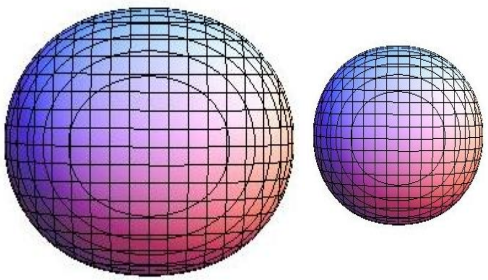
- Low negative parity rotational bands
- Strong dipole (*E1)* and octupole (*E3*) transitions connect negative parity states with members of the ground state band.

Conclusion:

Some nuclei (actinides and rare-earth nuclei) might have **reflection asymmetric shapes**

Octupole deformation **or** **Clustering**

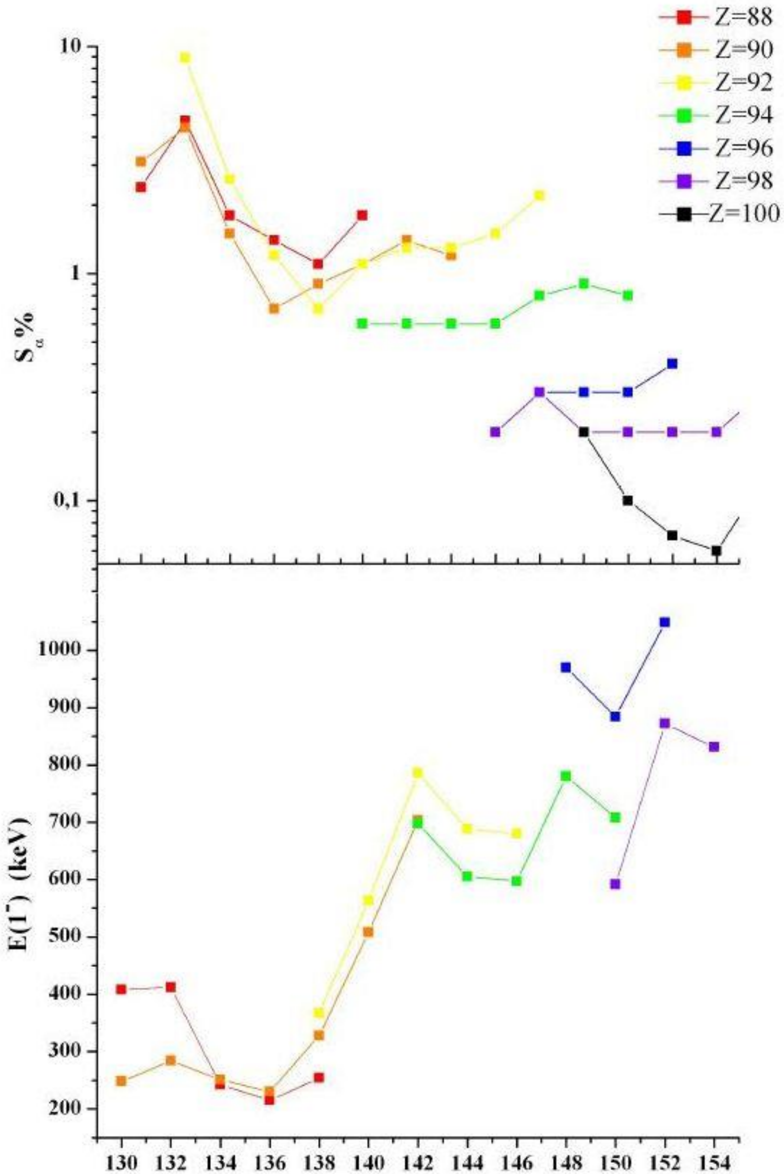




Estimates of dipole moments and $B(\mathrm{E1})$ values for clustering and octupole deformations

Nucleus	D (e fm)		$B(\mathrm{E1}; 0^+ \to 1^-)$ (e^2 fm^2)	
	α-clustering	octupole[a)]	α-clustering[b)]	octupole[a)]
$^{18}_{8}\mathrm{O}_{10}$	+0.90	+0 002	$5\ 8 \times 10^{-2}$	4.5×10^{-6}
$^{150}_{60}\mathrm{Nd}_{90}$	+3 29	+0 154	$7\ 7 \times 10^{-1}$	1.7×10^{-2}
$^{220}_{88}\mathrm{Ra}_{132}$	+3 64	+0 332	$9\ 5 \times 10^{-1}$	7.9×10^{-2}

Role of α–cluster



The value of α-particle preformation factor obtained from the experiment as:

$$S_\alpha^{exp} = T^{\alpha}_{1/2}/T^{exp}_{1/2}$$

$T^{\alpha}_{1/2}$ *– half-life of α-particle dinuclear system.*

Energies of $E(1^-)$– states as a function of neutron number.

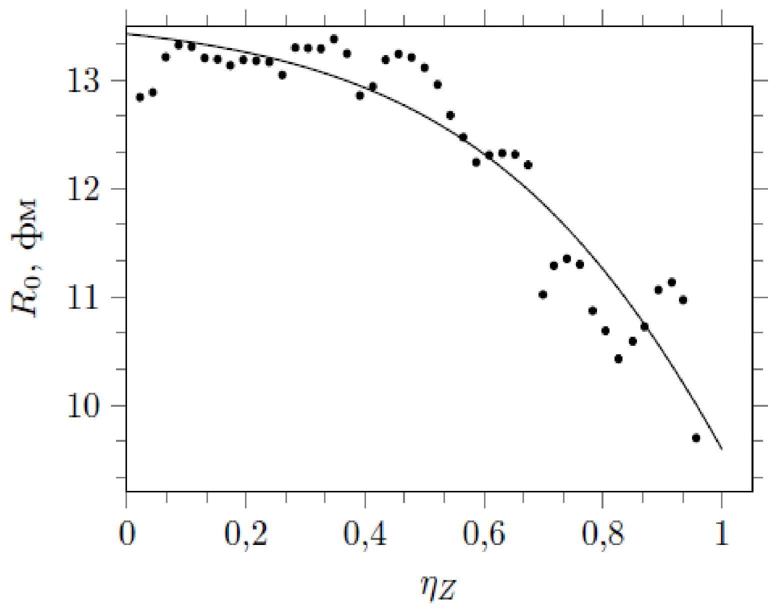
Degrees of Freedom of Dinuclear System

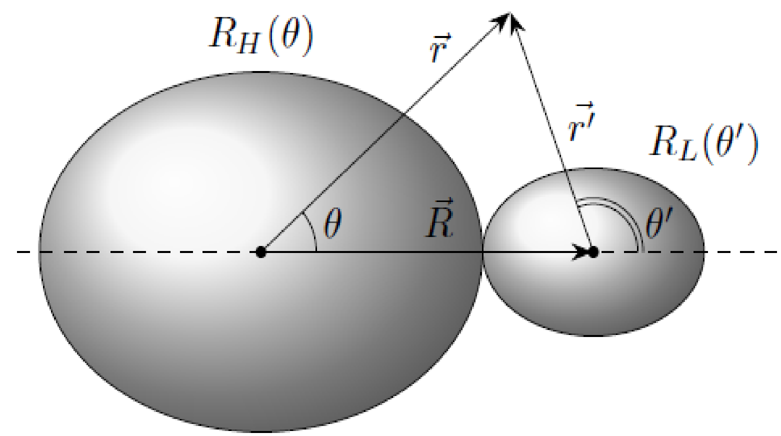
The dinuclear system (A,Z) consists of a configuration of two touching nuclei (clusters) (A_1,Z_1) and (A_2,Z_2) with $A = A_1 + A_2$ and $Z = Z_1 + Z_2$, which keep their individuality.

DNS has totally 15 collective degrees of freedom which govern its dynamics.

- *Relative motion of the clusters* $\mathbf{R} = (R, \theta_R, \phi_R)$
- *Rotation of the clusters* $\Omega_1 = (\phi_1, \theta_1, \chi_1)$, $\Omega_2 = (\phi_2, \theta_2, \chi_2)$
- *Intrinsic excitations of the clusters* (β_1, γ_1), (β_2, γ_2)
- *Nucleon transfer between the clusters* ξ, ξ_Z

Mass asymmetry $\xi = \frac{2A_2}{A_1+A_2}$. Charge asymmetry $\xi_Z = \frac{2Z_2}{Z_1+Z_2}$





$$\eta_Z = \frac{Z_H - Z_L}{Z_H + Z_L},$$

$$Z_{H,L}$$

Hamiltonian of the model

The kinetic energy operator of the DNS then becomes

$$\hat{T} = -\frac{\hbar^2}{2B(\xi_0)}\frac{1}{\mu^{3/2}(\xi)}\frac{\partial}{\partial\xi}\mu^{3/2}(\xi)\frac{\partial}{\partial\xi} - \frac{\hbar^2}{2\mu(\xi)}\frac{1}{R^2}\frac{\partial}{\partial R}R^2\frac{\partial}{\partial R}$$

$$+\frac{\hbar^2}{2\mu(\xi)R^2}\hat{l}_0^2 + \frac{\hbar^2}{2}\sum_{n=1}^{2}\sum_{k=1}^{3}\frac{\hat{l}_{(n)k}^2}{I_k^{(n)}(\beta_n,\gamma_n)} \qquad \left(\equiv \hat{T}_{rot}\right)$$

$$-\frac{\hbar^2}{2}\sum_{n=1}^{2}\frac{1}{D_n(\xi_0)}\left(\frac{1}{\beta_n^4}\frac{\partial}{\partial\beta_n}\beta_n^4\frac{\partial}{\partial\beta_n} + \frac{1}{\beta_n^2}\frac{1}{\sin 3\gamma_n}\frac{\partial}{\partial\gamma_n}\sin 3\gamma_n\frac{\partial}{\partial\gamma_n}\right)$$

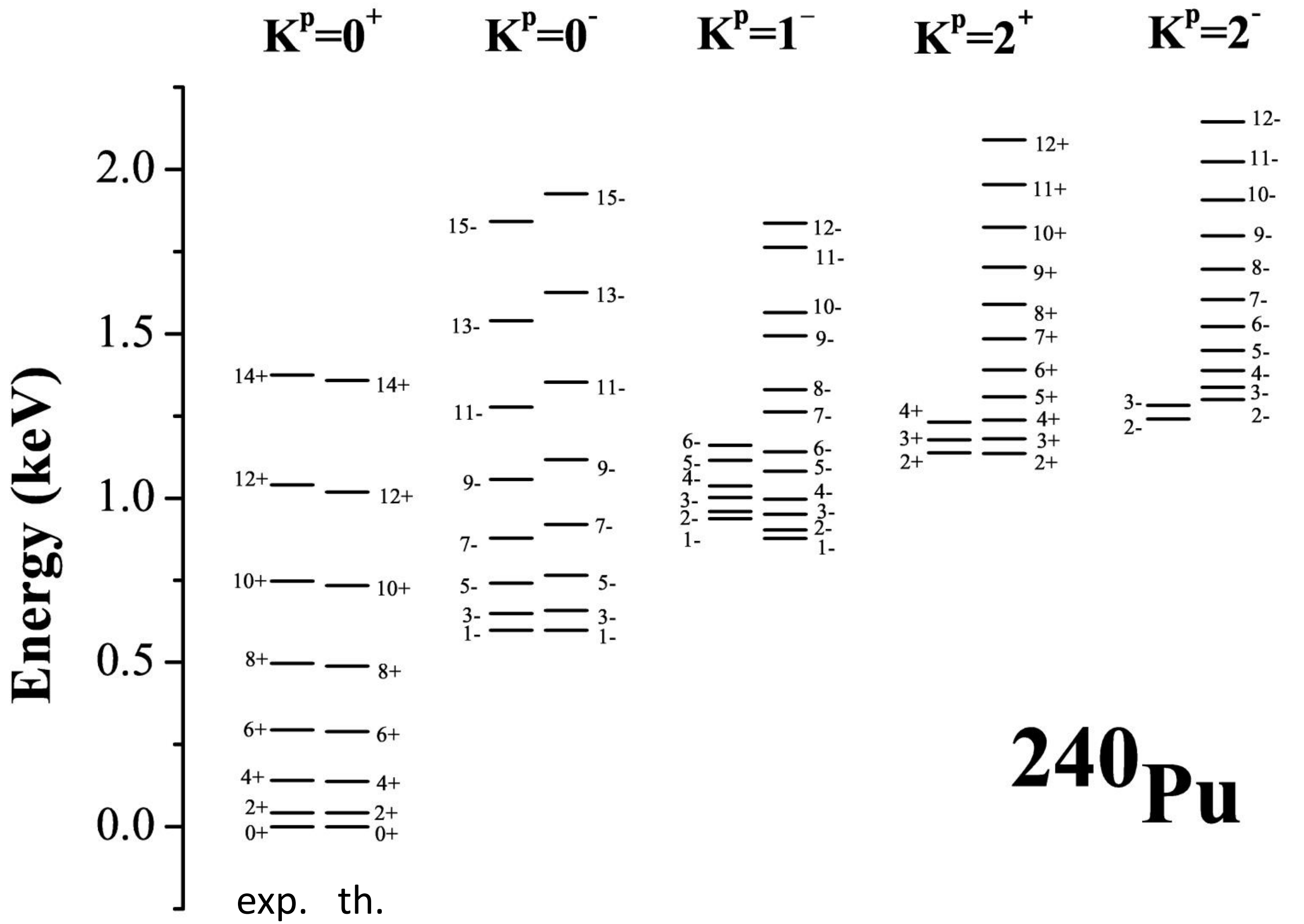
$$\left(\equiv \hat{T}_{\mathrm{int}\,r}\right)$$

The potential energy of the DNS is

$$V(\xi) = E_1(\xi,\beta_1,\gamma_1) + E_2(\xi,\beta_2,\gamma_2) + V_N(R,\xi,\beta_{\{1,2\}},\gamma_{\{1,2\}},\Omega_{\{1,2\}}) + V_C(R,\xi,\beta_{\{1,2\}},\gamma_{\{1,2\}},\Omega_{\{1,2\}})$$

Ground-State Well (^{240}Pu)

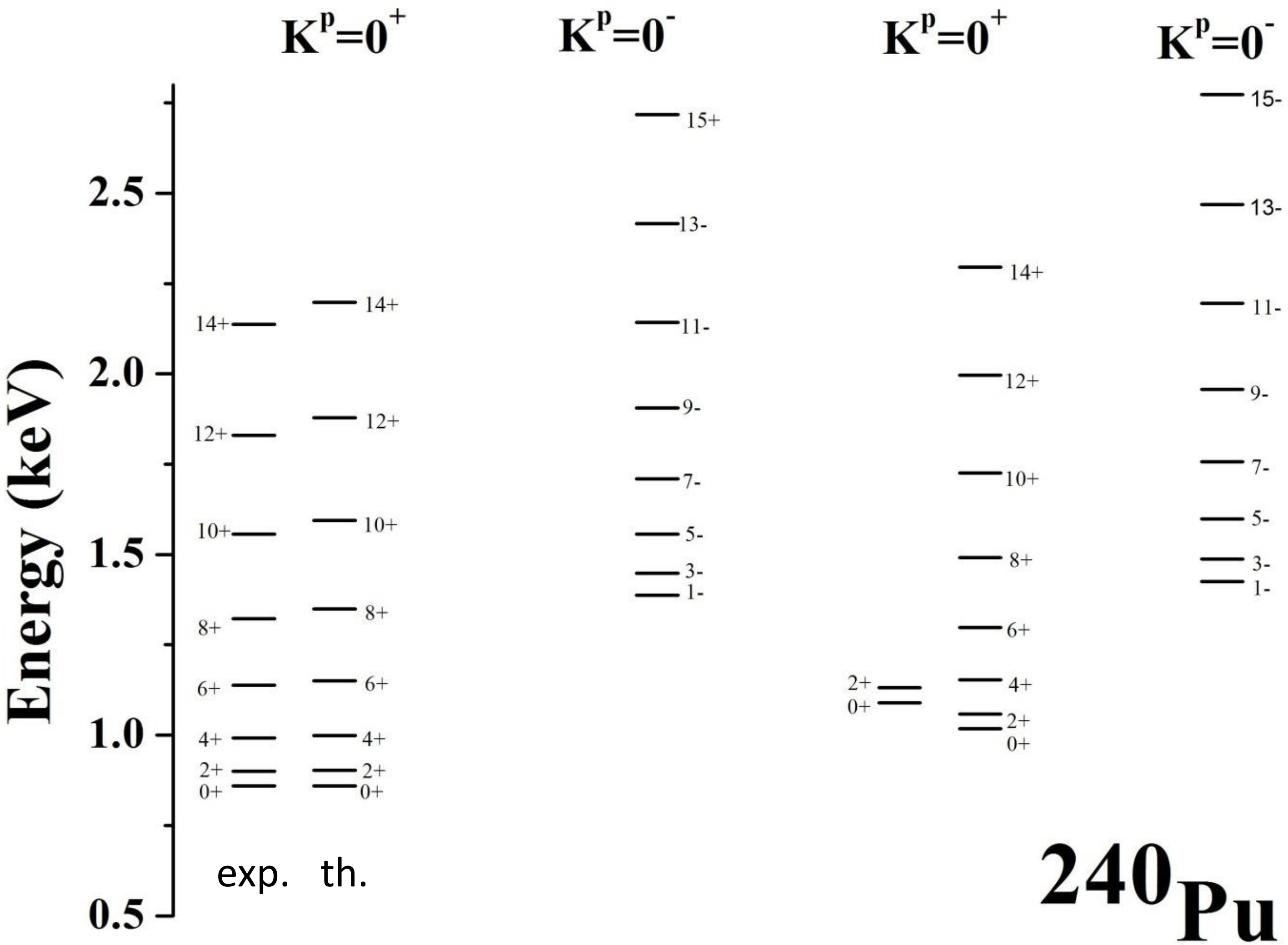
(Exp. data are taken from: http://www.nndc.bnl.gov/ensdf/)



PRC92, 034302 (2015)

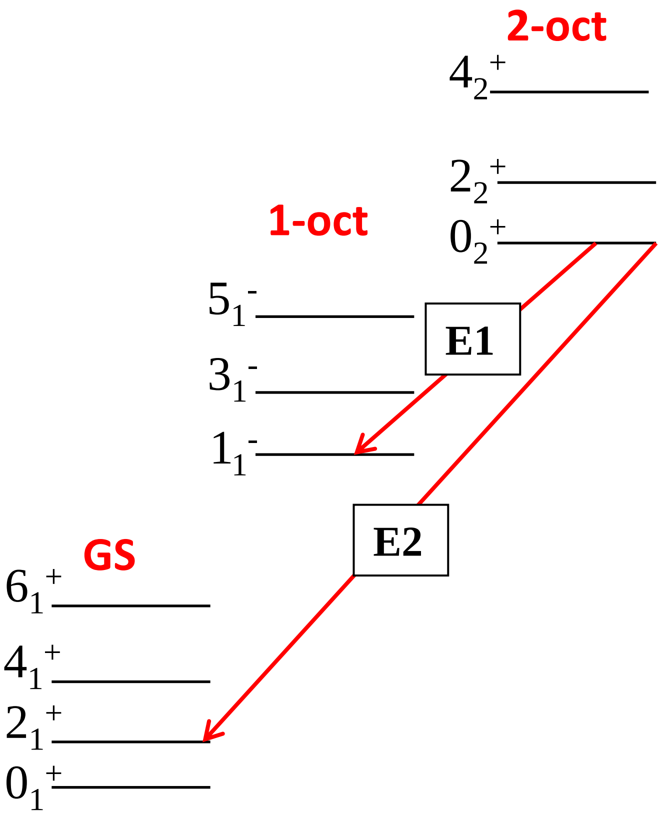
Ground-State Well (^{240}Pu) –continued

(Exp. data are taken from: http://www.nndc.bnl.gov/ensdf/)



PRC92, 034302 (2015)

Electromagnetic Transition in ^{240}Pu



Experimental B(E1)/B(E2) ratios (R_{exp}) are compared to the calculation of our model for the low-spin members of the $K\pi$ = 0+ 2 rotational band in ^{240}Pu.

I_i^π	$I_{f,E1}^\pi$	$I_{f,E2}^\pi$	R_{exp} (10^{-6} fm^{-2})	R_{DNS} (10^{-6} fm^{-2})
0_2^+	1_1^-	2_1^+	13.7(3)	19.17
2_2^+	1_1^-	0_1^+	99(15)	99.95
2_2^+	1_1^-	2_1^+	26(2)	39.15
2_2^+	1_1^-	4_1^+	5.9(3)	8.57
2_2^+	3_1^-	0_1^+	149(22)	165.60
2_2^+	3_1^-	2_1^+	39(2)	64.9
2_2^+	3_1^-	4_1^+	8.9(5)	14.2
4_2^+	3_1^-	6_1^+	4.4(11)	6.9
4_2^+	5_1^-	6_1^+	4.7(13)	10.59

(Exp. Data are from *M. Spieker et al., PRC88, 041303(R), (2013)*)

Proposed Experiments to study the role of the clustering in heavy nuclei

-- A natural tool to study the nature of nuclear reflection-asymmetry is provided by **inelastic photon scattering (γ,γ') experiments searching for all *E1* strength in the energy region E_x = (0– S_n) MeV** and also study possibly γ-decay behavior.

-- If enhanced E1 strength is generated due to the mass asymmetry **caused by α – or other cluster states, states of such a structure could strongly influence the outcome of (γ,α), (γ,cluster) reactions.**

Candidates: ^{150}Nd, ^{146}Sm, ^{150}Gd, $^{144-148}$Ba

Measurements:

E1-strength (**D_0**),

γ -decay behavior (**Γ_γ**)

(γ,α), (γ,cluster) (**Γ_α ,Γ_{cl}**)

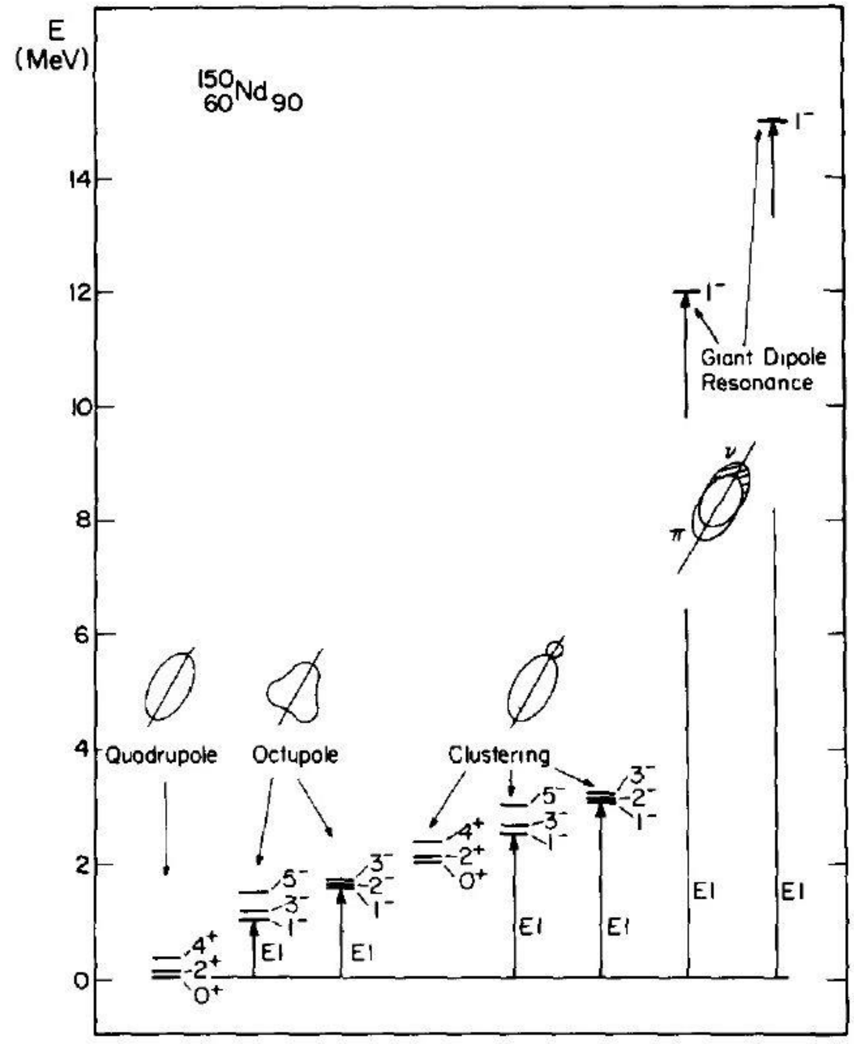
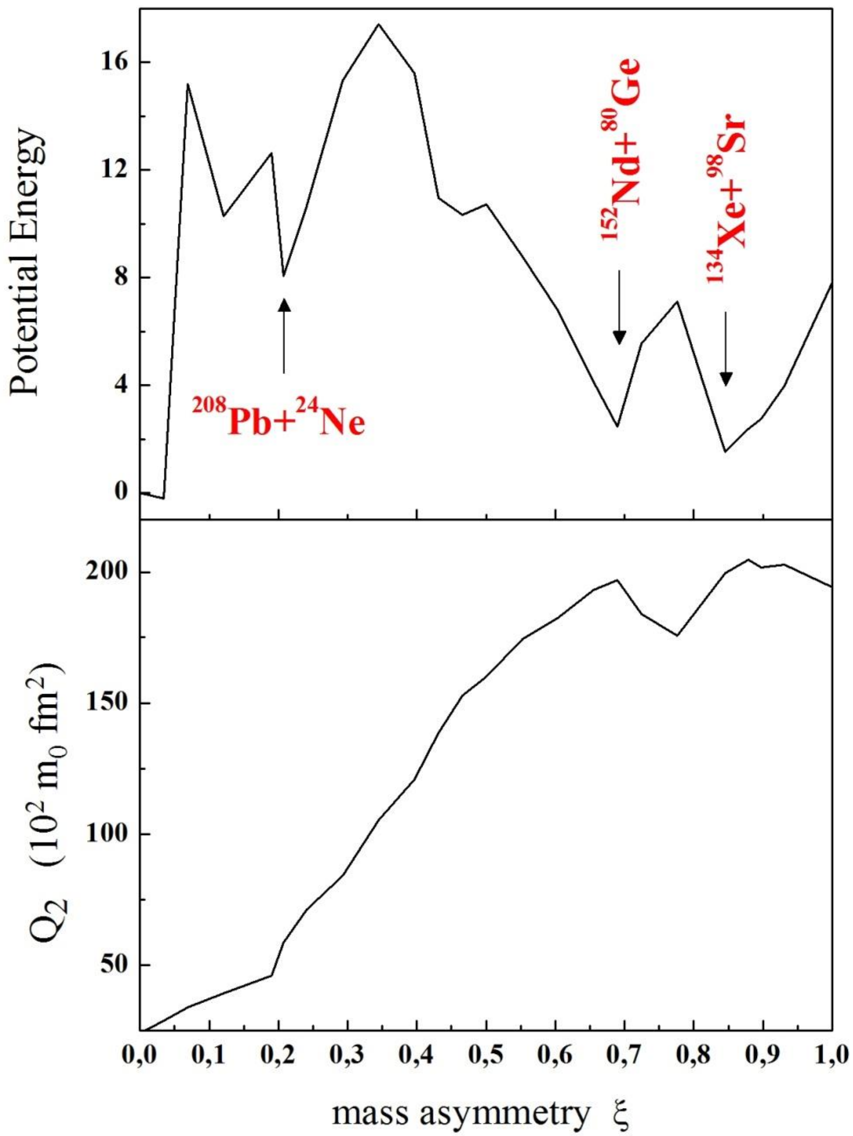


Fig 2 Schematic representation of the distribution of E1 strength expected in ^{150}Nd

Strongly Deformed States of Nuclei

Driving Potential for ^{232}U



The potential energy of the DNS

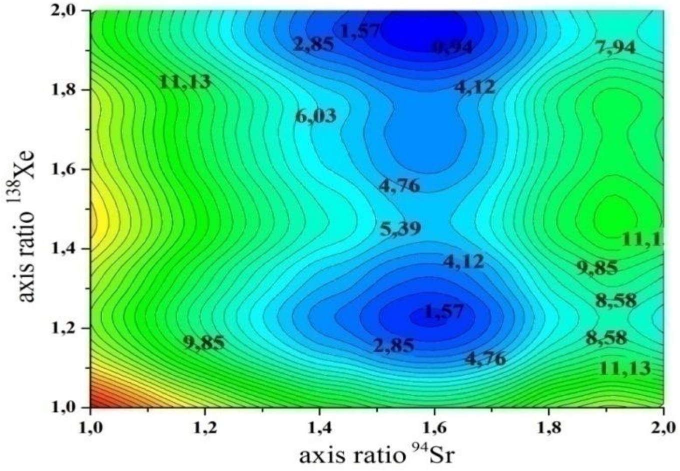
$$V(\xi) = E_1(\xi) + E_2(\xi) + V_N(R,\xi) + V_C(R,\xi)$$

Mass quadrupole moments of the DNS

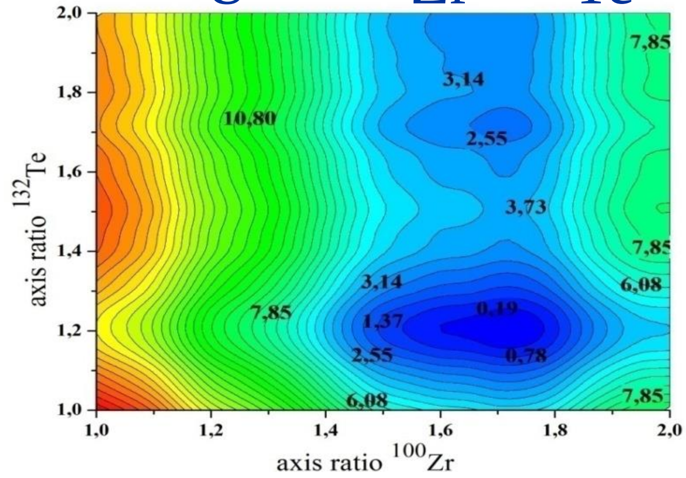
$$Q_2(\xi, R) = 2m_0 \frac{A_1 A_2}{A_1 + A_2} R^2 + Q_2(A_1) + Q_2(A_2)$$

Hyperdeformation and scission configurations

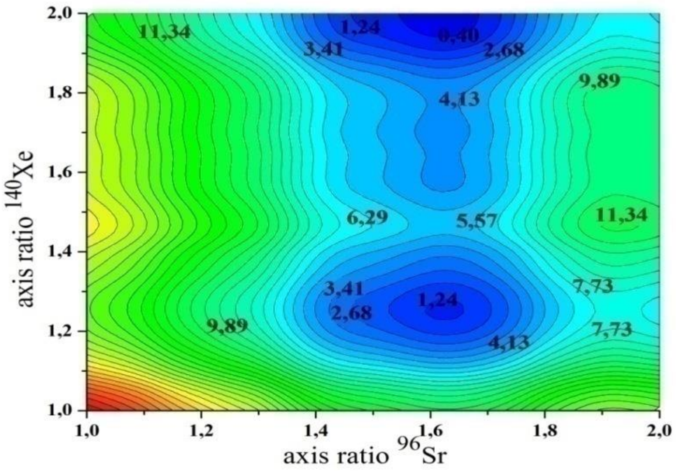
$^{232}U \rightarrow {}^{94}Sr + {}^{138}Xe$



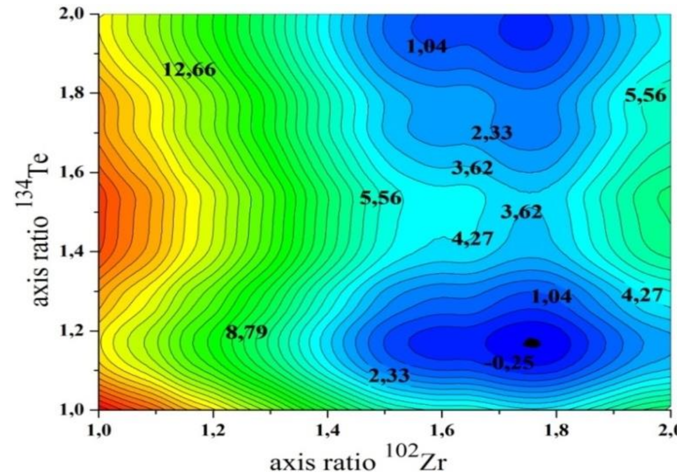
$^{232}U \rightarrow {}^{100}Zr + {}^{132}Te$



$^{236}U \rightarrow {}^{96}Sr + {}^{140}Xe$

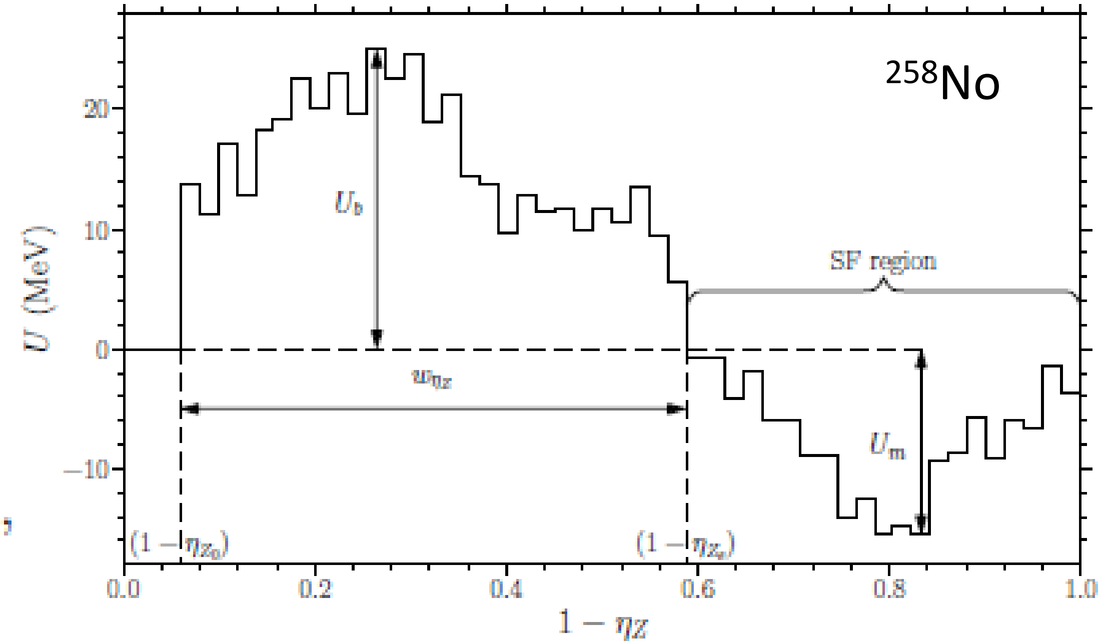


$^{236}U \rightarrow {}^{102}Zr + {}^{134}Te$



Spontaneous Fission

$$\hat{H}\Psi_n(\eta_Z) = E_n\Psi_n(\eta_Z),$$



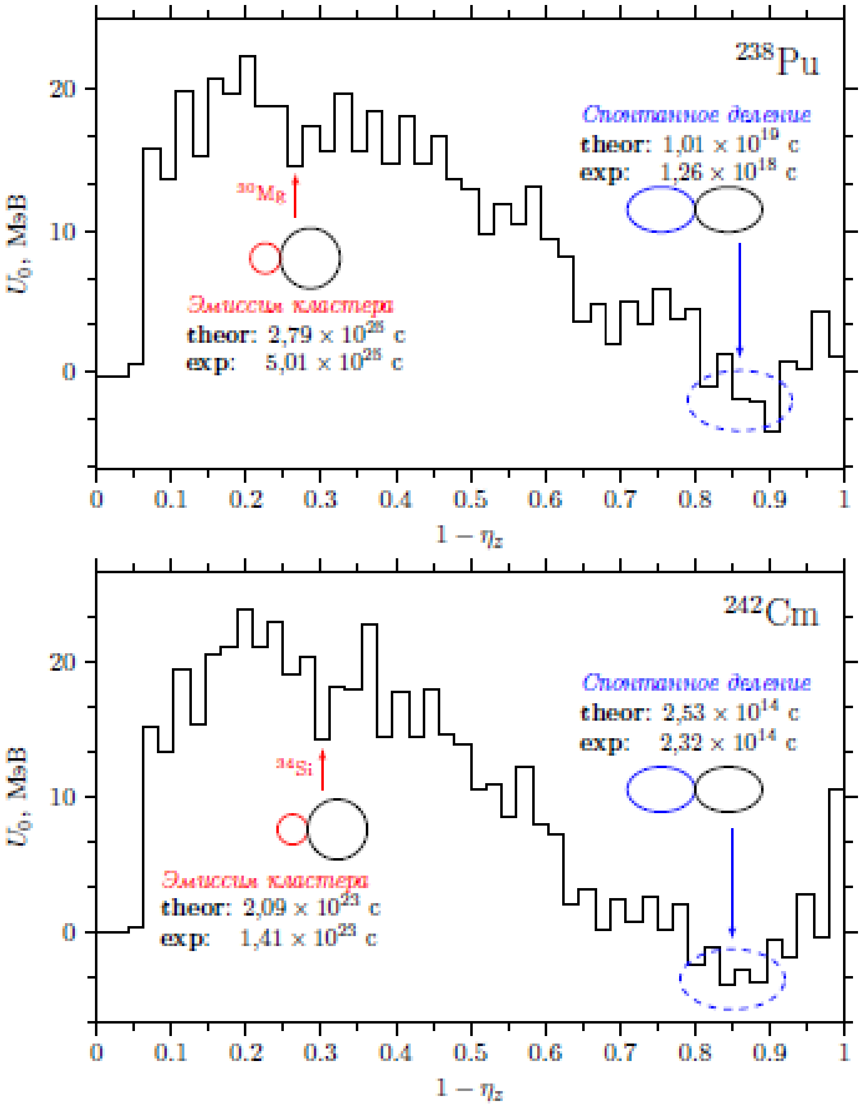
$$S_L = \int_{\eta_Z-1/Z}^{\eta_Z+1/Z} |\Psi(\eta_Z)|^2 d\eta_Z,$$

$$\eta_A = \frac{A_H - A_L}{A_H + A_L},$$

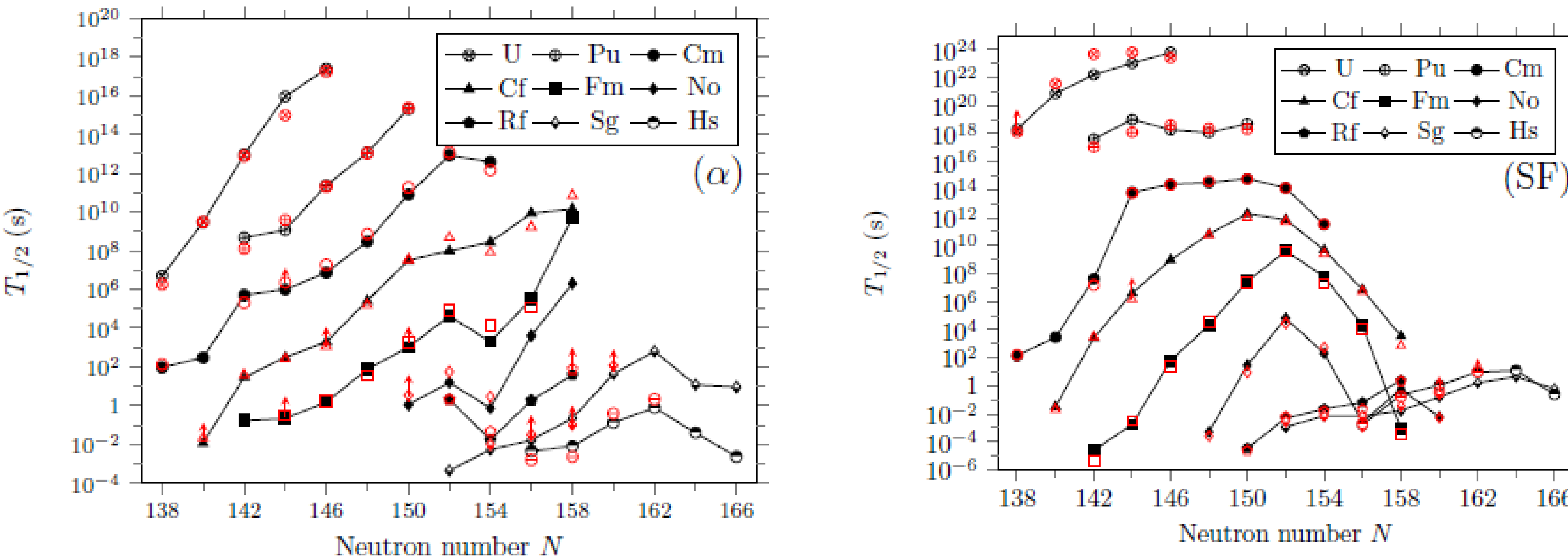
$$S_{SF} = \sum_L S_L,$$

$$\Gamma_L = \frac{\hbar\omega_0}{\pi} S_L P_L,$$

$\hbar\omega_0$– the energy of the first excited state in η_Z



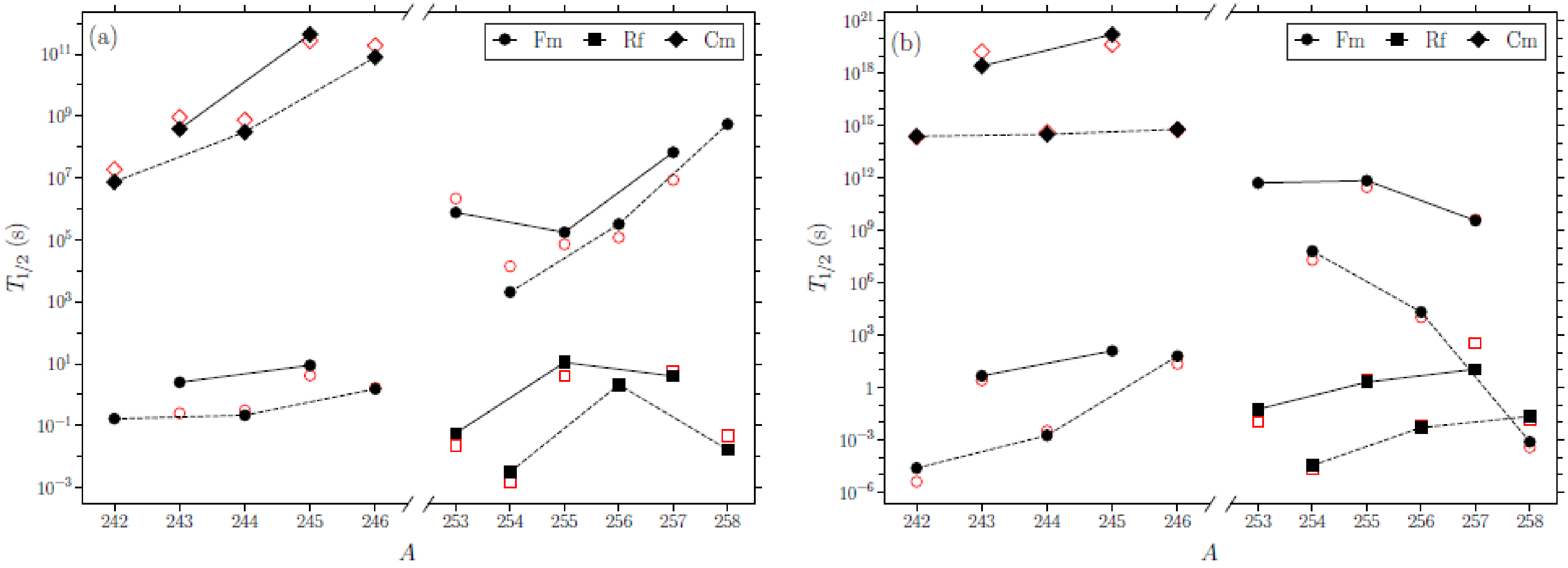
	$\hbar\omega_0$	S_{SF}	$T_{1/2}$	$T_{1/2}^{\exp}$
^{232}U	2,09	$9{,}19 \times 10^{-43}$	$7{,}43 \times 10^{20}$	$3{,}73 \times 10^{21}$
^{234}U	2,05	$4{,}34 \times 10^{-44}$	$1{,}61 \times 10^{22}$	$4{,}73 \times 10^{23}$
^{236}U	2,02	$6{,}45 \times 10^{-45}$	$1{,}10 \times 10^{23}$	$6{,}38 \times 10^{23}$
^{238}U	1,98	$3{,}62 \times 10^{-38}$	$2{,}00 \times 10^{16}$	$8{,}20 \times 10^{15}$
^{236}Pu	2,01	$1{,}68 \times 10^{-39}$	$4{,}24 \times 10^{17}$	$1{,}10 \times 10^{17}$
^{238}Pu	1,92	$7{,}39 \times 10^{-41}$	$1{,}01 \times 10^{19}$	$1{,}26 \times 10^{18}$
^{242}Cm	1,85	$3{,}06 \times 10^{-36}$	$2{,}53 \times 10^{14}$	$2{,}32 \times 10^{14}$
^{248}Cf	1,84	$3{,}97 \times 10^{-34}$	$1{,}96 \times 10^{12}$	$1{,}29 \times 10^{12}$



U
Pu
Cm
Cf
Fm
No
Rf
Sg
Hs
(α)
$T_{1/2}$ (s)
10^{20}
10^{18}
10^{16}
10^{14}
10^{12}
10^{10}
10^{8}
10^{6}
10^{4}
10^{2}
1
10^{-2}
10^{-4}
138
142
146
150
154
158
162
166
Neutron number N
(SF)
10^{24}
10^{22}
10^{20}
10^{18}
10^{16}
10^{14}
10^{12}
10^{10}
10^{8}
10^{6}
10^{4}
10^{2}
1
10^{-2}
10^{-4}
10^{-6}

Even-odd nuclei

^{257}Fm: 1.31×10^2 y – ^{256}Fm: 1.02×10^4 s
^{259}Fm: 1.5 s – ^{258}Fm: 0.37 ms



Proposed Experiments to study the role of the clustering in heavy nuclei

--- If cluster mechanism is responsible for spontaneous fission, the population of superdeformed or hyperdeformed cluster states with strongly influence the outcome of (γ,*SF*) channel.

Candidates:
^{240}U, ^{240}Pu
Measurements: E1-strength,
γ -decay behavior (Γ_γ)
(γ,*SF*), (Γ_α ,Γ_{cl})