# BSM - lecture 1

MUON

Budker INP

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HEPHY
INSTITUTE OF HIGH ENERGY PHYSICS

ÖAW AUSTRIAN ACADEMY OF SCIENCES

International school on muon dipole moments and hadronic effects

# Plan of the lectures (BSM = DM for the next 3h)

*Lecture 1*

- Fundamentals of standard cosmology
- Gravitational evidence for Dark Matter
- Basic properties of Dark Matter
- The Boltzmann equation and its application to the early Universe (explicit calculation)

*Lecture 2*

- Classification of Dark Matter models
- WIMP Dark Matter
- Laboratory Detection of WIMPs (explicit calculation)
- Light Dark Sectors with relevance for the intensity frontier

# Some scales

Earth's mass $6 \times 10^{27}\,\mathrm{g} \simeq 3 \times 10^{51}\,\mathrm{GeV} \simeq 3 \times 10^{-6} M_\odot$

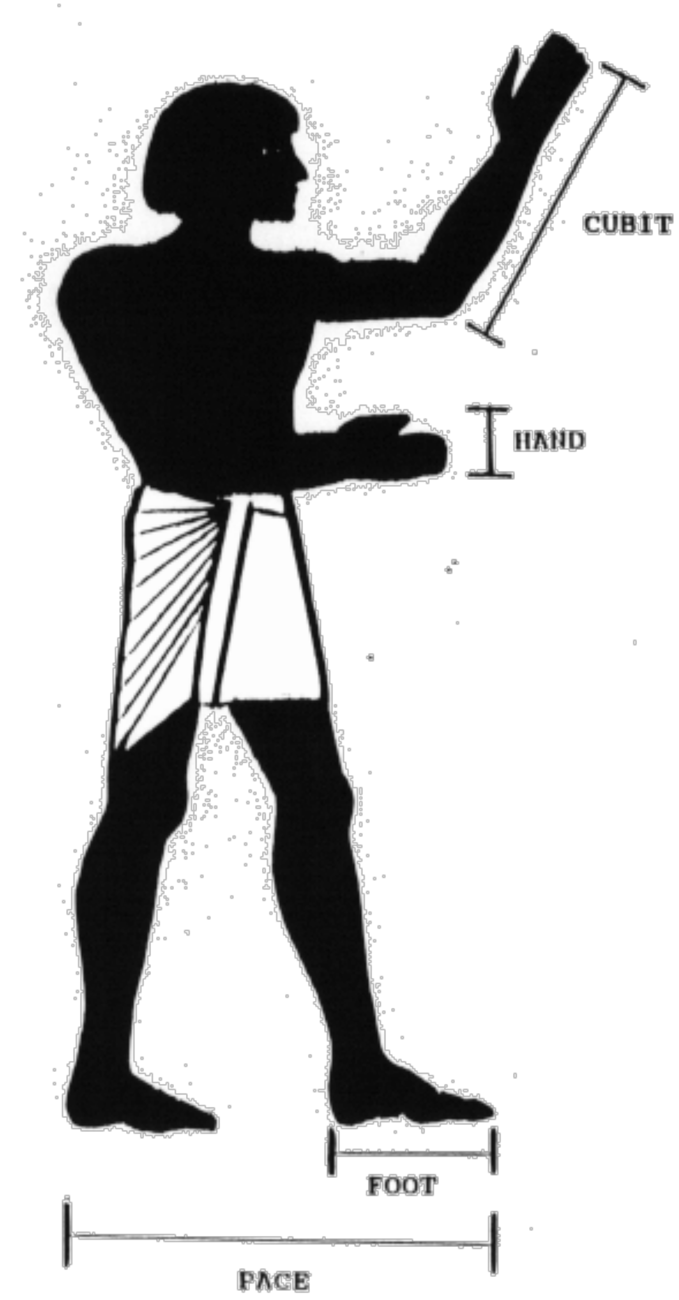
Earth's radius $6.4 \times 10^{8}\,\mathrm{cm}$

Earth's distance to sun $1.5 \times 10^{13}\,\mathrm{cm} = 1\,\mathrm{AU}$

Characteristic spacing of stars $1\,\mathrm{pc} = 3 \times 10^{18}\,\mathrm{cm}$

Distance to the Galactic center / Andromeda $8\,\mathrm{kpc}\,/\,50\,\mathrm{kpc}$

Spacing between Galaxies $\sim \mathrm{Mpc}$




# Useful approximate unit conversions and numbers

$200\,\mathrm{MeV\,fm} \simeq 1$ $1\,\mathrm{K} \simeq 10^{-4}\,\mathrm{eV}$ $n_\gamma \simeq 0.25 T^3$ $\alpha^2 \simeq 0.5 \times 10^{-4}$

$1\,\mathrm{eV} \simeq 10^{15}\,\mathrm{Hz}$ $m_A \simeq (A/N_A)\ g$ $M_P \simeq 2 \times 10^{18}\,\mathrm{GeV}$ $G_F^2 \simeq 10^{-10}/\mathrm{GeV}^4$

# Ground-breaking observations (that we shall discuss)

- The Universe is expanding (Hubble's law), isotropic and homogeneous on large scales
- The Universe is filled with photons with a close to perfect blackbody spectrum of temperature $T_0 = 2.7255(6)\,\mathrm{K}$ (Cosmic Microwave Background - CMB)
- Only a fraction of the Universe's mass is visible (there is evidence for new forms of matter - Dark Matter)
- The Universe is expanding at an accelerated pace (cosmological constant or some form of vacuum energy)
- The abundances of light elements (deuterium and helium) is abnormally high and not originating from stars

# Expansion of our Universe

Hubble (1920s): every galaxy moves away from us at a recessional velocity $v \simeq H_0 d_L$ proportional to its distance $d_L$ where $H_0 = 100h\,\mathrm{km/s/Mpc}$ where $h \simeq 0.7$

Hubble 1929

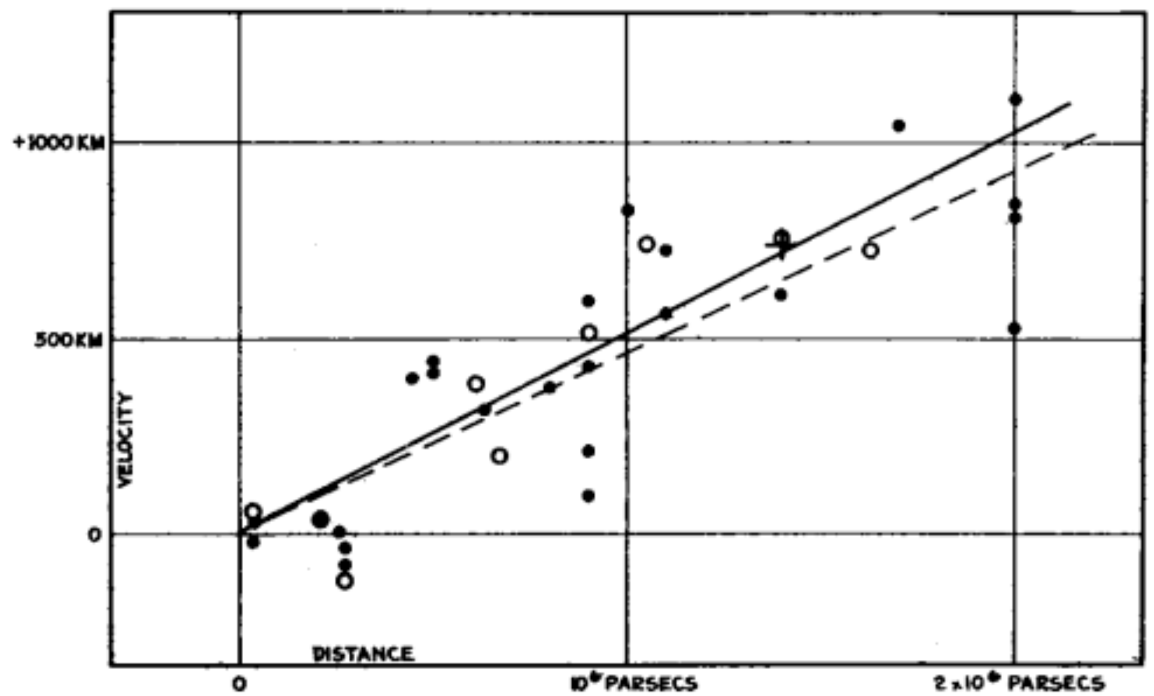
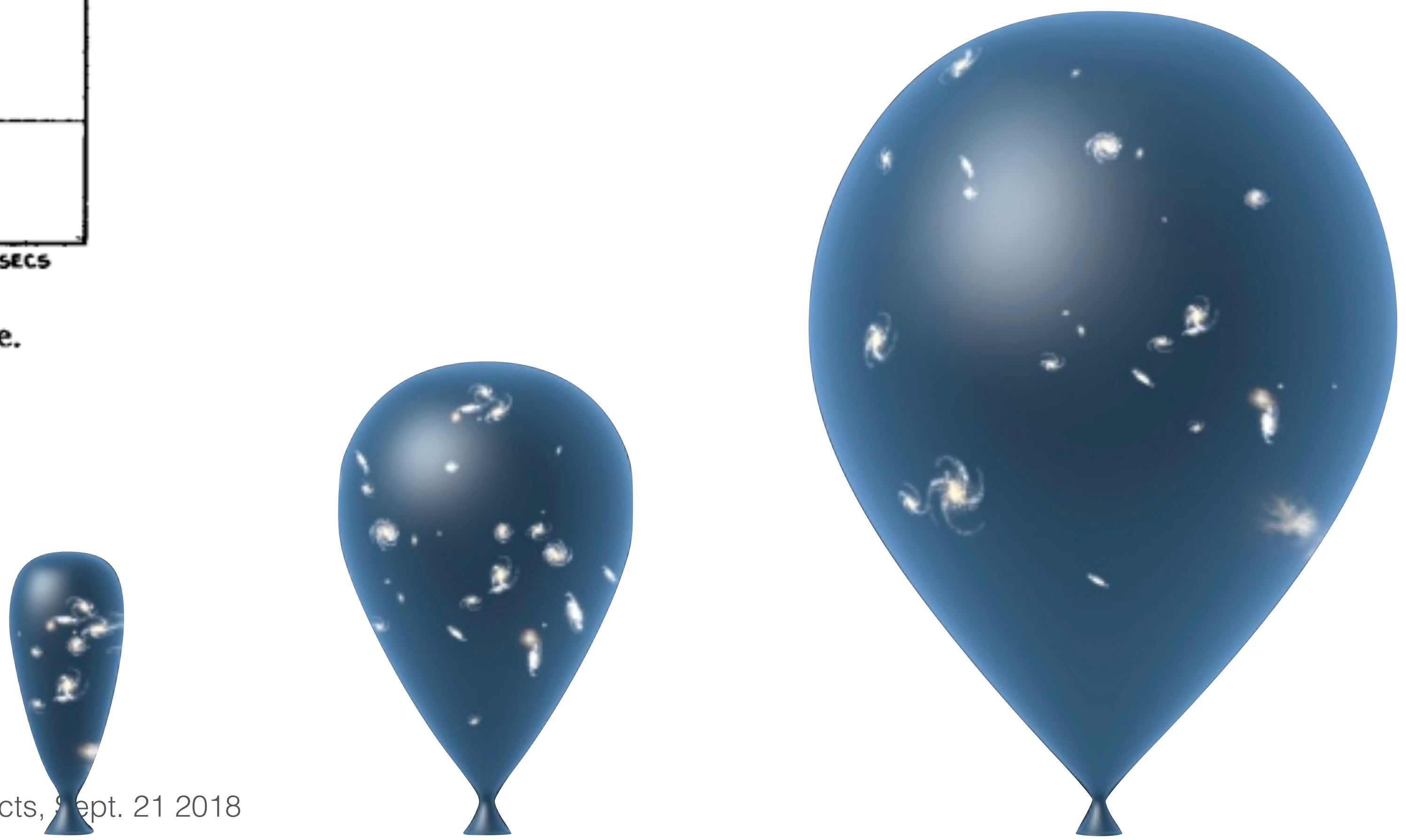



FIGURE 1

**Velocity-Distance Relation among Extra-Galactic Nebulae.**



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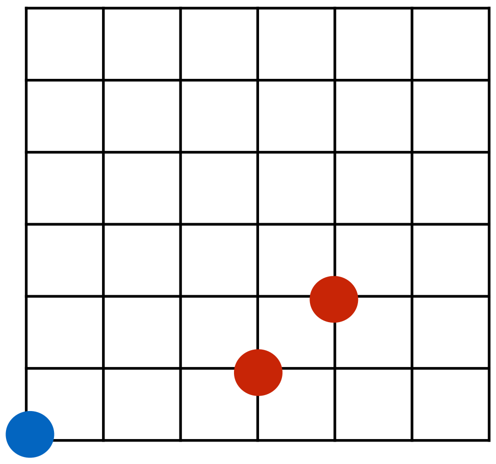

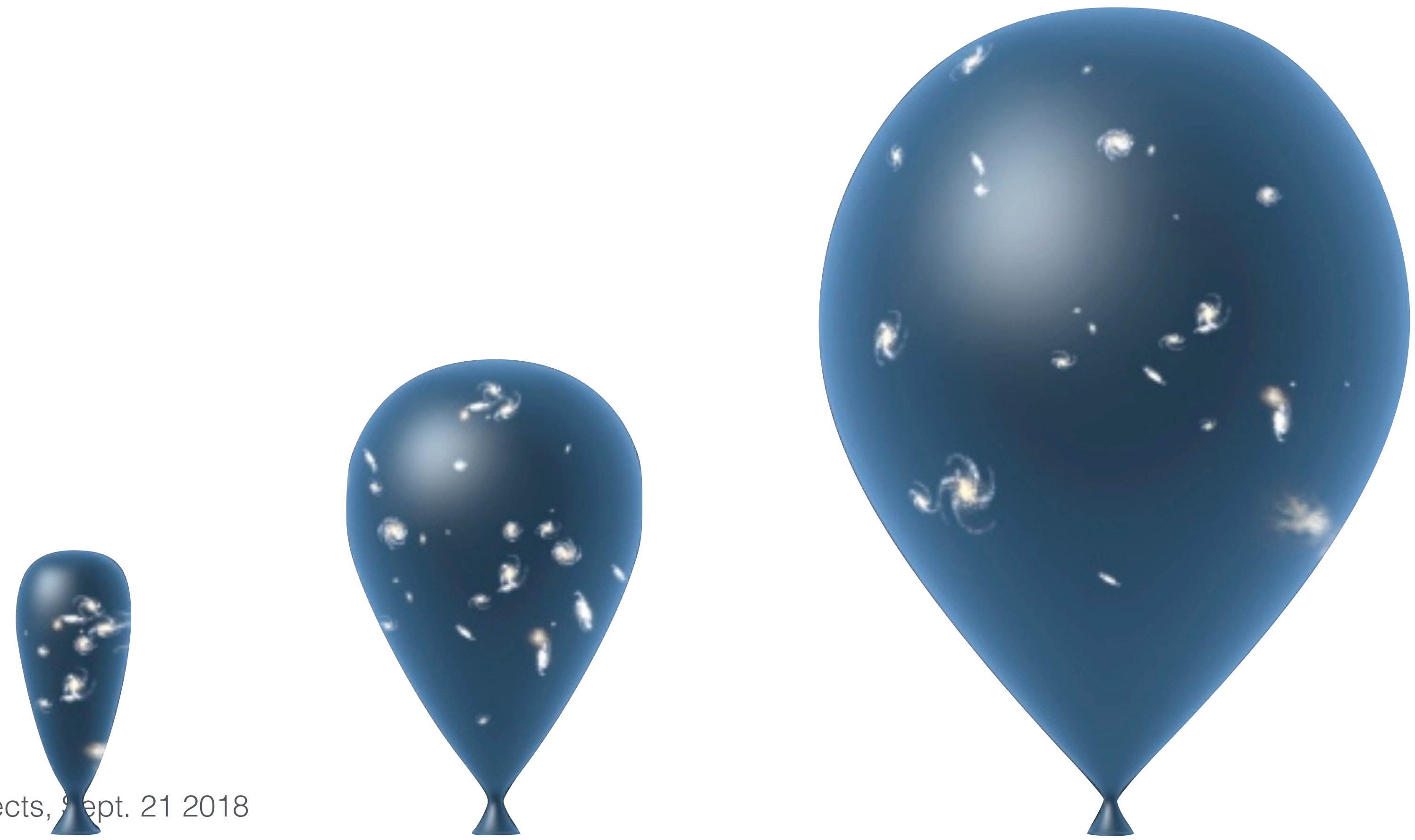

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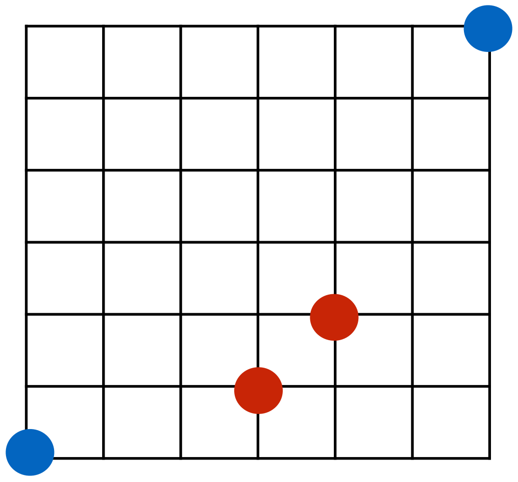

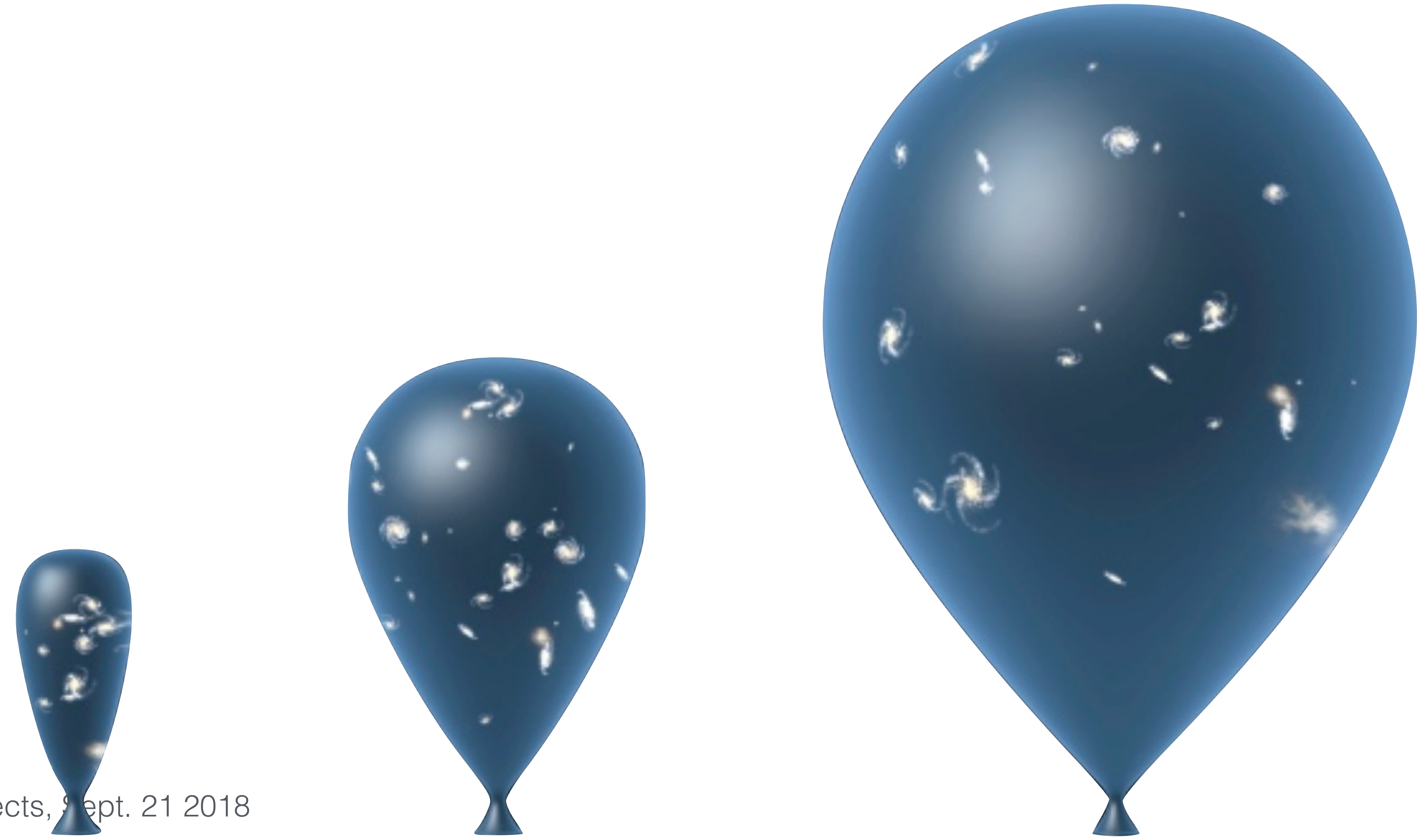

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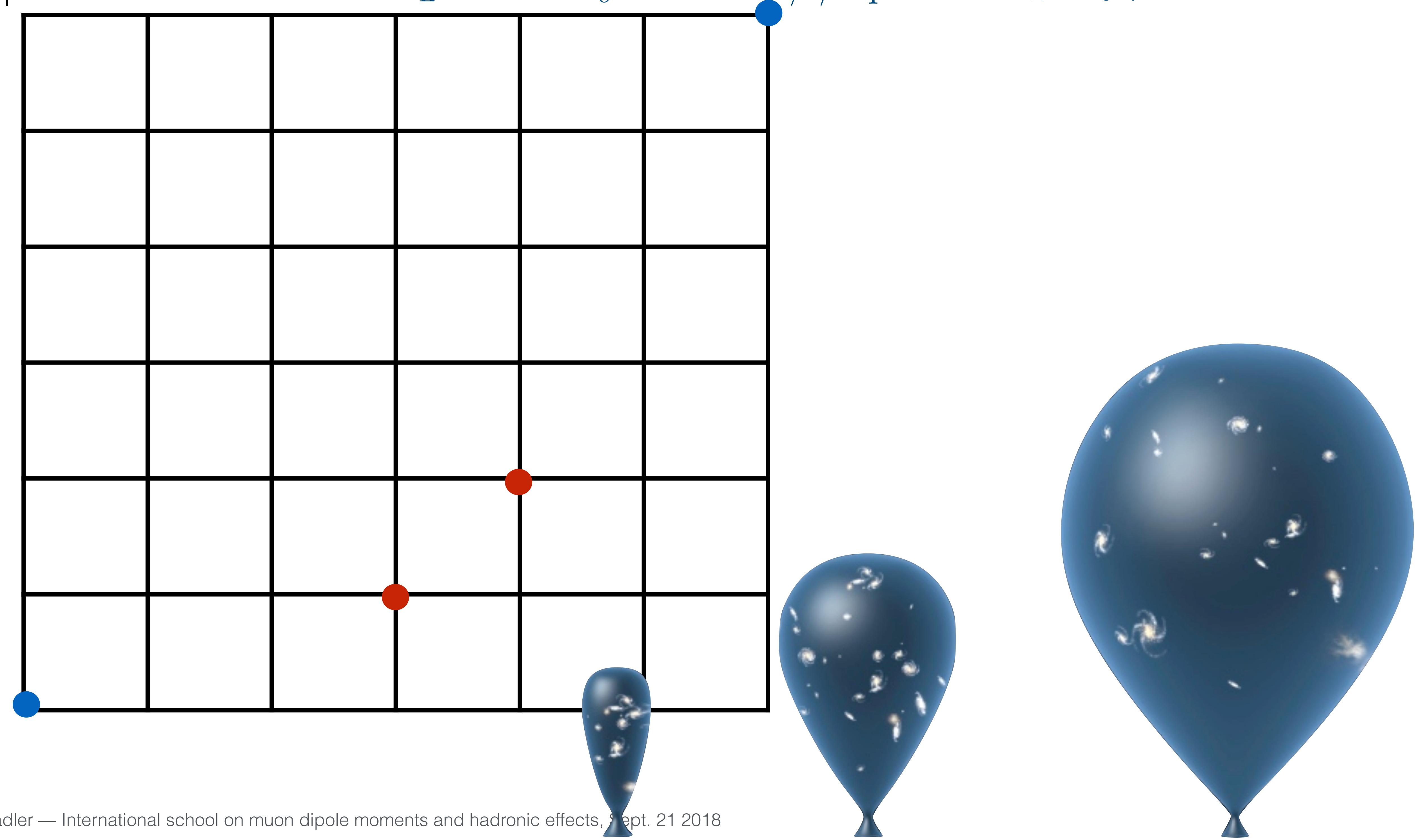
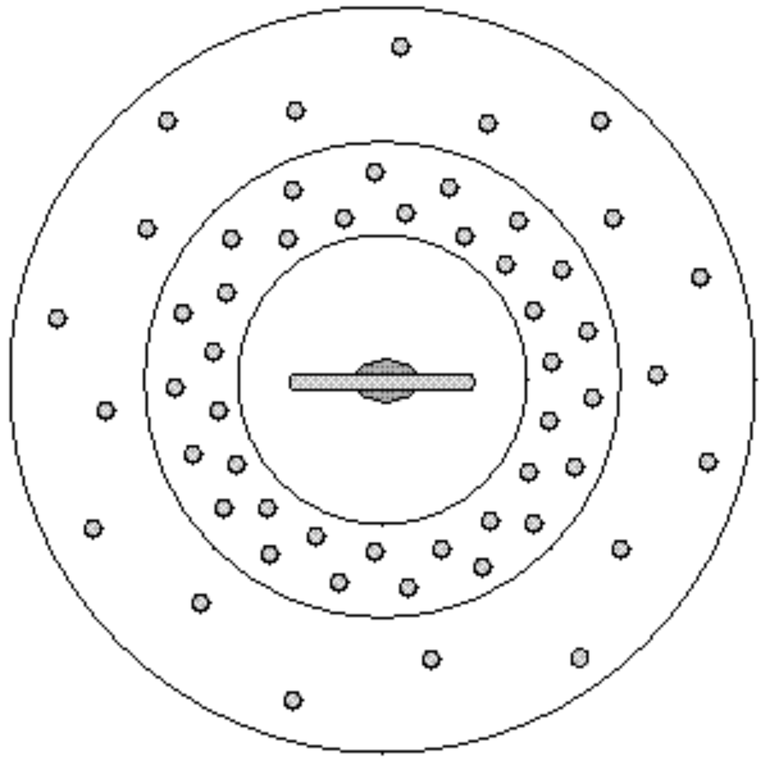


# Isotropy and Homogeneity of our Universe

On largest observable scales (100s to 1000s of Mpc) the galaxy distribution is roughly the same, independent on the direction where we look (isotropy).

With the assumption that we do not occupy a preferred position in the Universe (Copernican principle), it follows that any observer in another galaxy should also see a isotropic distribution of galaxies. This only works if the Universe is homogeneous.



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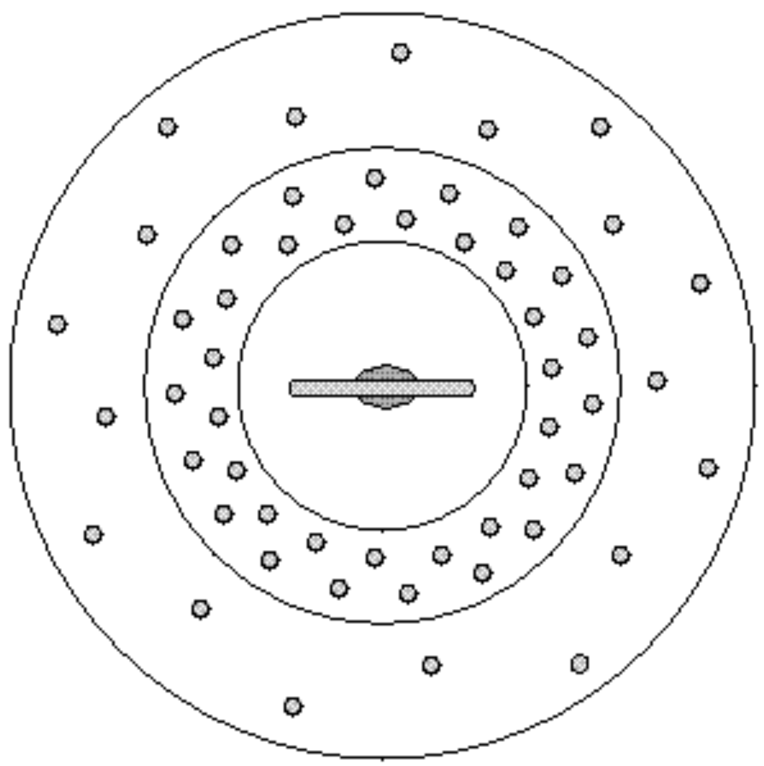
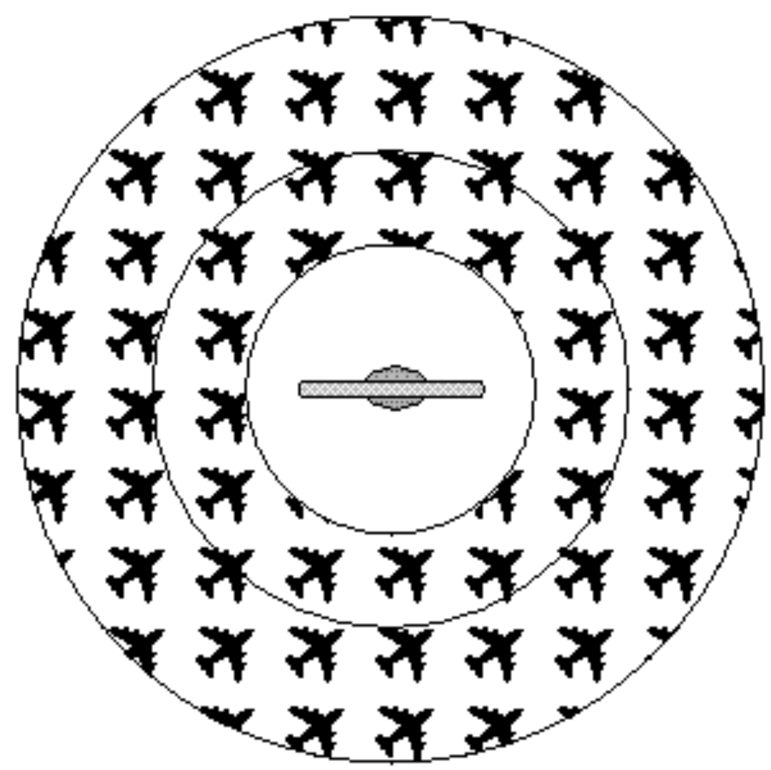




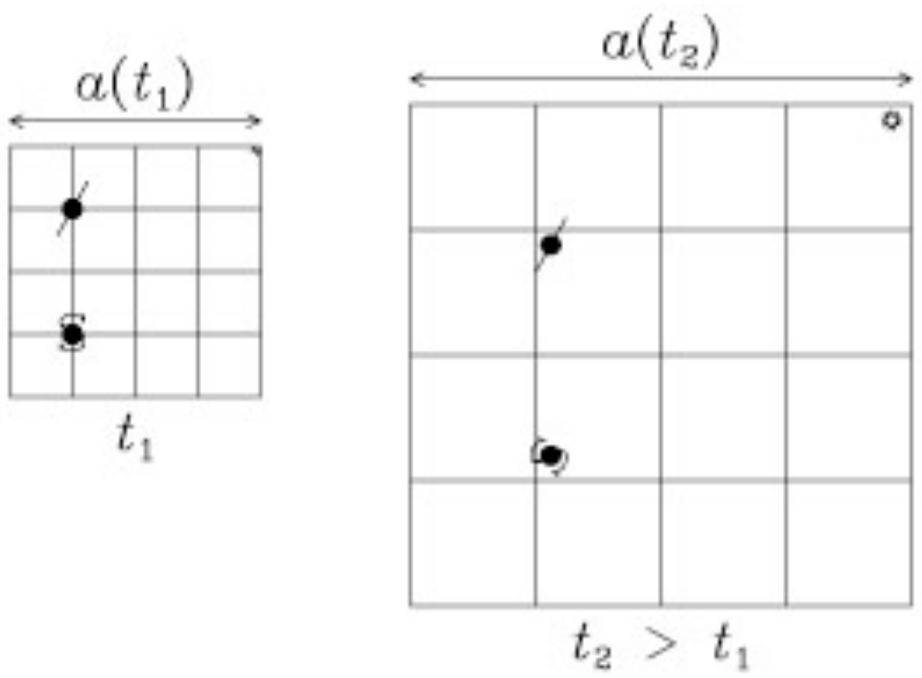
# Isotropy and Homogeneity of our Universe

Isotropy and homogeneity are reconciled with the observed Hubble expansion, if the Universe undergoes self-similar expansion

$\vec{r}_i(t) = a(t)\vec{x}_i$

$\vec{x}_i$ are *constant* coordinate labels of a galaxy i

$a(t)$ is called scale factor

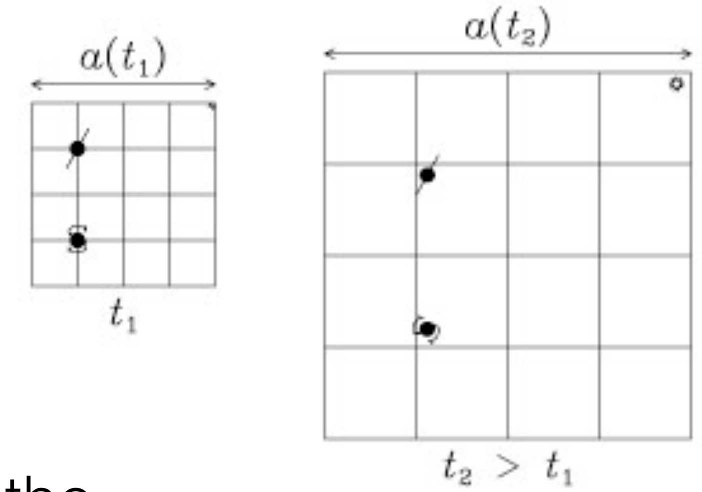



Relative velocity between two galaxies:

$$\vec{v} = \dot{\vec{r}}_i - \dot{\vec{r}}_j = \dot{a}(\vec{x}_i - \vec{x}_j) = \frac{\dot{a}}{a}(\vec{r}_i - \vec{r}_j)$$

$\frac{\dot{a}}{a} = H_0$ Hubble constant describes the rate of expansion of the Universe.

# The metric of an expanding Universe




Minkowski metric $ds^2 = dt^2 - d\vec{x}^2$

Expanding Universe $ds^2 = dt^2 - a(t)^2 d\vec{x}^2$

The metric with homogeneous and isotropic spatial sections is the Friedmann-Robertson-Walker (FRW) metric

$$ds^2 = g_{\mu\nu} dx^\mu dx^\nu = dt^2 - a^2(t) \left[ \frac{dr^2}{1-kr^2} + r^2 d\theta^2 + r^2 \sin^2\theta d\phi^2 \right]$$

$(t, r, \theta, \phi)$ are called “comoving" coordinates. $t$ is the proper time of an observer at rest in those coordinates. $r$ is dimensionless, $a$ has dimension of length; $k = 0, \pm 1$ is called the curvature parameter.

Physical spatial line element $d\vec{l}^2 = a^2(t) [\dots]$

$$g_{\mu\nu} = \begin{array}{cc} & \begin{array}{cccc} dt & dr & d\theta & d\phi \end{array} \\ \begin{array}{c} dt \\ dr \\ d\theta \\ d\phi \end{array} & \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & -\frac{a(t)^2}{1-kr^2} & 0 & 0 \\ 0 & 0 & -a(t)^2 r^2 & 0 \\ 0 & 0 & 0 & -a(t)^2 r^2 \sin^2\theta \end{pmatrix} \end{array}$$

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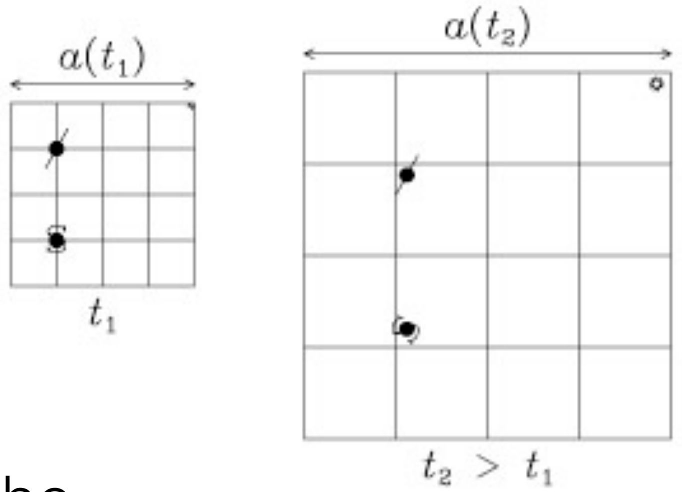


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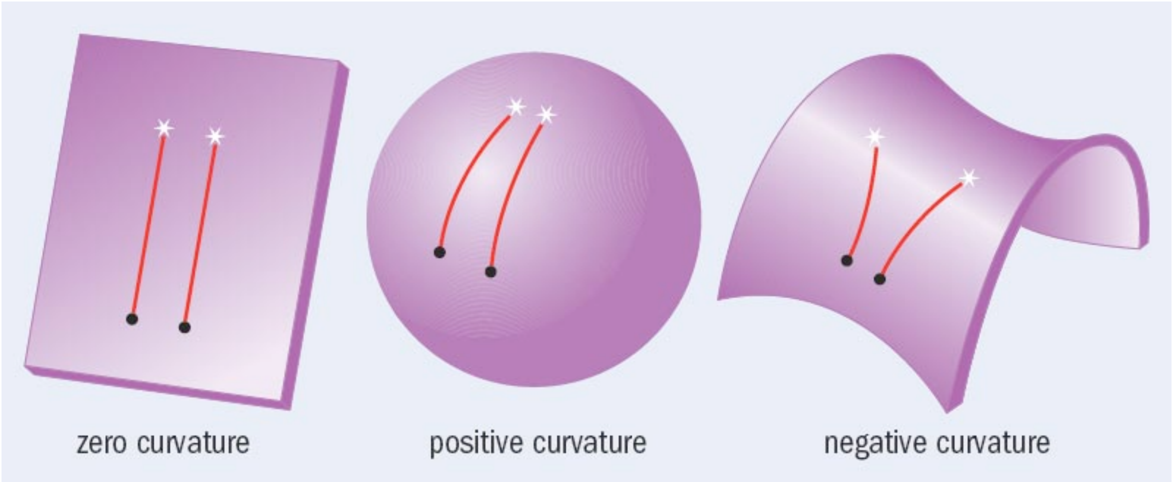


$k = 0$ $k = 1$ $k = -1$

Observations show that we live (to a high degree in a flat Universe, k=0)

# The metric of an expanding Universe

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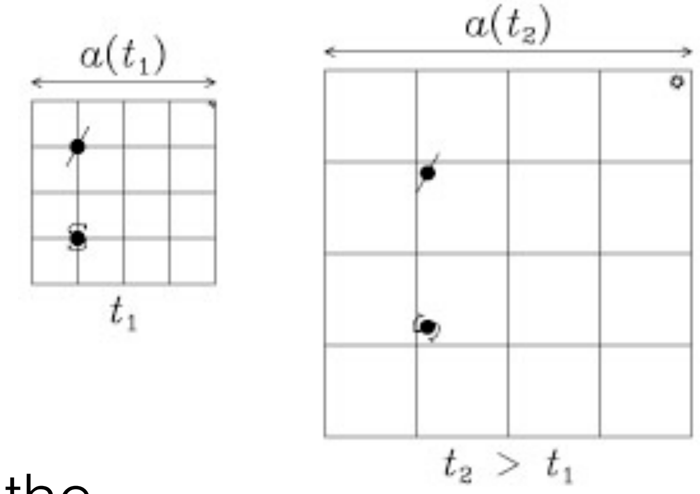


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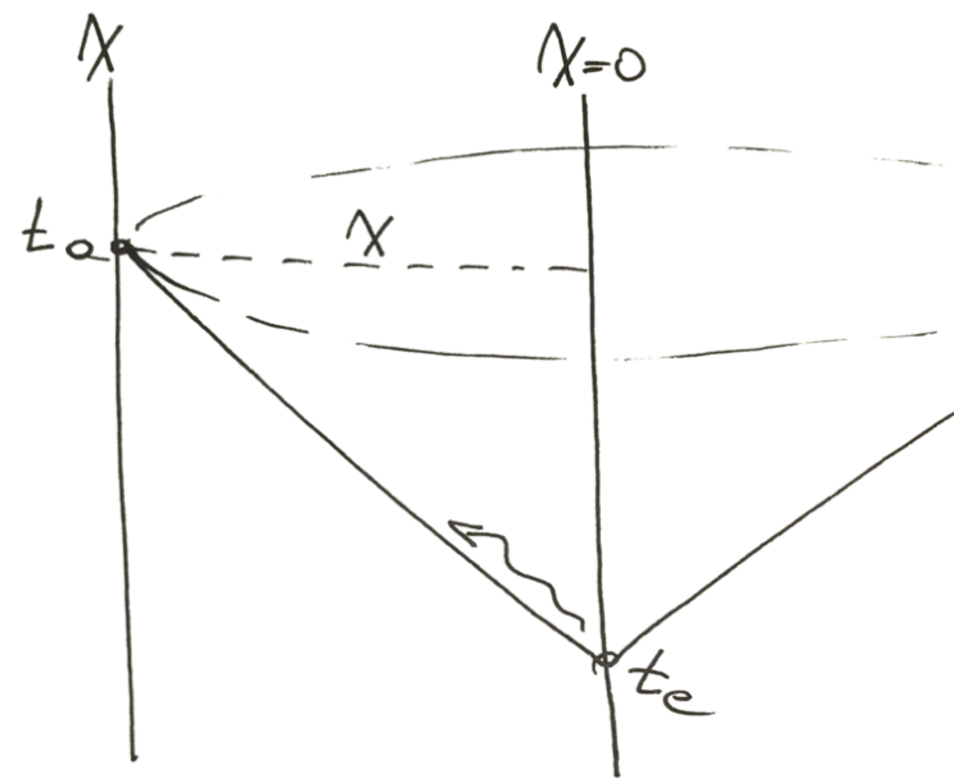
An alternative form of the metric is

$$ds^2 = dt^2 - a^2(t)\left[d\chi^2 + \begin{pmatrix} \sin^2\chi \\ \chi^2 \\ \sinh^2\chi \end{pmatrix} \left(d\theta^2 + r^2 \sin^2\theta d\phi^2\right)\right]$$

where the entries in the vector correspond to $k = +1, 0, -1$ .

# Horizons in an expanding Universe

*What part of the Universe is in causal contact?*




Consider a light ray emitted radially at $t = t_e$ in direction $\theta = \phi = 0$ from a comoving position $\chi = 0$ reaching us today at $t = t_0$

$ds^2 = 0$ Null geodesic (light ray)

$dt = a(t) d\chi$ => $\chi = \int_{t_e}^{t_0} \frac{dt}{a(t)}$ Coordinate distance traveled by a light ray

In a Universe filled with matter, $a \propto t^{2/3}$, and the physical distance traveled by a photon is

$$a_0 \chi = 3 t_0 \left[1 - (t_e/t_0)^{1/3}\right]$$

=> for a light ray emitted at $t = 0$: $a_0 \chi = 3 t_0 = 2/H_0 \sim 10\,\mathrm{Gpc}$

=> observable volume of the Universe is finite!

# Equations of motion for the scale factor

## Newtonian physics

$\nabla^2\phi = 4\pi G\rho$ Poisson Equation relates gravitational potential $\phi$ with the distribution of mass $\rho$

$\nabla\phi$ gradient gives the acceleration particles experience in the gravitational potential

## Homogeneous Universe $\rho = \rho(t)$. What is the equation of motion of $a(t)$ ?

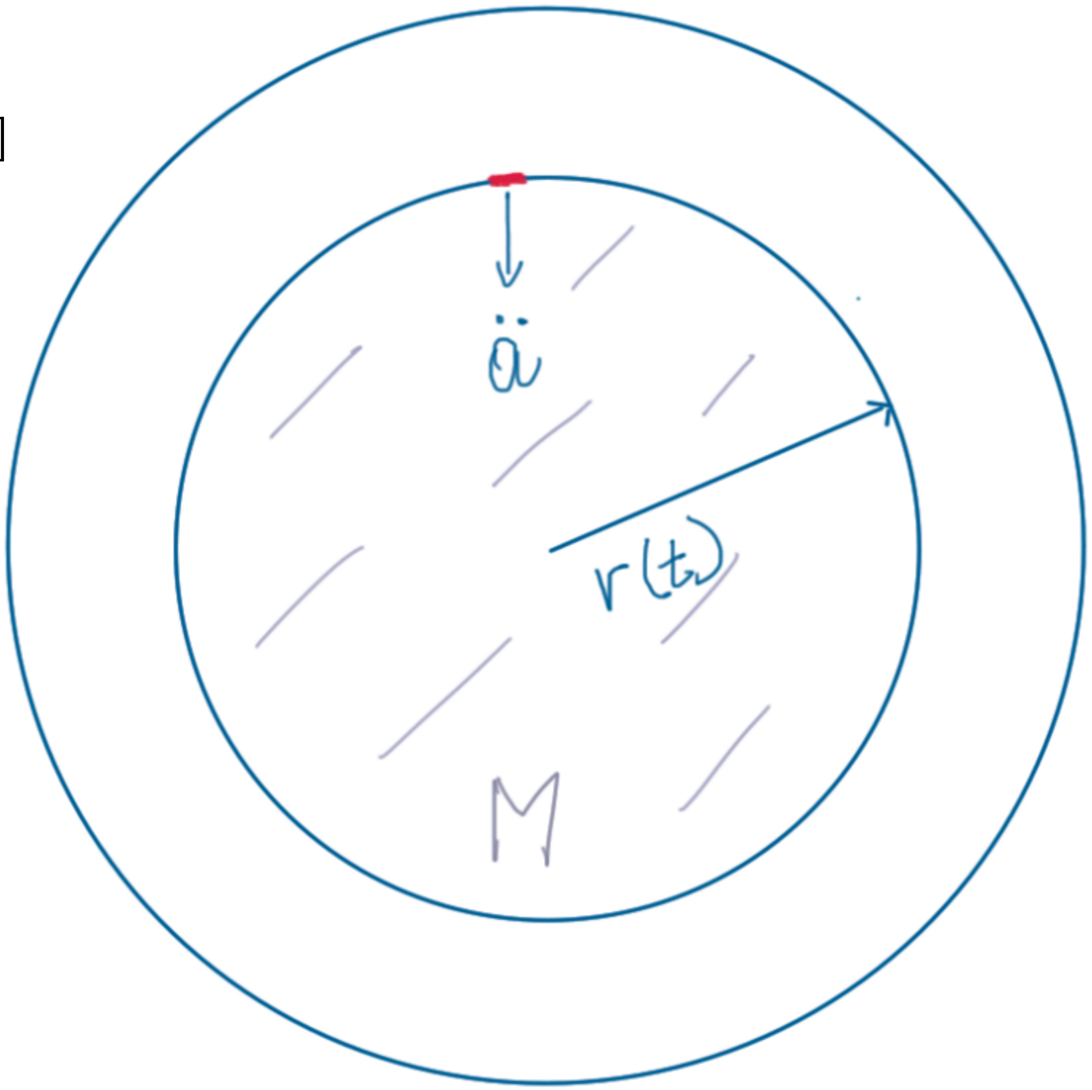
$r(t) = a(t)x$ spherical shell of radius a(t) [for x=1]

$\ddot{a}(t) = -GM/a^2$ acceleration toward center

$M = (4\pi/3)a^3\rho$ enclosed mass

$$\frac{\ddot{a}}{a} = -\frac{4\pi G}{3}\rho$$

Friedmann equation in
second form (p=0),
deceleration (ρ is positive definite)



# Equations of motion for the scale factor

We can also get the Friedmann equation in its “first form” in $\dot{a}$ rather than $\ddot{a}$

$$\ddot{a}(t) = -GM/a^2$$

from before; now multiply by $\dot{a}$ and integrate (M = const)

$$\frac{1}{2}\dot{a}^2 = \frac{GM}{a} + U$$

energy equation with integration constant U = total energy of the expanding sphere (kinetic + potential)

$$\left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G}{3}\rho - \frac{k}{a^2}$$

Friedmann equation in the first form

$k = -2U$ Integration constant is the curvature parameter;

U > 0 total energy is positive, sphere will expand forever (k < 0 and RHS will always be positive)

U < 0 total energy is negative, sphere will eventually collapse (k > 0 and RHS will eventually become negative)

$$H(t) = \frac{\dot{a}}{a}$$

Hubble rate


The Newtonian derivation of the Friedmann equation ultimately fails; it only applies only to non-relativistic (pressure-less) matter.

We may instead plug the FRW metric into Einstein equations.

<— Einstein Equations applied to a curved manifold

Left-hand side describes the geometry

$$R = R^{\mu}{}_{\mu}$$ Curvature scalar

$$R_{\sigma\nu} = R^{\alpha}{}_{\sigma\alpha\nu}$$ Ricci Tensor

$$R^{\rho}{}_{\sigma\mu\nu} = \partial_\mu \Gamma^{\rho}{}_{\nu\sigma} - \partial_\nu \Gamma^{\rho}{}_{\mu\sigma} + \Gamma^{\rho}{}_{\mu\lambda}\Gamma^{\lambda}{}_{\nu\sigma} - \Gamma^{\rho}{}_{\nu\lambda}\Gamma^{\lambda}{}_{\mu\sigma}$$ Riemann tensor

$$\Gamma^{\nu}{}_{\kappa\lambda} = \frac{1}{2} g^{\nu\mu}(g_{\mu\kappa,\lambda} + g_{\mu\lambda,\kappa} - g_{\kappa\lambda,\mu})$$ Christoffel symbols


The Newtonian derivation of the Friedmann equation ultimately fails; it only applies only to non-relativistic (pressure-less) matter.

We may instead plug the FRW metric into Einstein equations.

<— Einstein Equations applied to a curved manifold

Right-hand side describes the energy content

$$T_{0j} = 0, \quad T_{ij} = 0 \quad (i \neq j)$$

demanded by homogeneity and isotropy

$$T^{\mu}{}_{\nu} = \begin{pmatrix} \rho & 0 & 0 & 0 \\ 0 & -p & 0 & 0 \\ 0 & 0 & -p & 0 \\ 0 & 0 & 0 & -p \end{pmatrix}$$

Energy Momentum tensor of a perfect fluid (in the rest frame = comoving coordinates)

$\rho$ energy density

$p$ pressure

$$\left.\begin{array}{l} \bullet \text{ Photons, neutrinos} \\ \bullet \text{ Dark Matter, Atoms} \\ \bullet \text{ vacuum energy / dark energy} \\ \bullet \text{ Gravitational waves} \end{array}\right\} \quad T_{\mu\nu} = \sum_i T^i_{\mu\nu}$$

# Friedmann equation in GR

Now, let us evaluate the Einstein equations in the FRW Universe

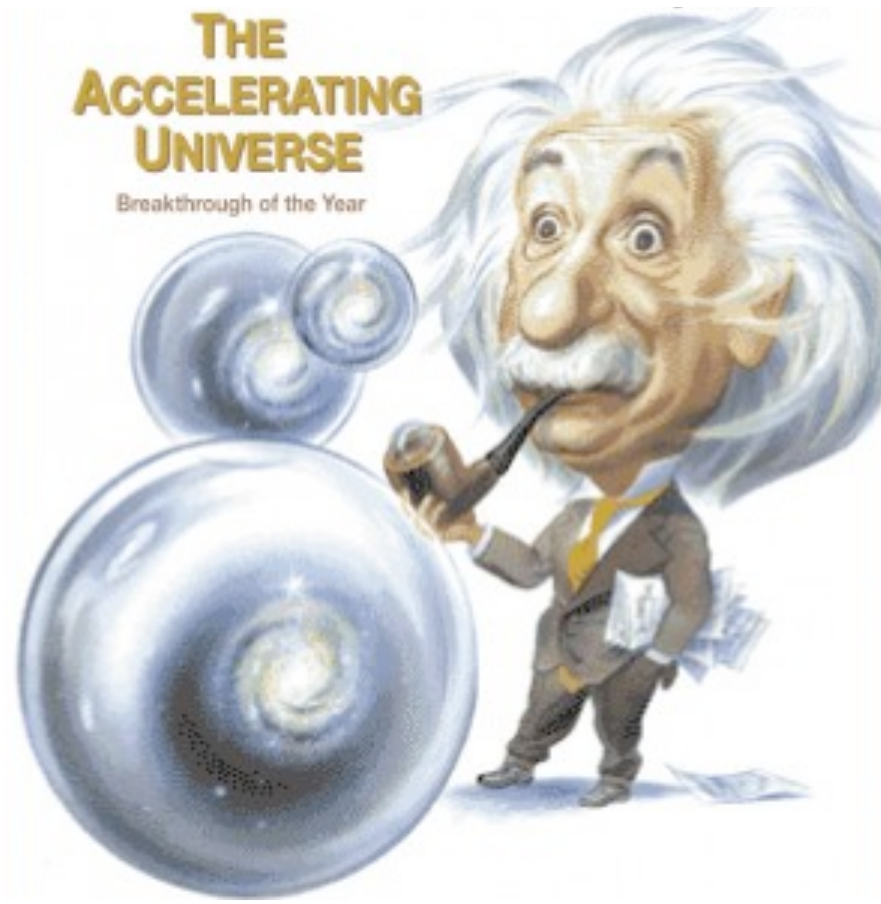
$$R_{00} - \frac{1}{2} R g_{00} = 8\pi G T_{00}$$ => $$H^2(t) \equiv \left(\frac{\dot{a}}{a}\right)^2 + \frac{k}{a^2} = \frac{8\pi G}{3} \sum_i \rho_i$$ **FR1**

$$R_{ij} - \frac{1}{2} R g_{ij} = 8\pi G T_{ij}$$ => $$2\frac{\ddot{a}}{a} + \left(\frac{\dot{a}}{a}\right)^2 + \frac{k}{a^2} = -8\pi G \sum_i p_i$$ FR2a

Plug FR1 into FR2a: $$\frac{\ddot{a}}{a} = -\frac{4\pi G}{3} \sum_i (\rho_i + 3p_i)$$ **FR2**

(Recall our Newtonian derivation?
It had no pressure term.)

=> for $p_i = -\rho_i/3$ the Universe may undergo *accelerated expansion!!*



# General Relativity - Matter

For each component local conservation of energy-momentum holds

$$\nabla_\lambda T^\mu{}_\nu \equiv \partial_\lambda T^\mu{}_\nu + \Gamma^\mu{}_{\lambda\rho} T^\rho{}_\nu - \Gamma^\rho{}_{\lambda\nu} T^\mu{}_\rho = 0$$

conservation of stress-energy

Using FRW and the energy-momentum tensor from before yields the continuity equation

$$\frac{d\rho_i}{dt} + 3\frac{\dot{a}}{a}(\rho_i + p_i) = 0$$

Continuity equation

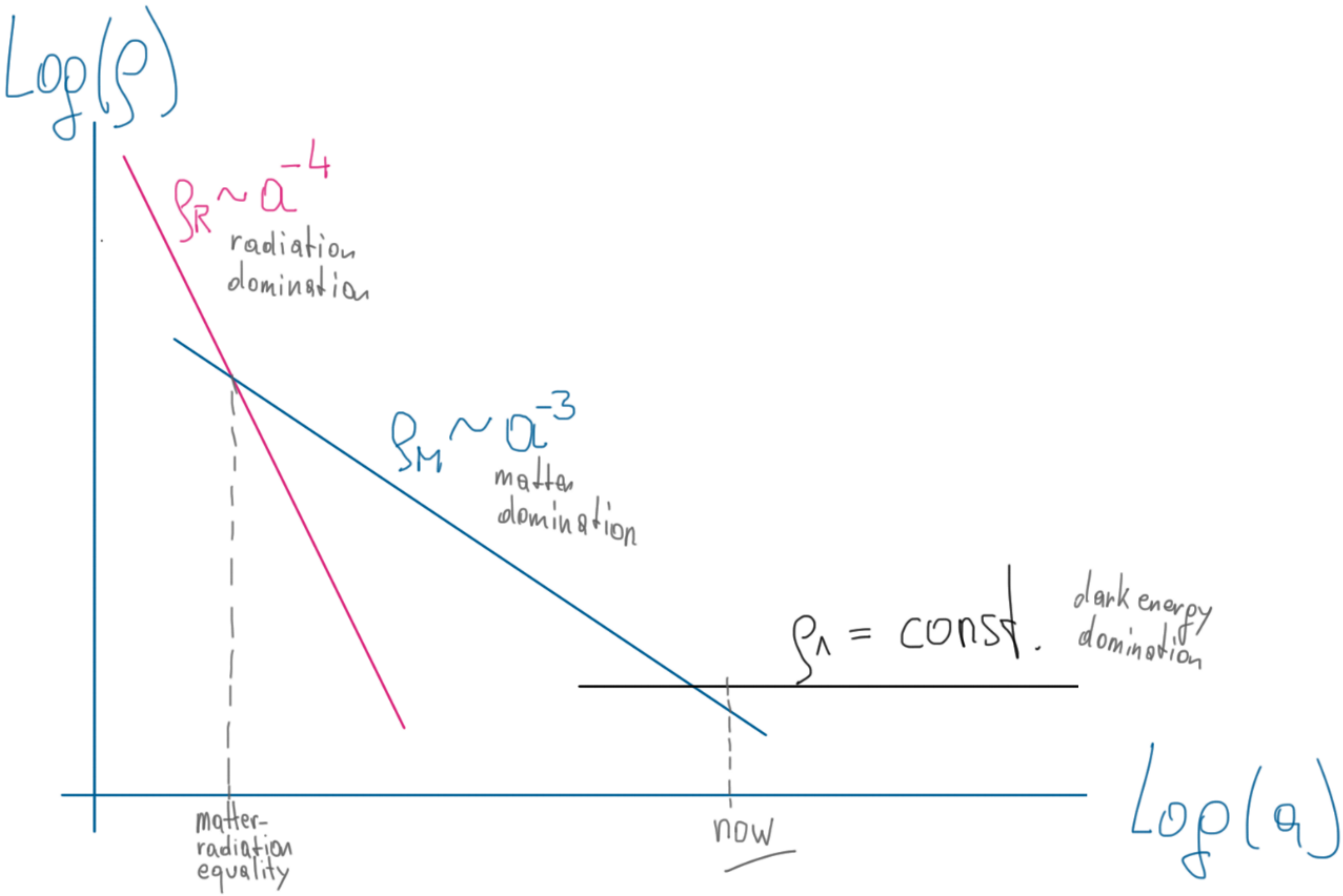
$$w_i \equiv \frac{p_i}{\rho_i}$$

Equation of state w relates pressure and energy density.

If w= const, then $\rho(t) \propto a^{-3(1+w)}$

- Non-relativistic matter w = 0
- Relativistic particles w = 1/3
- Vacuum energy w = -1
- Magnetic fields “w = -1/3”
- General case: w = w(t) e.g. when particles move in anharmonic potentials

# Dependence of components on scale factor



# Redshift

Momentum of a test particle:

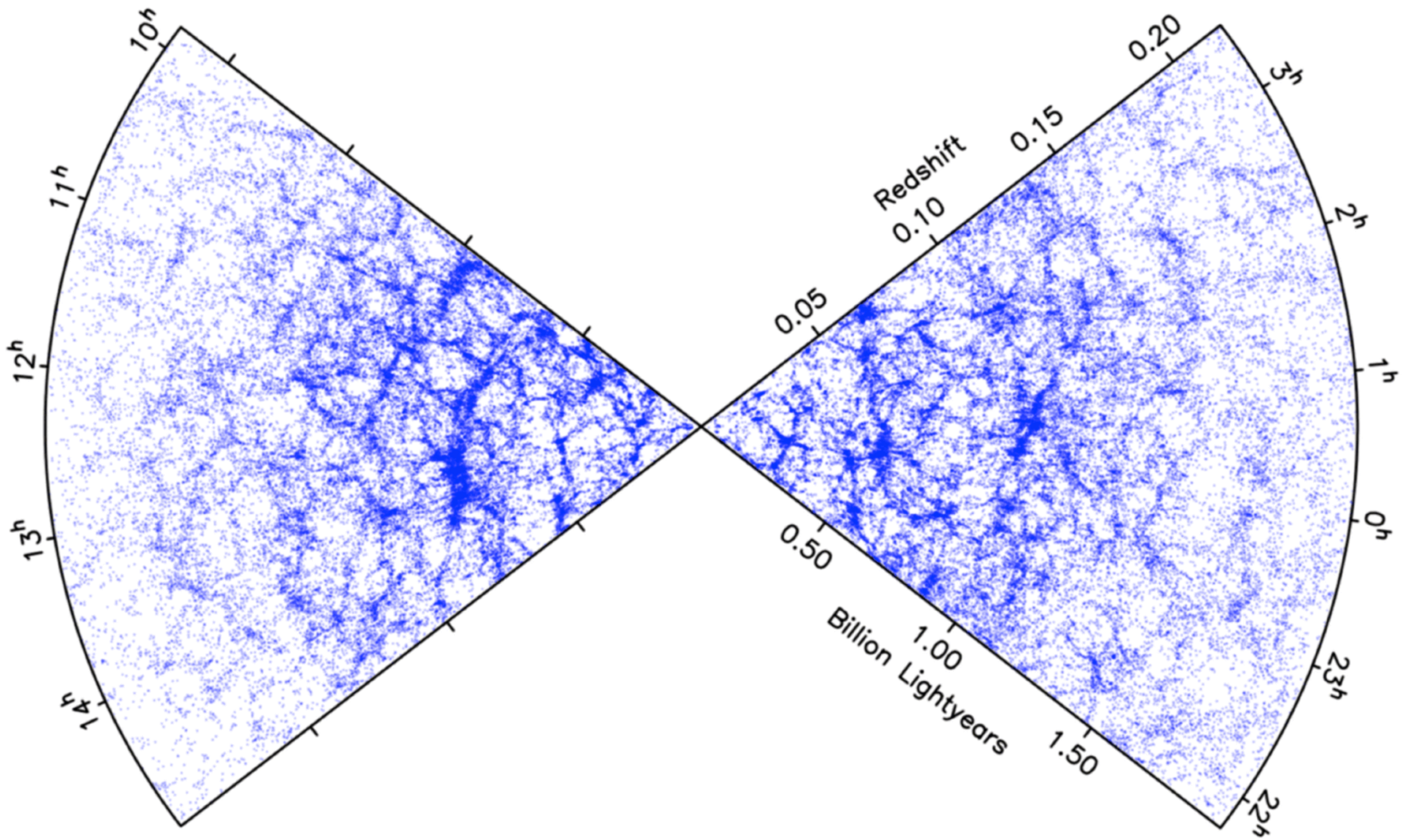
$$\frac{p(t_0)}{p(t_1)} = \frac{a(t_1)}{a(t_0)}$$

$$1 + z = \frac{\lambda_{\text{observed}}}{\lambda_{\text{emitted}}} = \frac{a(t_0)}{a(t_{\text{emitted}})}$$

$a(t_0) = 1$ customary normalization

cosmological redshift z

# Redshift




2dF large scale survey (state-of-the-art 2002)

# Density parameters

We may evaluate the Friedmann equation today

$$H_0^2 + \frac{k}{a_0^2} = \frac{1}{3M_P^2}\sum_i \rho_i(t_0)$$

$$M_P^{-2} = 8\pi G$$

$$M_P \simeq 2 \times 10^{18}\,\mathrm{GeV}$$ reduced Planck mass

$$H_0 \simeq 70\,\mathrm{km/s/Mpc} \simeq 10^{-33}\,\mathrm{eV}$$

Which can be written as

$$\frac{k}{H_0^2 a_0^2} = \sum_i \frac{\rho_i}{3M_P^2 H_0^2} - 1$$

$$\frac{k}{H_0^2 a_0^2} = \sum_i \Omega_i - 1$$

$$\rho_c = 3M_P^2 H_0^2 \simeq 5\,\mathrm{keV/cm^3} \simeq (2\,\mathrm{meV})^4$$ "critical density"

$$\Omega_i \equiv \frac{\rho_i}{\rho_c}$$ density parameter today

A flat Universe (k=0) is critical, $\sum_i \Omega_i = 1$ .

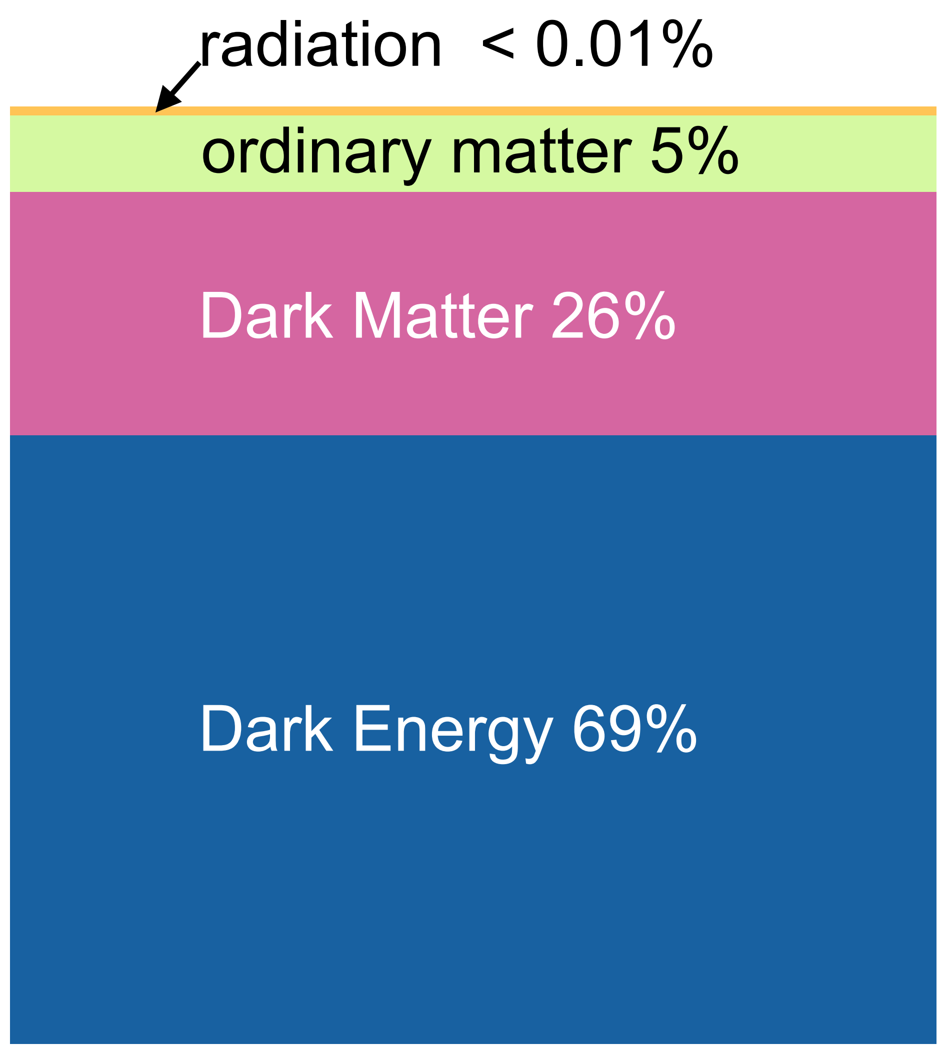
$$H = H_0\sqrt{\Omega_m (a_0/a)^3 + \Omega_{rad}(a_0/a)^4 + \Omega_\Lambda}$$

$$H = H_0\sqrt{\Omega_m (1+z)^3 + \Omega_{rad}(1+z)^4 + \Omega_\Lambda}$$

Planck: $|\Omega_k| < 0.005$

for a flat Universe with cosmological constant

# Energy content with %-precision

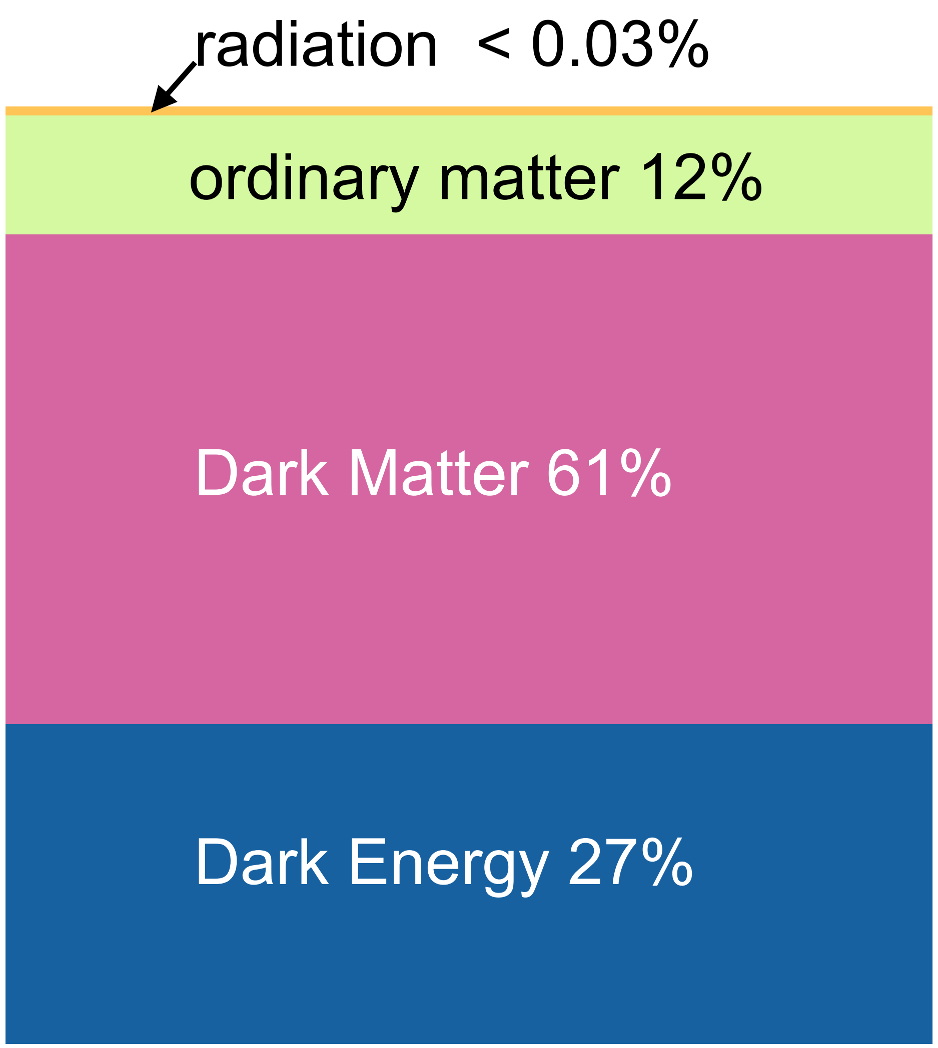



The energy content of the Universe today

PDG

| | |
|---|---|
| $\Omega_b = \rho_b/\rho_{crit}$ | $^{\ddagger}0.02207(27)\,h^{-2} \;=\; ^{\dagger}0.0499(22)$ |
| $\Omega_{cdm} = \rho_{cdm}/\rho_{crit}$ | $^{\ddagger}0.1198(26)\,h^{-2} \;=\; ^{\dagger}0.265(11)$ |
| $\Omega_\Lambda$ | $0.685^{+0.017}_{-0.016}$ |
| $\Omega_m = \Omega_{cdm} + \Omega_b$ | $0.315^{+0.016}_{-0.017}$ |
| $H_0$ | $100\,h$ km s$^{-1}$ Mpc$^{-1}$ |
| $h$ | $0.673(12)$ |

# Energy content with %-precision

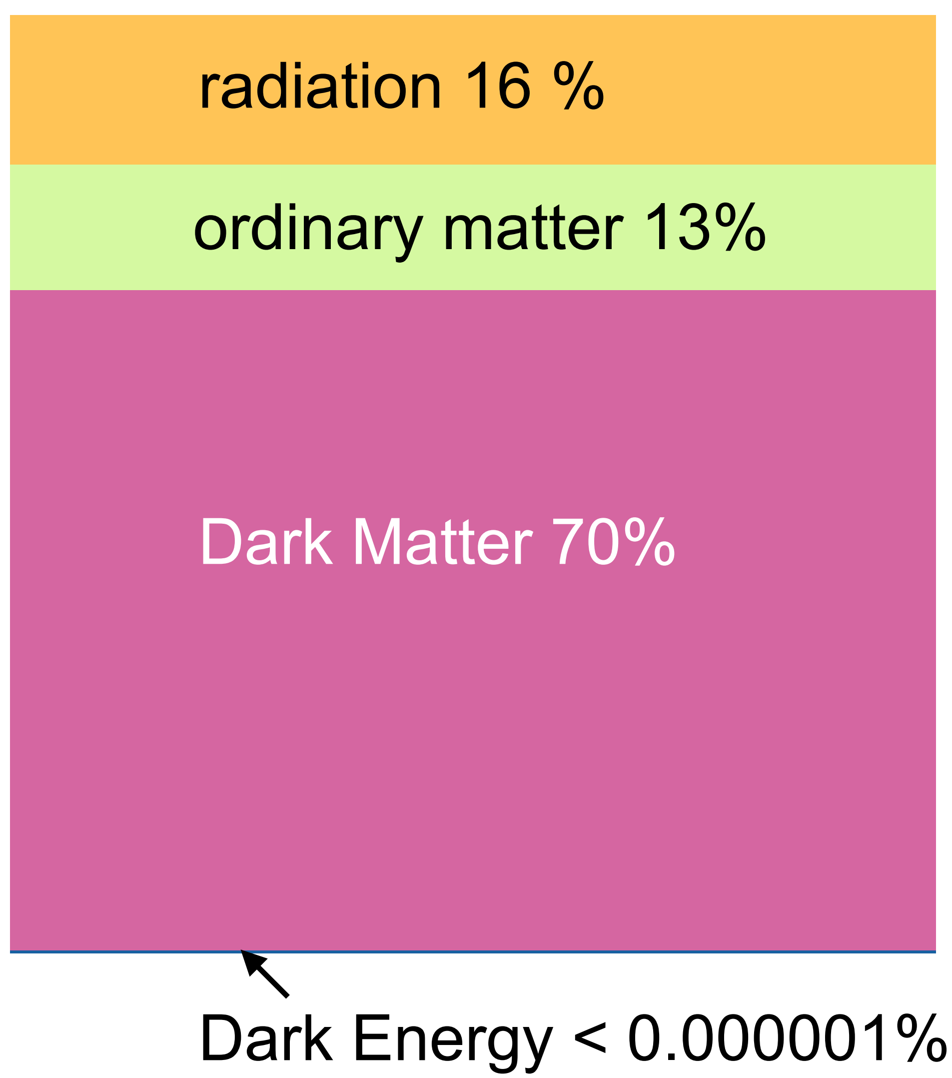



The energy content of the Universe

at half its age, i.e. 7 billion years ago

redshift $z = 0.8$

# Energy content with %-precision

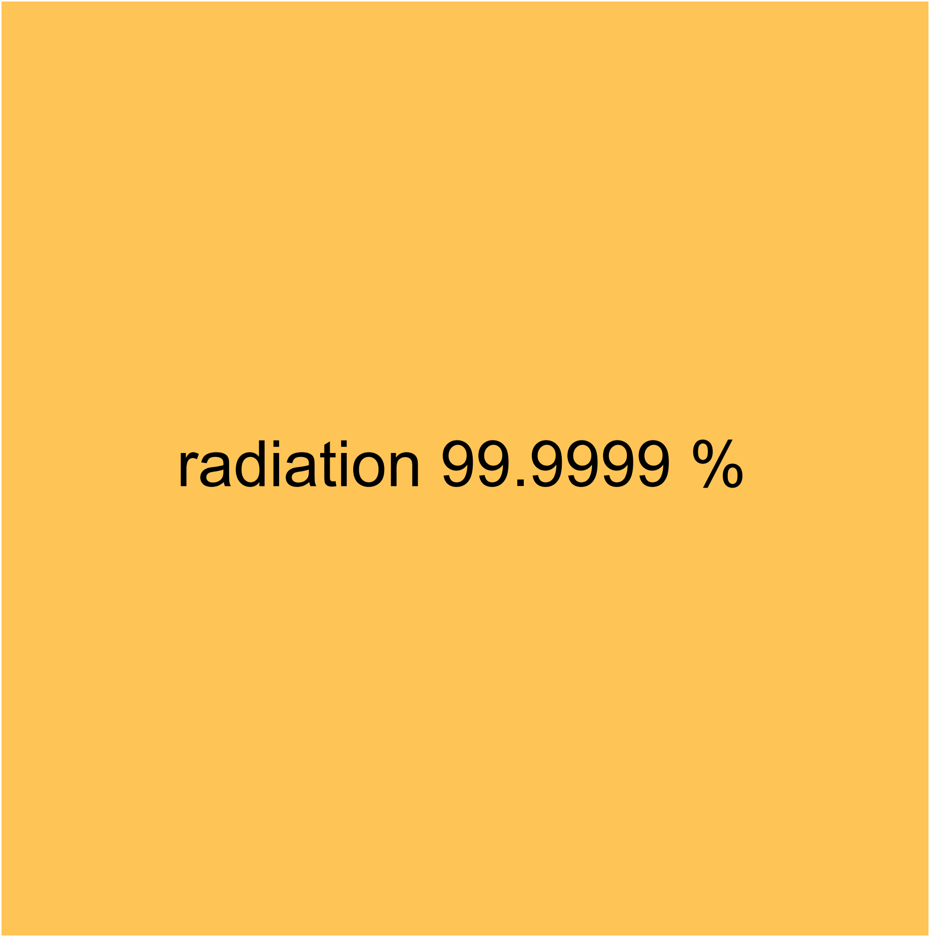



The energy content of the Universe

300.000 yrs after the big bang

redshift z = 1100

# Energy content with %-precision



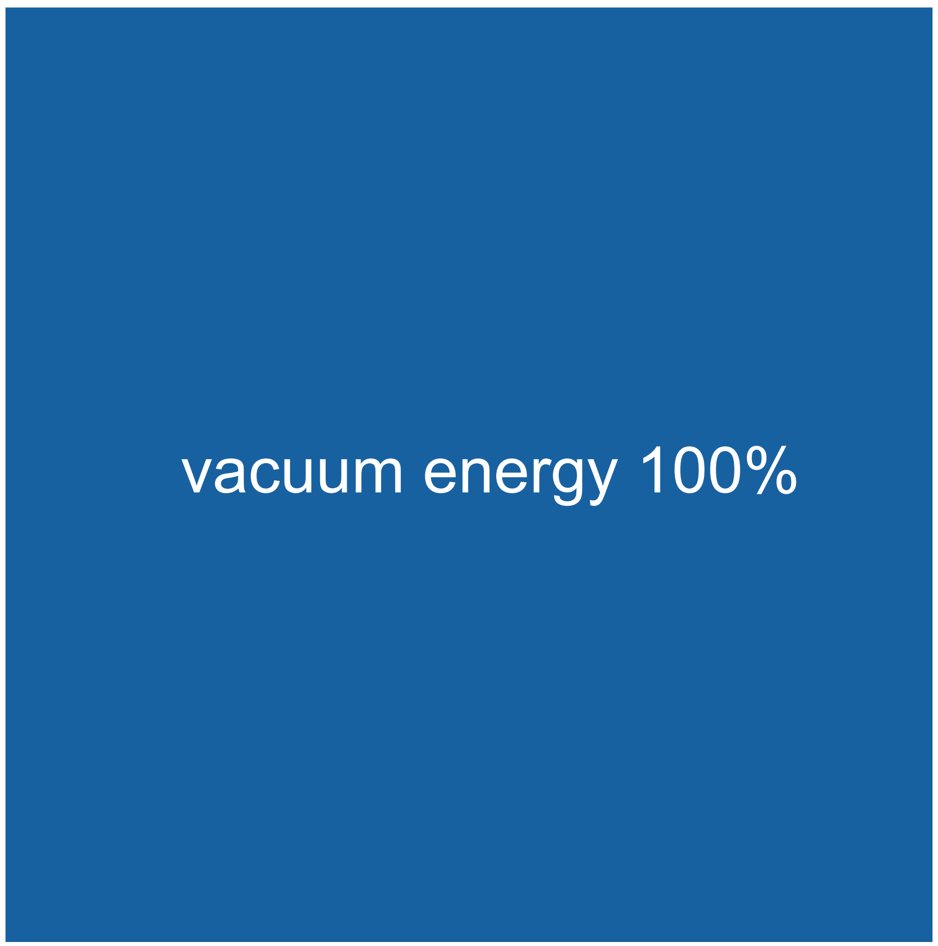

matter < 0.0001%

The energy content of the Universe

at 1 second

redshift z = 1 billion

# Energy content with %-precision

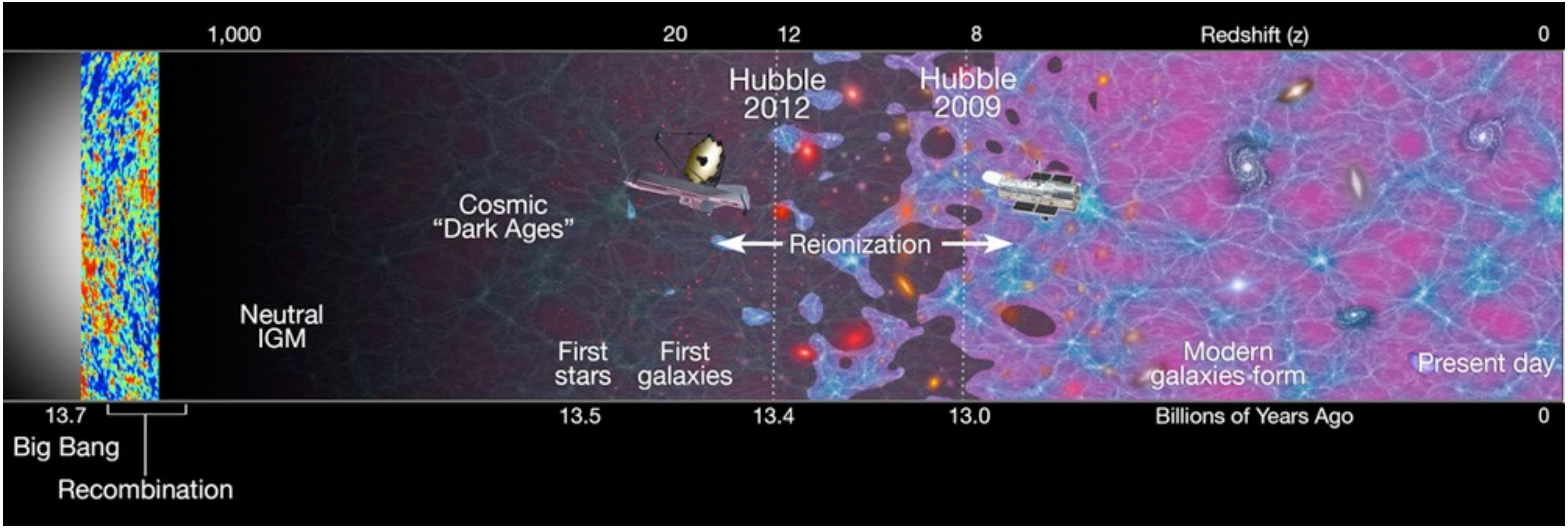



Possibly, the energy content of the Universe

when it “banged” (inflation)

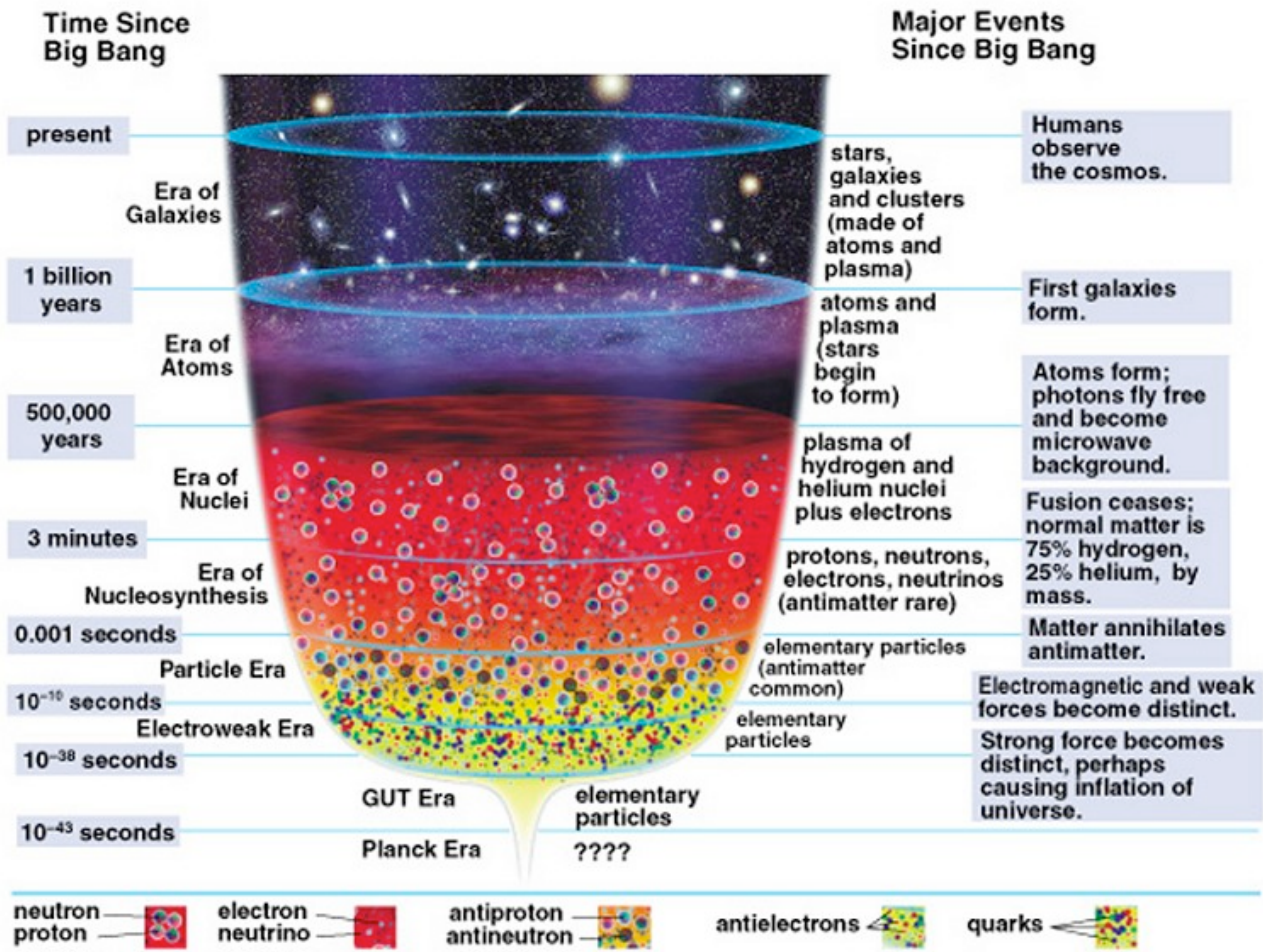
# Cosmic history

An astronomer's view:



# Cosmic history

A particle physicist's view



# Equilibrium Thermodynamics

Basic thermodynamic quantities

$f(\vec{p}, \vec{x}, t)$ *distribution function*

$$n_i = \frac{g_i}{(2\pi)^3} \int f_i(\vec{p}) d^3p$$ *number density*

$$\rho_i = \frac{g_i}{(2\pi)^3} \int E(\vec{p}) f_i(\vec{p}) d^3p$$ *energy density*

$$p_i = \frac{g_i}{(2\pi)^3} \int \frac{|\vec{p}|^2}{3E(\vec{p})} f_i(\vec{p}) d^3p$$ *pressure*

$$E^2(\vec{p}) = m^2 + |\vec{p}|^2$$

$g_i$ internal degrees of freedom of particle

$g_i = 1$ scalar particles

$g_i = 2$ photons (2 polarization degrees)

$g_i = 3$ massive vector particles (rho-meson, Z-boson)

$g_i = 4$ electrons + positrons (2 spin x 2 charges)

# Equilibrium Thermodynamics

Distribution function $f(\vec{p}, \vec{x}, t)$

Interaction rate of a particle species $\Gamma_{\rm int} \gg H$ Hubble rate

$$\Gamma_{\rm int} \sim n \times v \times \sigma$$

$$\frac{1}{\rm cm^3} \times \frac{\rm cm}{\rm s} \times {\rm cm}^2 = \frac{1}{\rm s}$$

If many interactions occur within the expansion time scale of the Universe

=> species has time to reach thermal equilibrium

$$f_i = \frac{1}{e^{(E_i - \mu_i)/T_i} \pm 1}$$

$+$ for fermions

$-$ for bosons

=> Maxwell-Boltzmann approximation: $f_i = e^{-(E_i - \mu_i)/T}$

# Kinetic vs. chemical equilibrium

Elastic scattering processes are generally much more frequent than inelastic reactions that change particle number.

$\gamma e^- \leftrightarrow \gamma e^-$

elastic; stops at recombination (T ~ eV)

$e^+ e^- \leftrightarrow \gamma\gamma$

inelastic; stops at ~20 keV

Elastic process i + j <—> i + j lead to equilibration of temperatures

$T_i = T_j$ *kinetic equilibrium*

Inelastic processes i + j <—> k + l lead to equilibration of chemical potentials

$\mu_i + \mu_j = \mu_k + \mu_l$ *chemical equilibrium*

Note that if $\chi\bar{\chi} \leftrightarrow e^+ e^- \leftrightarrow \gamma\gamma$ is in chemical equilibrium, $\mu_\chi + \mu_{\bar{\chi}} = 2\mu_\gamma = 0$, so that

$\mu_\chi = -\mu_{\bar{\chi}}$

(photons are their own antiparticles, so their chemical potential vanishes identically)

# Equilibrium Thermodynamics

Evaluating the integrals explicitly (for vanishing chemical potential)

| | Relativistic Bosons | Relativistic Fermions | Non-relativistic (Either) |
|---|---|---|---|
| $n_i$ | $\frac{\zeta(3)}{\pi^2} g_i T^3$ | $\left(\frac{3}{4}\right) \frac{\zeta(3)}{\pi^2} g_i T^3$ | $g_i \left(\frac{m_i T}{2\pi}\right)^{3/2} e^{-m_i/T}$ |
| $\rho_i$ | $\frac{\pi^2}{30} g_i T^4$ | $\left(\frac{7}{8}\right) \frac{\pi^2}{30} g_i T^4$ | $m_i n_i$ |
| $p_i$ | $\frac{1}{3}\rho_i$ | $\frac{1}{3}\rho_i$ | $n_i T \ll \rho_i$ |

Remember our estimate for the photon number density $n_\gamma = 0.25 T^3$ ? $\quad \frac{1}{4} \simeq 2 \times \frac{\zeta(3)}{\pi^2}$

If relativistic particles dominate the energy budget:

$$\rho = \sum_i \rho_i \simeq \frac{\pi^2}{30} g_{\text{eff}} T^4 \qquad\qquad g_{\text{eff}} = \sum_{\text{bosons}} g_i \left(\frac{T_i}{T}\right)^4 + \frac{7}{8} \sum_{\text{fermions}} g_i \left(\frac{T_i}{T}\right)^4$$

=> useful for parametric estimates: $H \sim \sqrt{\rho}/M_P \sim T^2/M_P$ (when radiation dominant)

# Entropy

Second law of thermodynamics => For the entropy density $s = S/a^3$ it follows

$$TdS = d(\rho V) + pdV \quad \Rightarrow \quad s = \frac{\rho + p}{T}$$

$$s = \frac{2\pi}{45} h_{\mathrm{eff}} T^3 \qquad h_{\mathrm{eff}} = \sum_{\mathrm{bosons}} g_i \left(\frac{T_i}{T}\right)^3 + \frac{7}{8} \sum_{\mathrm{fermions}} g_i \left(\frac{T_i}{T}\right)^3$$

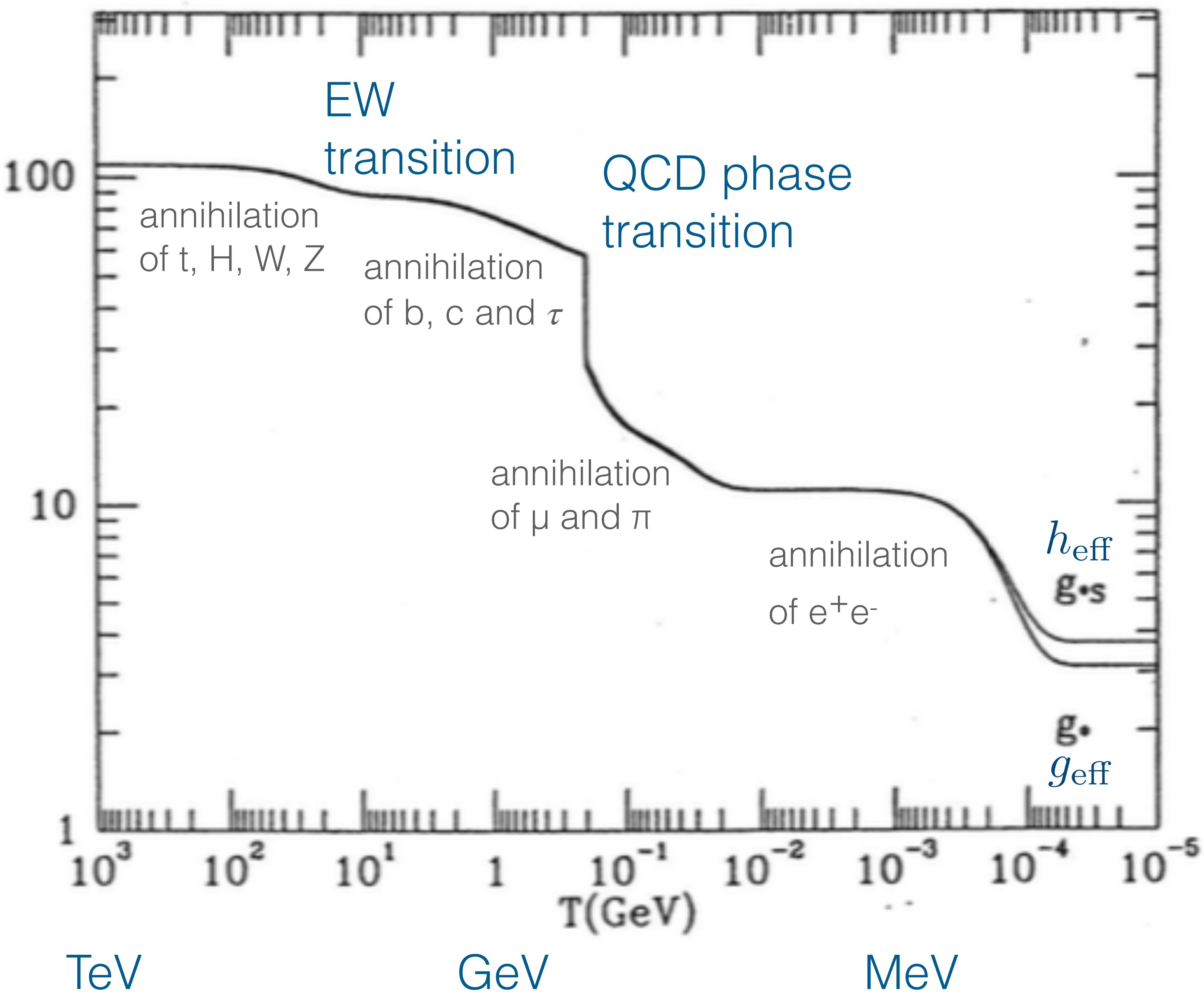
We get an expression for the evolution of photon temperature as a function of scale factor when comoving entropy is conserved (Universe's expansion is adiabatic)

$$\frac{d(a^3 s)}{dt} = 0 \quad \Rightarrow \quad s \propto a^{-3}$$

Comparing entropy with energy density yields evolution of the photon temperature $T = T_\gamma$

$$T \propto h_{\mathrm{eff}}^{-1/3} a^{-1}$$

# Evolution of relativistic degrees of freedom

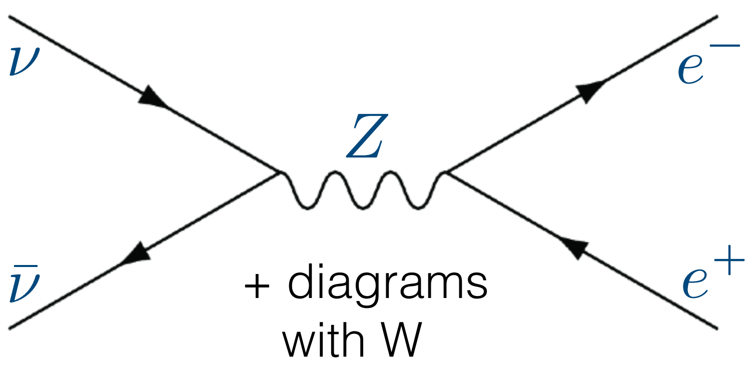



Plot assumes Standard Model field content

< — What happened there?

# Neutrino decoupling

Our assumption for thermal equilibrium was $\Gamma_{\text{int}} \gg H$




Let us evaluate $\Gamma_{\text{int}} \sim n \times v \times \sigma$ for $\nu e \leftrightarrow \nu e,\ \bar{\nu}\nu \leftrightarrow e^+ e^-$

$$\left.\begin{aligned} &\sigma_{\text{weak}} \sim G_F^2 E_{\text{cm}}^2 \sim G_F^2 T^2 \\ &v = 1 \\ &n \sim T^3 \end{aligned}\right\} \quad \frac{\Gamma_{\text{int}}}{H} \sim G_F^2 T^5 \times \frac{M_P}{T^2} \sim O(1) \times \left(\frac{T}{1\,\text{MeV}}\right)^3$$

Neutrino interactions “freeze out” at T<1MeV

# Relic Neutrinos

After neutrino decoupling, entropies in the respective fluids ($e\gamma$ and $\nu$) are conserved separately

$$a_1^3 s_{e\gamma} = a_2^3 s_{e\gamma}$$

| before electron-positron annihliation | after electron-positron annihilation |
|---|---|

$$s_{e\gamma}(a_1) \propto \left(g_\gamma + \frac{7}{8} g_e\right) T_\gamma^3(a_1) = \frac{11}{2} T_\gamma^3(a_1)$$

$$s_{e\gamma}(a_2) \propto g_\gamma T_\gamma^3(a_2) = 2 T_\gamma^3(a_2)$$

$$\frac{T_\gamma(a_1)}{T_\gamma(a_2)} = \left(\frac{4}{11}\right)^{1/3} \frac{a_2}{a_1}$$

$$a_1^3 s_\nu = a_2^3 s_\nu$$

$$\frac{a_2}{a_1} = \frac{T_\nu(a_1)}{T_\nu(a_2)}$$

Before decoupling, neutrinos and photons shared the same temperature $T_\gamma(a_1) = T_\nu(a_1)$

It follows that $T_\nu(a_2) = \left(\frac{4}{11}\right)^{1/3} T_\gamma(a_2)$

$T_\gamma(\text{today}) = 2.7\,\text{K}$ CMB

$T_\nu(\text{today}) = 1.95\,\text{K}$ CνB!

# Relic Neutrinos

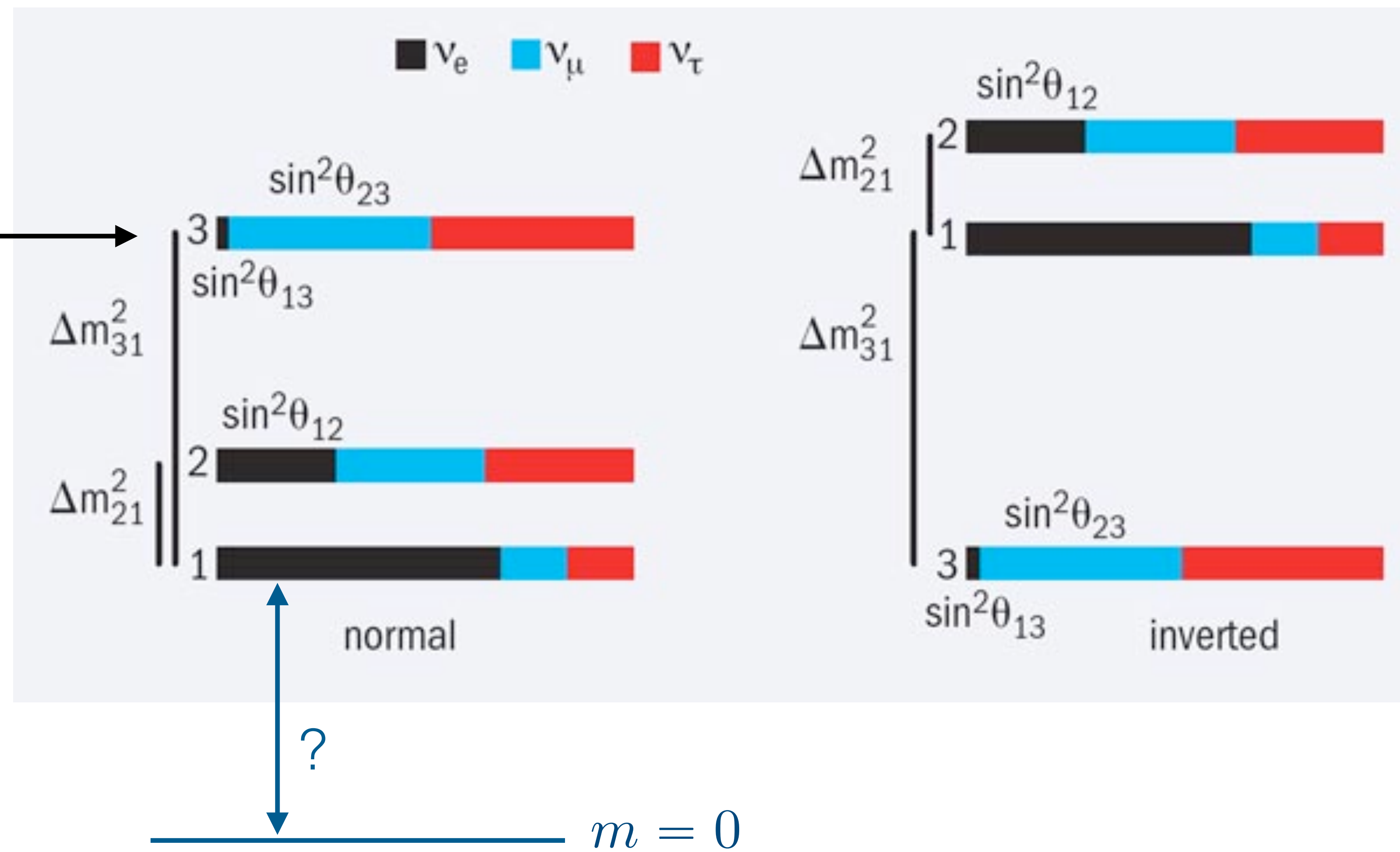
What is the energy density of neutrinos today?

There is at least one neutrino with mass

$$\sqrt{(\Delta m)^2_{\rm atm}} \sim 0.05\,{\rm eV}$$

$$(\sim 600\,{\rm K})$$

and it is non-relativistic.



# Relic Neutrinos

What is the energy density of neutrinos today?

$$m_{\text{tot}} = \sum_{\nu} (g_\nu/2) m_\nu$$

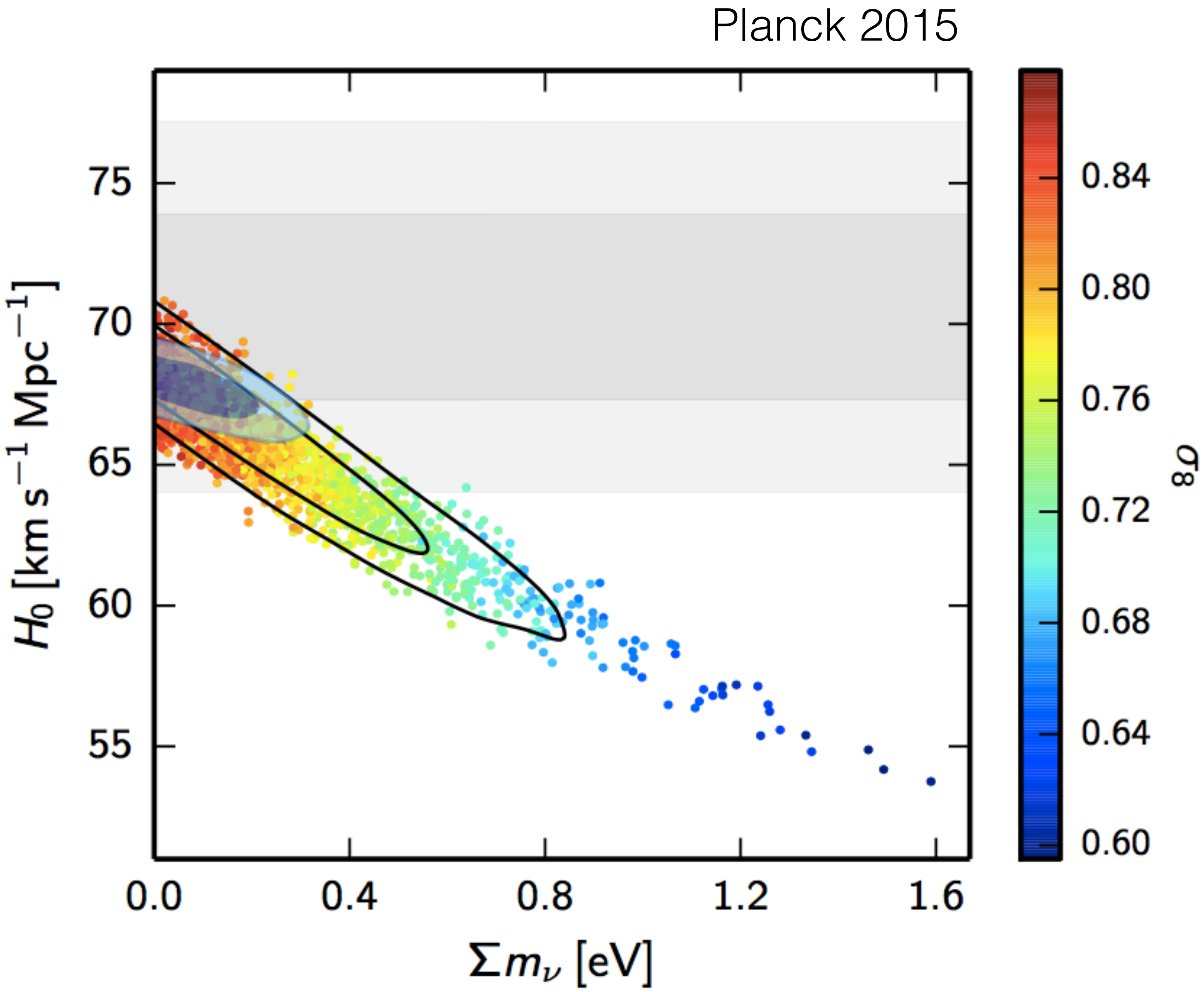
$$\rho_\nu = m_{\text{tot}} \times (3/11) n_\gamma(t_0)$$

$$\Omega_\nu = \frac{\rho_\nu(t_0)}{\rho_c} = \frac{m_{\text{tot}} n_{\nu_i}(t_0)}{\rho_c} = \frac{m_{\text{tot}}}{94 h^2\,\text{eV}}$$

Best limits from CMB, clustering of galaxies, Lyman-α forest, BAO

$$m_{\text{tot}} \lesssim 0.2\,\text{eV}$$

=> neutrinos cannot be Dark Matter



# Evidence for Dark Matter

# Stellar motions (kpc)

Follow the motion of stars
off the galactic disc


- follow the kinematics of stars

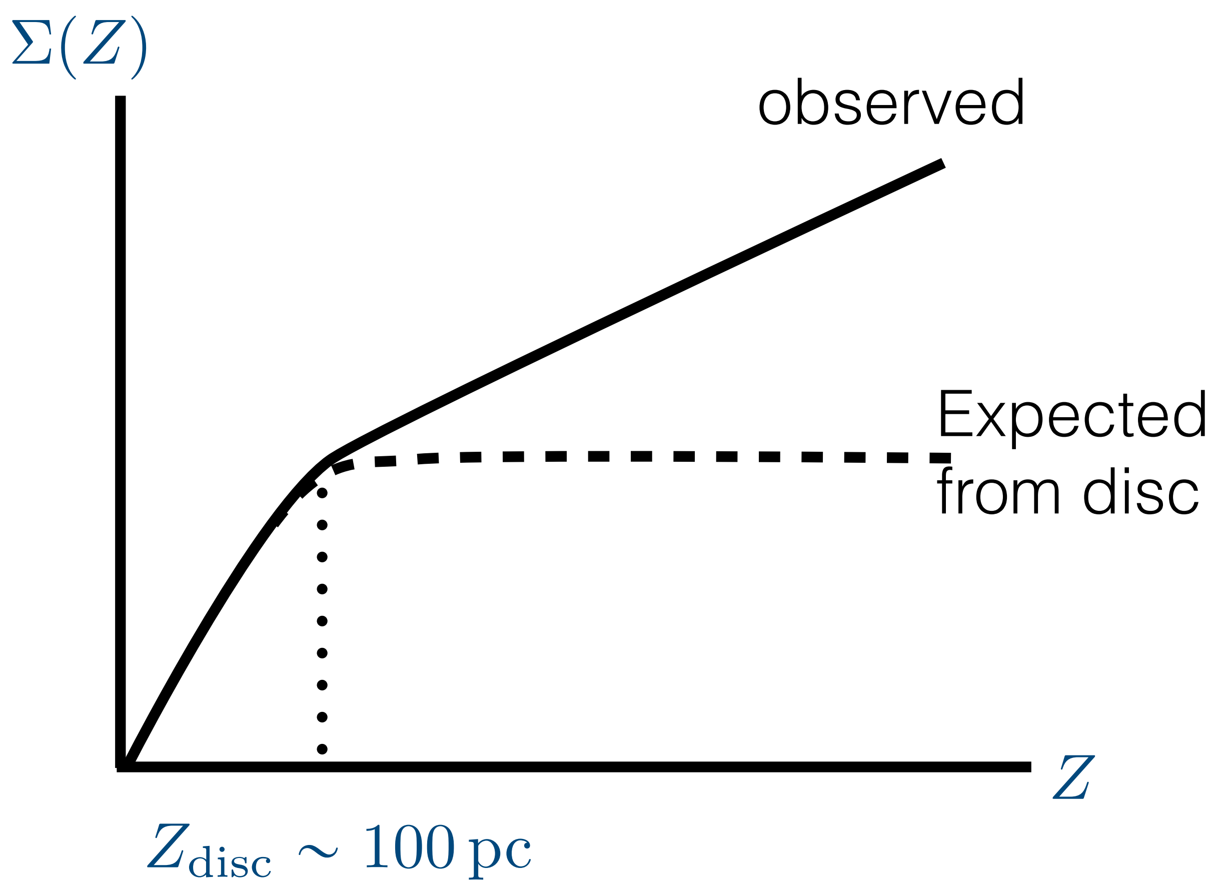
$$\Sigma(Z) = \int_{-Z}^{Z} dz \rho(z)$$

- local dark matter density

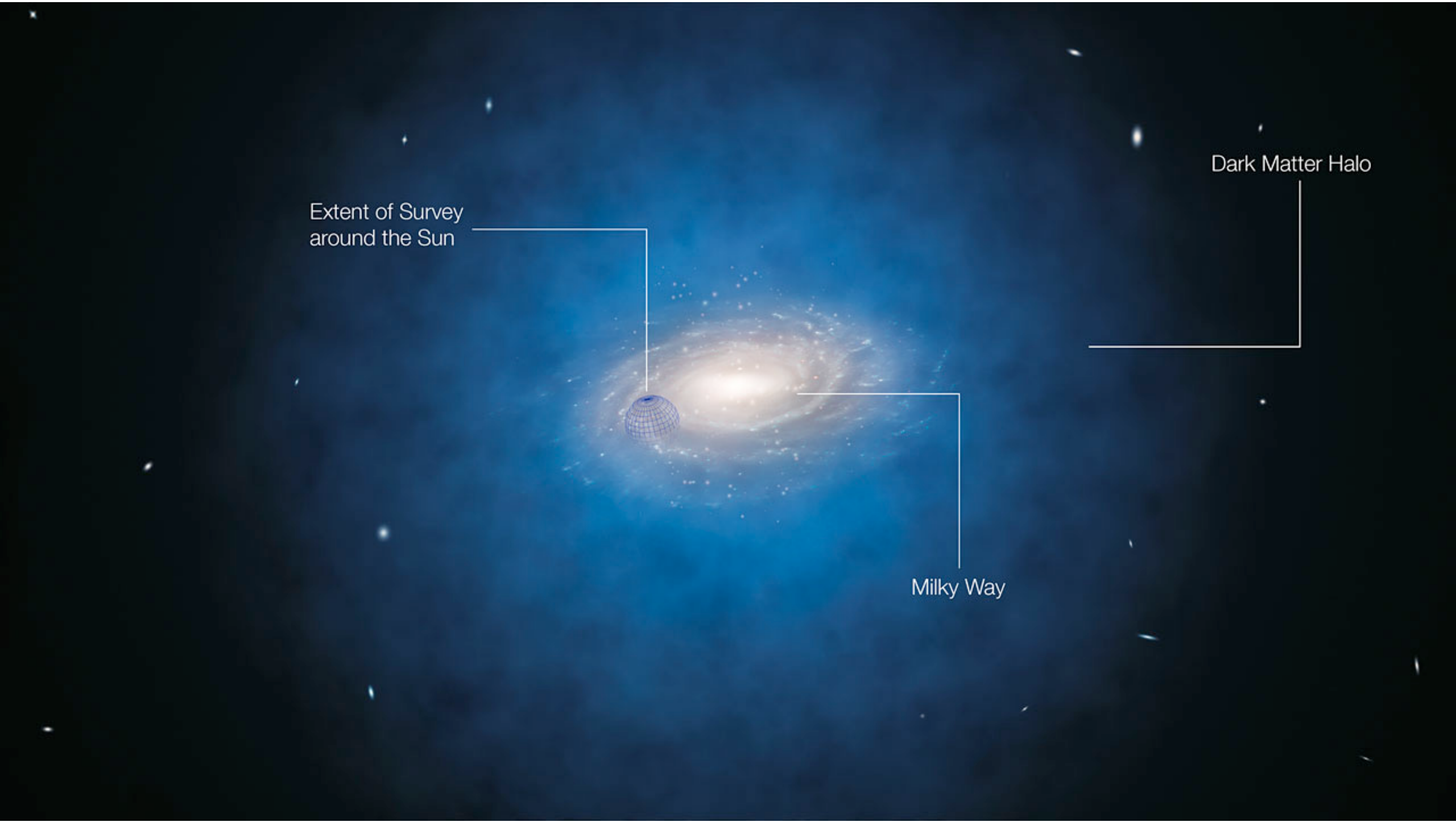
$$\rho_{\mathrm{dm}} = (0.3 \pm 0.1)\,\mathrm{GeV/cm^3}$$

Bovy, Tremaine 2012

- evidence for a halo

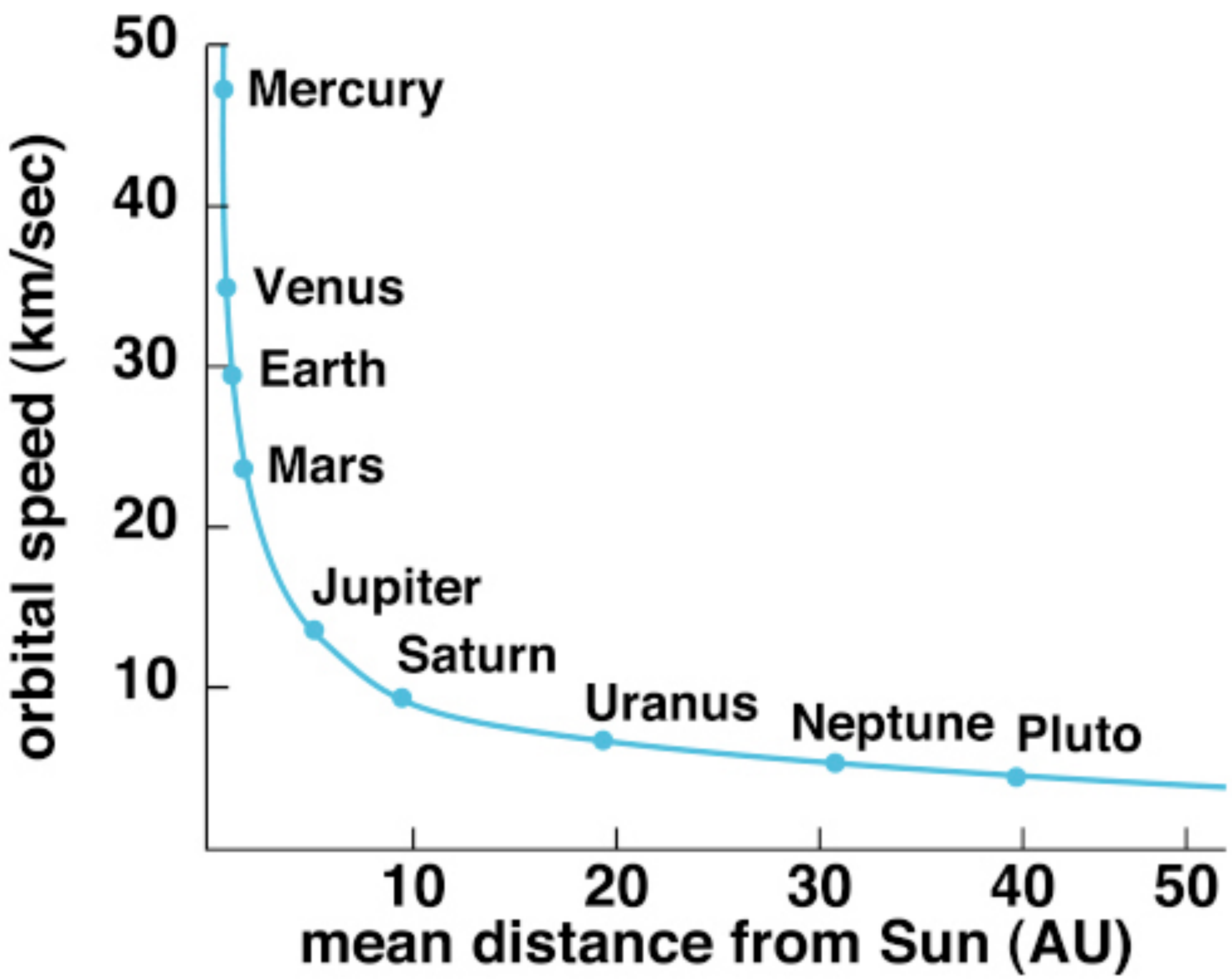


# Stellar motions (kpc)



# Rotation curves (10's kpc)

Consider the solar system

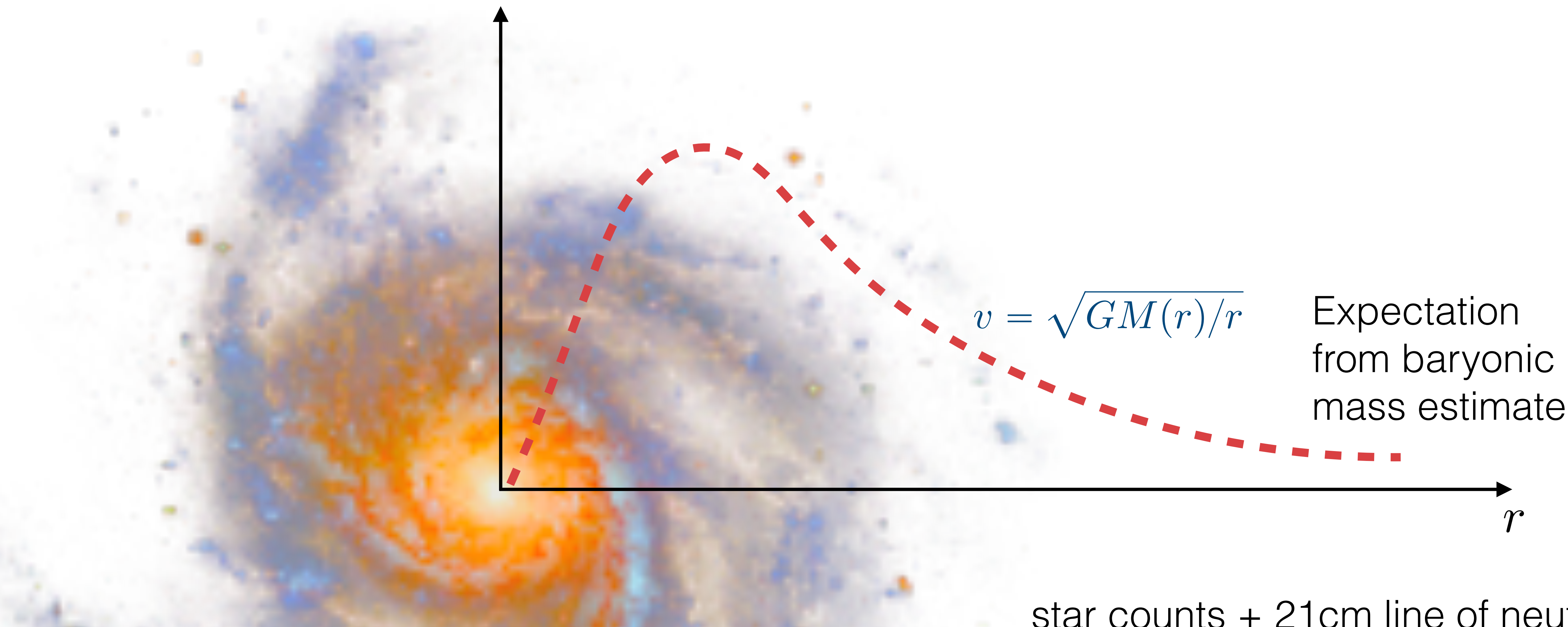



Orbit: $\dfrac{GmM(r)}{r^2} = \dfrac{mv^2}{r}$

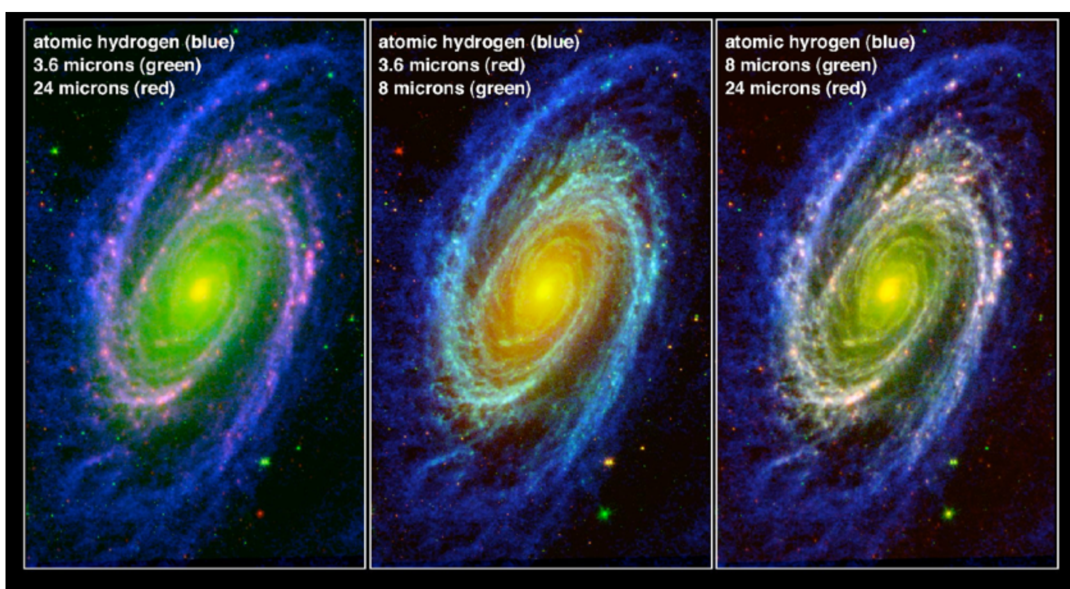
=> $v = \sqrt{GM(r)/r}$

Observations are explained with $M = M_\odot \simeq 10^{30}\,\mathrm{kg}$

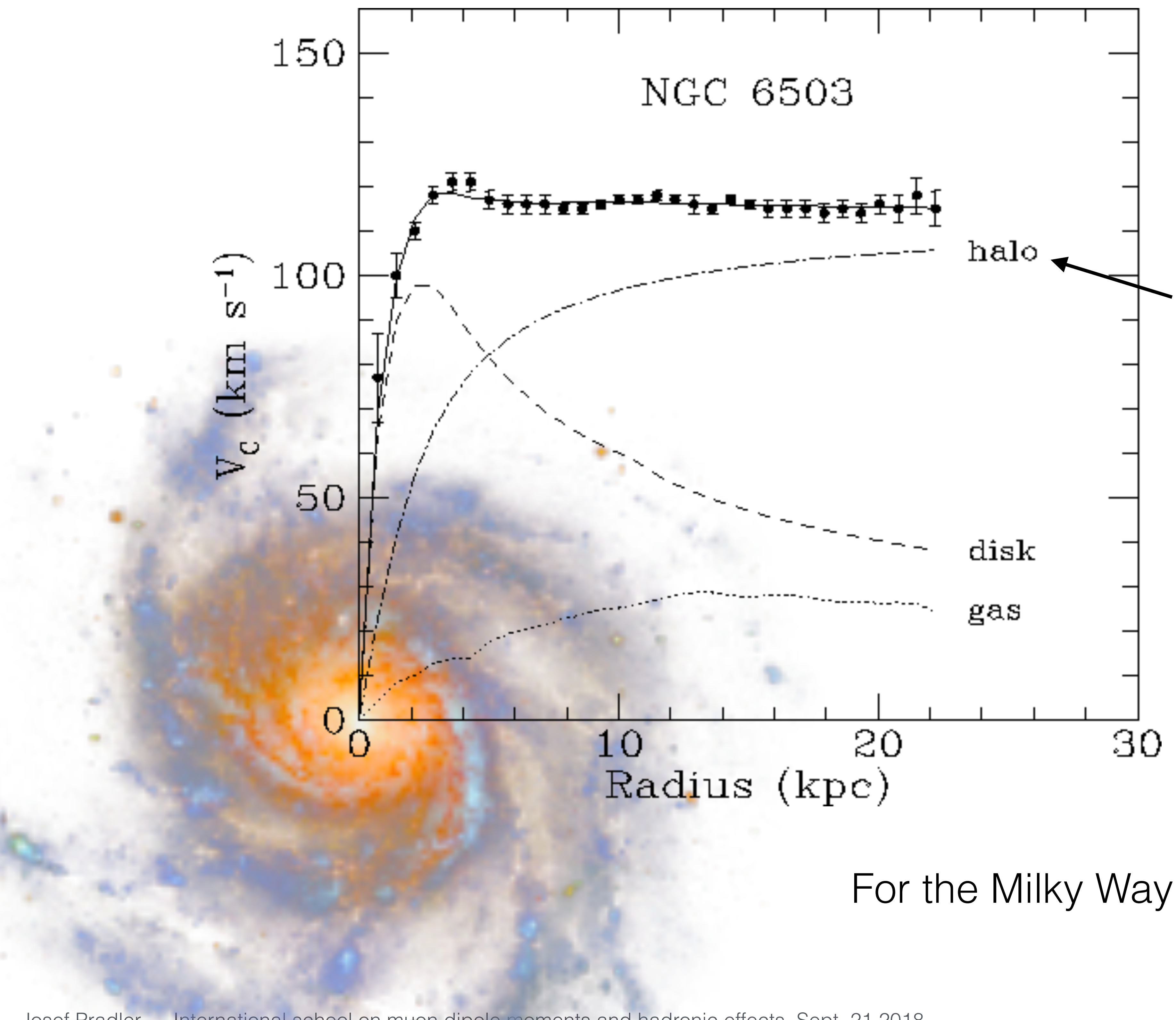
# Rotation curves (10’s kpc)




star counts + 21cm line of neutral H



# Rotation curves (10’s kpc)




observation

missing mass

expectation

For the Milky Way fit yields at sun’s position

$$\rho_\odot \simeq (0.3 \pm 0.1)\,\mathrm{GeV/cm^3}$$

# Weighing galaxies (10's kpc)

- satellite galaxies as "test particles"

$$M_{\rm halo} = (5-30)\times 10^{11}\,M_\odot$$

Watkins, Evans, 2010
Deason et al., 2012

- Milky Way luminosity

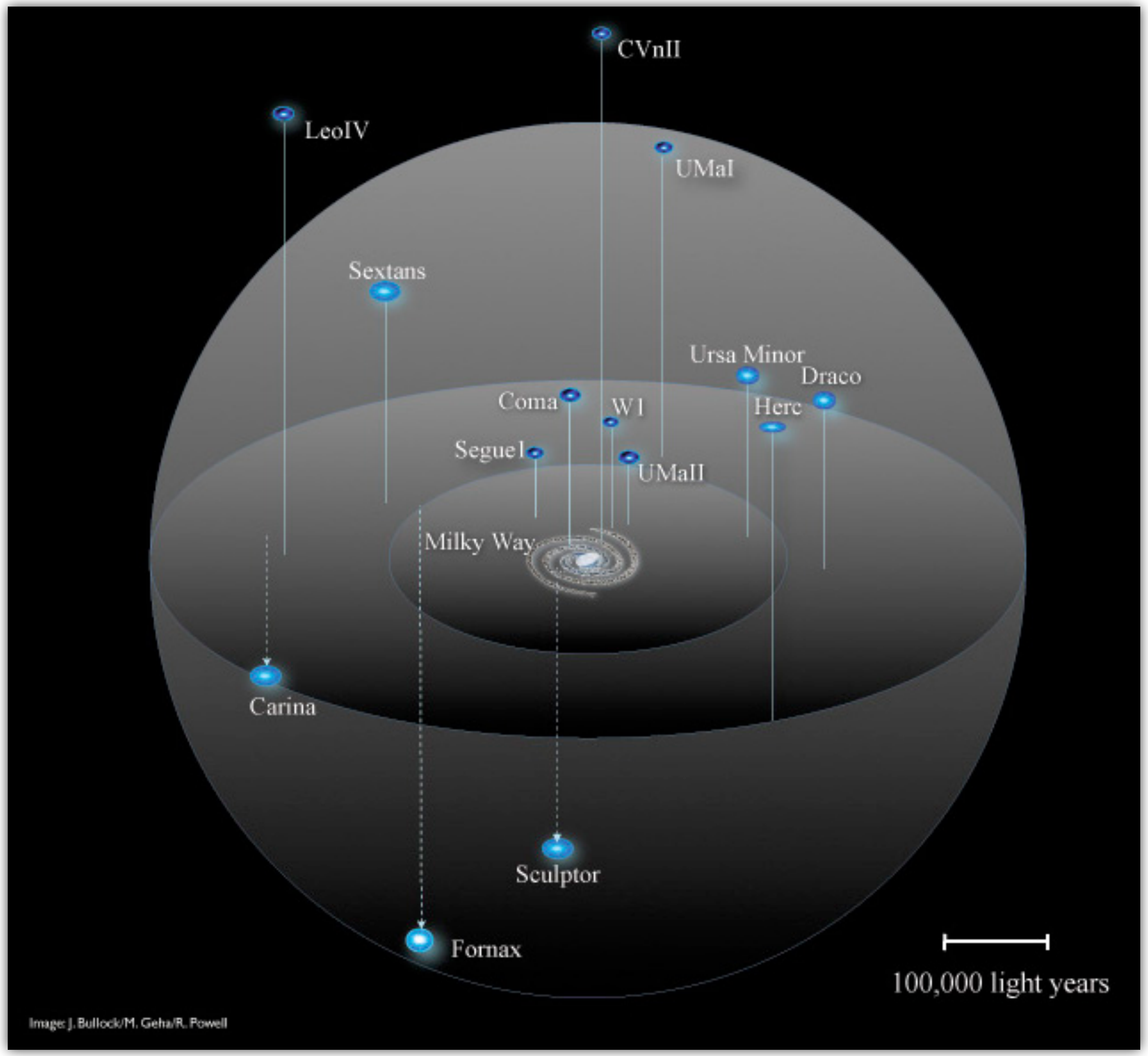
$$L \simeq 2\times 10^{10} L_\odot$$

e.g. Faber Gallagher 1997

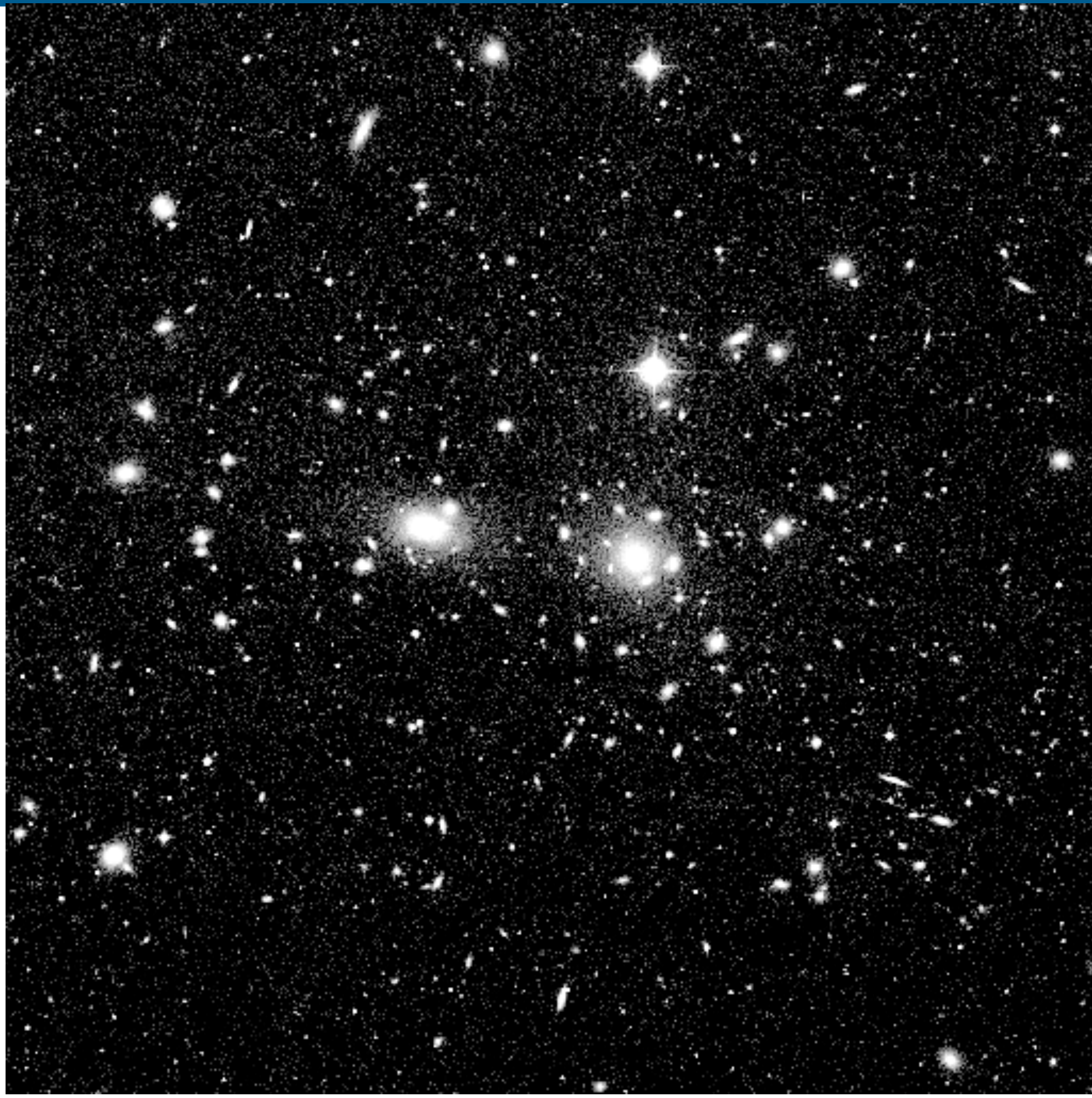
- Dwarf spheroidal galaxies

$$\left.\frac{M}{L}\right|_{\rm dSph} = O(100)\frac{M_\odot}{L_\odot}$$

e.g. Wu 2007



# Motion of galaxies (Mpc)



Coma cluster

In an equilibrated cluster of galaxies

- Virial theorem connects average kinetic and gravitational binding energy

$$\langle E_{\rm kin}\rangle = -\frac{1}{2}\langle E_{\rm grav}\rangle$$

can be inferred from the motion of galaxies

depends on the mass of the entire cluster of galaxies

- Zwicky first observed 1932 that there are 1000s of galaxies in Coma Cluster and measured radial velocities and their dispersion
=> "*kalte dunkle Materie*"

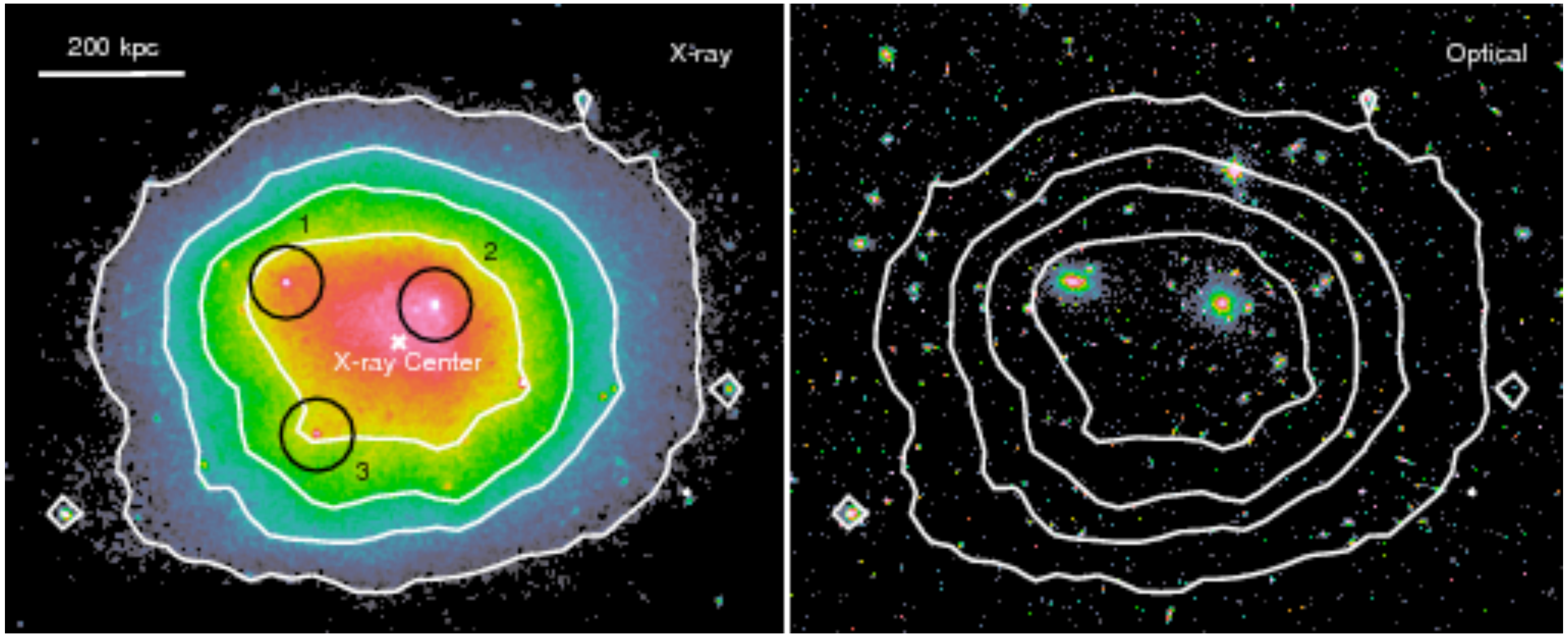
$$\left.\frac{M}{L}\right|_{\rm Coma} \simeq 160\frac{M_\odot}{L_\odot}$$

Fusco-Femiano, Hughes 1994

Fritz Zwicky

# Cluster observations today

- The Coma cluster again




Most of the baryonic mass in clusters is in form of hot, intergalactic gas which can be observed in X rays (=keV energies).

Bolometric X-ray luminosity <=> baryonic gas density

X-ray spectrum <=> pressure <=> potential depth

$$\frac{dP_{\mathrm{gas}}}{dr} = -\frac{GM(r)\rho_{\mathrm{gas}}(r)}{r^2}$$

Equation of hydrostatic equilibrium
(gravity is balanced by pressure gradient force)

*=> Cluster is dominated by non-luminous matter (by a factor of ~7)*

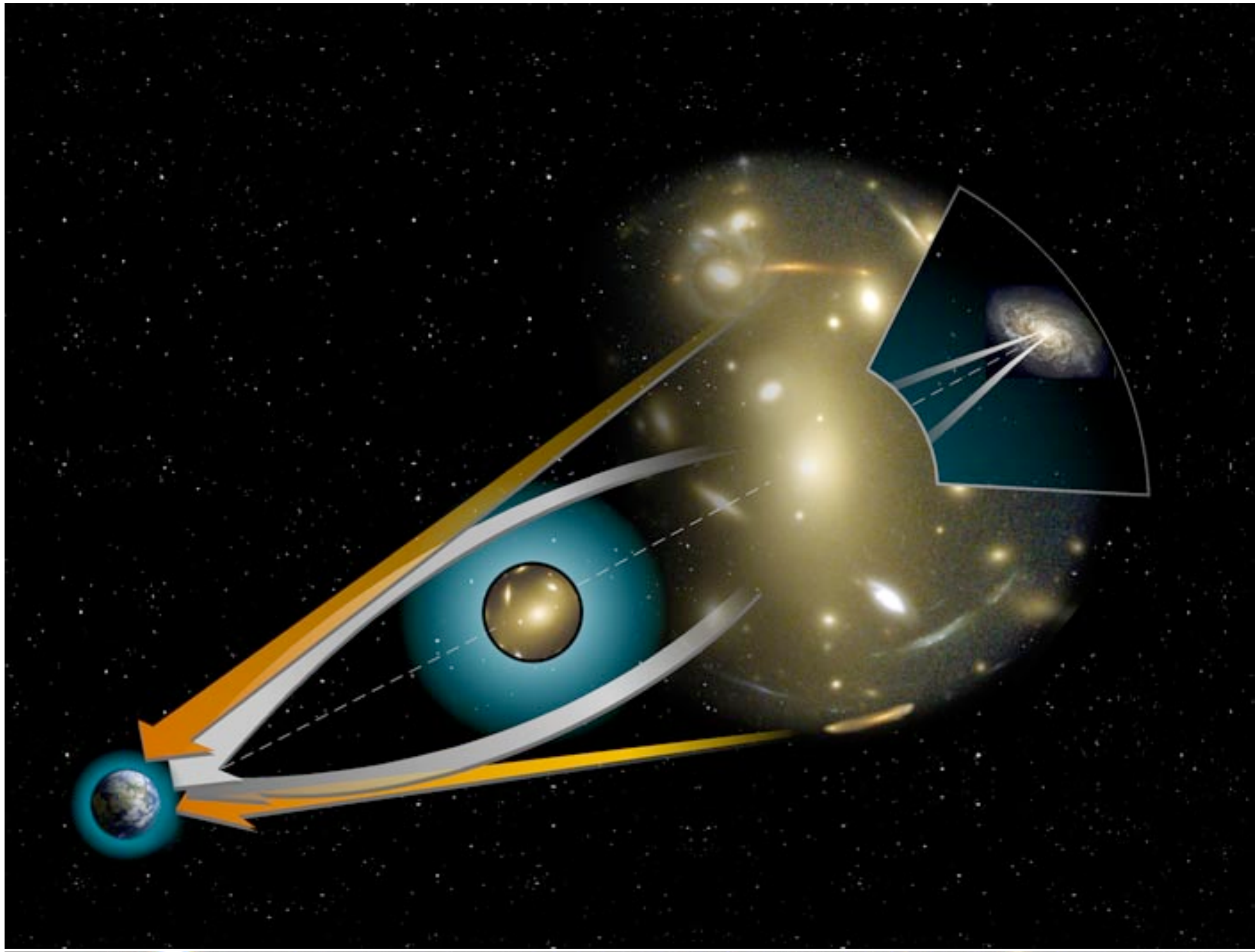
# Gravitational lensing

- Gravitational lensing magnifies and distorts the images of background galaxies or quasars (weak lensing) or makes multiple images of galaxies (strong lensing)

Deflection angle $\alpha$ from a spherical mass distribution

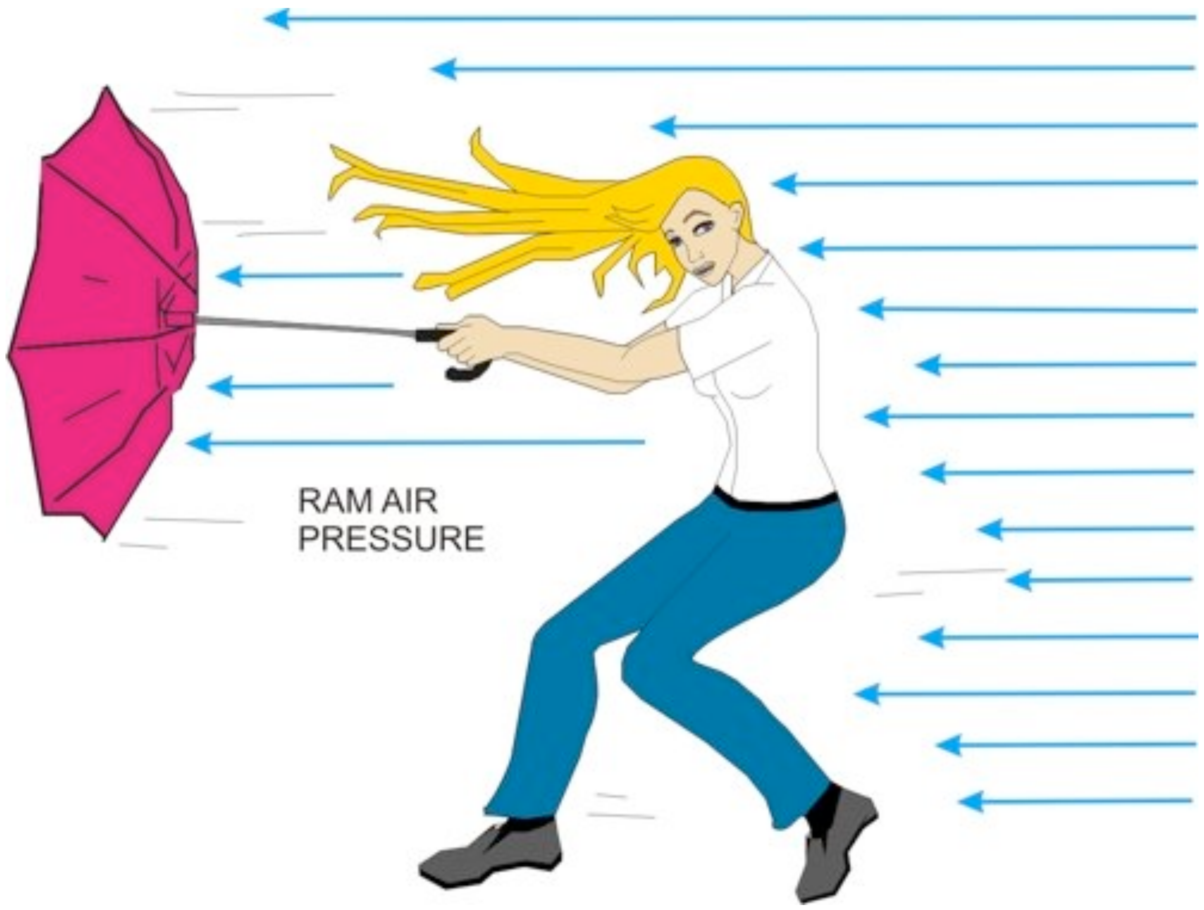
$$\alpha \simeq \frac{4GM(b)}{b}$$

=> gravitational potential is dominated by unseen matter

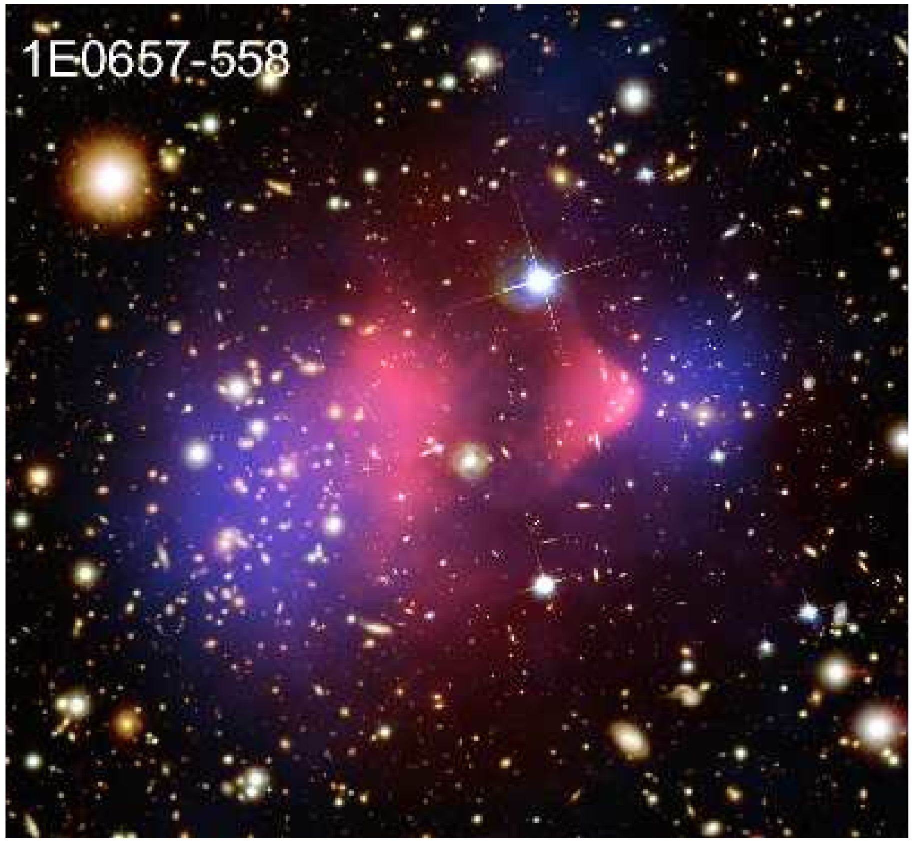


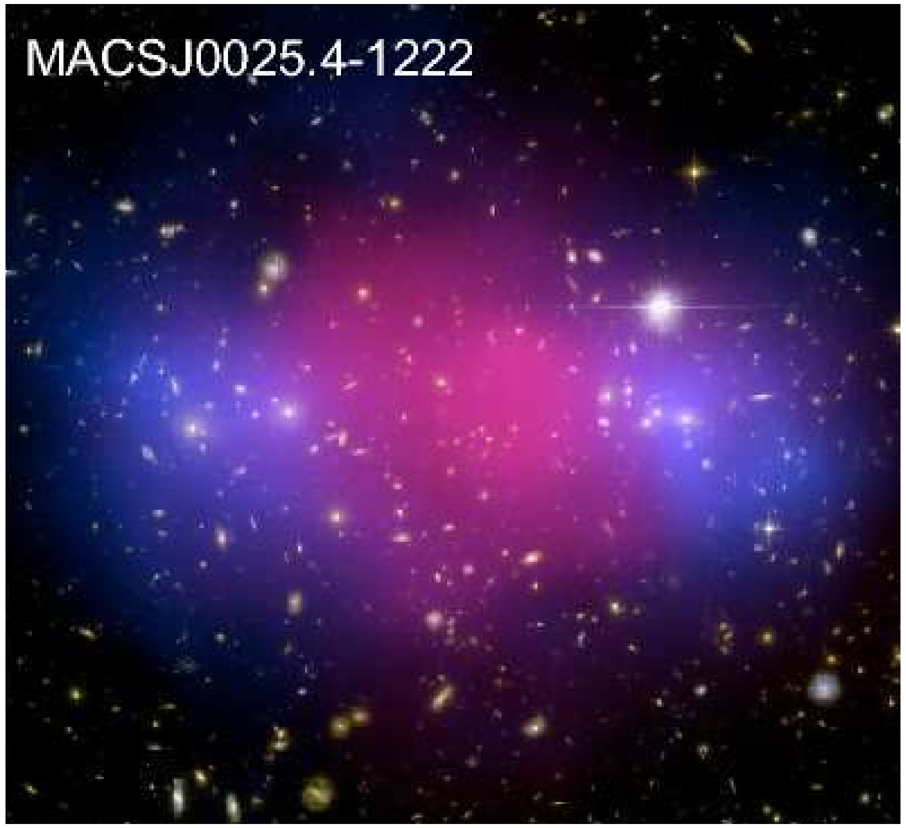
# Collisions of clusters

- When two clusters collide, the baryonic gas experiences RAM pressure and gets stripped.




- However, most of the matter is collisionless and passes through (Red: hot gas, Blue: matter inferred from gravitational lensing)
- famous example:  Bullet cluster

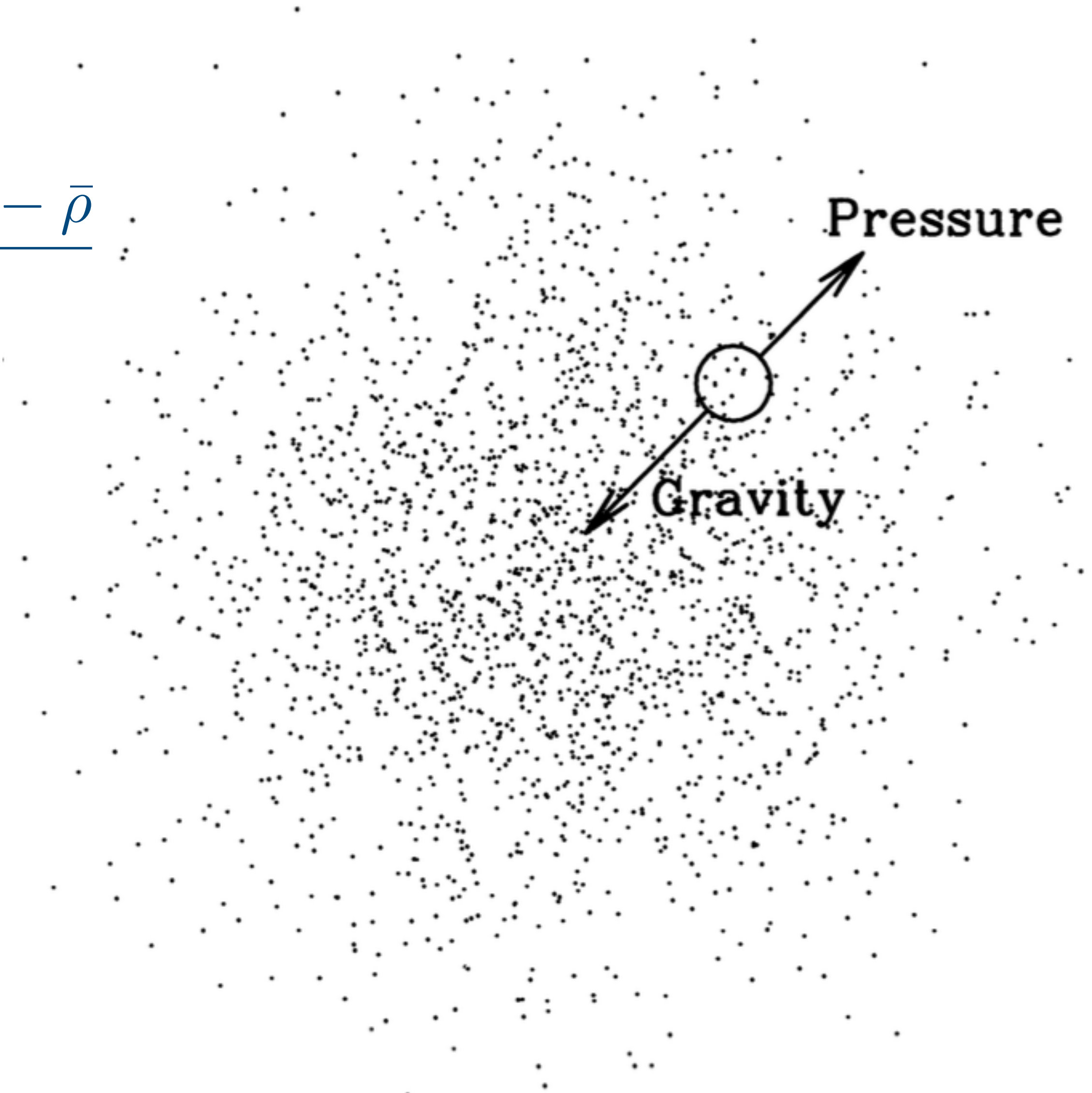


# Growth of structure

Equation of motion for the overdensity $\delta(\vec{x}) \equiv \dfrac{\rho(\vec{x}) - \bar{\rho}}{\bar{\rho}}$

$$\ddot{\delta} + [\text{Pressure} - \text{Gravity}]\,\delta = 0$$




*Pressure small*: exponential solution
*Pressure large*: oscillating solution

In an expanding Universe, the combination of continuity-, Euler-, and Poisson equation yields

$$\ddot{\delta}_k + 2\frac{\dot{a}}{a}\dot{\delta}_k + (k^2 - k_J^2)c_s^2\delta = 0$$

k is the comoving wavenumber of a perturbation $\delta(\vec{x}, t) = \delta_k \sin(\vec{k}\cdot\vec{x})$

The "Jeans scale" $k_J^2 = 4\pi G\bar{\rho}a^2/c_s^2$ separates gravitationally stable from unstable modes

$$\tau_{\text{grav}} \sim (G\bar{\rho})^{-1/2}$$

$$\tau_{\text{pressure}} \sim \lambda/c_s$$

$$\tau_{\text{pressure}} \gtrsim \tau_{\text{grav}} \sim \lambda_J$$

condition for collapse

# Growth of structure

Equation of motion for the overdensity

$$\ddot{\delta}_k + 2\frac{\dot{a}}{a}\dot{\delta}_k + (k^2 - k_J^2)c_s^2\delta = 0 \qquad\qquad k_J^2 = 4\pi G\bar{\rho}a^2/c_s^2$$

In absence of expansion $\dot{a} = 0$, $\delta \sim e^{\pm t/\tau}$ $\qquad \tau = (a/c_s)(k_J^2 - k^2)^{-1/2}$

Large perturbations with wavelength $\lambda = 2\pi/k > \lambda_J$ grow (or decay) exponentially

Small perturbations with wavelength $\lambda = 2\pi/k < \lambda_J$ oscillate (sound waves)

In an expanding Universe there are two significant changes:

1. the Jeans scale becomes a function of time (with drastic change at recombination)

2. growth is “quenched” by expansion: $\delta \sim a$ matter domination

   $\delta \sim \ln a$ radiation domination

# Growth of structure

Radiation dominated epoch $c_s \simeq c/\sqrt{3}$

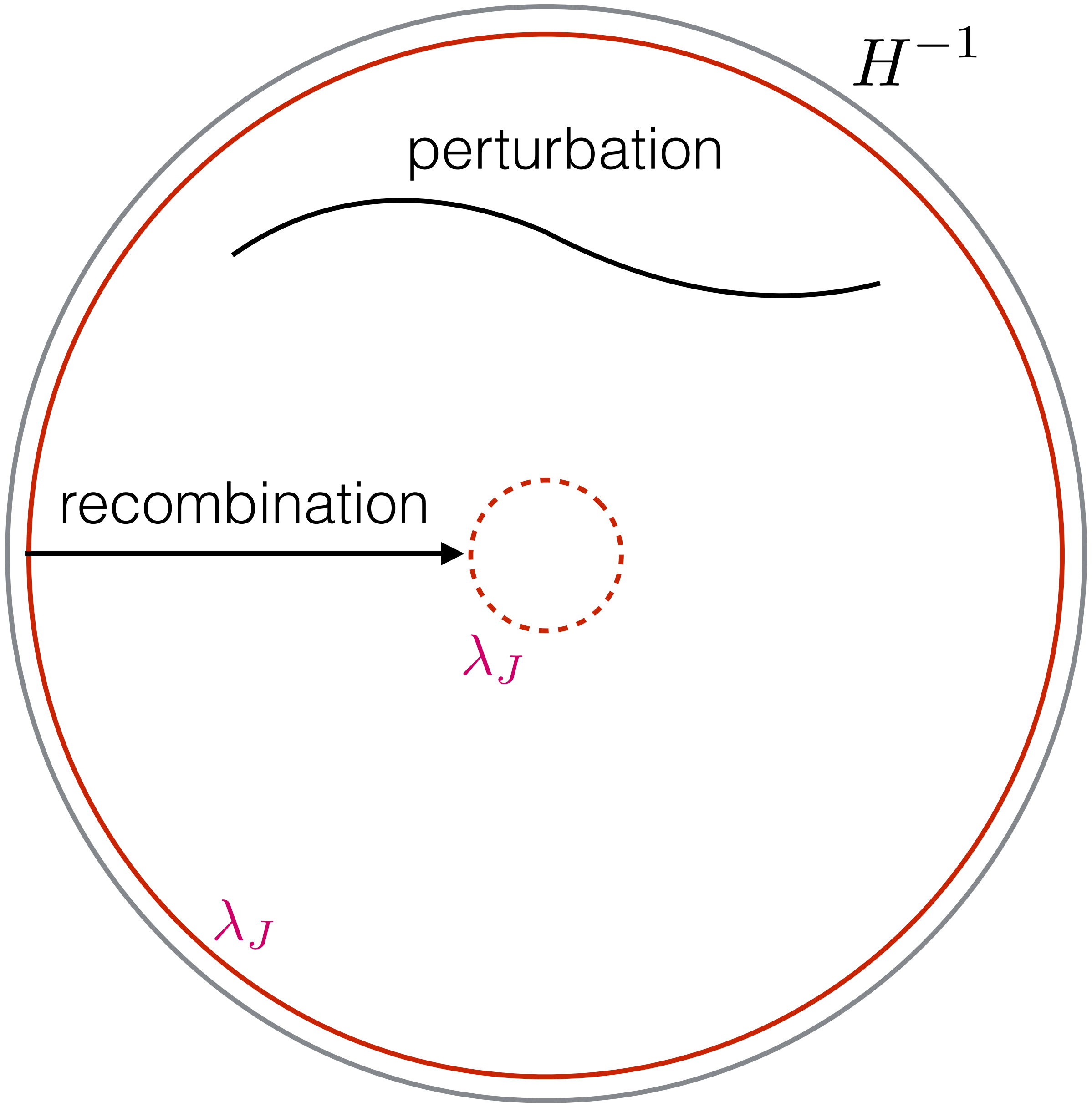
$$\lambda_J = \frac{2\pi}{k_{J,\text{phys}}} = \frac{2\pi a}{k_J} = c_s \sqrt{\frac{\pi}{G\bar{\rho}}} \sim H^{-1}$$

close to the size of the horizon

*No sub-horizon growth of perturbations is possible!*

After recombination, $c_s$ becomes very small, and so does $\lambda_J$

Perturbations grow linearly then.



# The seeds of structure

What was the size of perturbations at recombination, when baryons could collapse?
A view on the cosmic microwave background (CMB) reveals it!

$$\frac{\delta T}{T} < 10^{-4}$$

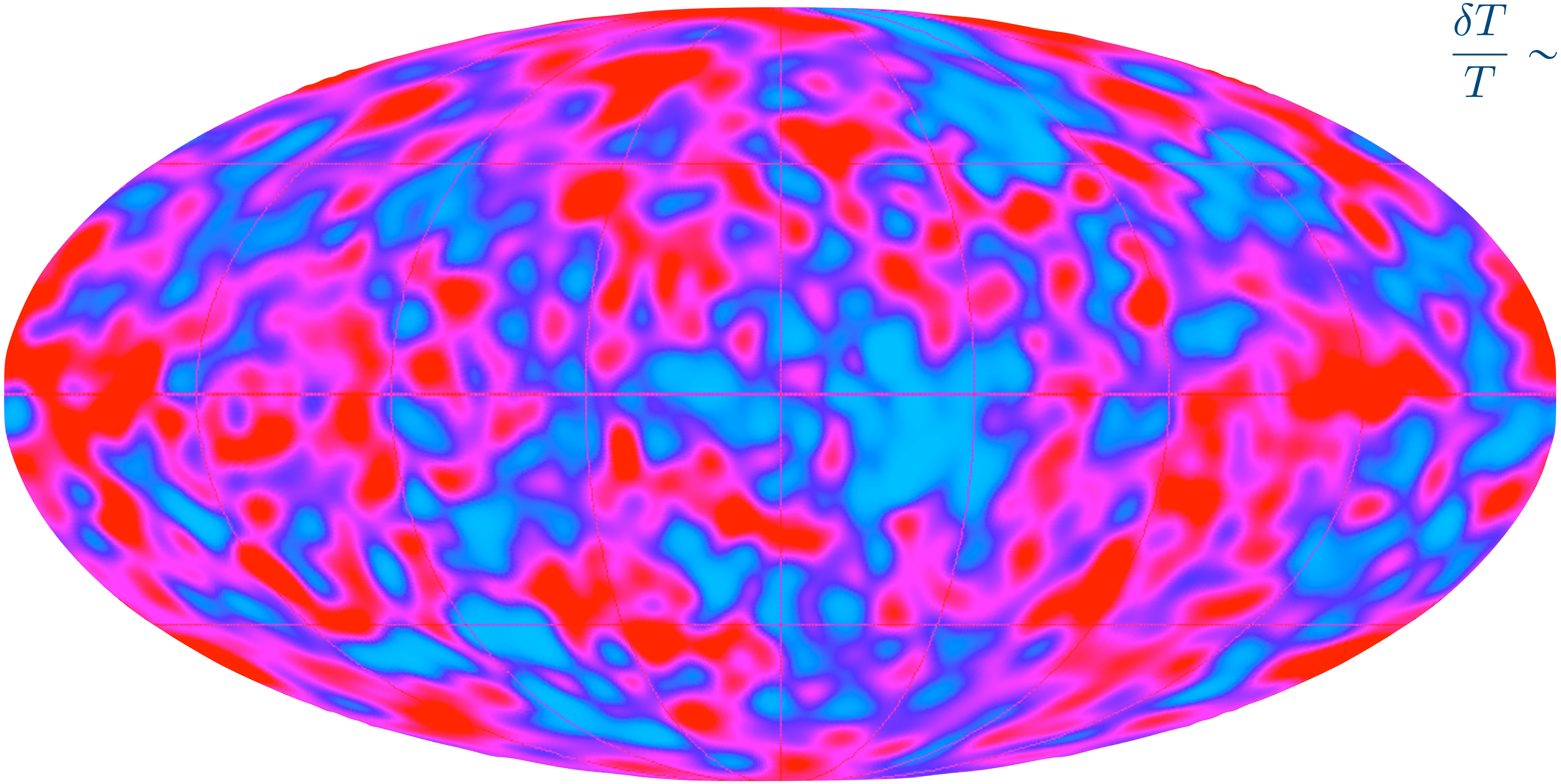
The CMB with an accuracy of 1/10000.

# The seeds of structure

What was the size of perturbations at recombination, when baryons could collapse?
A view on the cosmic microwave background (CMB) reveals it!

NASA Cosmic Background Explorer (COBE 1991-1994)

$$\frac{\delta T}{T} \sim 10^{-5}$$

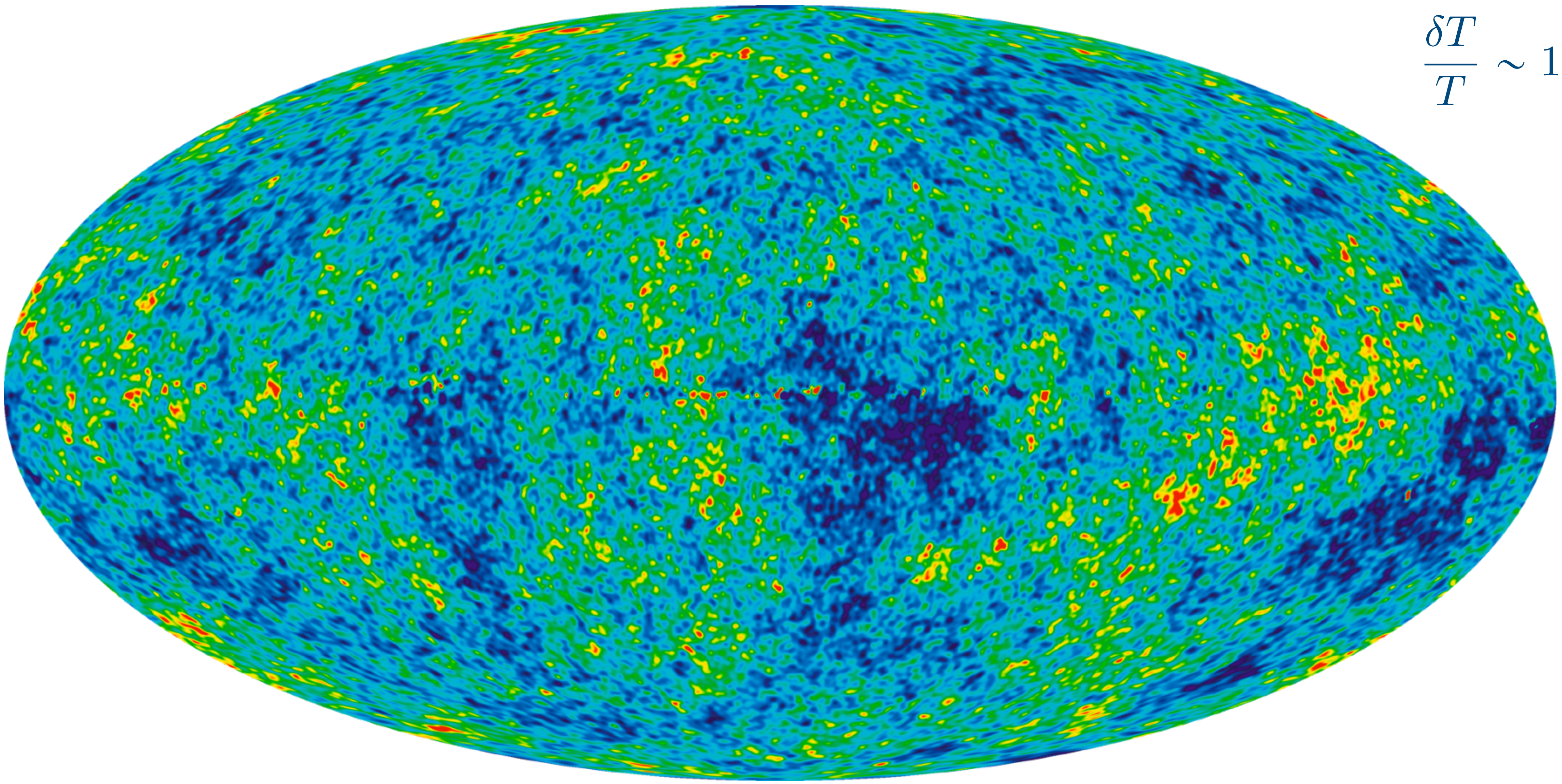


The CMB with an accuracy better then 1/100000.

# The seeds of structure

What was the size of perturbations at recombination, when baryons could collapse?
A view on the cosmic microwave background (CMB) reveals it!

NASA Wilkinson Microwave Anisotropy Probe (WMAP 2001-2009)



$$\frac{\delta T}{T} \sim 10^{-5}$$

The CMB with an accuracy better then 1/100000.

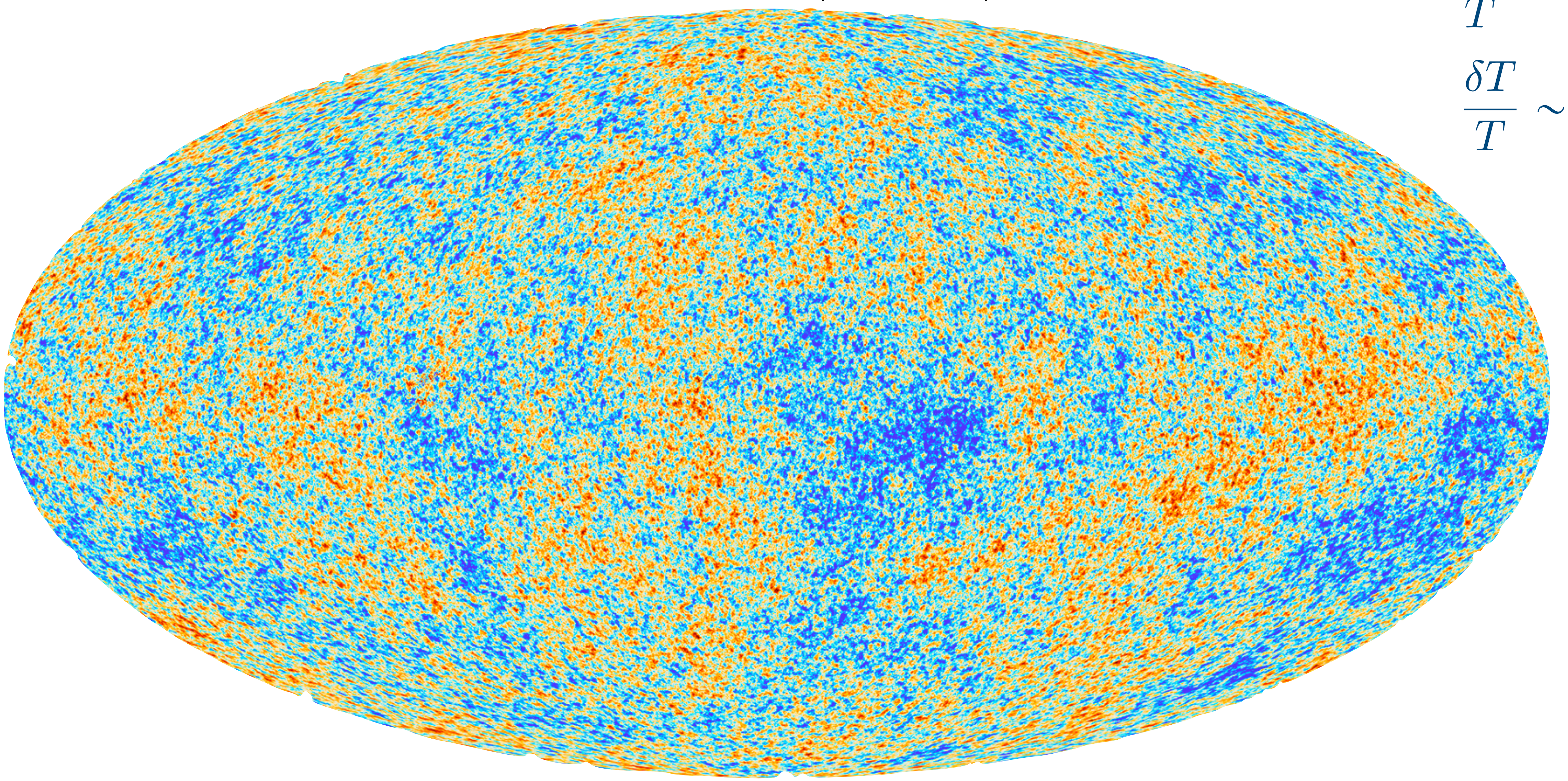
# The seeds of structure

What was the size of perturbations at recombination, when baryons could collapse?
A view on the cosmic microwave background (CMB) reveals it!

$$\frac{\delta T}{T} \sim 10^{-5}$$

$$\frac{\delta T}{T} \sim \frac{\delta \rho_b}{\rho_b}$$

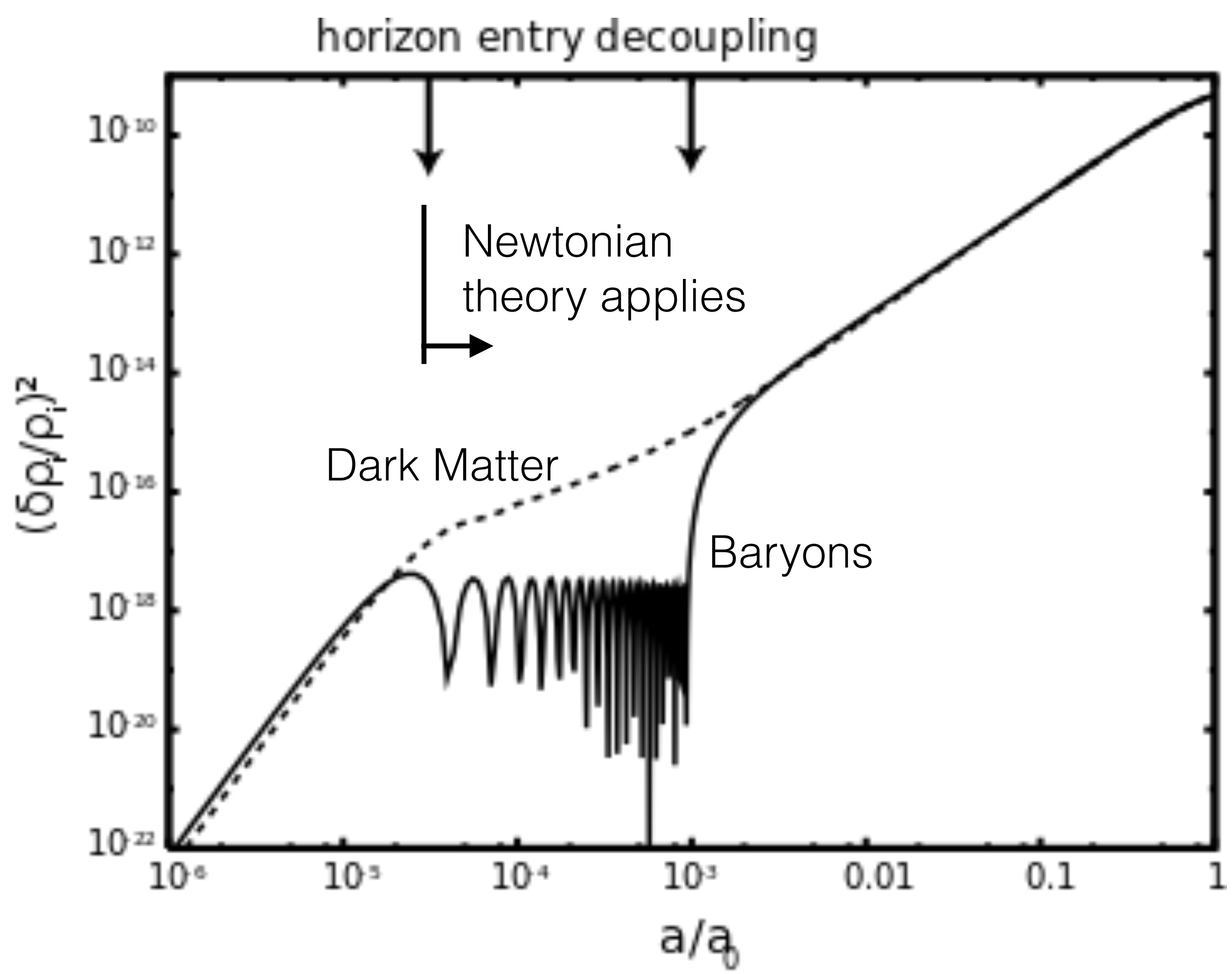
ESA Planck Satellite (2009-2013)

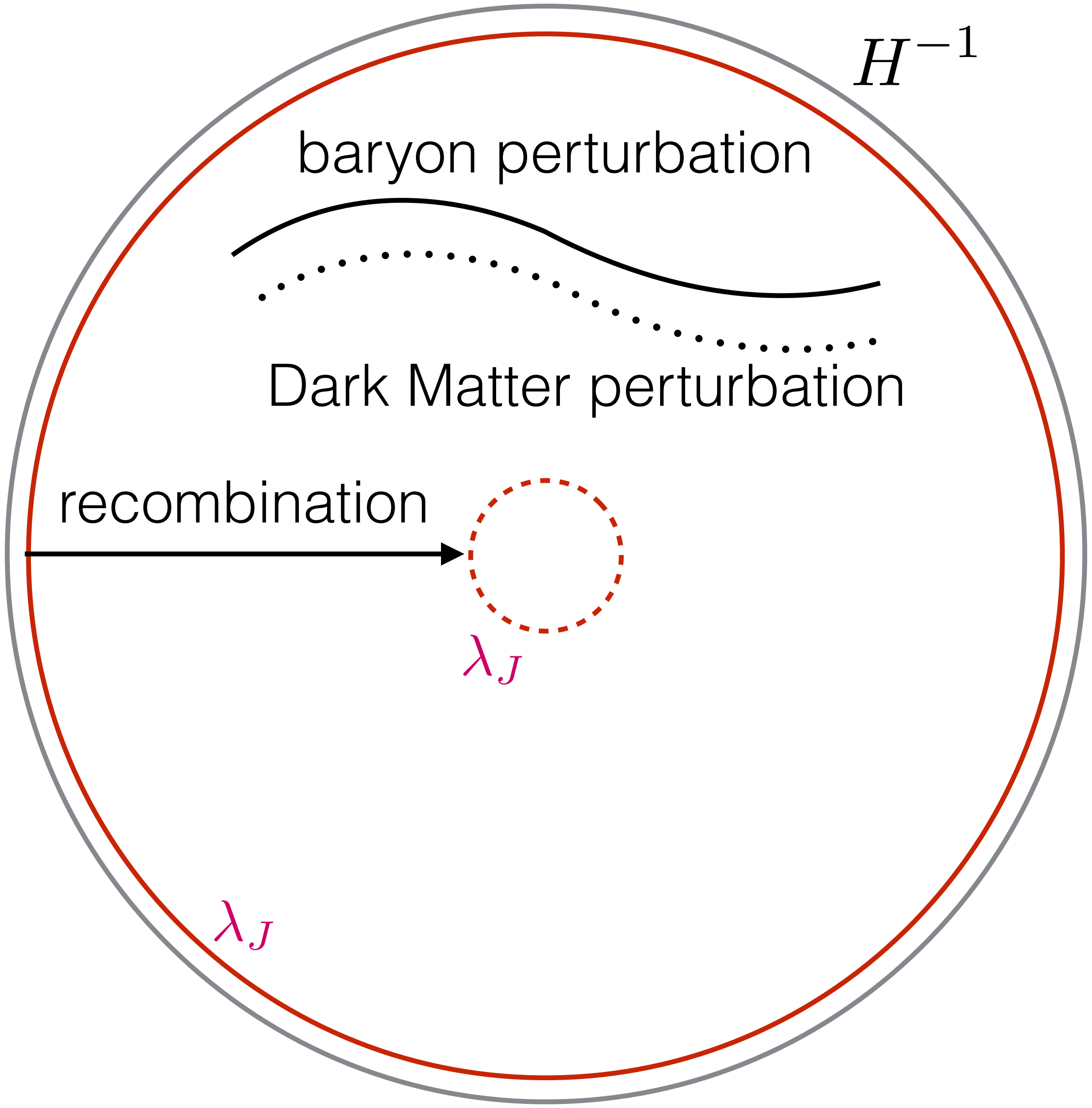


The CMB with an accuracy better then 1/100000.

# Dark Matter assisted growth of structure

A perturbation of a certain wavenumber k enters the horizon during the radiation dominated epoch; baryons undergo oscillations, Dark Matter can grow; at recombination, the Jeans length changes drastically, and baryons fall into the potential wells created by dark matter.



# Particle Dark Matter

- An electrically neutral particle, stable on cosmological timescales is most successful in explaining all those gravitational “missing mass” anomalies
- Given that evidence is in form of gravity, microscopic DM properties remain largely unknown
- Dark Matter can be a single particle, or there could be an entire hidden sector of particles and forces—it is on us to find out.


# “Standard Cosmological Model”

- A charge neutral Universe filled with baryons and photons, dark matter, neutrinos, and a cosmological constant fits the data.

# Properties of Dark Matter

# The “missing mass” - what is it?

Modified Newtonian Dynamics? $\vec{F} = m\vec{a} \times \mu(a/a_0)$

$$\mu \to \begin{cases} 1 & a \gg a_0 \\ a/a_0 & a \ll a_0 \end{cases}$$

*Are the gravitational anomalies a refutation of the laws of gravitation or are they an indication of the existence of unseen particles/objects?*

$$a_0 \simeq 10^{-8}\,\mathrm{cm/s^2}$$

There are of course examples from history on either side:

Dark Matter: Neptune was postulated to explain the anomalous motion of Uranus

Modified Gravity: planet Vulcan was postulated to explain the anomalous motion of Mercury which due to a failure of Newtonian dynamics and explained by GR

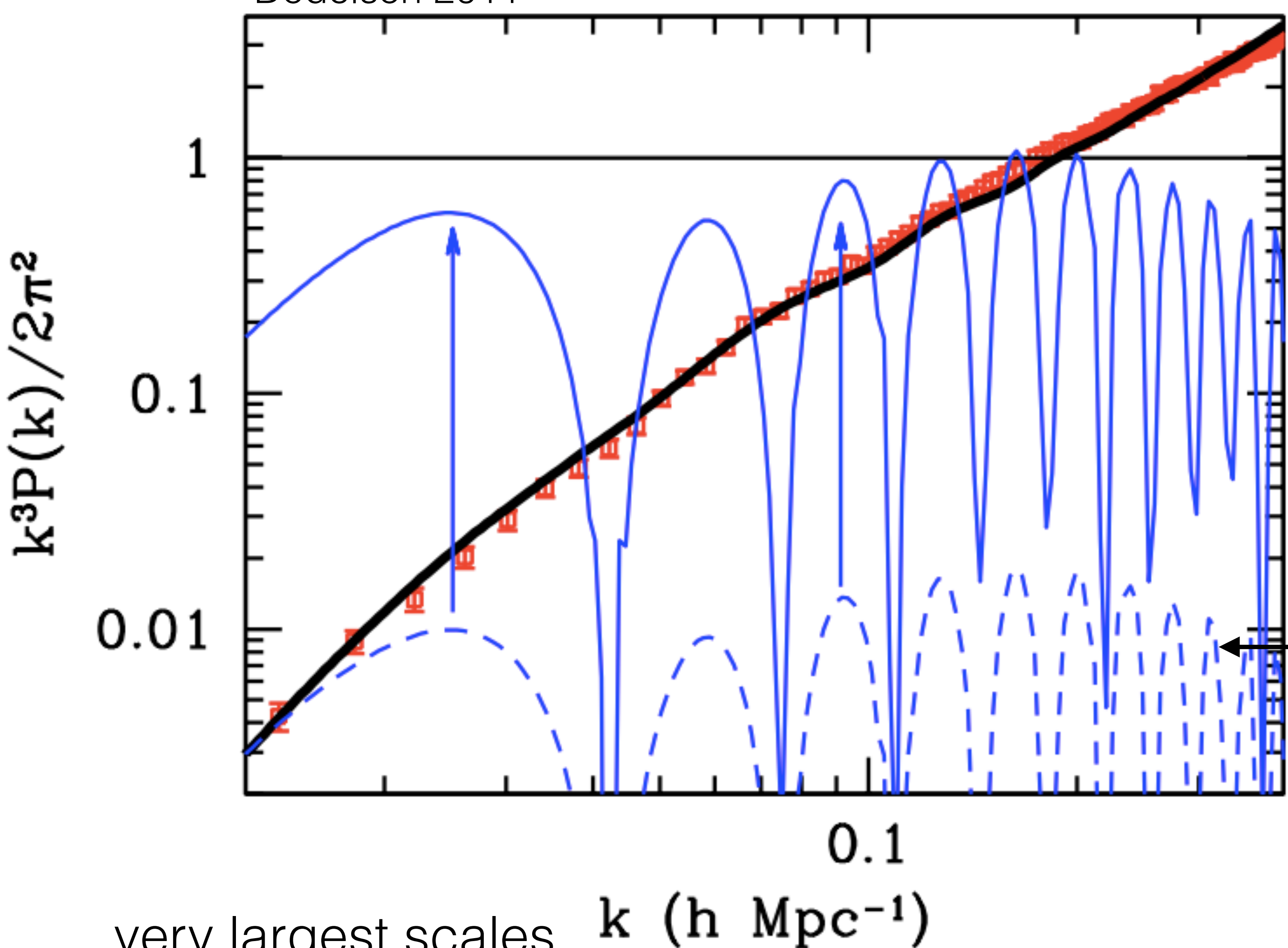
MOND explains flat rotation curves but is challenged by dynamics of clusters, gravitational lensing, growth of structure, CMB,…

NB: some will surely insist that Vulcan exists

# Growth of structure

Growth of structure is quantified by the power spectrum $$\frac{k^3 P(k)}{2\pi^2} = \left(\frac{\delta\rho}{\rho}\right)_k^2$$

Dodelson 2011




solid: with Dark Matter

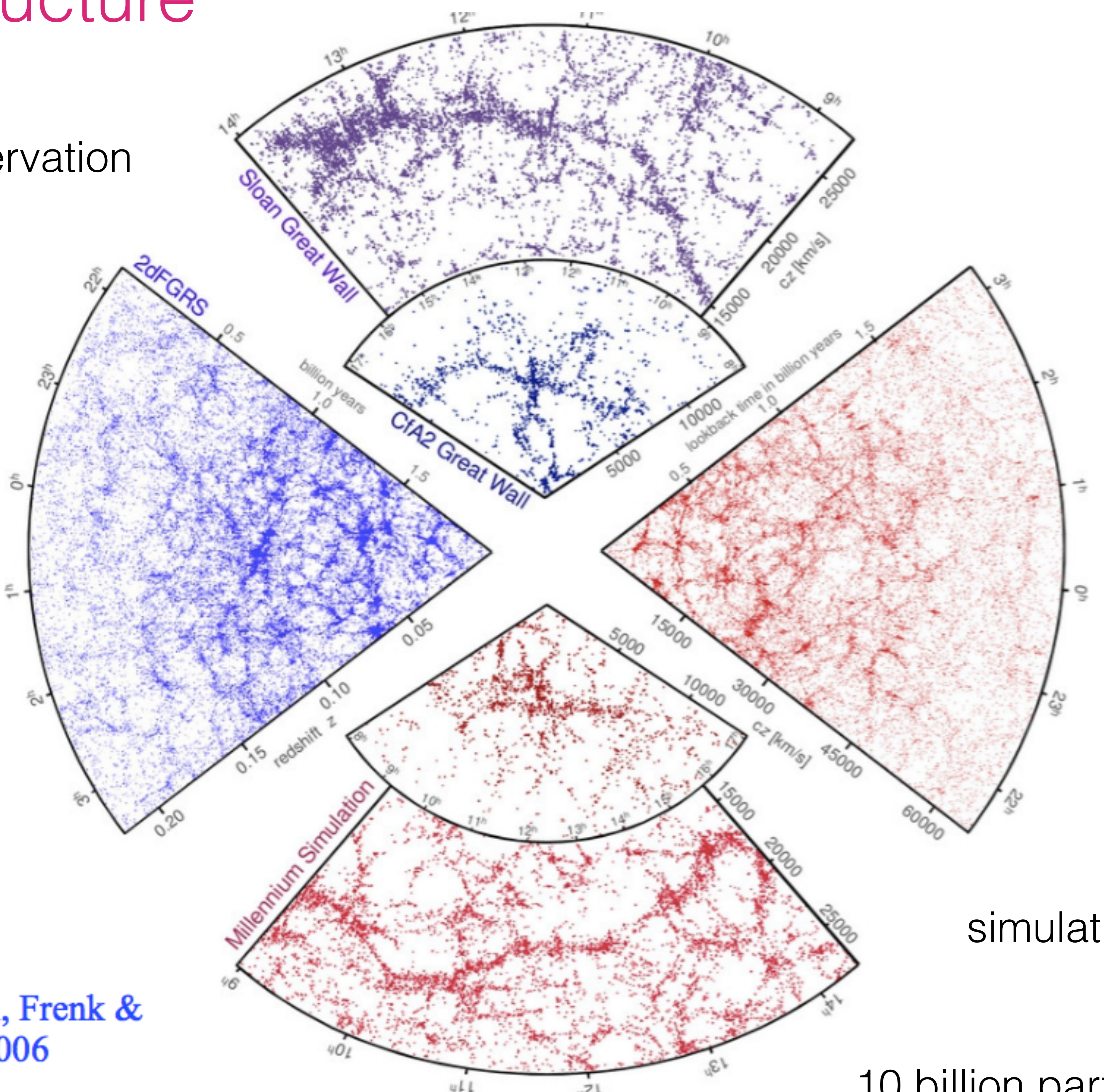
growth has become non-linear

dashed: baryon-only Universe

very largest scales

# Large scale structure

observation



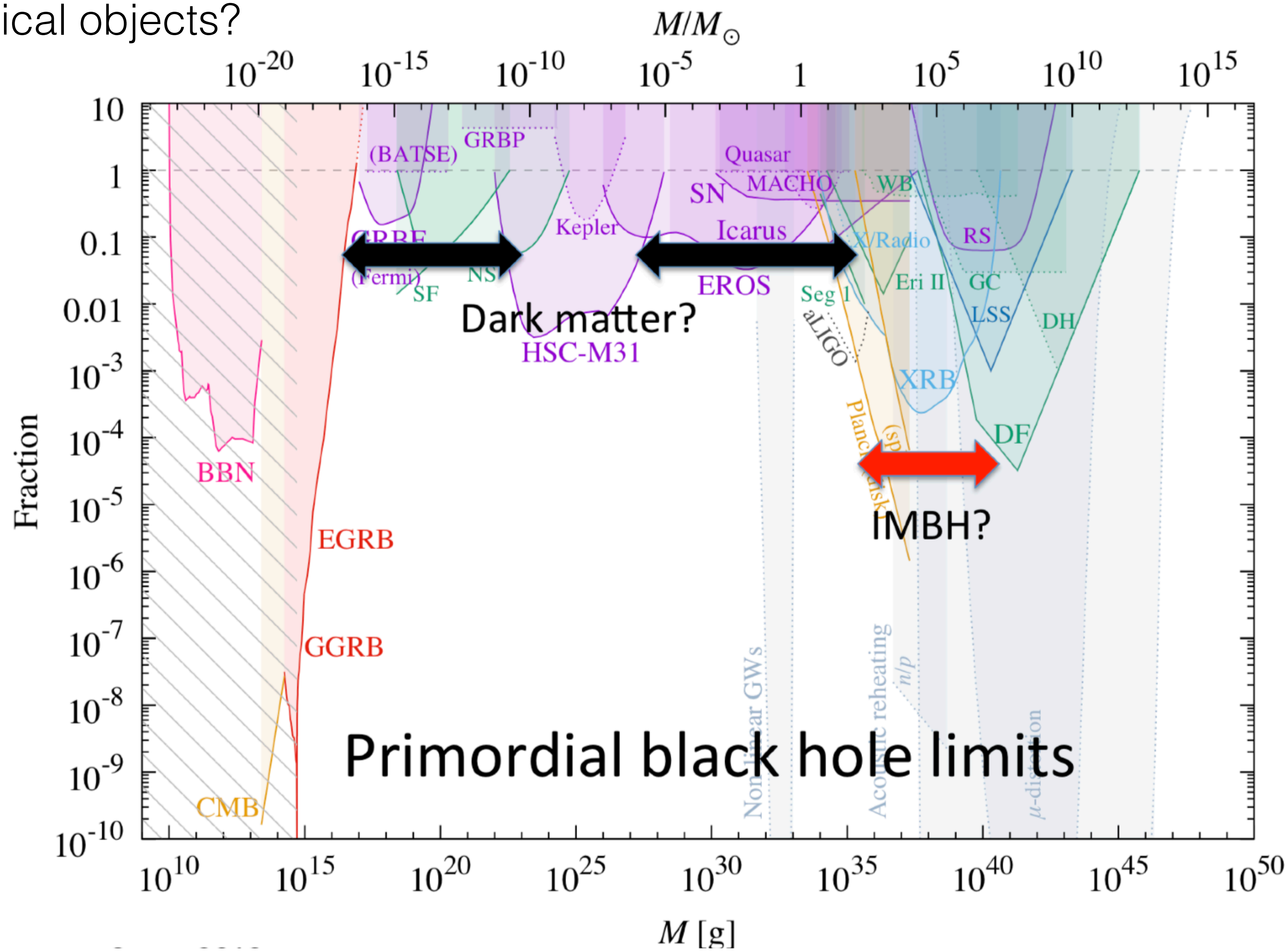

simulation

Springel, Frenk & White 2006

10 billion particles of mass $10^7\,M_\odot$

# The “missing mass” - what is it?

Astrophysical objects?




Carr et al 2018

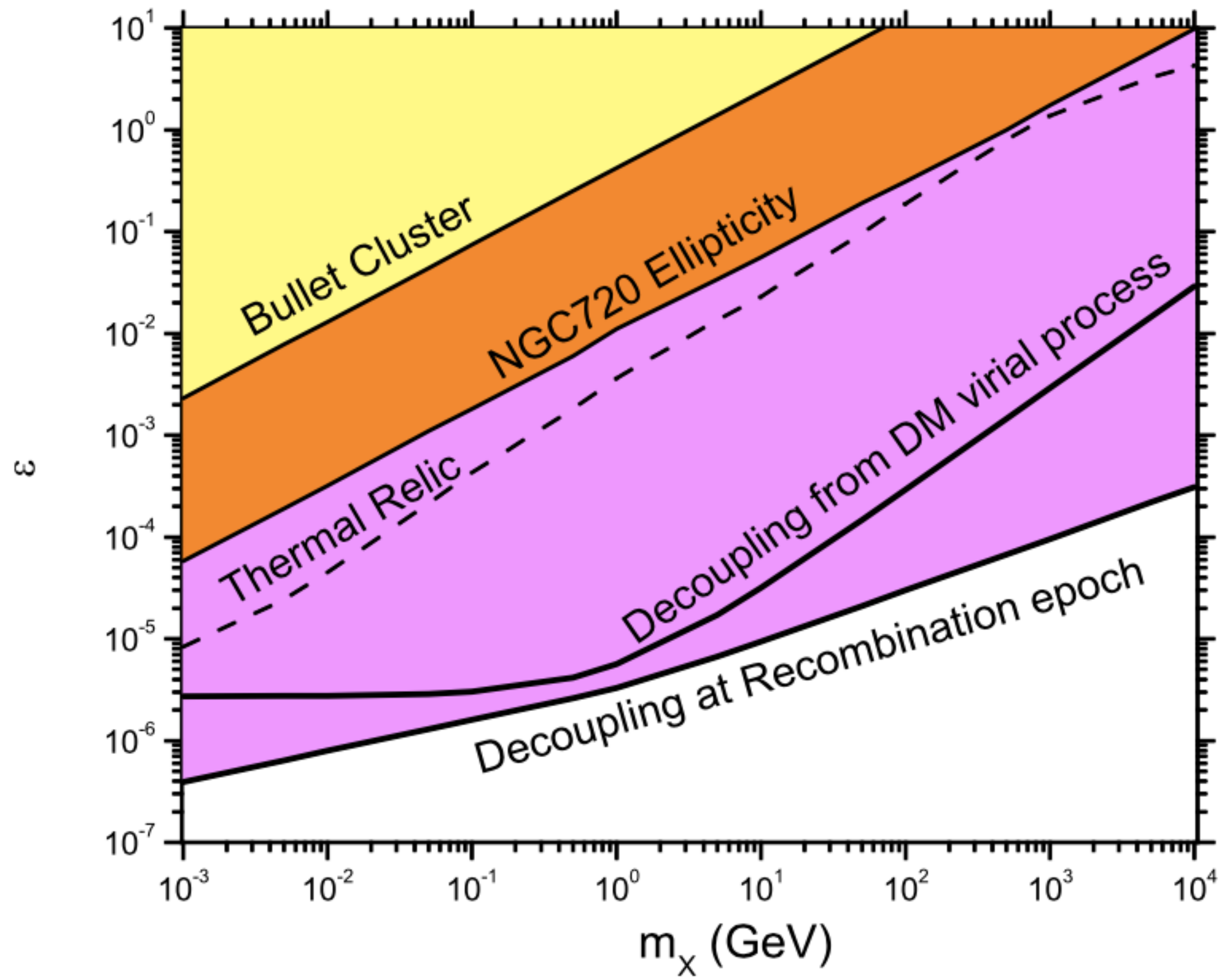
=> seems rather unlikely that DM = compact objects. Also, DM *obeys a “fluid limit” on astrophysical scales => most think it’s particle physics*

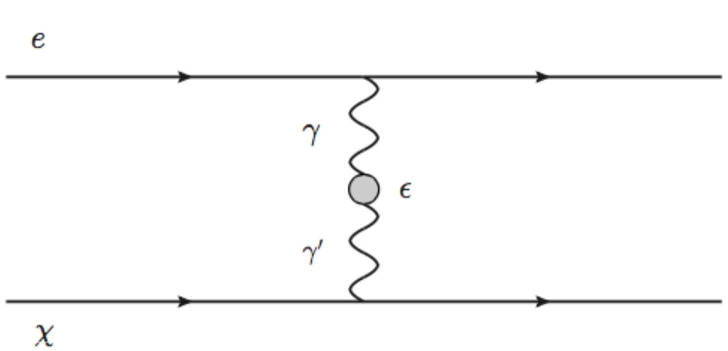
# Dark Matter - electromagnetic properties

Dark Matter is “dark”, i.e. non-luminous at the current level of observability

- Integral electric charges (charged massive particles, CHAMPs) are strongly constrained from non-observation of anomalous heavy nuclei (bound states)
- Millicharged DM is a possibility, but $\varepsilon e$ constrained from CMB acoustic peaks, DM heating by baryons, …
- DM may in fact possess electromagnetic form factors like a magnetic moment

McDermott et al 2010



# Dark Matter - dissipationless

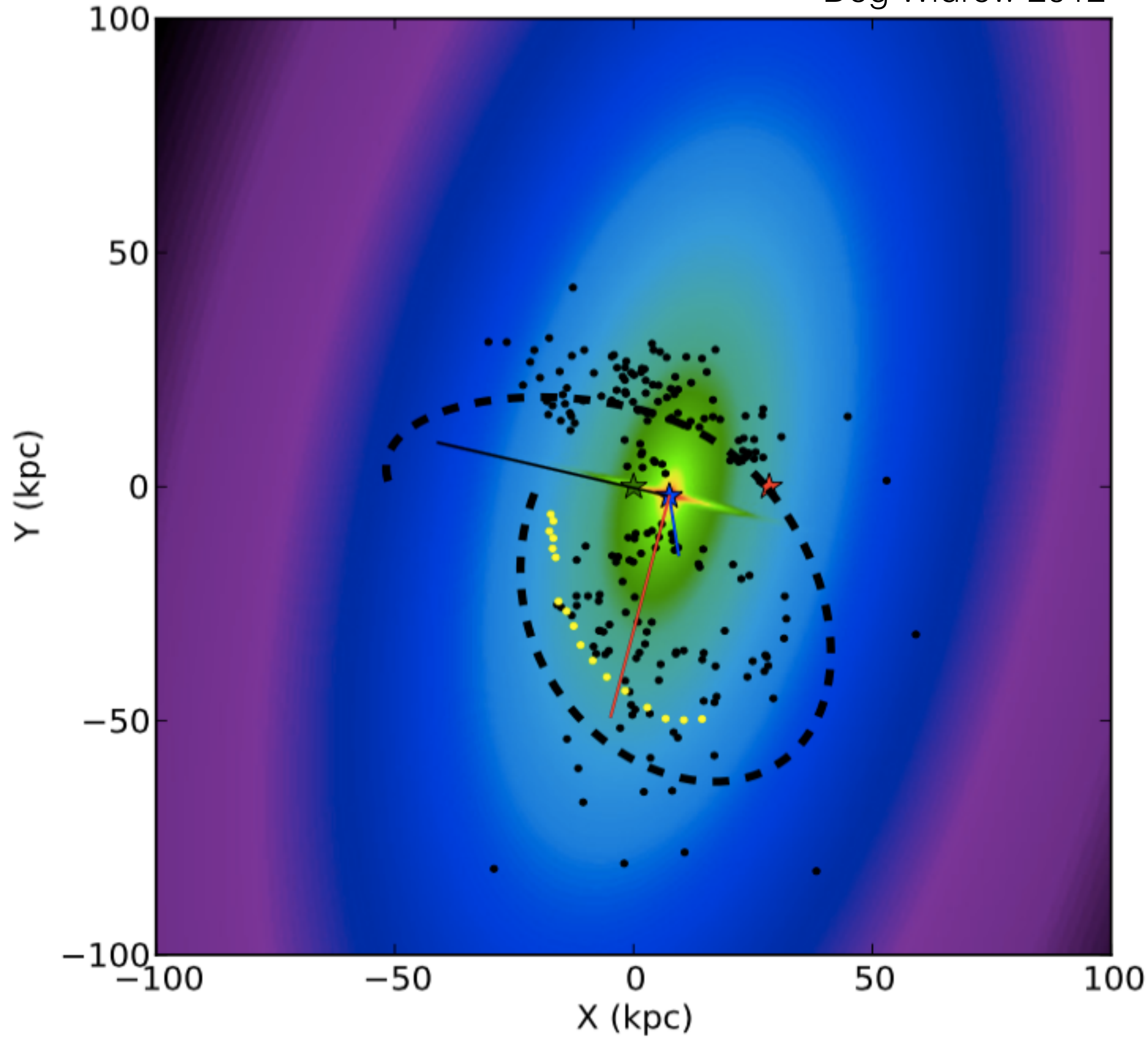
Dominant component of DM is dissipationless.

Ordinary matter “clumps” because it can loose angular momentum through dissipative processes (bremsstrahlung, molecular excitations,…) and can form e.g. discs

DM forms triaxial halos in which baryons settle in the inner parts

Example right: a tidal stream of stars, produced by the destruction of the Sagittarius dwarf spheroidal constrains the Galactic gravitational potential

Deg Widrow 2012

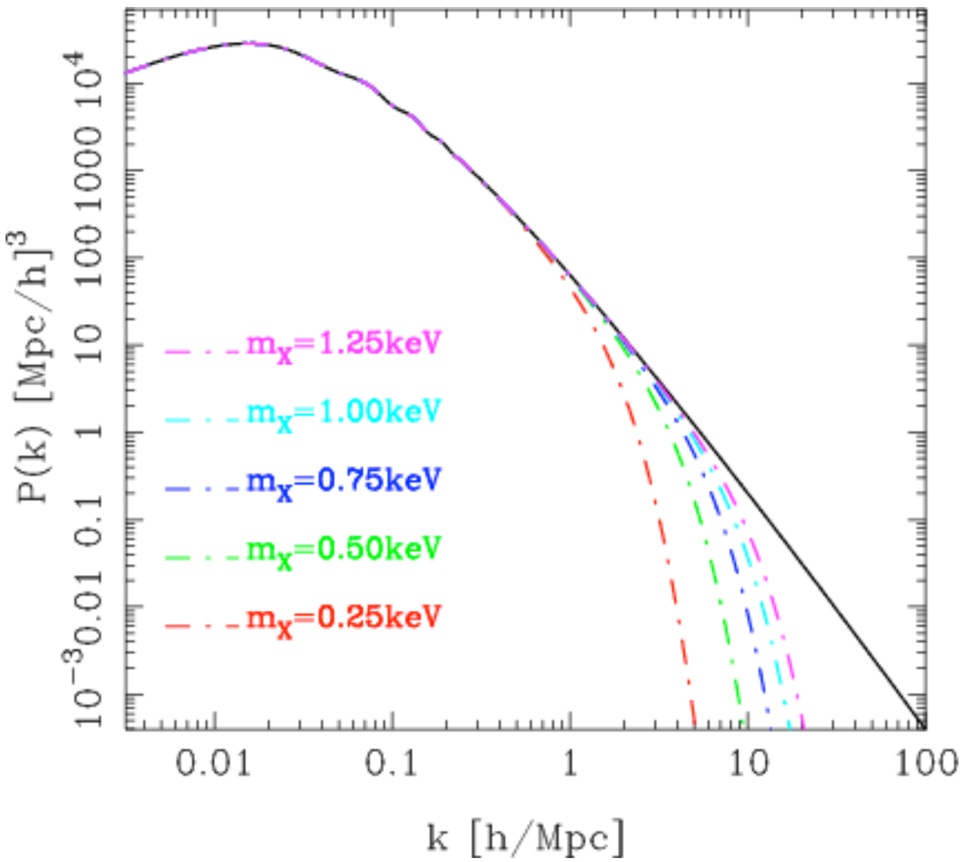


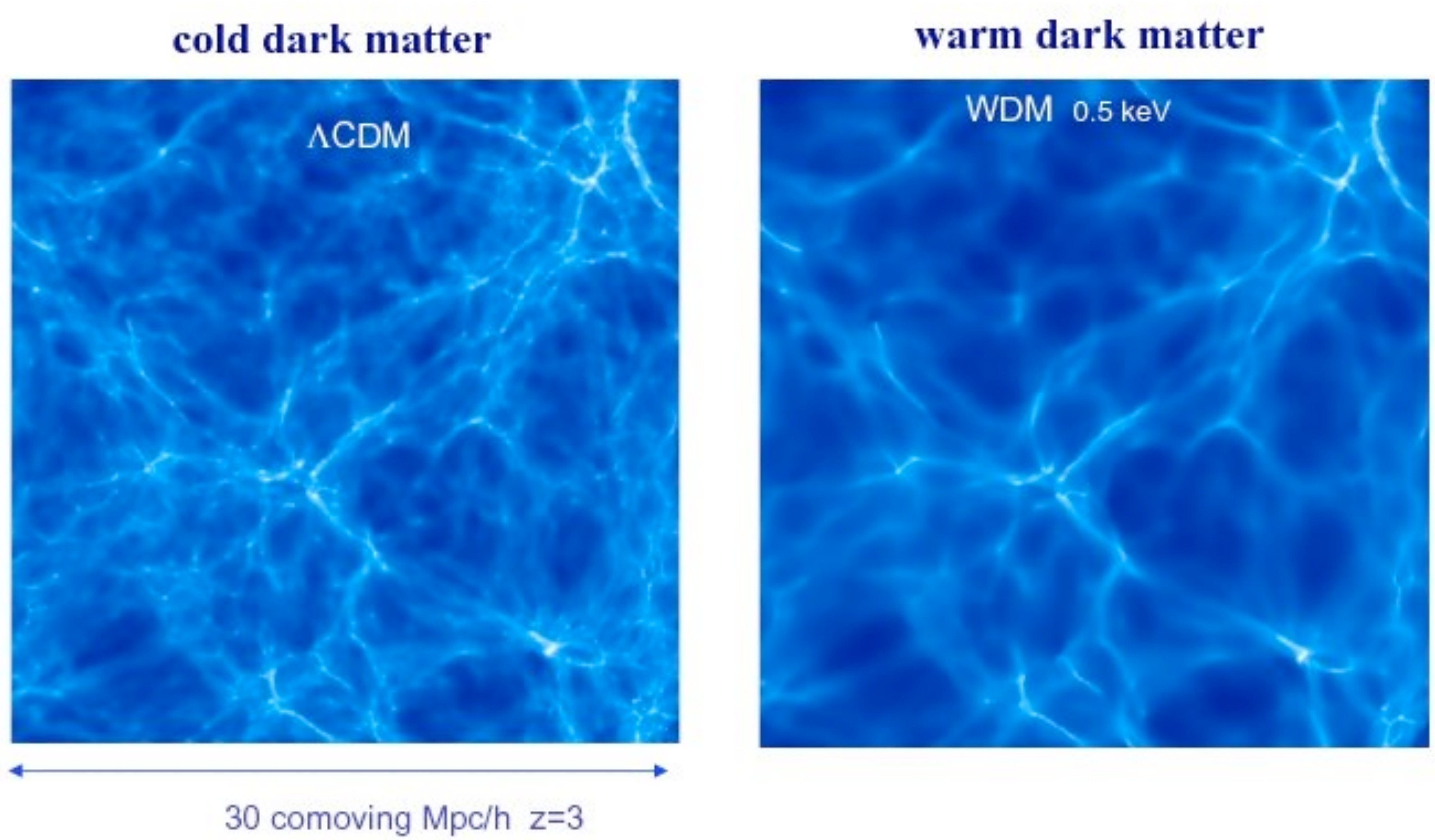
# Dark Matter is cold

The *dominant* fraction of DM must have non-relativistic velocity during structure formation

- Free-streaming of DM suppresses matter power spectrum at small scales where it can stream out of overdense regions (wipes out substructure; halo abundances for masses below the free-streaming mass-scale are suppressed relative to CDM.)
- Warm DM remains a possibility and is constrained by small-scale observations (Ly-alpha forest); typically, $m_{\rm wdm} \gtrsim 1\,{\rm keV}$

e.g. Smith, Markovic 2011

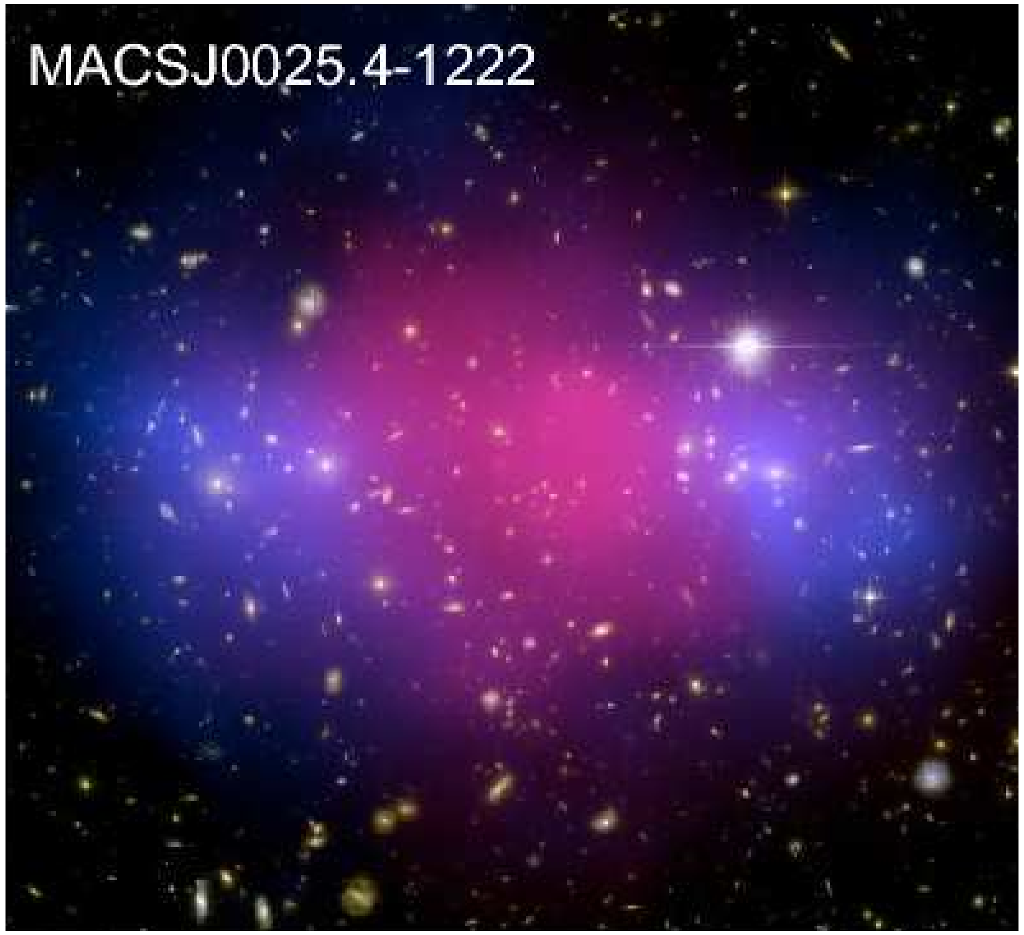


# Dark Matter - self-interactions

Dark Matter self-interactions are constrained by cluster collisions and from the fact that the position of the dark mass remains closely aligned with associated stars

From the particle physics viewpoint these are weak constraints
$\sigma_\chi/m_\chi \lesssim 1\,\mathrm{cm/g} \simeq 1\,\mathrm{bn/GeV}$




NB: there are tensions between cold DM simulations and observed structure on small scales (cored density profiles in dwarf spheroidal galaxies, abundance of satellite galaxies) and self-interactions can play a role in ameliorating these tensions.

# Dark Matter Abundance

# Liouville Equation

Fundamental object in statistical physics is the distribution function $f(\vec{x}, \vec{p}, t)$

Evolution of $f$ is governed by the Boltzmann equation

$$L[f] = C[f]$$

Liouville operator (gravitational effects) — Collision operator (non-gravitational effects)

$$p^\mu \frac{\partial f}{\partial x^\mu} - \Gamma^\mu_{\alpha\beta} p^\alpha p^\beta \frac{\partial f}{\partial p^\mu} = C[f]$$

In a FRW Universe $f(\vec{x}, \vec{p}, t) = f(|\vec{p}|, t)$

$$\frac{\partial f}{\partial t} - \frac{\dot{a}}{a} |\vec{p}| \frac{\partial f}{\partial |\vec{p}|} = \frac{1}{E} C[f]$$

# Integrated Boltzmann equation

We are often not interested in the momentum distribution of particles, therefore we integrate the Boltzmann equation over momenta $d^3\vec{p}$

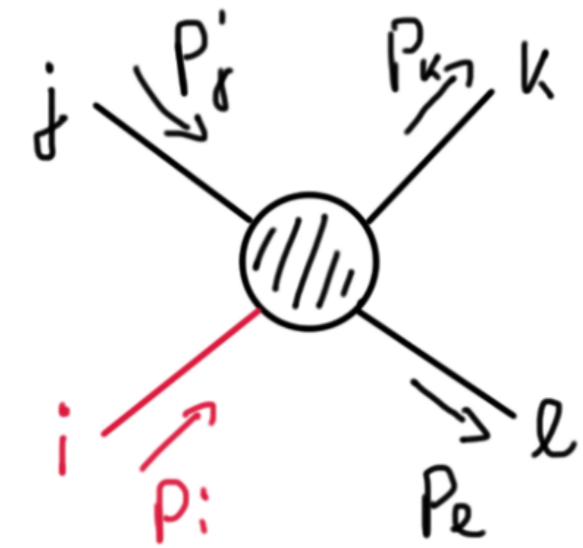
$$\frac{g}{(2\pi)^3}\int d^3\vec{p}\left[\frac{\partial f}{\partial t} - \frac{\dot{a}}{a}|\vec{p}|\frac{\partial f}{\partial|\vec{p}|}\right] = \frac{g}{(2\pi)^3}\int d^3\vec{p}\,\frac{1}{E}C[f]$$

This yields a rate equation for the number density (also often called “Boltzmann equation”)

$$\frac{dn}{dt} + 3\frac{\dot{a}}{a}n = \frac{g}{(2\pi)^3}\int \frac{d^3\vec{p}}{E}C[f] \qquad\qquad n = \frac{g}{(2\pi)^3}\int d^3\vec{p}\, f(\vec{x},\vec{p},t)$$

If no number changing collisions are present, RHS = 0, and $n \propto a^{-3}$ .

# Collision Integral




Consider the scattering i + j <-> k + l

$$\frac{g_i}{(2\pi)^3}\int d^3\vec{p}_i \frac{C[f_i]}{E_i} = -\int d\Pi_i d\Pi_j d\Pi_k d\Pi_l \, (2\pi)^4 \delta^{(4)}(p_i + p_j - p_k - p_l)$$
$$\times \left[|M|^2_{i+j\to k+l} f_i f_j (1 \pm f_k)(1 \pm f_l) - |M|^2_{k+l\to i+j} f_k f_l (1 \pm f_i)(1 \pm f_j)\right]$$

$+$ for bosons (Bose-enhancement)

$-$ for fermions (Pauli-blocking)

$$d\Pi_i \equiv \frac{g_i}{(2\pi)^3}\frac{d^3\vec{p}_i}{2E_i}$$

$|M|^2$ squared matrix element (averaged over initial and final spins)

If the scattering is CP or T-invariant, then $|M|^2_{i+j\to k+l} = |M|^2_{k+l\to i+j} = |M|^2$

Neglecting quantum statistics: $1 + f_i \to 1$

# Kinetic vs. chemical equilibrium

Elastic scattering processes are generally much more frequent than inelastic reactions that change particle number. Only the latter are part in C of the integrated Boltzmann equation.

$$\gamma e^- \leftrightarrow \gamma e^-$$

elastic; stops at recombination (T ~ eV)

$$e^+ e^- \leftrightarrow \gamma\gamma$$

inelastic; stops at ~20 keV

Kinetic equilibrium can be assumed: $T_i = T_j = T_k = T_l = T$

Chemical equilibrium must not hold: $\mu_i + \mu_j \neq \mu_k + \mu_l$

The distribution functions are then given by

$$f_i = \frac{1}{e^{(E_i - \mu_i)/T} \pm 1}$$

=> Maxwell-Boltzmann approximation $f_i = e^{-(E_i - \mu_i)/T}$

# Collision integral simplified

Kinetic equilibrium allows us to simplify the Boltzmann equation:

$$n_i^{(0)} = g_i \int \frac{d^3\vec{p}}{(2\pi)^3}\, e^{-E_i/T} \qquad \Rightarrow \qquad \frac{n_i}{n_i^{(0)}} = \frac{g_i \int \frac{d^3\vec{p}}{(2\pi)^3}\, e^{-(E_i-\mu_i)/T}}{g_i \int \frac{d^3\vec{p}}{(2\pi)^3}\, e^{-E_i/T}} = e^{\mu_i/T}$$

$$f_i = e^{-(E_i-\mu_i)/T} \qquad \Rightarrow \qquad f_i = \frac{n_i}{n_i^{(0)}} e^{-E_i/T}$$

---

$$\text{coll. integral} = -\int d\Pi_i d\Pi_j d\Pi_k d\Pi_l\, (2\pi)^4 \delta^{(4)}(p_i + p_j - p_k - p_l)|M|^2 \, [f_i f_j - f_k f_l]$$

$$[f_i f_j - f_k f_l] = \left[ \frac{n_i}{n_i^{(0)}} \frac{n_j}{n_j^{(0)}} e^{-(E_i+E_j)/T} - \frac{n_k}{n_k^{(0)}} \frac{n_l}{n_l^{(0)}} e^{-(E_k+E_l)/T} \right]$$

same (delta-function)

$$= e^{-(E_i+E_j)/T} \left[ \frac{n_i}{n_i^{(0)}} \frac{n_j}{n_j^{(0)}} - \frac{n_k}{n_k^{(0)}} \frac{n_l}{n_l^{(0)}} \right]$$

# Boltzmann equation in usual form

$$\frac{dn_i}{dt} + 3\frac{\dot{a}}{a}n_i = -n_i^{(0)}n_j^{(0)}\langle\sigma v\rangle\left[\frac{n_i}{n_i^{(0)}}\frac{n_j}{n_j^{(0)}} - \frac{n_k}{n_k^{(0)}}\frac{n_l}{n_l^{(0)}}\right]$$

$$\langle\sigma v\rangle \equiv \frac{1}{n_i^{(0)}n_j^{(0)}}\int d\Pi_i d\Pi_j d\Pi_k d\Pi_l\, e^{-(E_i+E_j)/T}(2\pi)^4\delta^{(4)}(p_i+p_j-p_k-p_l)|M|^2$$

Thermally averaged cross section

This equation (and generalized versions for 2 <—> 3 processes, etc…) forms the basis of practically any particle physics investigations of early Universe processes. [Selected problems require the use of distribution functions of course.]

# Thermally averaged cross section

We had

$$\langle \sigma v \rangle \equiv \frac{1}{n_i^{(0)} n_j^{(0)}} \int d\Pi_i d\Pi_j d\Pi_k d\Pi_l \, e^{-(E_i+E_j)/T} (2\pi)^4 \delta^{(4)}(p_i + p_j - p_k - p_l)|M|^2$$

We can make the relation to the "normal" cross section more explicit. Note that,

$$4 g_i g_j F \sigma_{i+j\to k+l} = \int d\Pi_k d\Pi_l \, (2\pi)^4 \delta^{(4)}(p_i + p_j - p_k - p_l)|M_{i+j\to k+l}|^2$$

Flux factor $F = \left[p_i \cdot p_j - m_i^2 m_j^2\right]^{1/2}$

$$v_{\mathrm{Møl}} = \frac{F}{E_i E_j}$$ Møller velocity (relativistic generalization of the relative velocity)

$$\frac{dN}{dV\,dt} = \sigma v_{\mathrm{Møl}} n_i n_j$$ this is a Lorentz invariant reaction rate per volume

# WIMP Dark Matter

Let us calculate the abundance of dark matter particles $\chi$ for which $\chi\bar{\chi} \leftrightarrow X\bar{X}$ is the lowest order total number changing process

- Assume that X are Standard Model particles that have other interactions that keep them in thermal equilibrium,

$$n_X = n_X^{(0)}, \quad n_{\bar{X}} = n_{\bar{X}}^{(0)}$$

- Assume no asymmetry in the Dark Matter sector, $n_\chi = n_{\bar{\chi}}$ .

$$\frac{dn_\chi}{dt} + 3\frac{\dot{a}}{a}n_\chi = -\sum_X \langle \sigma_{\chi\bar{\chi}\to X\bar{X}} v\rangle \left[n_\chi^2 - (n_\chi^{\rm eq})^2\right]$$

It is convenient to *scale out the expansion* term, by normalizing to a comoving volume

$$Y \equiv \frac{N_\chi}{S} = \frac{a^3 n_\chi}{a^3 s} = \frac{n_\chi}{s}$$

$Y = const.$ if no annihilation of creation

# WIMP freeze out

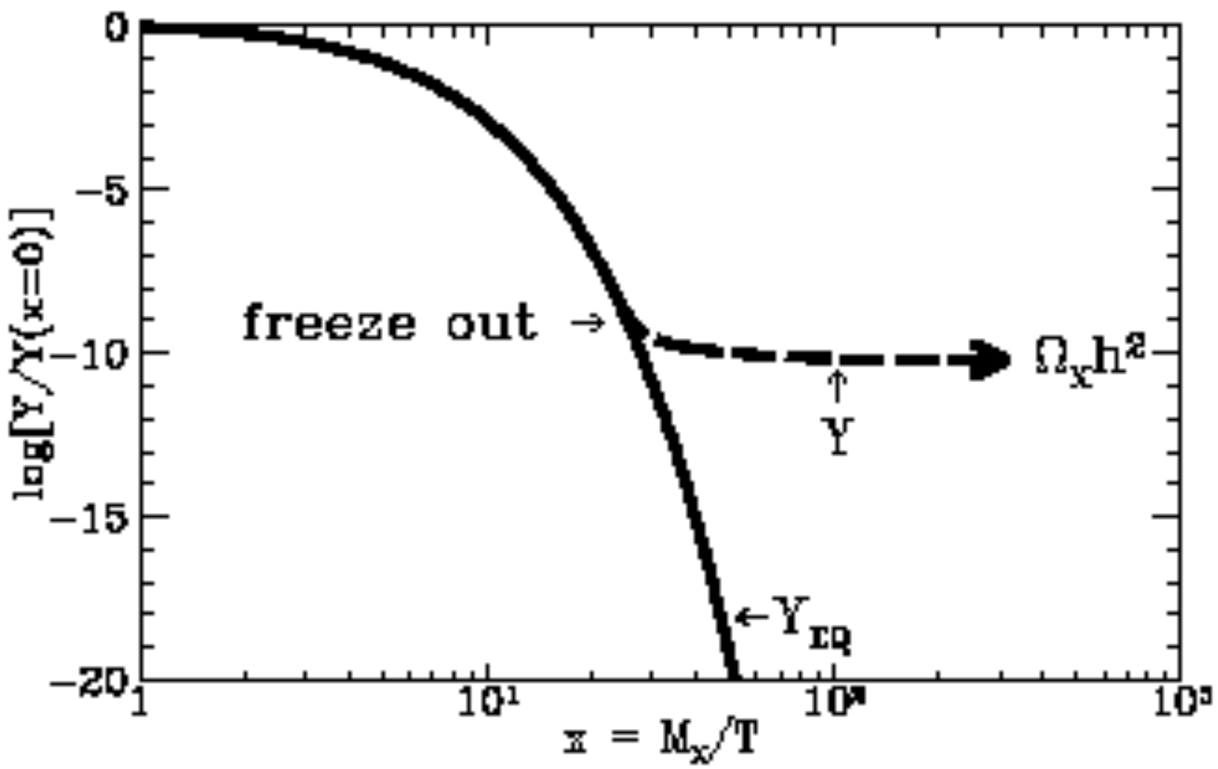
In terms of the "yield" Y

$$\dot{Y} = -\langle\sigma v\rangle s \left[Y^2 - Y_{\mathrm{eq}}^2\right]$$

Changing variables $x = m_\chi/T$

and for $h_{\mathrm{eff}} = const.$

$$\frac{x}{Y_{\mathrm{eq}}}\frac{dY}{dx} = -\frac{\Gamma_{\mathrm{int}}}{H}\left[Y^2 - Y_{\mathrm{eq}}^2\right]$$




$\Gamma_{\mathrm{int}} = n_{\chi,\mathrm{eq}}\langle\sigma v\rangle$ and we can find the point when Dark Matter "freezes out" from $\Gamma_{\mathrm{int}}/H \simeq 1$

WIMPs $x_F \simeq 20$

# Thermally averaged cross section

For practical purposes one makes a velocity expansion of the cross section

$$s = 4m_\chi^2 + v^2 m_\chi^2$$

One gets $\sigma v \approx a + bv^2$ => $\langle \sigma v \rangle \approx a + (3/2)b(T/m)$

s-wave p-wave

An inelastic cross section that is constant at small relative velocities (no charge repulsion assumed) has the typical property ("Bethe's law")

$$\sigma v \approx const.$$

e.g. capture of slow neutrons by nucleus

=> simplifies the estimation of relic abundance

# Precise calculation of yield

Accurate treatment

$$\sqrt{g_*} = \frac{h_{\rm eff}}{\sqrt{g_{\rm eff}}}\left[1+\frac{1}{3}\frac{d\ln h_{\rm eff}}{d\ln T}\right]$$

$$\frac{dY}{dx} = -\sqrt{\frac{8\pi^2 g_*}{45}}\,\frac{m_\chi M_P\langle\sigma v\rangle}{x^2}\left[Y^2 - Y_{\rm eq}^2\right]$$

$$Y_{\rm eq} = \frac{45 g}{4\pi^4}\,\frac{x^2 K_2(x)}{h_{\rm eff}(m/x)}$$

valid for relativistic and non-relativistic particles

1. Thermal average

$$\langle\sigma v\rangle = \frac{1}{8m_a^4 T K_2(m_a/T)^2}\int_{4m_a^2}^{\infty} ds\,(s-4m_a^2)\,\sigma(s)\sqrt{s}\,K_1\left(\frac{\sqrt{s}}{T}\right)$$

2. Find freeze out point $x_F$ (there are various recipes to do so.)

3. Neglect $Y_{\rm eq}^2$ and integrate the equation

$$\frac{1}{Y_0} = \frac{1}{Y_f} + \sqrt{\frac{8\pi^2}{45}}\,M_P\int_{T_0}^{T_f} dT\,\sqrt{g_*}\langle\sigma v\rangle$$

# Relic abundance - the quick way

If $\langle \sigma v \rangle \simeq const$

$$Y_0 \simeq \sqrt{\frac{45}{8\pi^2 g_*(x_F)}} \frac{x_F}{M_P m_\chi} \frac{1}{\langle \sigma v \rangle}$$

Relic density parameter today

$$\Omega h^2 = \frac{\rho_0 h^2}{\rho_c} = \frac{s_0 Y_0 m_\chi h^2}{\rho_c}$$

=>

$$\Omega h^2 \simeq \frac{3 \times 10^{-38}\,\mathrm{cm}^2}{\langle \sigma v \rangle} \frac{x_F}{\sqrt{g_*(x_F)}}$$

Let's take an electroweak-scale WIMP, $x_F \simeq 20$, $m_\chi \sim 100\,\mathrm{GeV}$,

$$\frac{x_F}{\sqrt{g_*(x_F)}} \simeq \frac{20}{\sqrt{80-90}}$$

$$\Omega h^2 = \frac{10^{-37}\,\mathrm{cm}^2}{\langle \sigma v \rangle}$$

The larger the annihilation cross section, the smaller the abundance.

# WIMP miracle

Observations determine the cold dark matter abundance with %-precision

$$\Omega_{\rm dm} h^2 = \frac{10^{-37}\,\mathrm{cm}^2}{\langle \sigma v \rangle} = 0.1197 \pm 0.0022 \qquad h^2 = 0.67^2 \approx \frac{1}{2}$$

Planck

*=> we need an annihilation cross section of* $\langle \sigma v \rangle \approx 10^{-36}\,\mathrm{cm}^2 = 1\,\mathrm{pb}$

*=> That points towards the electroweak scale, and fuels the hope to detect DM in direct detection experiments and at the collider*

- Annihilation through s-wave of a DM candidate $\chi$ through some mediator $\phi$

$$\langle \sigma v \rangle \sim \frac{g^4}{\pi} \frac{m_\chi^2}{m_\phi^4} \sim 10^{-36}\,\mathrm{cm}^2 g^4 \left( \frac{m_\chi}{100\,\mathrm{GeV}} \right)^2 \left( \frac{1\,\mathrm{TeV}}{m_\phi} \right)^4 \qquad m_\phi \gg m_\chi$$

$$\langle \sigma v \rangle \sim \frac{g^4}{\pi} \frac{1}{m_\chi^2} \sim 10^{-36}\,\mathrm{cm}^2 \left( \frac{g}{0.1} \right)^4 \left( \frac{100\,\mathrm{GeV}}{m_\chi} \right)^4 \qquad m_\phi \ll m_\chi$$

# Summary

- The standard cosmological model is built on the observational evidence that the universe homogenous and isotropic on large scales; derived the Friedmann equation that connects the expansion rate H of the Universe with the energy components it contains
- Showed how matter and radiation redshift. Going back in time, the Universe enters the radiation dominated epoch and with it “particle era”.
- There is a severe problem of missing mass on all scales relevant to astronomy and cosmology, from the motion of stars in the solar neighbourhood to the horizon of the visible Universe.
- Particle Dark Matter solves the missing mass problem. DM can be a single particle or be part of a larger sector of yet undiscovered states.

# Summary

- Established the basic properties of DM (stability, neutrality, dissipationless…)
  - DM has a fluid limit, it probably cannot consist of macroscopic bodies. It is likely a new particle(s).
  - DM must be stable on cosmological timescales, electrically neutral, dissipationless, cold, and to a degree collisionless.

- Relic density requirement
  - DM must comprise today 26% of the Universe’s mass. That puts a restriction on its interactions with other particles and its mass.
  - Assuming a standard cosmological history and particle content, the DM abundance becomes a prediction

  => tool for making such prediction is the Boltzmann equation. WIMPS regulate their abundance through the equation

$$\frac{dn_\chi}{dt} + 3\frac{\dot{a}}{a}n_\chi = -\sum_X \langle \sigma_{\chi\bar{\chi}\to X\bar{X}} v\rangle \left[n_\chi^2 - (n_\chi^{\rm eq})^2\right]$$

# BSM - lecture 2

MUON

Budker INP

Josef Pradler
Sept 21 2018

HEPHY
INSTITUTE OF HIGH ENERGY PHYSICS

ÖAW
AUSTRIAN ACADEMY OF SCIENCES

International school on muon dipole moments and hadronic effects

# Plan of the lectures (BSM = DM for the next 3h)

*Lecture 1*

- Fundamentals of standard cosmology
- Gravitational evidence for Dark Matter
- Basic properties of Dark Matter
- The Boltzmann equation and its application to the early Universe (explicit calculation)

*Lecture 2*

- Classification of Dark Matter models
- WIMP Dark Matter
- Laboratory Detection of WIMPs (explicit calculation)
- Light Dark Sectors with relevance for the intensity frontier

# Dark Matter zoology

WIMP DM is an attractive possibility, but there are many candidates for Dark Matter.

- electroweak scale WIMPs, GeV-scale DM
- axion, ALPs
- keV sterile neutrinos
- gravitinos
- other super-WIMPs such as Dark Photons

# Two prospective models of Dark Matter

$$\mathcal{L} = \frac{1}{2}(\partial_\mu S)^2 - \frac{1}{2}m_S^2 S^2 - \lambda S^2 (H^\dagger H)$$

$$\begin{aligned}
\frac{1}{e}\mathscr{L}_{\mathrm{sugra}} = & -\frac{M_P^2}{2}R + g_{ij^*}\tilde{\mathscr{D}}_\mu \phi^i \tilde{\mathscr{D}}^\mu \phi^{*j} - \frac{1}{2}g^2\left[(\mathrm{Re}f)^{-1}\right]^{ab} D_{(a)}D_{(b)} \\
& + i g_{ij^*}\overline{\chi}_L^j \gamma^\mu \tilde{\mathscr{D}}_\mu \chi_L^i + \varepsilon^{\mu\nu\rho\sigma}\overline{\psi}_{L\mu}\gamma_\nu \tilde{\mathscr{D}}_\rho \psi_{L\sigma} \\
& - \frac{1}{4}\mathrm{Re}f_{ab}F_{\mu\nu}^{(a)}F^{\mu\nu(b)} + \frac{1}{8}\varepsilon^{\mu\nu\rho\sigma}\mathrm{Im}f_{ab}F_{\mu\nu}^{(a)}F_{\rho\sigma}^{(b)} \\
& + \frac{i}{2}\mathrm{Re}f_{ab}\overline{\lambda}^a\gamma^\mu \tilde{\mathscr{D}}_\mu \lambda^b - e^{-1}\frac{1}{2}\mathrm{Im}f_{ab}\tilde{\mathscr{D}}_\mu\left[e\overline{\lambda}_R^a\gamma^\mu\lambda_R^b\right] \\
& + \Bigg[ -\sqrt{2}g\partial_i D_{(a)}\overline{\lambda}^a\chi_L^i + \frac{1}{4}\sqrt{2}g\left[(\mathrm{Re}f)^{-1}\right]^{ab}\partial_i f_{bc}D_{(a)}\overline{\lambda}^c\chi_L^i \\
& + \frac{i}{16}\sqrt{2}\partial_i f_{ab}\overline{\lambda}^a[\gamma^\mu,\gamma^\nu]\chi_L^i F_{\mu\nu}^{(b)} - \frac{1}{2M_P}gD_{(a)}\overline{\lambda}_R^a\gamma^\mu\psi_\mu \\
& - \frac{i}{2M_P}\sqrt{2}g_{ij^*}\tilde{\mathscr{D}}_\mu\phi^{*j}\overline{\psi}_\nu\gamma^\mu\gamma^\nu\chi_L^i + \mathrm{h.c.}\Bigg] \\
& - \frac{i}{8M_P}\mathrm{Re}f_{ab}\overline{\psi}_\mu[\gamma^m,\gamma^n]\gamma^\mu\lambda^a F_{mn}^{(b)} \\
& - e^{K/2M_P^2}\Bigg[\frac{1}{4M_P^2}W^*\overline{\psi}_{R\mu}[\gamma^\mu,\gamma^\nu]\psi_{L\nu} - \frac{1}{2M_P}\sqrt{2}D_iW\overline{\psi}_\mu\gamma^\mu\chi_L^i \\
& + \frac{1}{2}\mathscr{D}_iD_jW\overline{\chi_L^c}^i\chi_L^j + \frac{1}{4}g^{ij^*}D_{j^*}W^*\partial_i f_{ab}\overline{\lambda}_R^a\lambda_L^b + \mathrm{h.c.}\Bigg] \\
& - e^{K/M_P^2}\left[g^{ij^*}(D_iW)(D_{j^*}W^*) - 3\frac{|W|^2}{M_P^2}\right] + \mathcal{O}(M_P^{-2}),
\end{aligned}$$

=> we need a classification scheme

# Classification of Dark Matter

What is the abundance of $\chi$ for $T \gg m_\chi$?

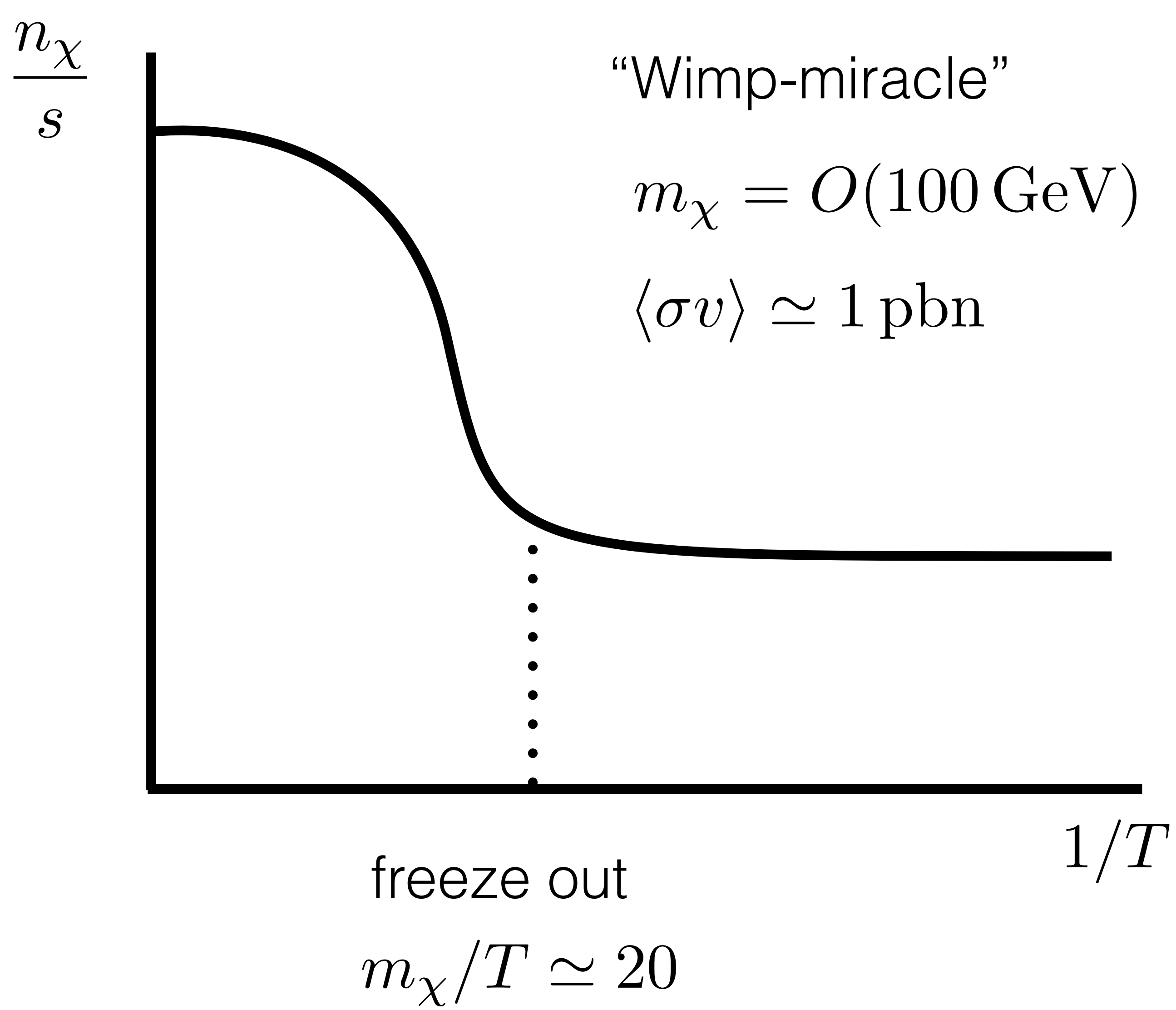
**NORMAL** $N_\chi/N_\gamma \sim 1$

$\Gamma_\chi \gg H(T)$

$\Gamma_\chi = n_\chi \langle\sigma v\rangle$

"WIMPs"
(weakly massive
interacting particles)

e.g. SUSY neutralino,
Higgs-Portal models, …



# Classification of Dark Matter

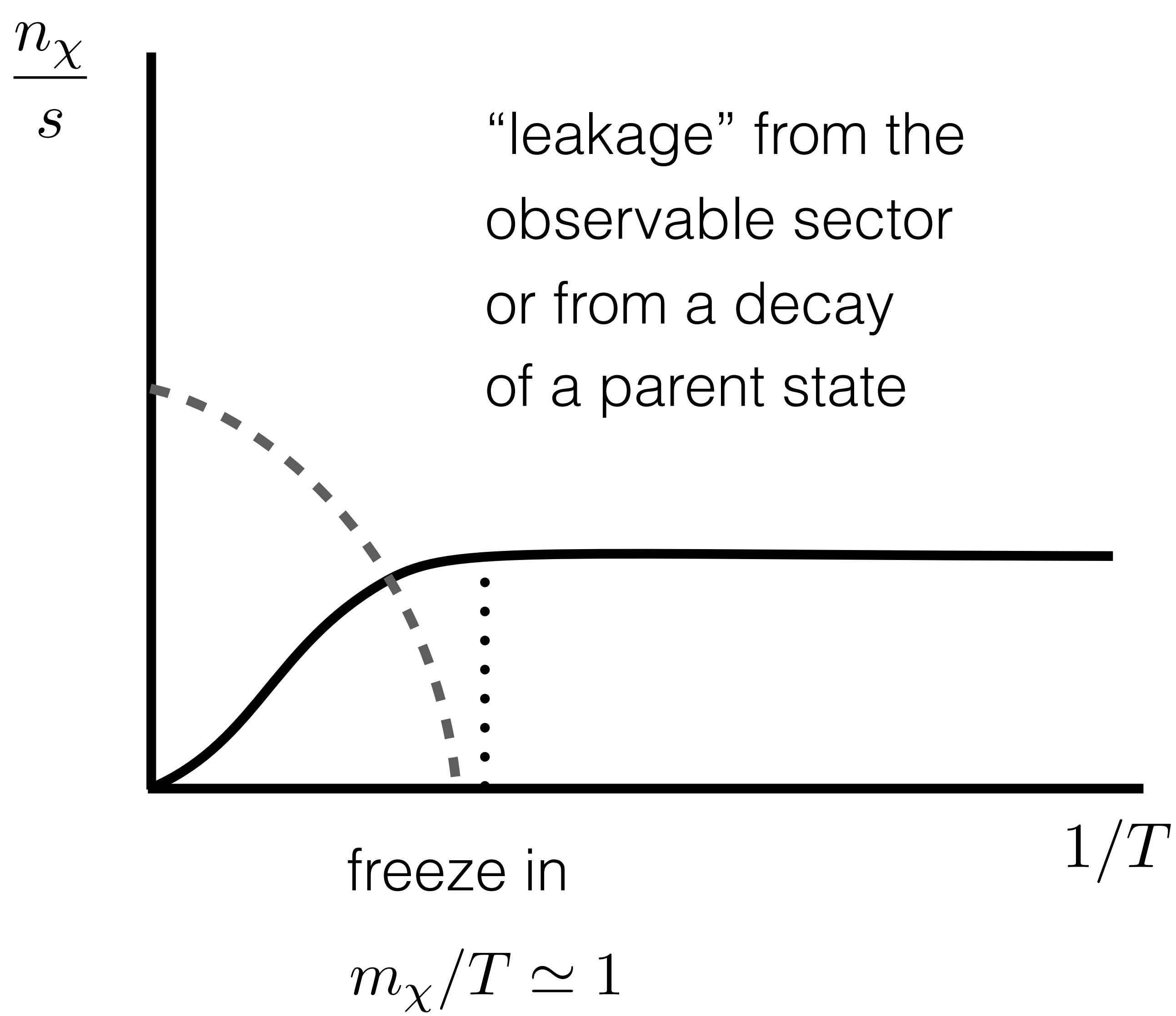
What is the abundance of $\chi$ for $T \gg m_\chi$?

**TINY** $N_\chi/N_\gamma \ll 1$

$\Gamma_\chi \ll H(T)$

"super-WIMPs"

Gravitinos,
sterile neutrinos,…




Their weak link to SM makes
them hard to detect

# Classification of Dark Matter

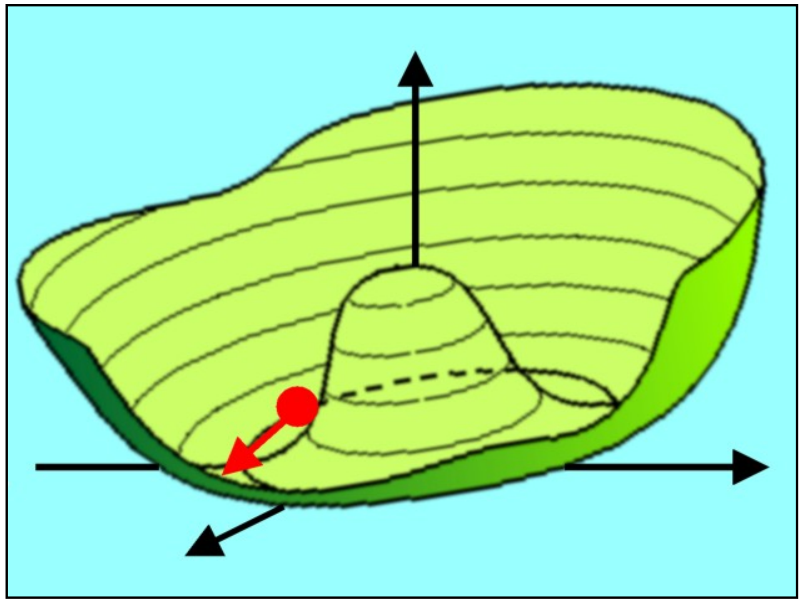
What is the abundance of $\chi$ for $T \gg m_\chi$?

**HUGE** $N_\chi / N_\gamma \gg 1$

Axions, PNGBs

Bosonic, initially displaced field;
Potential at low-energies/T provides mass

$$m_a(T) \simeq H(T)$$



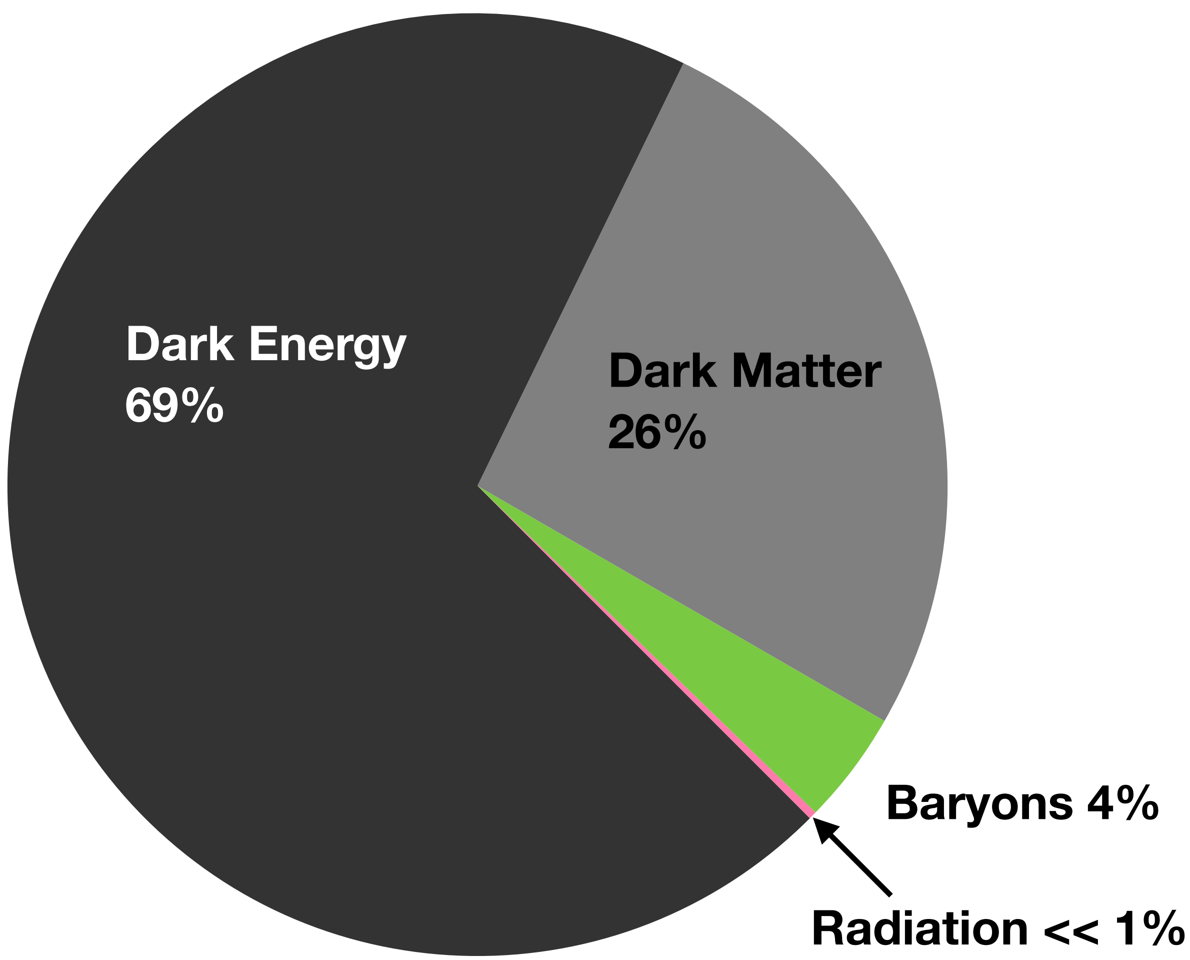
classical field oscillations populate zero mode

"super-cold" Dark Matter

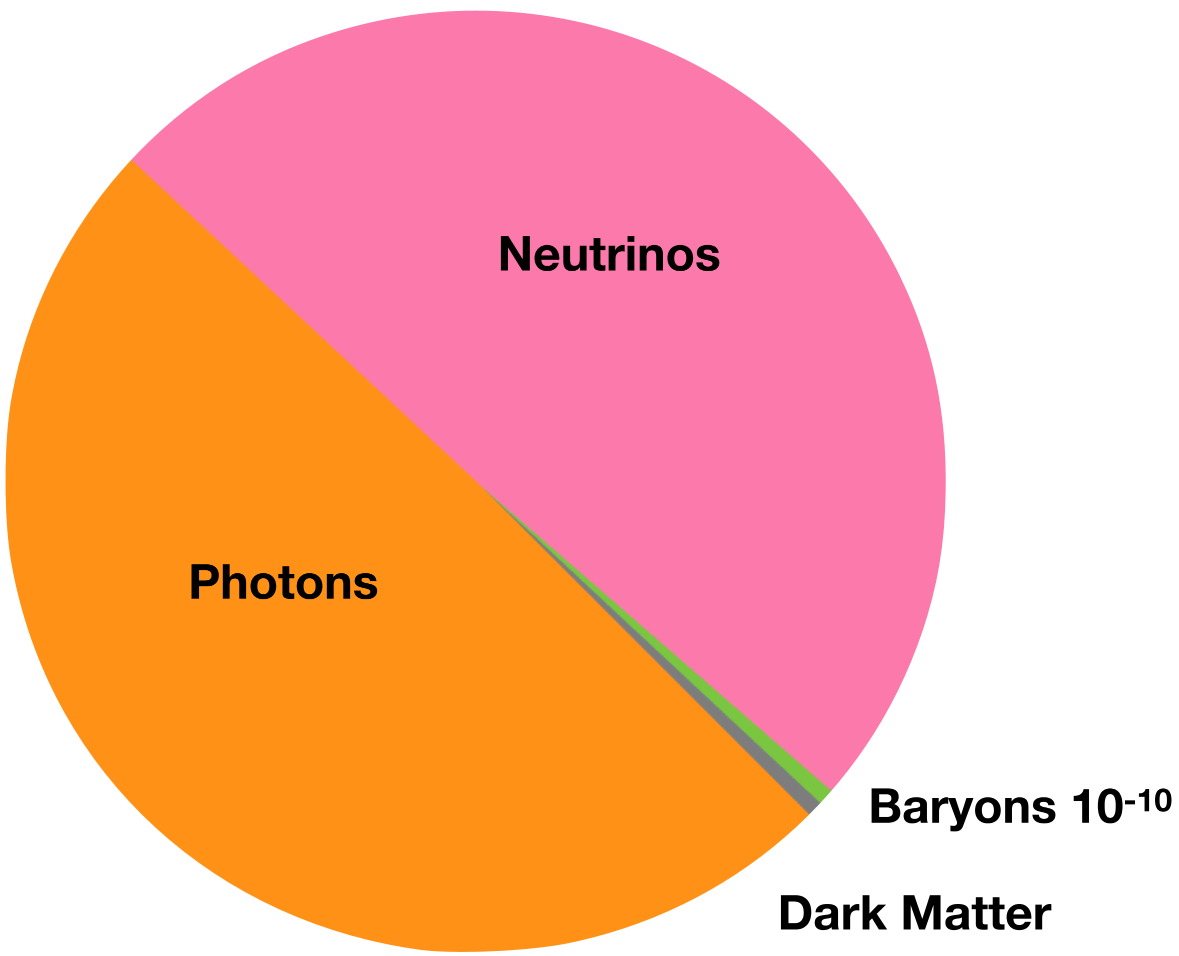
NB: super-WIMPs can at times be super-cold

# The universe in “numbers”

WIMPs or super-WIMPs



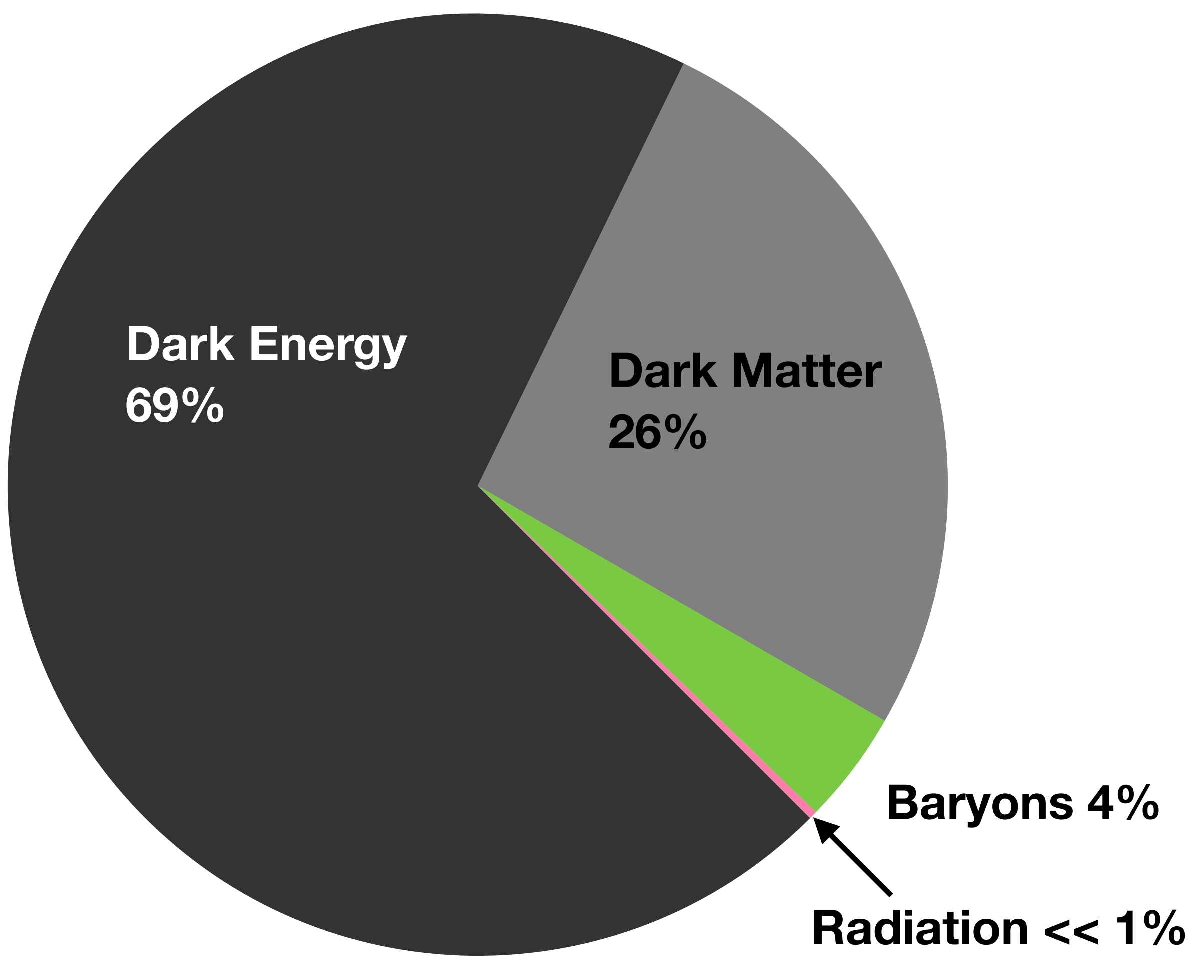

in terms energy densities




in terms of number densities

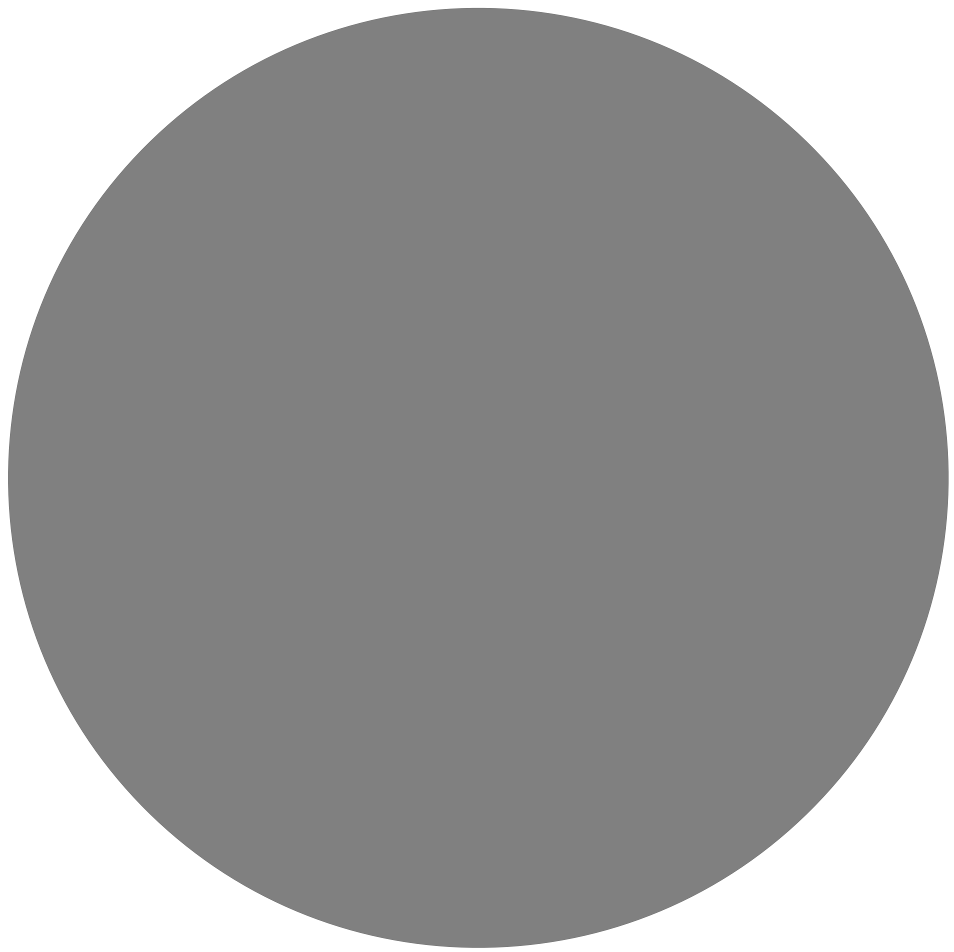
$$\rho_{\rm DM} = m_{\rm DM} n_{\rm DM}$$

# The universe in “numbers”




in terms energy densities

axions

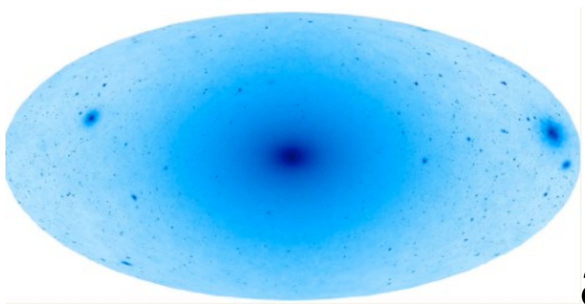


in terms of number densities

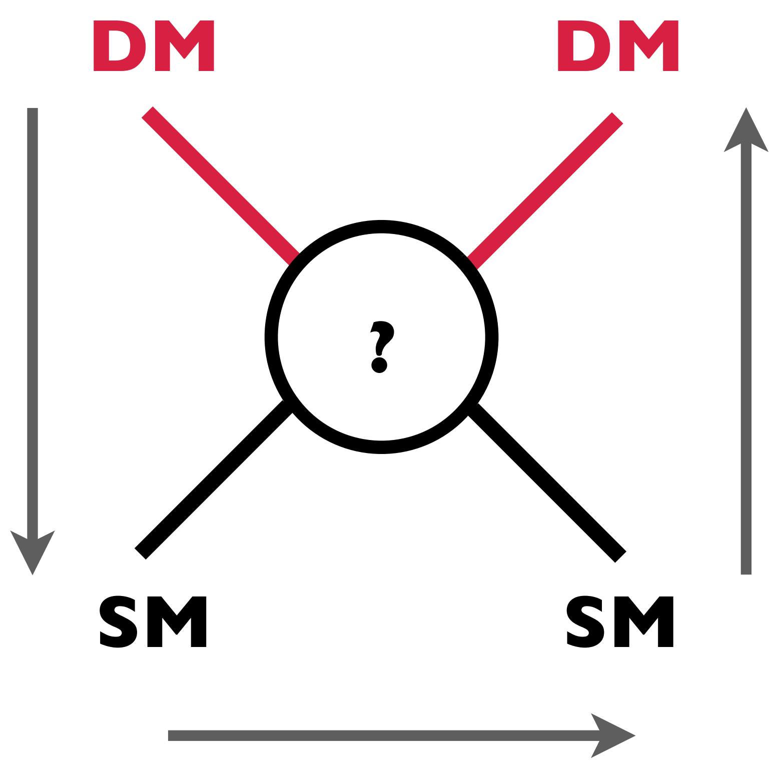
$$\rho_{\rm DM} = m_{\rm DM} n_{\rm DM}$$

# WIMP Phenomenology

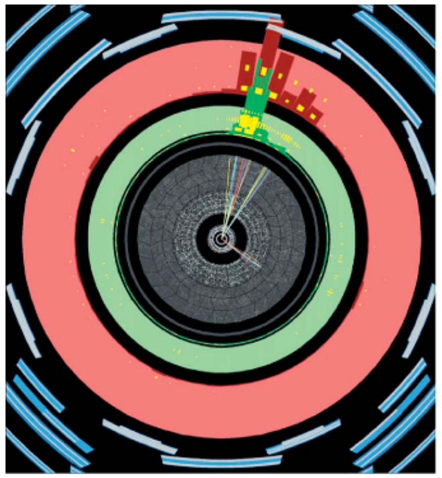
Multi-messenger approach to unraveling the particle nature of WIMP DM



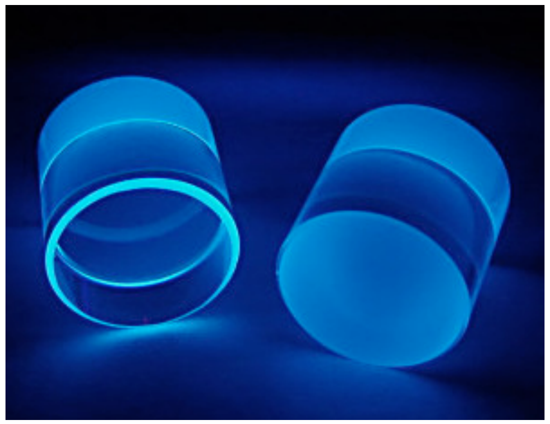
primordial
& galactic
annihilation




production
at colliders

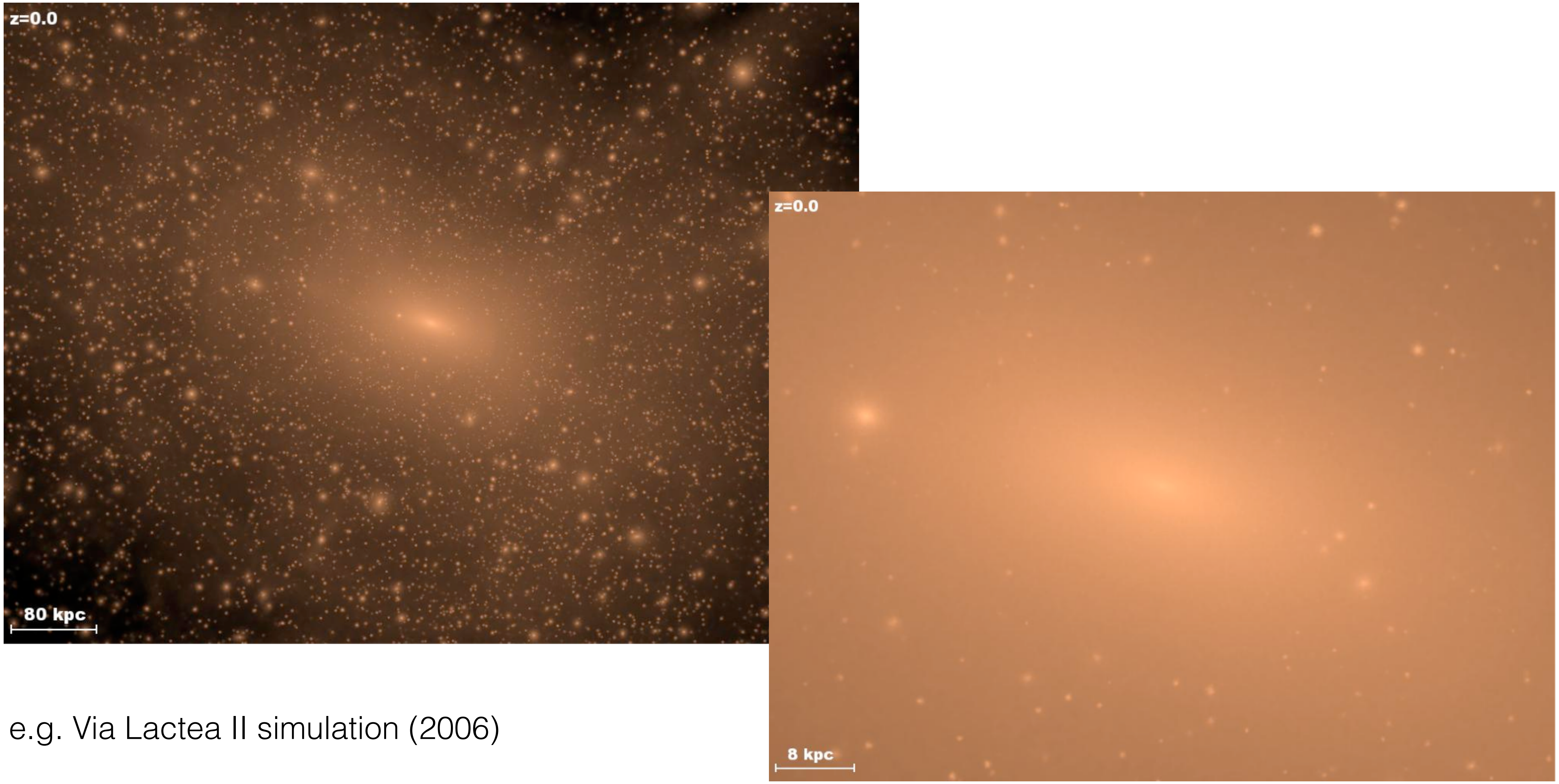


WIMP-nucleus scattering



# Direct Detection of the DM Halo

N-body simulations inform us about the expected phase space distribution of Dark Matter in a Milky Way-type galaxy




e.g. Via Lactea II simulation (2006)

# Direct Detection of the DM Halo

Crucial ingredients for direct detection are

- local density of DM $\rho_0 \simeq (0.3 \pm 0.1)\,\mathrm{GeV/cm^3}$
- velocity dispersion of DM particles $\sigma_{ij}^2 = \int d^3\vec{v}(v_i - \bar{v}_i)(v_j - \bar{v}_j)f(\vec{v}) = \sigma_{ii}^2 \delta_{ij}$

---

Consider a simple density profile $\rho(r) = \dfrac{\sigma^2}{2\pi G r^2}$ "isothermal sphere"

It can be shown that an isothermal sphere of a gas is identical with a collisionless system of particles with Maxwell Boltzmann distribution; it does not matter if the particles collide or not; it also produces flat rotation curves.

$$dn \propto \exp\left(-\frac{|\vec{v}|^2}{2\sigma^2}\right) d^3\vec{v}$$

Maxwell Boltzmann distribution

A "test-particle" on orbit has a circular velocity $v_0^2 = 2\sigma^2$

$$dn \propto \exp\left(-\frac{|\vec{v}|^2}{v_0^2}\right) d^3\vec{v}$$

Velocity of the sun around the galactic center is $v_0 \simeq 220\,\mathrm{km/s}$

# Direct Detection of the DM Halo

- Density profiles in N-body simulations are found to be approximated well by

$$\rho_{\mathrm{NFW}}(r) = \frac{\rho_0}{\frac{r}{R_s}\left(1 + \frac{r}{R_s}\right)^2}$$

Navarro-Frenk-White profile

Milky Way halo parameters

$\rho_s = 0.26\,\mathrm{GeV\,cm^{-3}}$

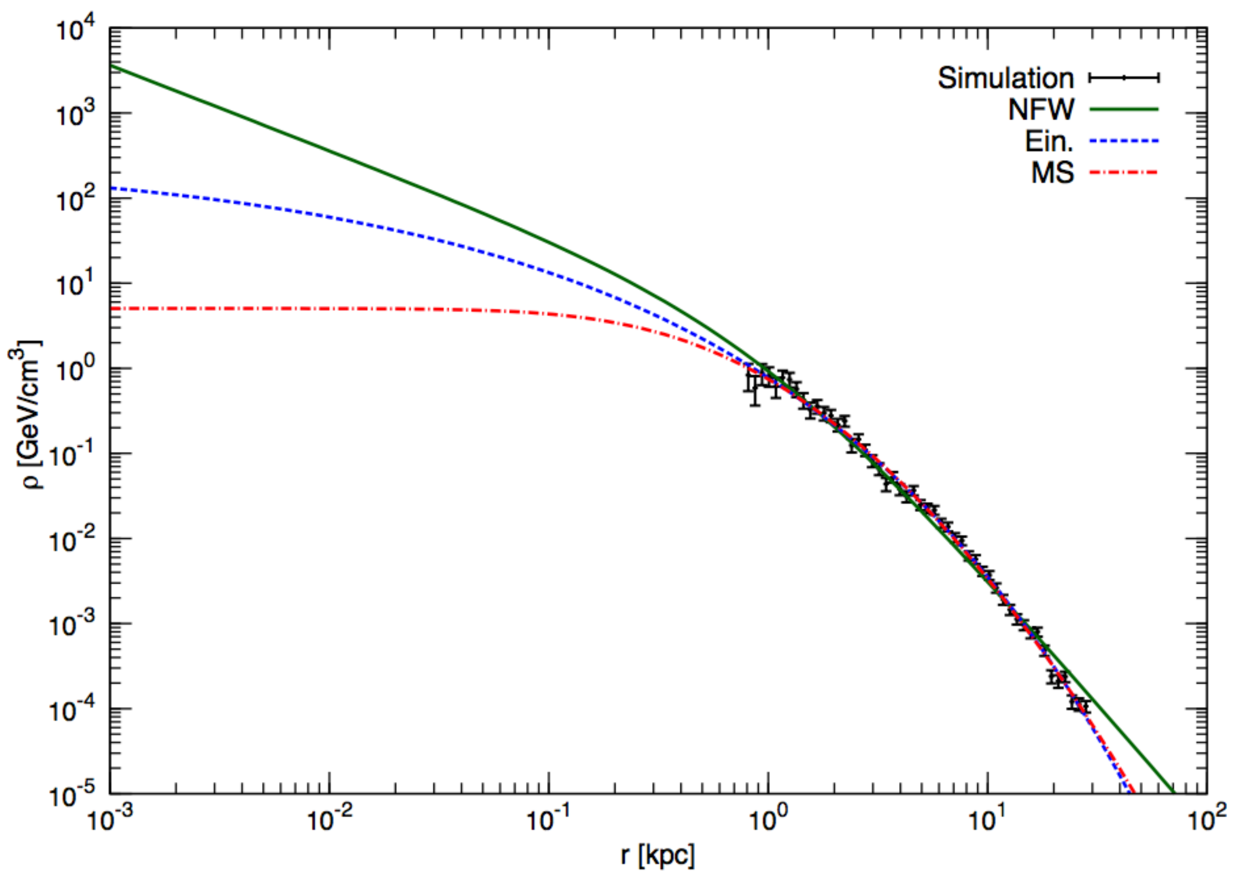
$r_s = 20\,\mathrm{kpc}$

yield

$\rho_0 = 0.3\,\mathrm{GeV\,cm^{-3}}$

$$\rho_{\mathrm{Ein}}(r) = \rho_s \exp\left\{-\frac{2}{\alpha}\left[\left(\frac{r}{r_s}\right)^{\alpha} - 1\right]\right\}$$

Einasto profile




from Calore et al 2014

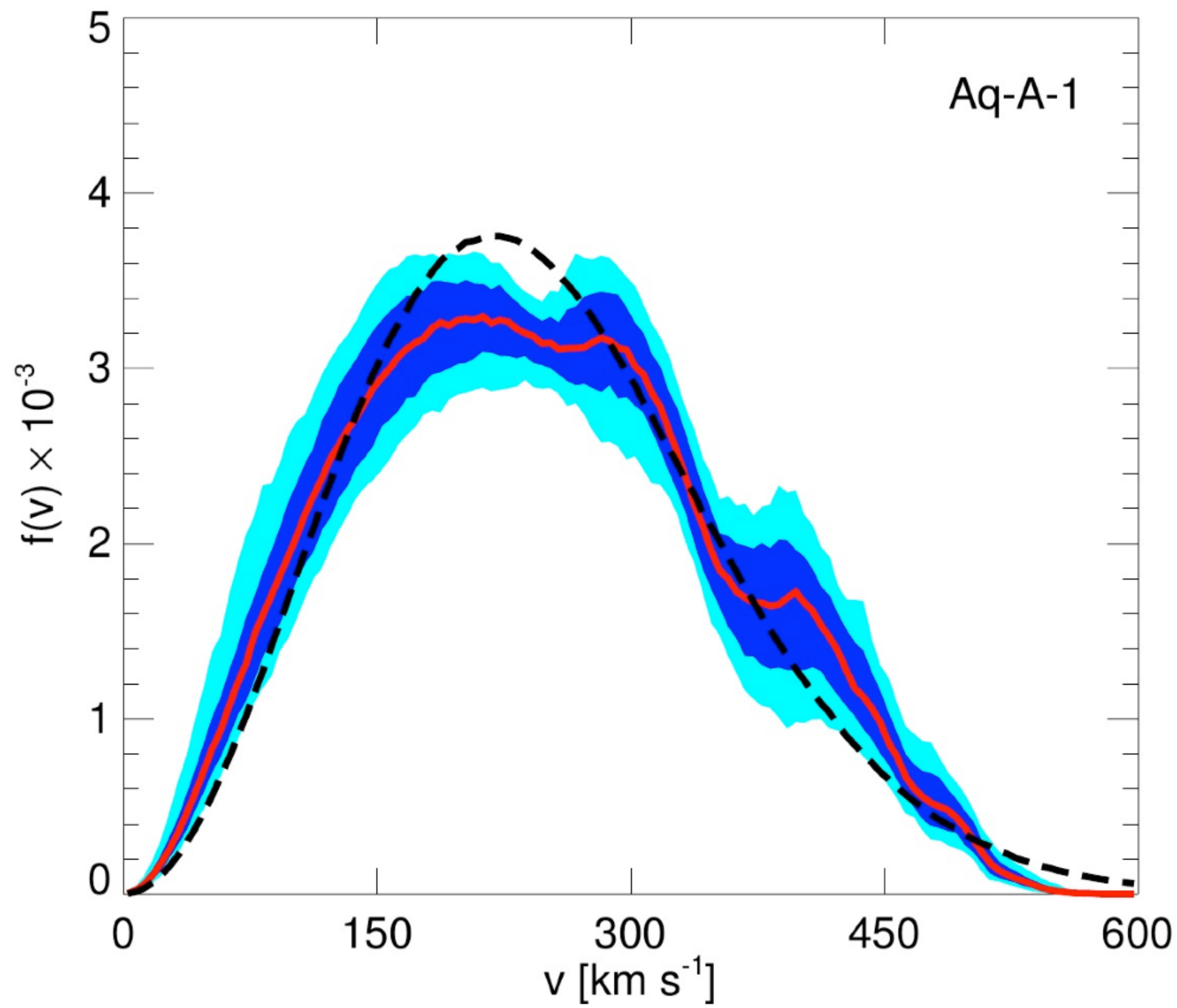
# Direct Detection of the DM Halo

- Maxwellian velocity distribution is found in N-body simulations

- Note: not all particles of arbitrary velocity can be gravitationally bound to the halo

$$f_{\mathrm{gal}}(\vec{v}) \approx \begin{cases} N \exp\left(-|\vec{v}|^2/v_0^2\right) & v < v_{\mathrm{esc}} \\ 0 & v > v_{\mathrm{esc}} \end{cases}$$

$$v_{\mathrm{esc}} \simeq 650\,\mathrm{km/s}$$

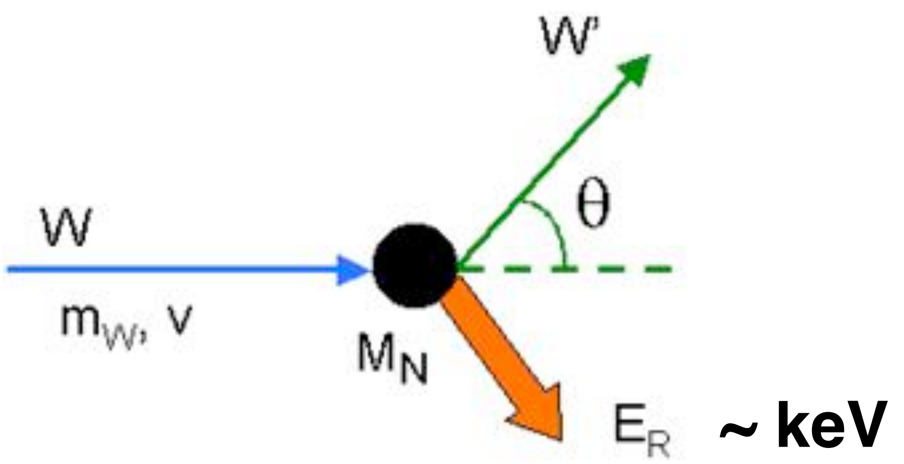



Vogelsberger et al 2009

- Local DM flux is $(v_\chi \sim 10^{-3} c)$

$$\phi_\chi \sim \frac{\rho_0 v_\chi}{m_\chi} \sim 10^5\,/\mathrm{cm}^2/\mathrm{s} \left(\frac{100\,\mathrm{GeV}}{m_\chi}\right)$$

# Direct Detection




Detection Rate = particle flux x cross section

$$\frac{dR(t)}{dE_R} = N_T \frac{\rho_0}{m_{\rm DM}} \int_{v \geqslant v_{\rm min}} d^3\mathbf{v}\, v f_{\rm LAB}(\mathbf{v}) \frac{d\sigma}{dE_R} \qquad [\text{cpd/kg/keV}]$$

*Astrophysics*

local DM density

DM velocity distribution in the LAB frame

recoil cross section

*Particle Physics*

Contributions to f(v)

- isotropic smooth (Maxwellian)
- substructure $(p < 10^{-4})$
- debris flow, streams

Very little is known

- largely dissipationless
- stable on cosmological timescales

# Direct Detection - Astrophysics

- Changing frames

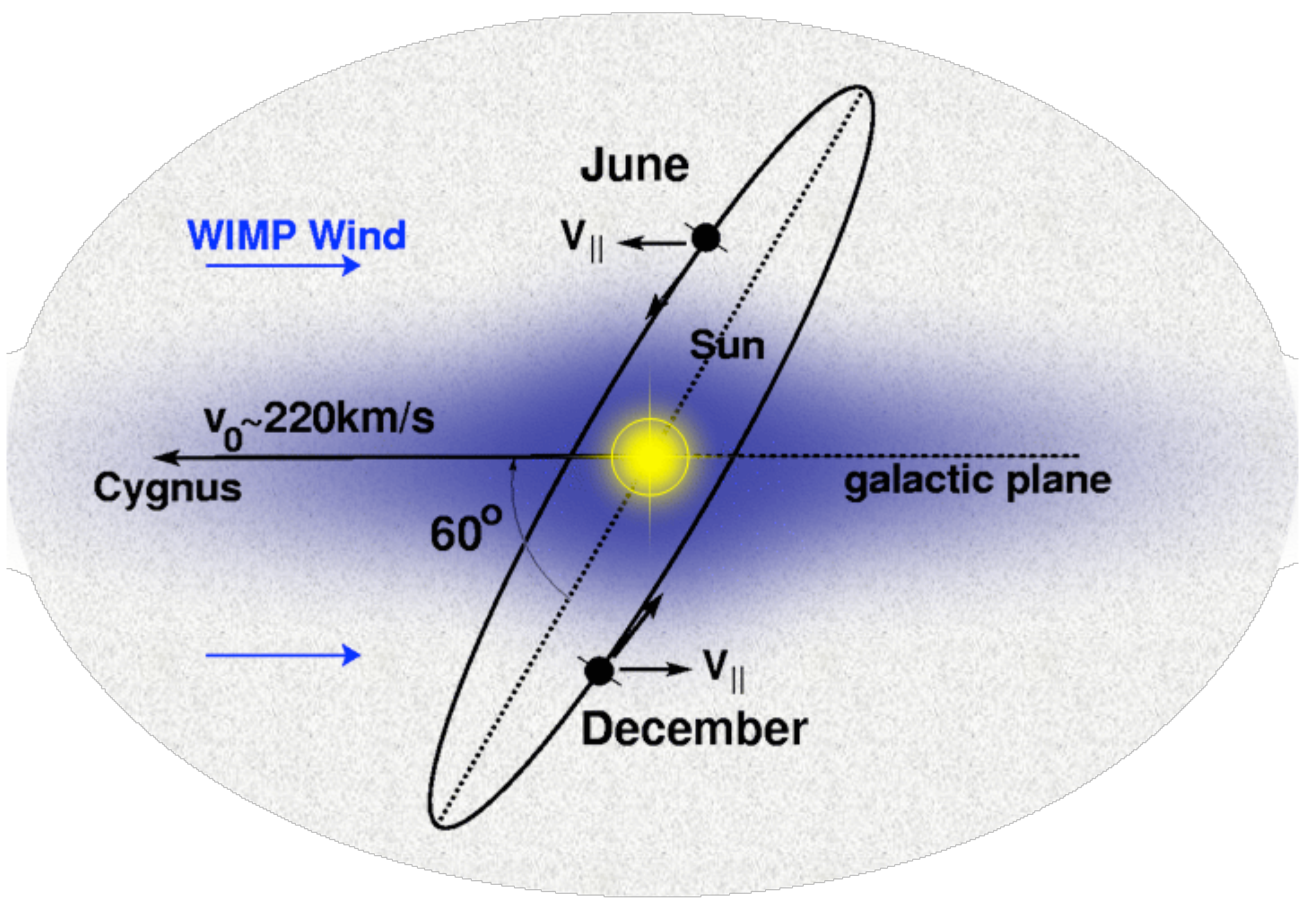
$$\frac{dR(t)}{dE_R} = N_T \frac{\rho_0}{m_{\rm DM}} \int_{v \geqslant v_{\rm min}} d^3\mathbf{v}\, v\, f_{\rm LAB}(\mathbf{v}) \frac{d\sigma}{dE_R} \qquad [\mathrm{cpd/kg/keV}]$$

$$f_{\rm LAB}(\mathbf{v}) \longrightarrow f_{\rm GAL}(\mathbf{v}_{\rm obs} + \mathbf{u})$$

$$|\mathbf{v}_{\rm obs}| = |\mathbf{v}_\odot| + \frac{1}{2} V_\oplus \cos\omega(t - t_0)$$

$$\boxed{t_0 \simeq 152\,\text{days} \quad \text{(June 2nd)}}$$

*geometric prediction*
for an isotropic velocity distribution!



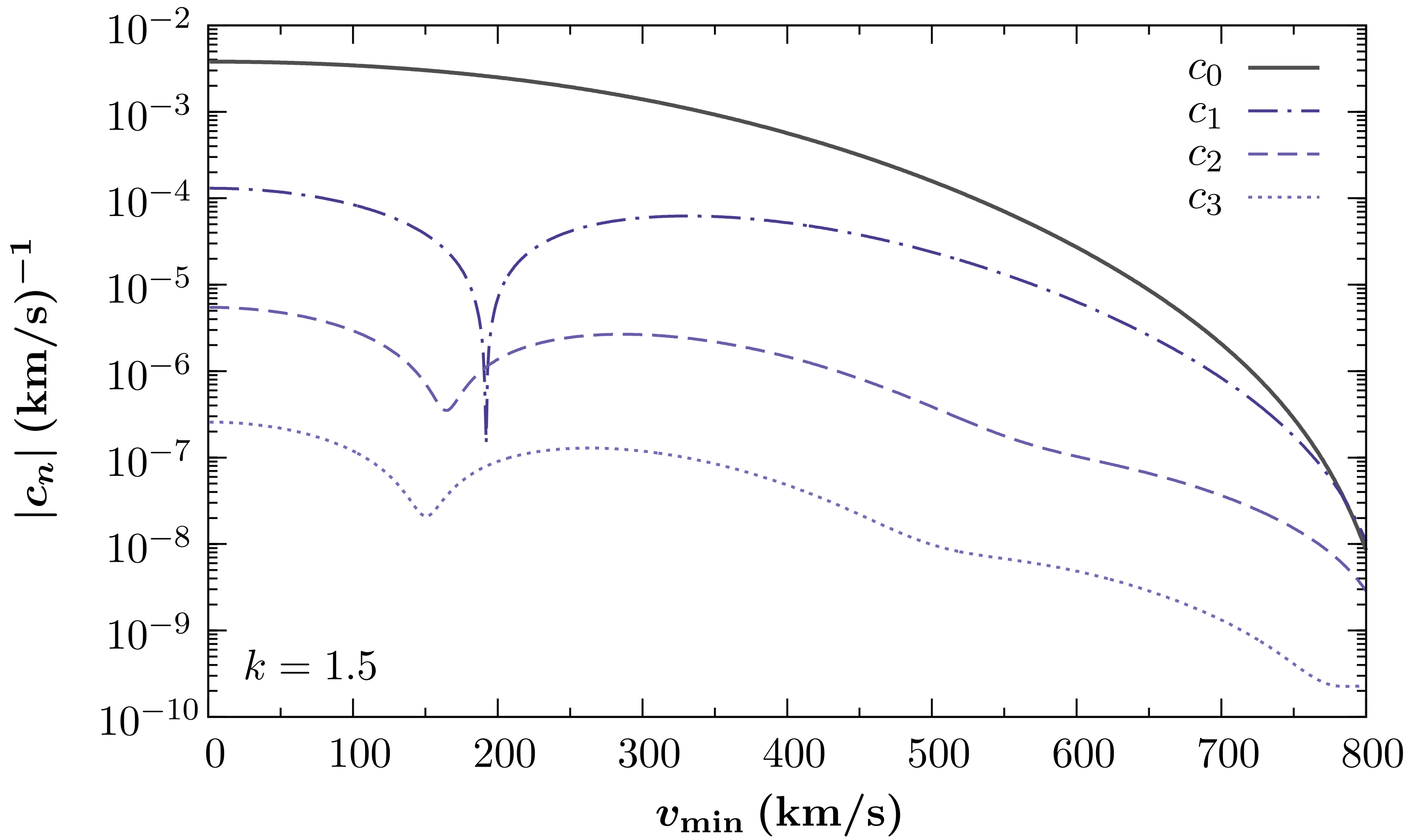
# Direct Detection - Astrophysics

- recoil spectrum

$$\frac{dR(t)}{dE_R} \sim c_0(v_{\rm min}) + c_1(v_{\rm min}) \cos\left[\omega(t - t_0)\right] \quad + \text{higher harmonics}$$

Chang, JP, Yavin (2012)

**annual modulation**




$$v_{min} = \frac{1}{\sqrt{2 m_N E_R}} \left( \frac{m_N E_R}{\mu_{N\chi}} + \delta \right)$$

# Direct Detection - Astrophysics

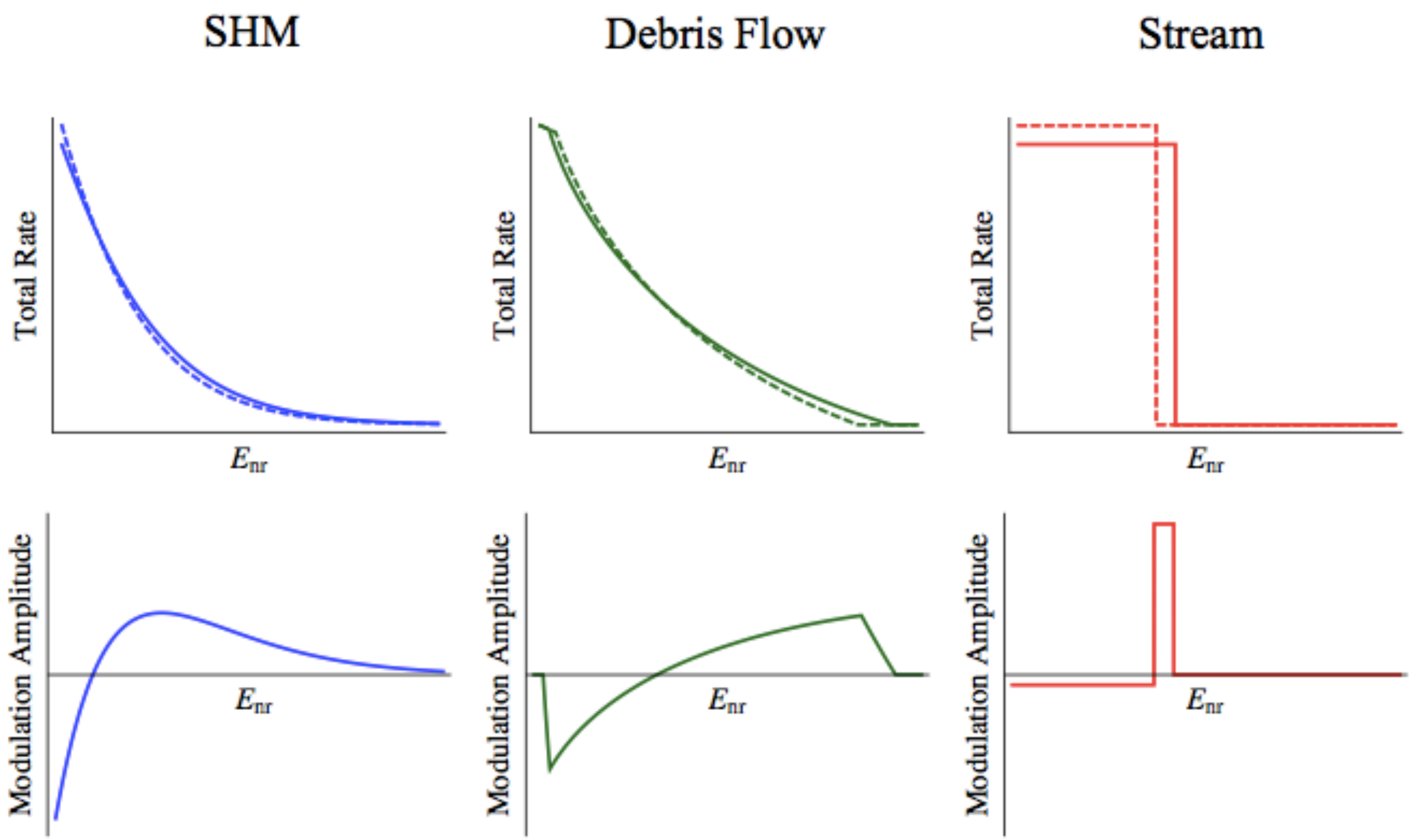



Fig. from Freese, Lisanti, Savage (2012)

Affects phase of annual modulation as well as higher harmonics

# An explicit example

Scalar Dark Matter candidate S, that couples to the Higgs via $\lambda S^2(H^\dagger H)$

$$\mathcal{L}_{\rm int} = -\lambda v h S S - \sum_q \frac{m_q}{v} h \bar{q} q \qquad\qquad H_I = -\int d^3x \mathcal{L}_{\rm int}$$

Nucleon-WIMP scattering $\quad \mathbf{p}_n + \mathbf{p}_S \to \mathbf{p}'_n + \mathbf{p}'_S$

$$\left\langle \mathbf{p}'_n \mathbf{p}'_S | i\mathcal{T} | \mathbf{p}_n \mathbf{p}_S \right\rangle = \left\langle \mathbf{p}'_n \mathbf{p}'_S | T \left\{ e^{-i\int dt H_I(t)} \right\} | \mathbf{p}_n \mathbf{p}_S \right\rangle$$

$$= \sum_q \left\langle \mathbf{p}'_n \mathbf{p}'_S | T \left\{ 2 \frac{(-i)^2}{2!} \frac{\lambda v m_q}{v} \int d^4x h S S \int d^4y h \bar{q} q \right\} | \mathbf{p}_n \mathbf{p}_S \right\rangle$$

$$= -2\lambda \int d^4x \int d^4y \, D_F^{(h)}(x-y) e^{-i(p_S - p'_S)\cdot x} \sum_q \left\langle \mathbf{p}'_n | m_q \bar{q}(y) q(y) | \mathbf{p}_n \right\rangle$$

Higgs propagator — nucleon matrix element

# An explicit example - WIMP nucleon interaction

Nucleon Matrix element

$$\left\langle \mathbf{p}'_n | m_q \bar{q}(y) q(y) | \mathbf{p}_n \right\rangle = e^{-i(p_n - p'_n)\cdot y} \bar{u}(\mathbf{p}'_n) \Gamma_q u(\mathbf{p}_n) \qquad \Gamma_q = \langle n | m_q \bar{q}(0) q(0) | n \rangle$$

$$f_n \equiv \sum_q \langle n | m_q \bar{q} q | n \rangle$$ => low energy input $f_{p,n} \simeq 0.52\,\mathrm{GeV}$

Matrix element for scattering on protons or neutrons is

$$\left\langle \mathbf{p}'_n \mathbf{p}'_S | i\mathcal{T} | \mathbf{p}_n \mathbf{p}_S \right\rangle = i(2\pi)^4 \delta^{(4)}(p_n + p_S - p'_n - p'_S) \times \frac{-2\lambda}{q^2 - m_h^2} \times f_n u(\mathbf{p}'_n) u(\mathbf{p}_n)$$

From this we get an effective WIMP-nucleon Lagrangian $(q^2 \ll m_h^2)$

$$\mathcal{L}_{\text{eff}} = +\frac{\lambda f_n}{m_h^2} S^2 \bar{n} n$$ or $$\mathcal{L}_{\text{eff}} = -\lambda v h S S - \frac{f_n}{v} h \bar{n} n$$

# An explicit example - WIMP nucleus interaction

Now we proceed and calculate the *Nucleus*-WIMP matrix element

$$\left\langle \mathbf{p}'_N \mathbf{p}'_S | i\mathcal{T} | \mathbf{p}_N \mathbf{p}_S \right\rangle = -2\lambda \int d^4x \int d^4y \, D_F^{(h)}(y-x) e^{-i(p_S - p_{S'})\cdot y} \sum_n f_n \left\langle \mathbf{p}'_N | \bar{n}(x) n(x) | \mathbf{p}_N \right\rangle$$

The piece we are really interested in

$$\sum_n f_n \left\langle \mathbf{p}'_N | \bar{n}(x) n(x) | \mathbf{p}_N \right\rangle = 2m_N \times e^{-i(E_N - E'_N)\cdot t} \left\langle JM \middle| \sum_n f_n \bar{n}(\mathbf{x}) n(\mathbf{x}) \middle| JM' \right\rangle_{\mathrm{NR}}$$

switching to non-relativistic normalization $\langle \mathbf{p} | \mathbf{q} \rangle_{\mathrm{NR}} = (2\pi)^3 \delta^{(3)}(\mathbf{p} - \mathbf{q})$

(relativistic: $\langle \mathbf{p} | \mathbf{q} \rangle = 2E_{\mathbf{p}} (2\pi)^3 \delta^{(3)}(\mathbf{p} - \mathbf{q})$ )

$$\mathcal{M}_N = +\frac{4\lambda m_N}{m_h^2} \int d^3\mathbf{x} \, e^{+i\mathbf{q}\cdot\mathbf{x}} \left\langle JM \middle| \sum_n f_n \bar{n}(\mathbf{x}) n(\mathbf{x}) \middle| JM' \right\rangle \equiv +\frac{4\lambda m_N}{m_h^2} \rho(\mathbf{q})$$

scalar nuclear current

# An explicit example - nuclear form factors

Evaluate the scalar current

$$\rho(\mathbf{q}) = \int d^3\mathbf{x}\, e^{+i\mathbf{q}\cdot\mathbf{x}} \langle J_f M_f | \hat{\rho}(\mathbf{x}) | J_i M_i \rangle$$

The interaction that probes the nucleus depends on the model

In our case it is simple: $\bar{n}n = n^\dagger \gamma^0 n \sim a^\dagger a$ the operator is the number operator and counts nucleons

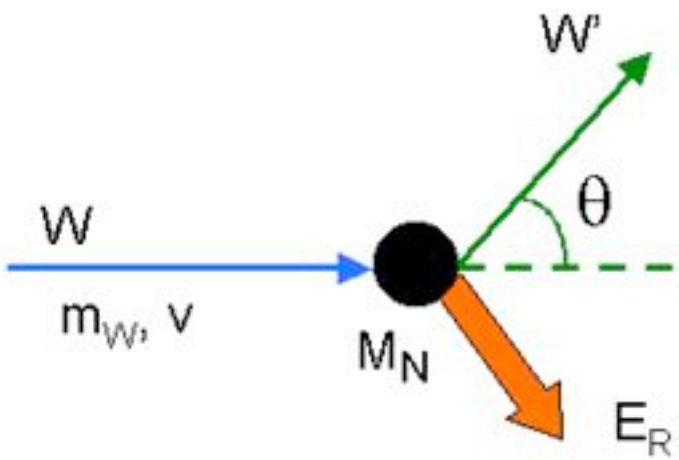
$$\sum_{M_i M_j} |\rho(\mathbf{q})|^2 = F^2(q) \times \frac{2J+1}{4\pi} \left[ Z f_p + (A - Z) f_n \right]^2$$

number of protons

number of neutrons

form factor for finite momentum transfer

# An explicit example - recoil cross section




WIMP nucleus cross section

$$\frac{d\sigma}{dt} = \frac{d\sigma}{d|\mathbf{q}|^2} = \frac{|\overline{\mathcal{M}}_N|^2}{64\pi m_N^2 m_\chi^2 v^2} \qquad |\overline{\mathcal{M}}_N|^2 = \frac{1}{(2s_\chi+1)(2J+1)} \sum_{\text{spins}} |\mathcal{M}_N|^2$$

We turn this into a *recoil cross section* ($E_R$ is the kinetic energy of the recoiling nucleus)

$$E_R = \frac{-t}{2m_N} = \frac{|\mathbf{q}|^2}{2m_N} = \frac{\mu_N^2 v^2}{m_N}(1-\cos\theta_*)$$

$$\sigma_0 \equiv \int_0^{4\mu_N^2 v^2} d|\mathbf{q}|^2 \, \frac{d\sigma(|\mathbf{q}|^2=0)}{d|\mathbf{q}|^2} = \frac{\mu_N^2 |\overline{\mathcal{M}}_N(|\mathbf{q}|^2=0)|^2}{16\pi m_N^2 m_\chi^2}$$

"total cross section at zero momentum transfer"

$$\frac{d\sigma}{dE_R} = 2m_N \frac{d\sigma_{\text{tot}}}{d|\mathbf{q}|^2} = \frac{m_N}{2\mu_N^2 v^2}\left[\sigma_0^{\text{SI}} F^2(|\mathbf{q}|) + \sigma_0^{\text{SD}} S(|\mathbf{q}|)/S(0)\right]$$

spin-independent scattering     spin-dependent scattering

# An explicit example - recoil cross section

WIMP nucleus cross section

$$\frac{d\sigma}{dE_R} = 2m_N \frac{d\sigma_{\rm tot}}{d|\mathbf{q}|^2} = \frac{m_N}{2\mu_N^2 v^2}\left[\sigma_0^{\rm SI} F^2(|\mathbf{q}|) + \sigma_0^{\rm SD} S(|\mathbf{q}|)/S(0)\right]$$

For our example of Higgs mediated scattering

$$\sigma_0^{\rm SI} = \frac{\mu_N^2 \lambda^2}{\pi m_\chi^2 m_h^4}\left[Z f_p + (A-Z) f_n\right]^2 \qquad \frac{d\sigma}{dE_R} = \frac{\lambda^2 m_N}{2\pi v^2 m_\chi^2 m_h^4}\left[Z f_p + (A-Z) f_n\right]^2 F^2(|\mathbf{q}|)$$

coherent scattering $\sigma \propto A^2 \quad (f_n = f_p)$

Experiments report their results often in terms *WIMP-nucleon* cross section $\sigma_n$

$$\sigma_0^{\rm SI} = \frac{\sigma_n}{f_n^2}\left(\frac{\mu_N}{\mu_n}\right)^2 \left[Z f_p + (A-Z) f_n\right]^2$$

# An explicit example - form factor

The maximum recoil is reached in a back-to-back scattering in the center of momentum frame

$$E_R^{\max} = \frac{|\mathbf{q}_{\max}|^2}{2m_N} = \frac{(2\mu_N v)^2}{2m_N} \sim \begin{cases} 20\,\mathrm{keV}\left(\frac{A}{20}\right) & (m_N \ll m_{DM}) \\ 4\,\mathrm{keV}\left(\frac{m_{DM}}{20\,\mathrm{GeV}}\right)^2\left(\frac{100}{A}\right) & (m_{DM} \ll m_N) \end{cases}$$

The hard scatterings resolve nuclear scales (few fm)

$$|\mathbf{q}| = \sqrt{2m_N E_R} \lesssim 100\,\mathrm{MeV}$$

=> loss of coherence

$$F(q) = \frac{1}{M}\int \rho(r) e^{-i\vec{q}\cdot\vec{r}}$$

(matter ~ charge distribution)

e.g. Duda et al 2007

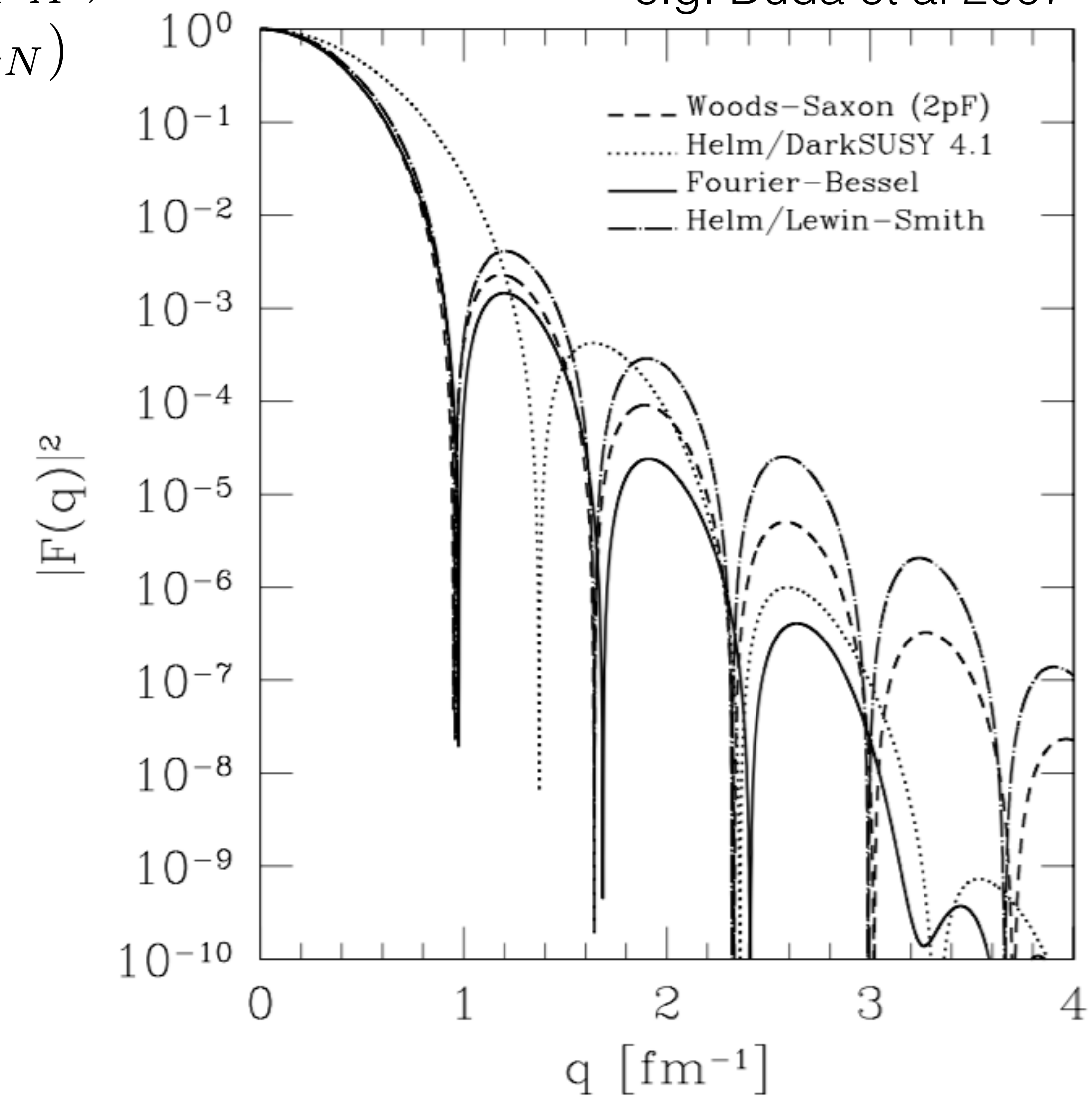
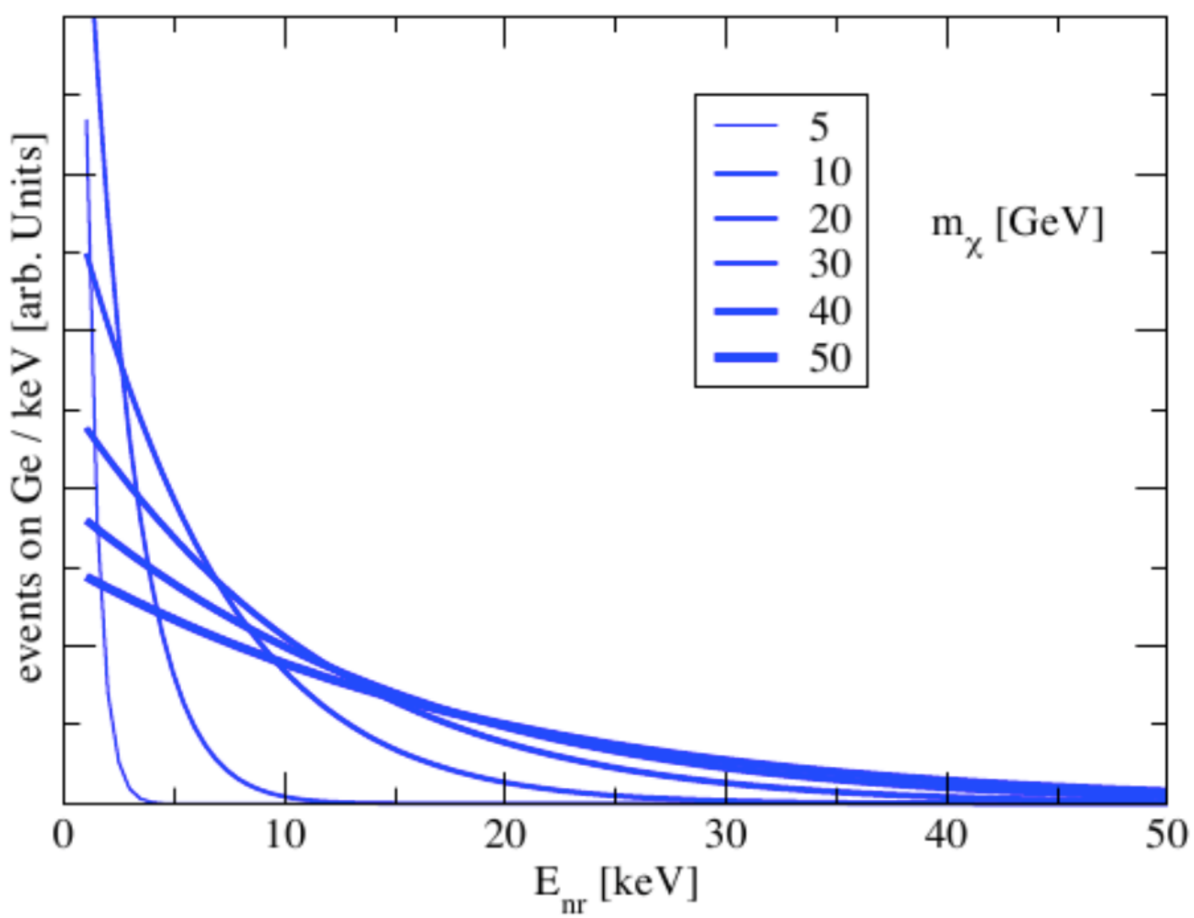


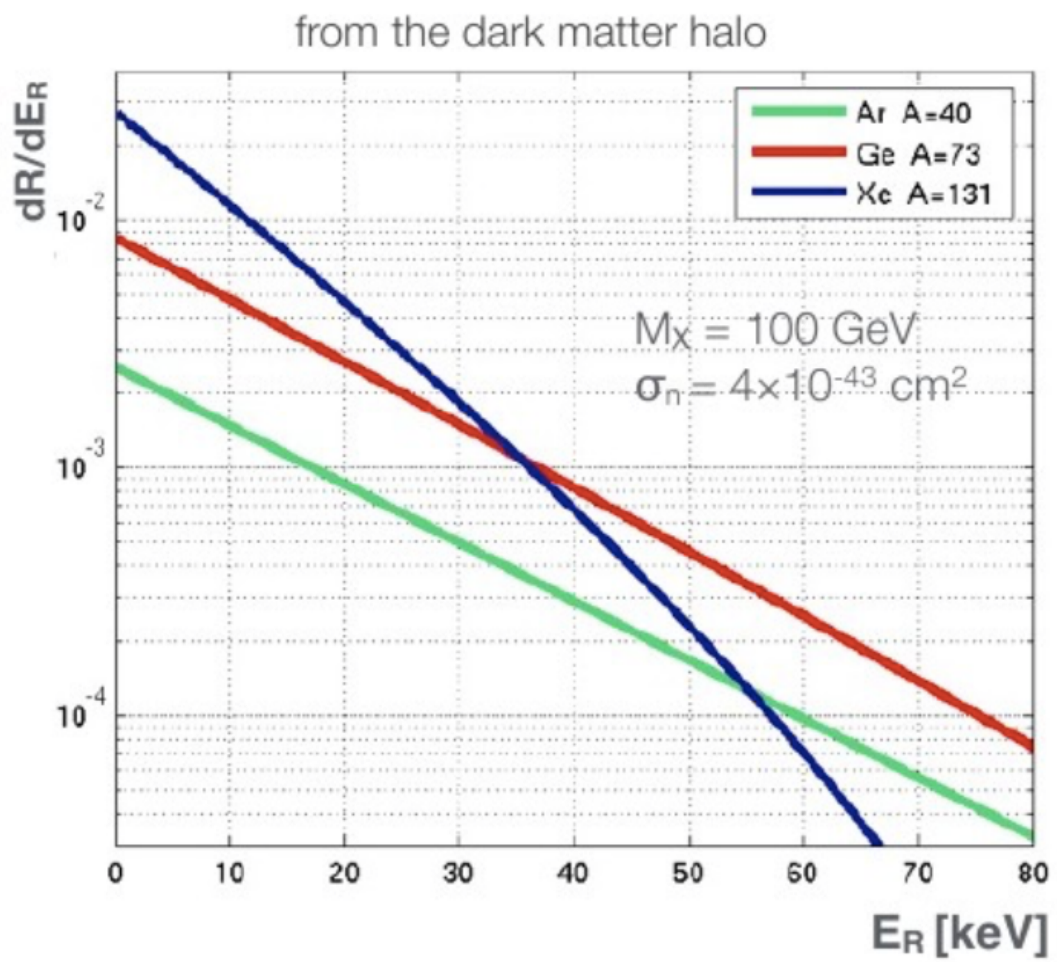

FIG. 4: Helm and FB form factors for $^{70}$Ge.

# Event spectrum

Recoil spectrum $$\frac{dR(t)}{dE_R} = N_T \frac{\rho_0}{m_{\rm DM}} \int_{v \geqslant v_{\rm min}} d^3\mathbf{v}\, v f_{\rm LAB}(\mathbf{v}) \frac{d\sigma}{dE_R}$$




Various DM masses

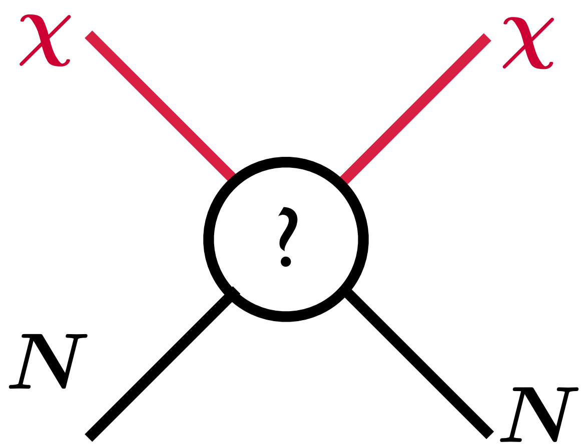



Various targets

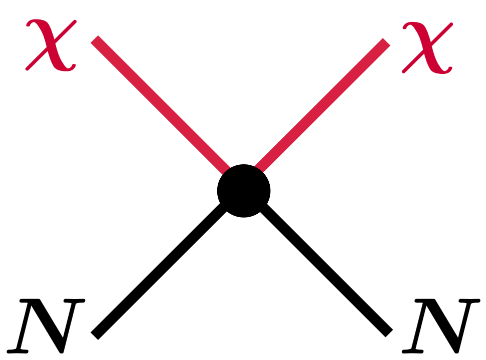
# Effective Operators

see, in particular,
Fan, Reece, Wang (2010)
Fitzpatrick et al. (2013)

- Effective operator approach between DM $\chi$ and nucleons $N$ often holds as $|q_{\max}| \sim O(100\,\mathrm{MeV})$




$\Rightarrow$




$$\mathcal{L}_{\rm eff} = \frac{1}{\Lambda^2}(\bar\chi\Gamma_\chi\chi)(\bar q\Gamma_q q)$$

- For example, fermionic DM

| | S | P | V | A | T | AT |
|---|---|---|---|---|---|---|
| $\Gamma_{\chi,q}$ | $1$ | $\gamma^5$ | $\gamma^\mu$ | $\gamma^\mu\gamma^5$ | $\sigma^{\mu\nu}$ | $\sigma^{\mu\nu}\gamma^5$ |

Non-relativistic limit (on tree level):

$S \times S$ or $V \times V$ coherent, spin-independent scattering

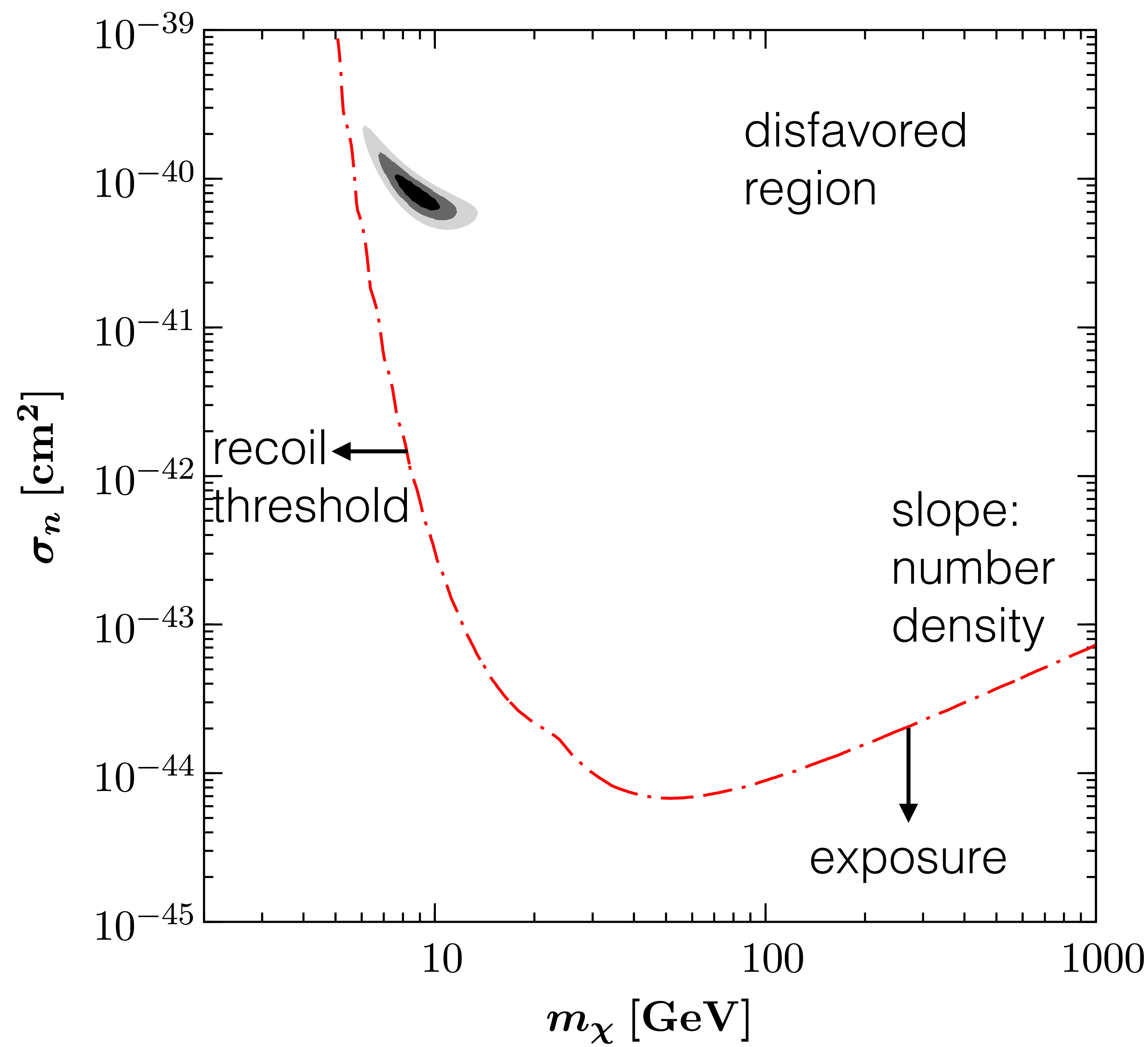
$A \times A$ or $T \times T$ spin-dependent, coupling to the unpaired nucleon

other combinations are suppressed by $v^2, \mathbf{q}^2/m_N^2 \sim 10^{-6}$

# Exclusion limits

Shown here is a typical situation in this field:

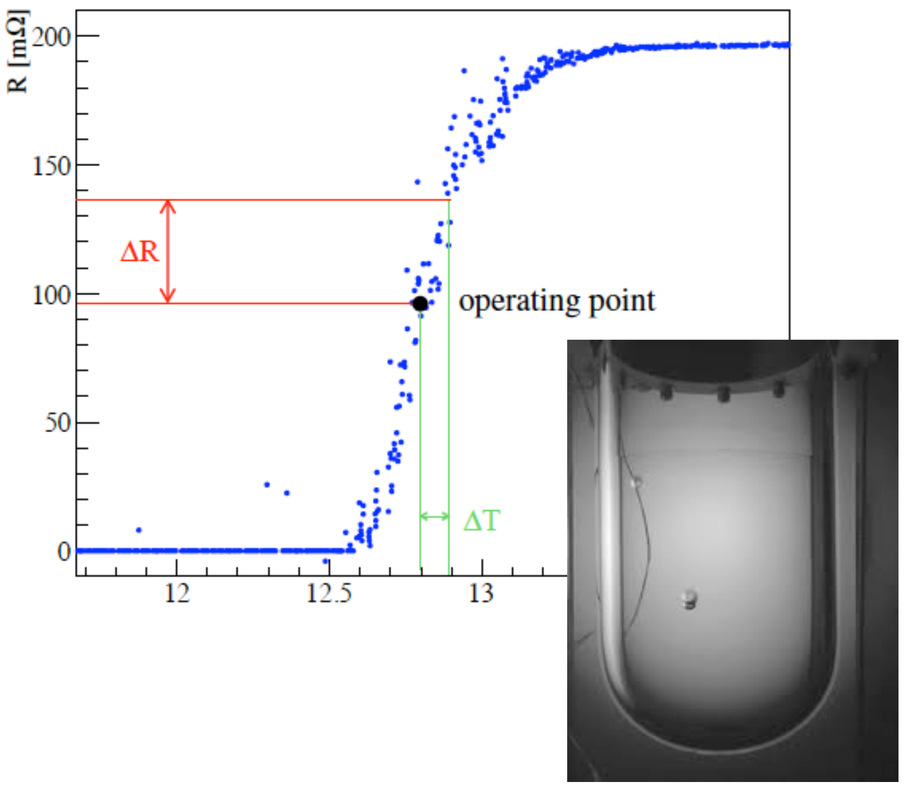
an anomaly in one experiment which is in tension with null results from another experiment(s)




$$\frac{dR(t)}{dE_R} = N_T \frac{\rho_0}{m_{\rm DM}} \int_{v \geqslant v_{\rm min}} d^3\mathbf{v}\, v f_{\rm LAB}(\mathbf{v}) \frac{d\sigma}{dE_R}$$

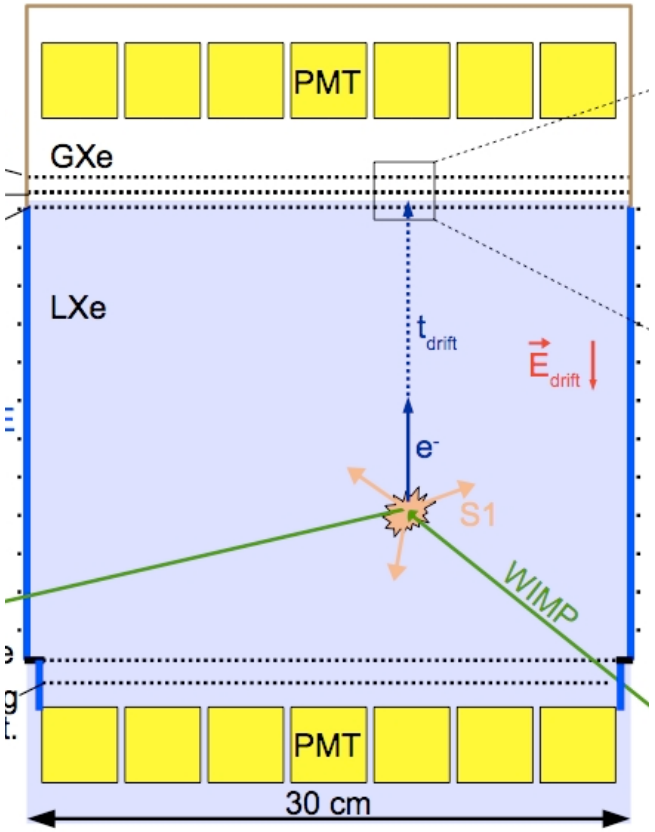
# Observables

Experimental techniques to detect keV recoiling nucleus




**heat**
CRESSST-II, (super)CDMS, EDELWEISS,..

**scintillation**
DAMA/LIBRA, XMASS, DEAP, XENON, DarkSide,…




**ionization**
CoGeNT, CDEX, DAMIC, PICO,…

*Most experiments utilize 2 channels* to discriminate between nuclear recoils (WIMPs or, unfortunately, neutrons) and electron recoils (gamma-rays, beta-radioactivity, Compton backgrounds,…)

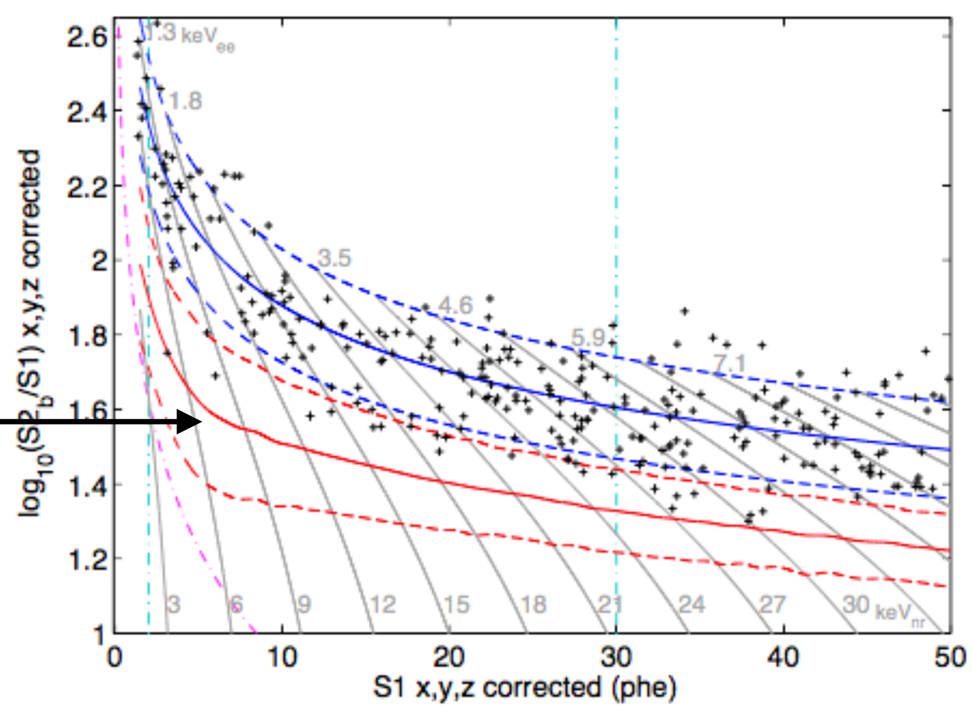
# Exposure driven experiments (more kg - days)

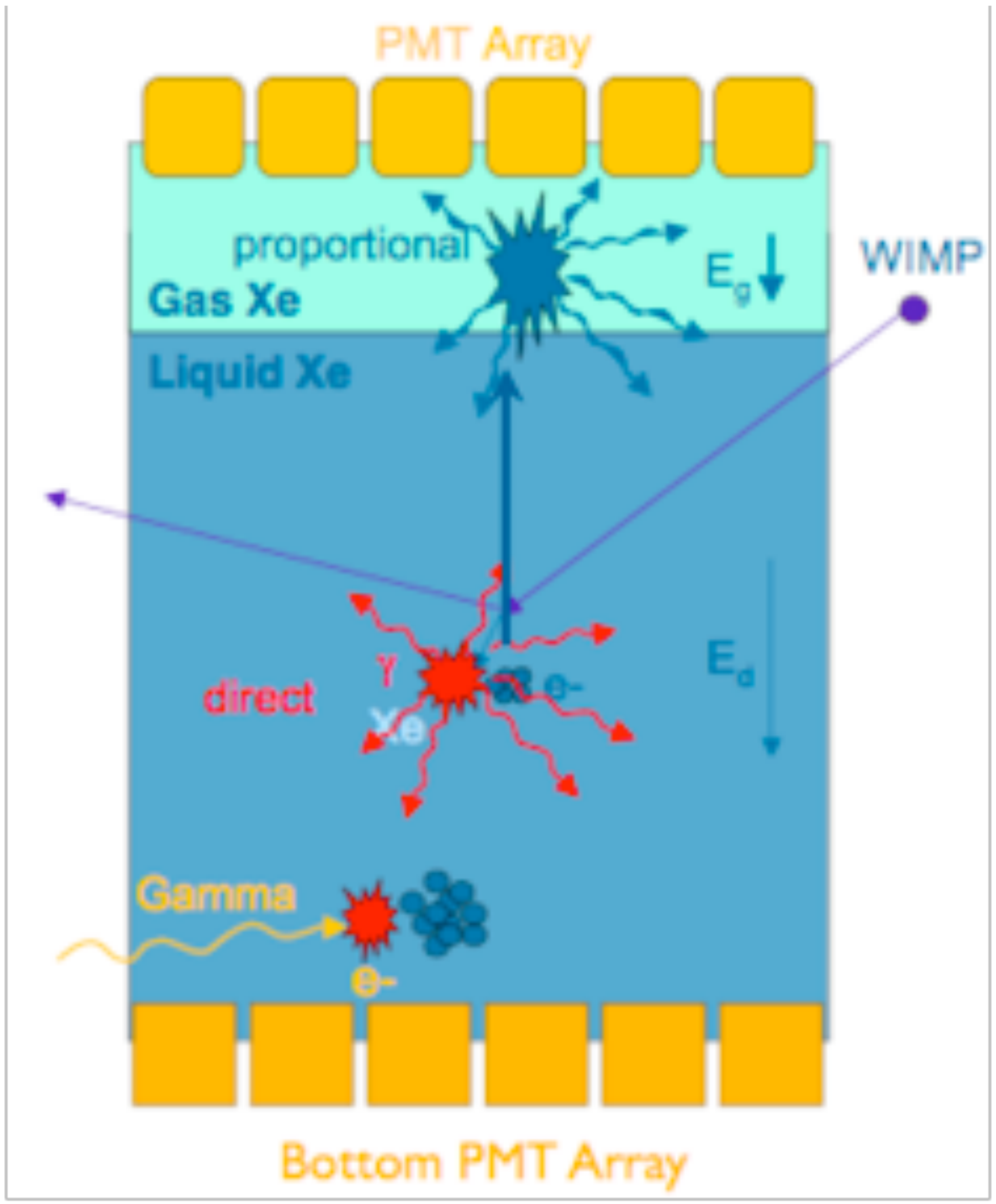
Leaders: Liquid scintillators (inexpensive, scalable, dense, and can be purified.)

current: LUX, XENON100, XMASS, …)
future: LZ, XENON1T, DEAP, …

High *scintillation yields* without absorbing own scintillation light; drifting charges (*ionization*) in an electric field is a powerful amplification mechanism

discrimination



# Threshold driven experiments

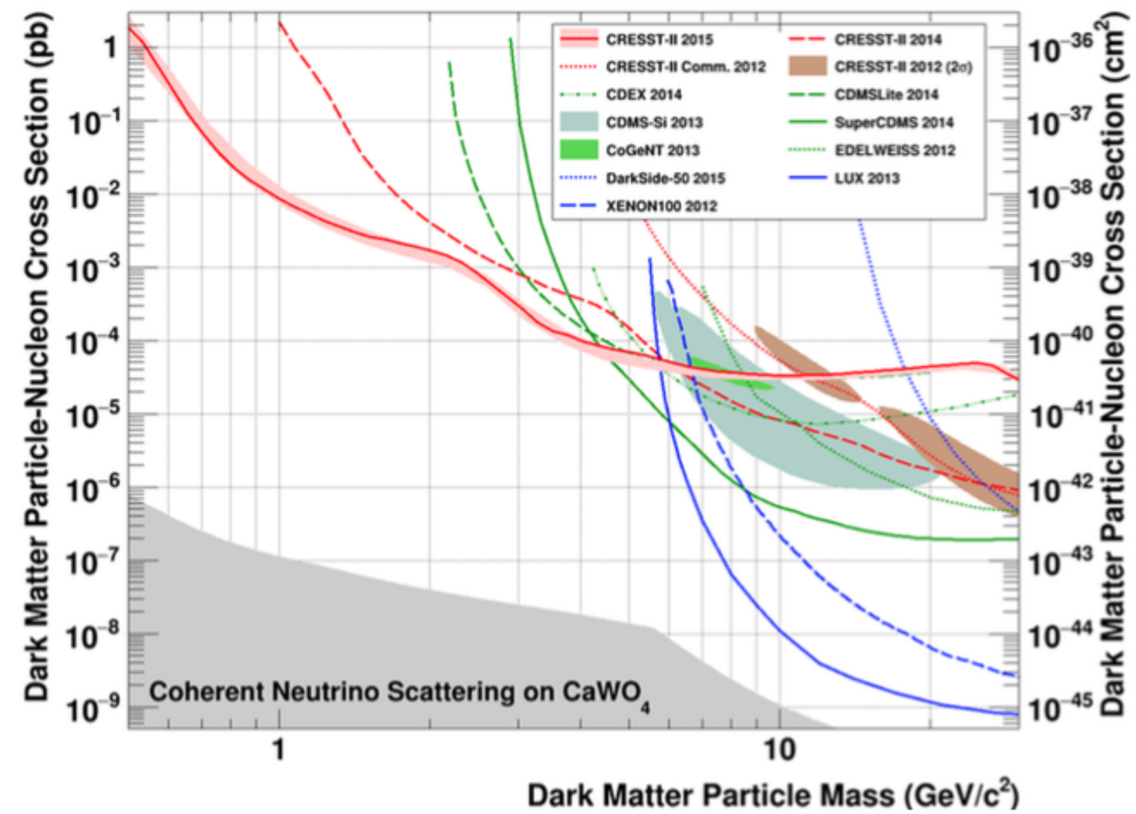
Lowering the threshold, one may gain quasi-exponentially in the rate (or, in turn, access lower DM masses.)

Prominent players:

CRESST, CDMS, DAMIC [detector thresholds O(100 eV)]

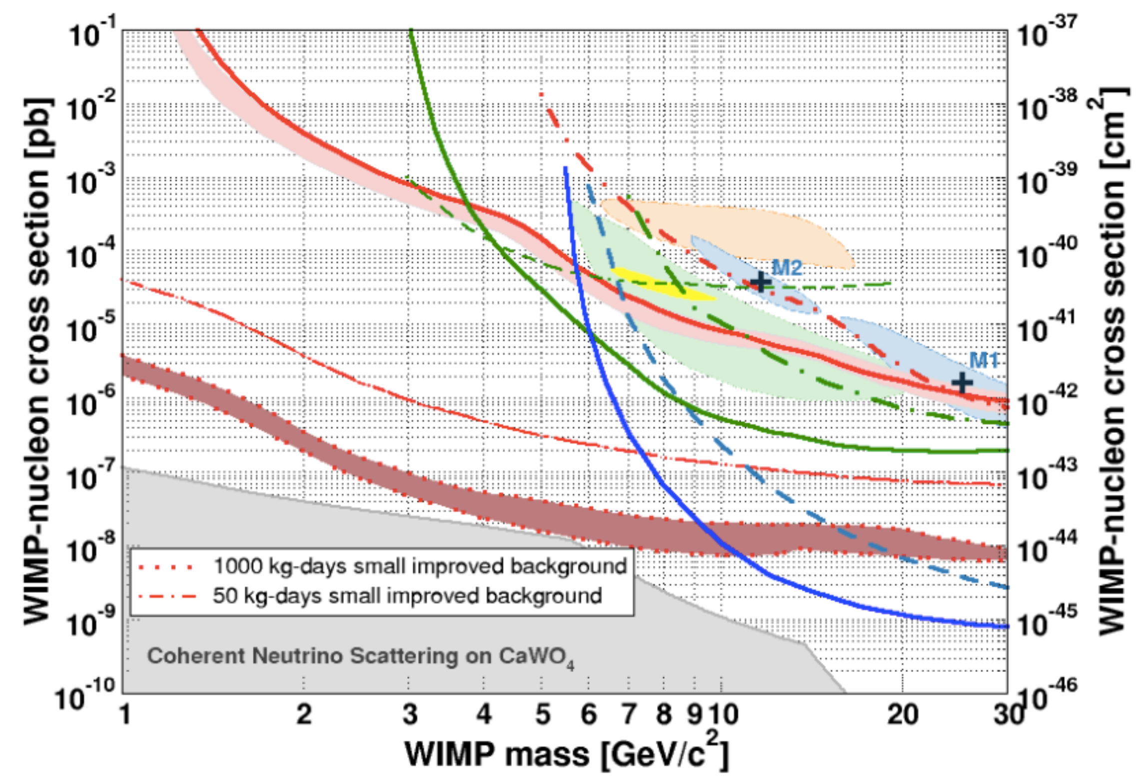
Going beyond:

- look for scattering on electrons (analyses are already performed)
- bridging the few eV band gap in semiconductors, or meV in superconductors by scattering off electrons,
- breaking chemical bond in crystals (creating color centers),




future

CRESST 2015



# WIMPs vs. solar neutrinos

Flux

$$\Phi_{DM} = \frac{\rho_0 v}{m_{DM}} \sim 10^5\,\mathrm{cm}^{-2}\mathrm{s}^{-1}\left(\frac{100\,\mathrm{GeV}}{m_{DM}}\right)$$

Cross section

$$\sigma = 10^{-44}\,\mathrm{cm}^2 \times \sigma_{44} A^2 \left(\frac{\mu_N}{\mu_n}\right)^2$$

Recoil

$$E_R^{\max} = \frac{(2\mu_N v)^2}{2m_N} \sim \begin{cases} 20\,\mathrm{keV}\left(\frac{A}{20}\right) \\ \quad (m_N \ll m_{DM}) \\ 4\,\mathrm{keV}\left(\frac{m_{DM}}{20\,\mathrm{GeV}}\right)^2\left(\frac{100}{A}\right) \\ \quad (m_{DM} \ll m_N) \end{cases}$$

Spectrum

Flux

$$\Phi_{pp} = 6 \times 10^{10}\,\mathrm{cm}^{-2}\mathrm{s}^{-1}$$

$$\Phi_{^8\mathrm{B}} = 6 \times 10^{6}\,\mathrm{cm}^{-2}\mathrm{s}^{-1}$$

Cross section

$$\sigma \simeq 10^{-44}\,\mathrm{cm}^2 \times N^2 \left(\frac{E_\nu}{1\,\mathrm{MeV}}\right)^2$$

Recoil

$$E_R^{\max} = \frac{(2E_\nu)^2}{2m_N}$$

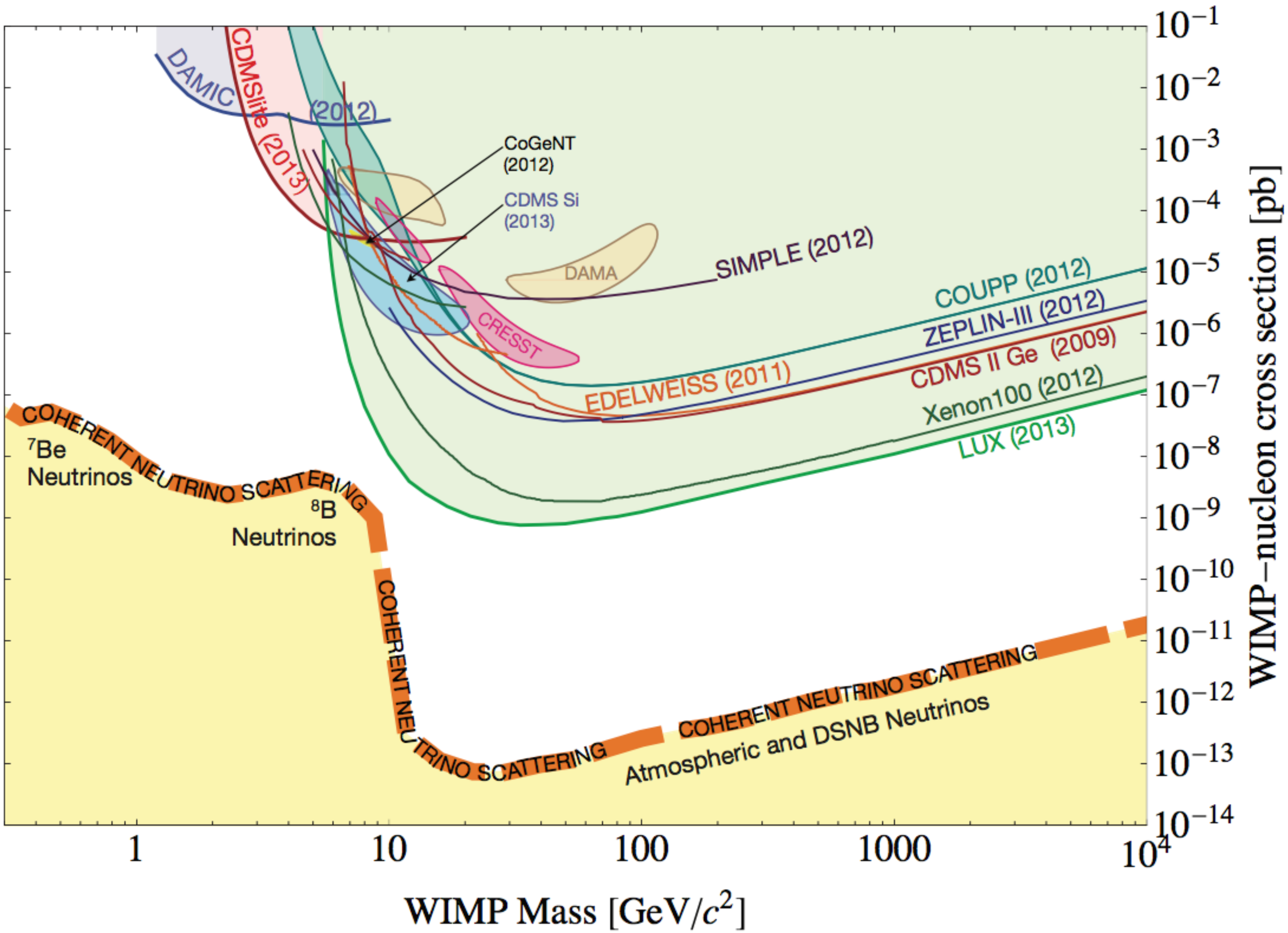
$$\sim 0.1\,\mathrm{keV}\left(\frac{20}{A}\right)\left(\frac{E_\nu}{1\,\mathrm{MeV}}\right)^2$$

Spectrum of a light WIMP almost identical to boron-8 neutrino

# Dark Matter - The End Game?

Coherent neutrino-nucleus scattering is an irreducible background in directionless DM searches.

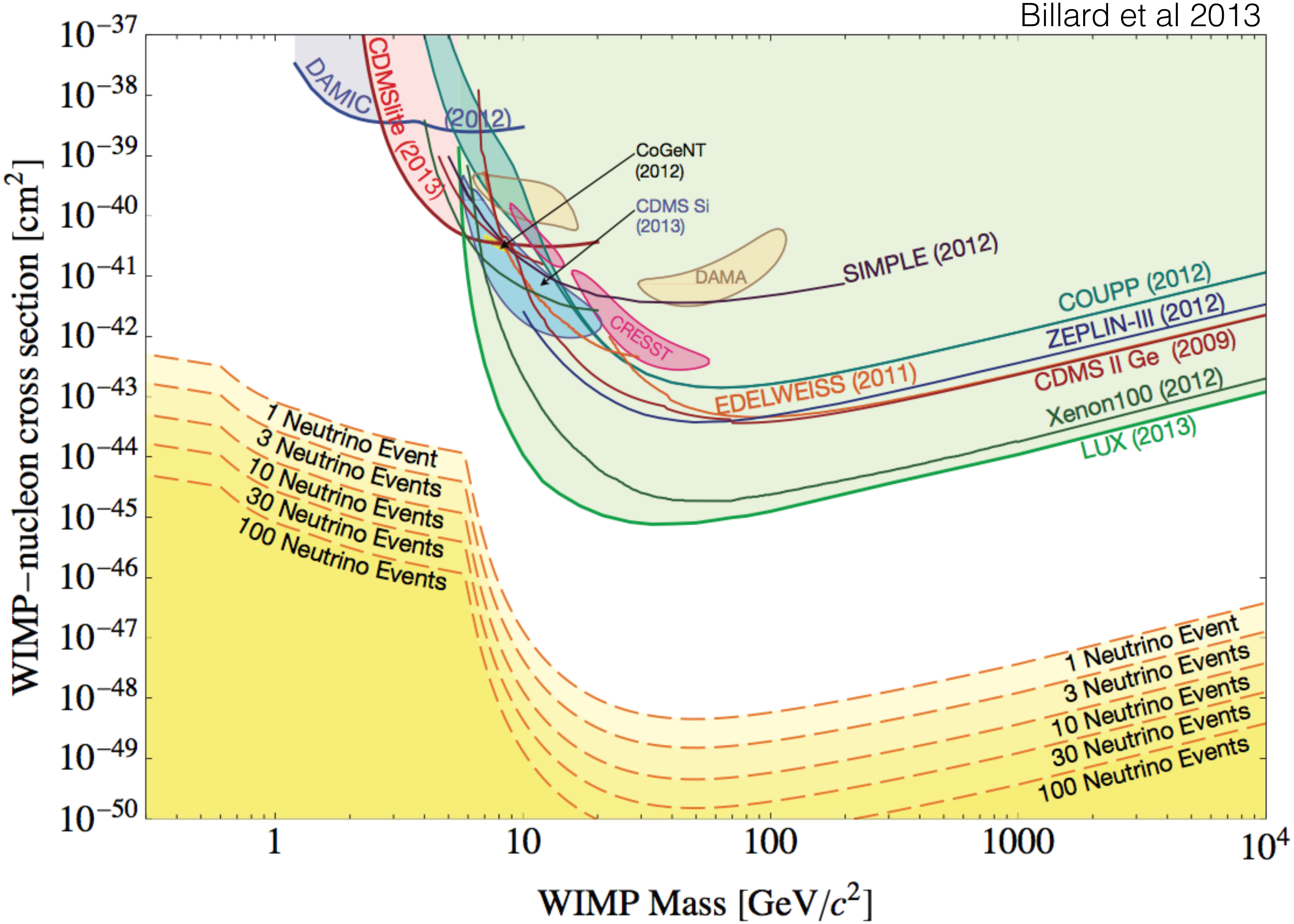
brick wall?



# Dark Matter - The End Game?

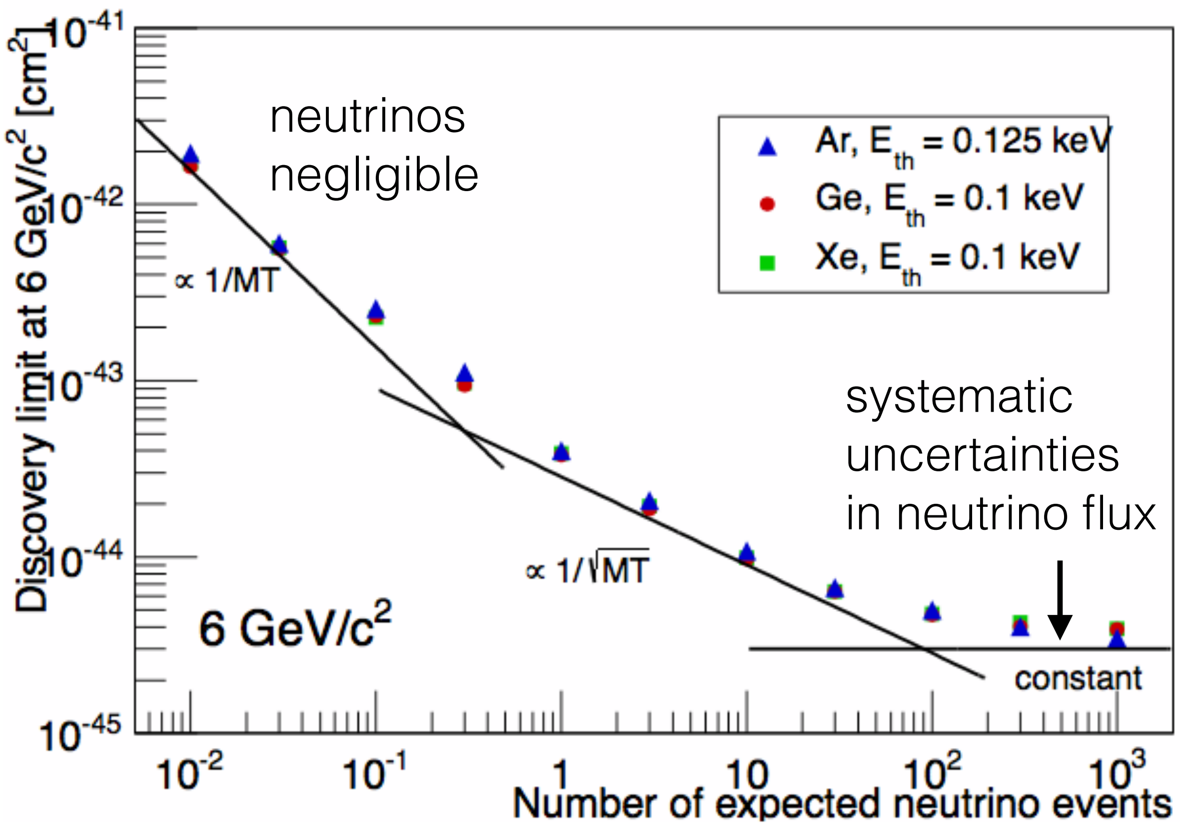
Coherent neutrino-nucleus scattering is an irreducible background in directionless DM searches.

Billard et al 2013

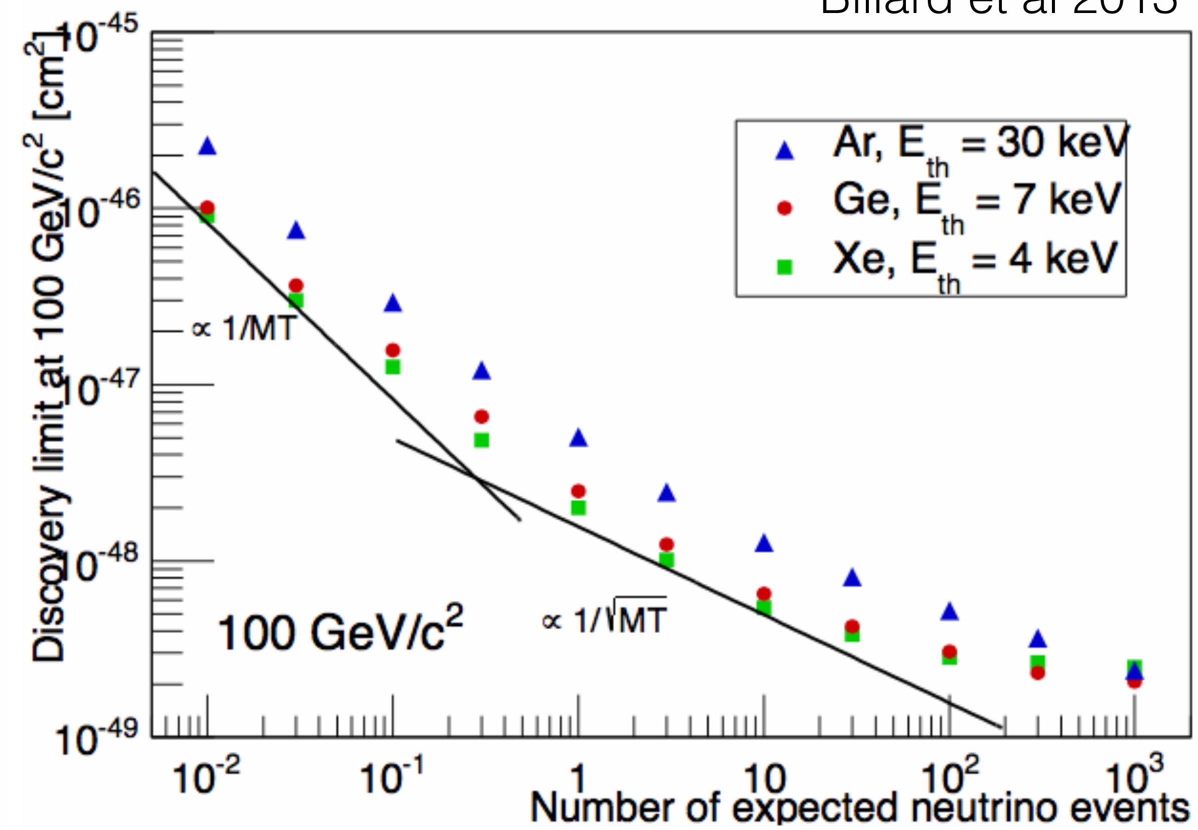


# Dark Matter - The End Game?

Discovery limits in the presence of neutrino events




Billard et al 2013

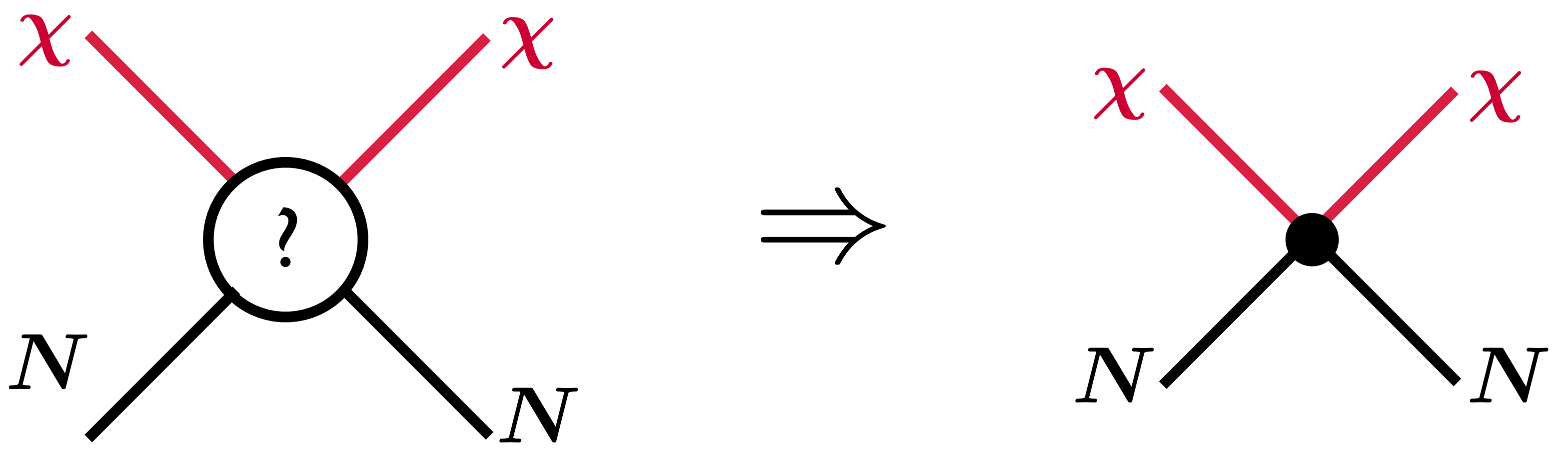



- roughly 3-4 orders of magnitude improvement potential in sensitivity possible
- directionality is a game changer, but hard to reach exposure (gaseous targets)
- measurement of annual modulation of the event rate would help (solar neutrinos are anti-modulated—earth sun are closest in January!)

# Effective Operators in direct detection

see, in particular,
Fan, Reece, Wang (2010)
Fitzpatrick et al. (2013)

- Effective operator approach between DM $\chi$ and nucleons $N$ often holds as $|q_{\max}| \sim O(100\,\mathrm{MeV})$




$$\mathcal{L}_{\rm eff} = \frac{1}{\Lambda^2}(\bar\chi\Gamma_\chi\chi)(\bar q\Gamma_q q)$$

- For example, fermionic DM

| | S | P | V | A | T | AT |
|---|---|---|---|---|---|---|
| $\Gamma_{\chi,q}$ | $1$ | $\gamma^5$ | $\gamma^\mu$ | $\gamma^\mu\gamma^5$ | $\sigma^{\mu\nu}$ | $\sigma^{\mu\nu}\gamma^5$ |

Non-relativistic limit (on tree level):

$S\times S$ or $V\times V$ coherent, spin-independent scattering

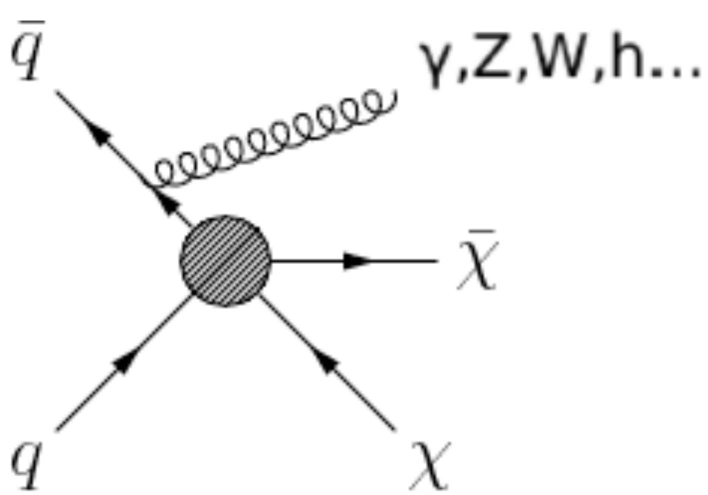
$A\times A$ or $T\times T$ spin-dependent, coupling to the unpaired nucleon

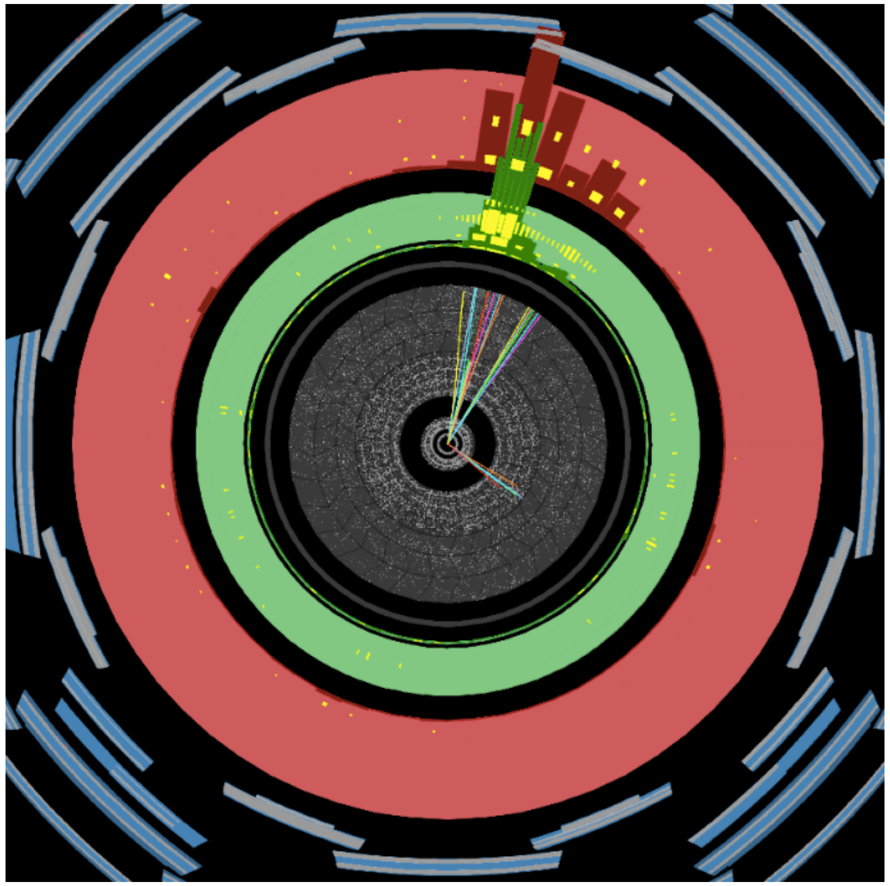
other combinations are suppressed by $v^2, \mathbf{q}^2/m_N^2 \sim 10^{-6}$

# Dark Matter at the LHC

- Effective theory in which only the DM and SM fields appear (=contact) provide the simplest parameterization of new physics

  => mono-jet/photon/W/Z + missing momentum

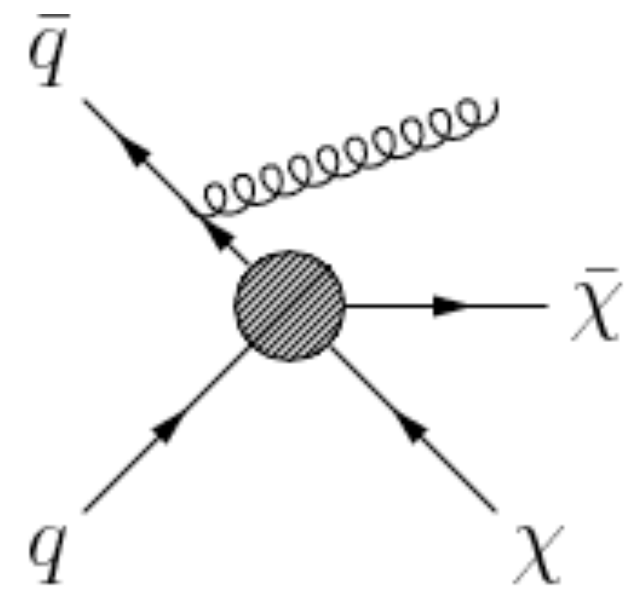


| Name | Operator | Coefficient |
|---|---|---|
| D1 | $\bar{\chi}\chi\bar{q}q$ | $m_q/M_*^3$ |
| D2 | $\bar{\chi}\gamma^5\chi\bar{q}q$ | $im_q/M_*^3$ |
| D3 | $\bar{\chi}\chi\bar{q}\gamma^5 q$ | $im_q/M_*^3$ |
| D4 | $\bar{\chi}\gamma^5\chi\bar{q}\gamma^5 q$ | $m_q/M_*^3$ |
| D5 | $\bar{\chi}\gamma^\mu\chi\bar{q}\gamma_\mu q$ | $1/M_*^2$ |
| D6 | $\bar{\chi}\gamma^\mu\gamma^5\chi\bar{q}\gamma_\mu q$ | $1/M_*^2$ |
| D7 | $\bar{\chi}\gamma^\mu\chi\bar{q}\gamma_\mu\gamma^5 q$ | $1/M_*^2$ |
| D8 | $\bar{\chi}\gamma^\mu\gamma^5\chi\bar{q}\gamma_\mu\gamma^5 q$ | $1/M_*^2$ |
| D9 | $\bar{\chi}\sigma^{\mu\nu}\chi\bar{q}\sigma_{\mu\nu} q$ | $1/M_*^2$ |
| D10 | $\bar{\chi}\sigma_{\mu\nu}\gamma^5\chi\bar{q}\sigma_{\alpha\beta} q$ | $i/M_*^2$ |
| D11 | $\bar{\chi}\chi G_{\mu\nu}G^{\mu\nu}$ | $\alpha_s/4M_*^3$ |
| D12 | $\bar{\chi}\gamma^5\chi G_{\mu\nu}G^{\mu\nu}$ | $i\alpha_s/4M_*^3$ |
| D13 | $\bar{\chi}\chi G_{\mu\nu}\tilde{G}^{\mu\nu}$ | $i\alpha_s/4M_*^3$ |
| D14 | $\bar{\chi}\gamma^5\chi G_{\mu\nu}\tilde{G}^{\mu\nu}$ | $\alpha_s/4M_*^3$ |

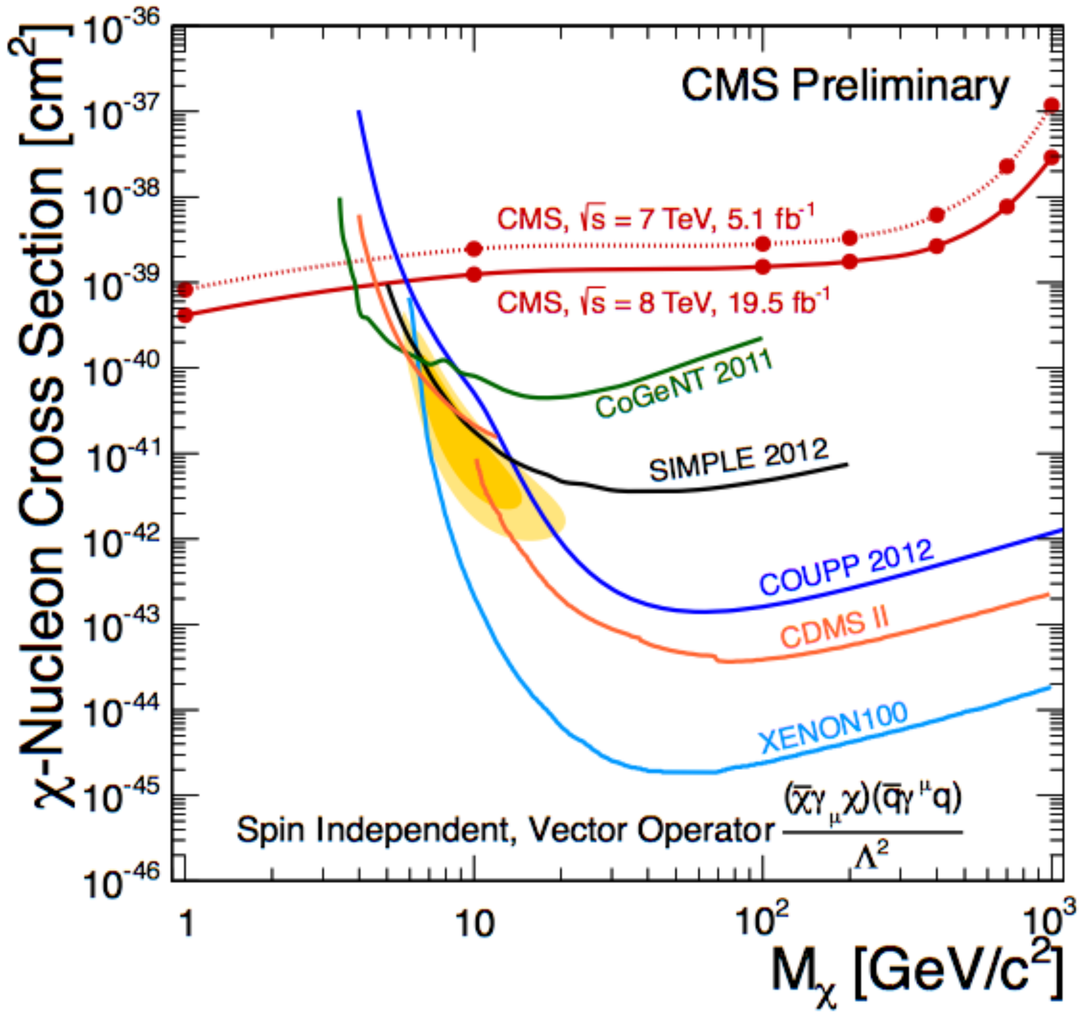
e.g. Goodman et al 2010

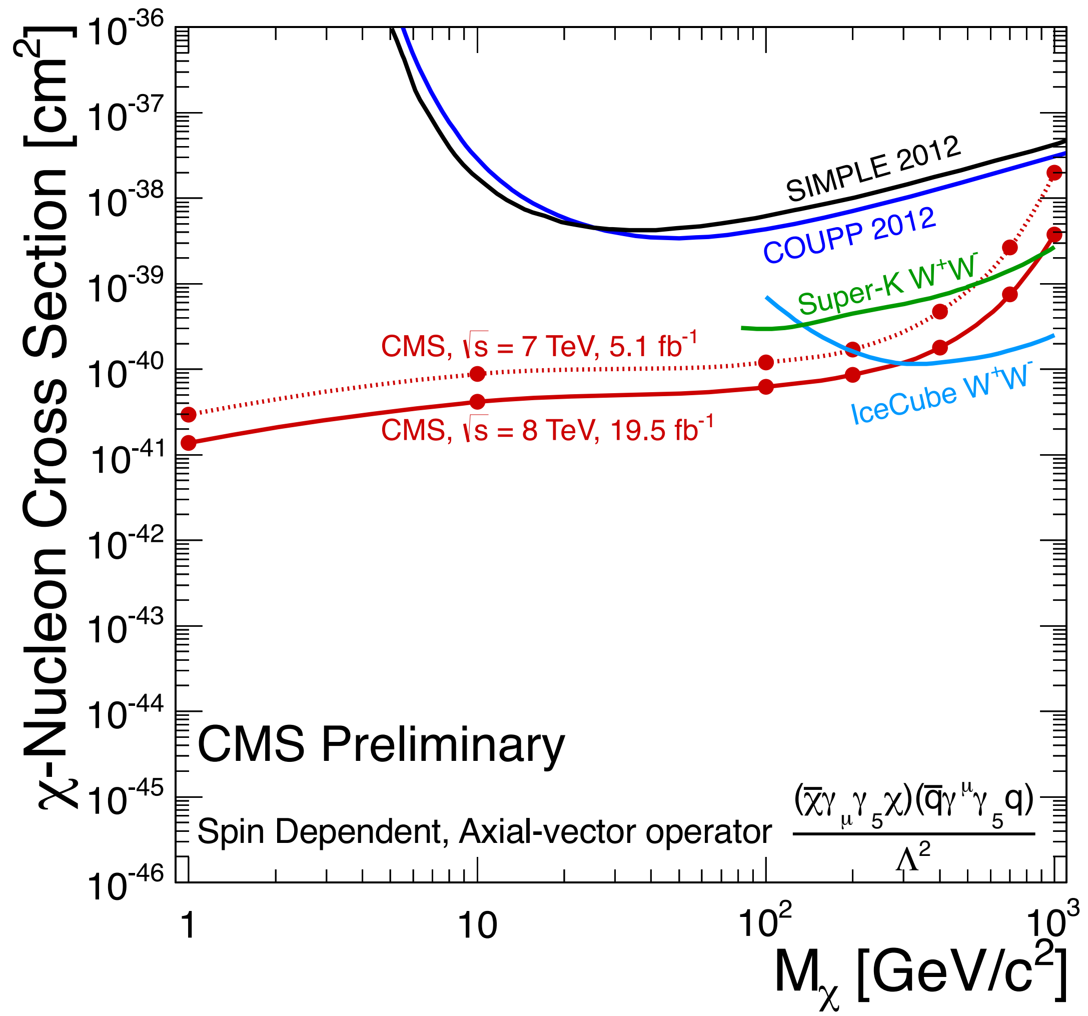
# DM at the LHC




LHC does exquisitely well for

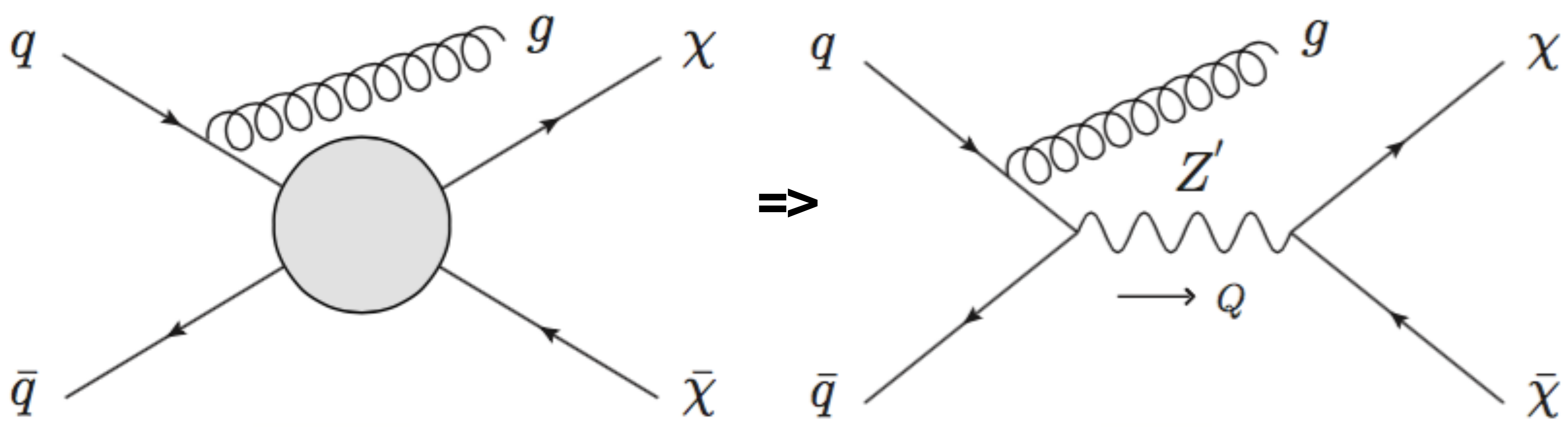
- low WIMP masses $m_\chi \lesssim 10\,\mathrm{GeV}$
- operators that are velocity suppressed in direct detection



# DM at the LHC

Effective field theory approach breaks down, once $q^2 \simeq m^2_{\rm mediator}$




$$\frac{g_q\, g_\chi}{Q^2 - m^2_{\rm med}} \approx -\frac{g_q\, g_\chi}{m^2_{\rm med}} \quad \text{=>} \quad \frac{g_q\, g_\chi}{Q^2 - m^2_{\rm med}} \approx \frac{g_q\, g_\chi}{Q^2}$$

$$\Lambda \equiv \frac{m_{\rm med}}{\sqrt{g_q\, g_\chi}}$$ contact interaction

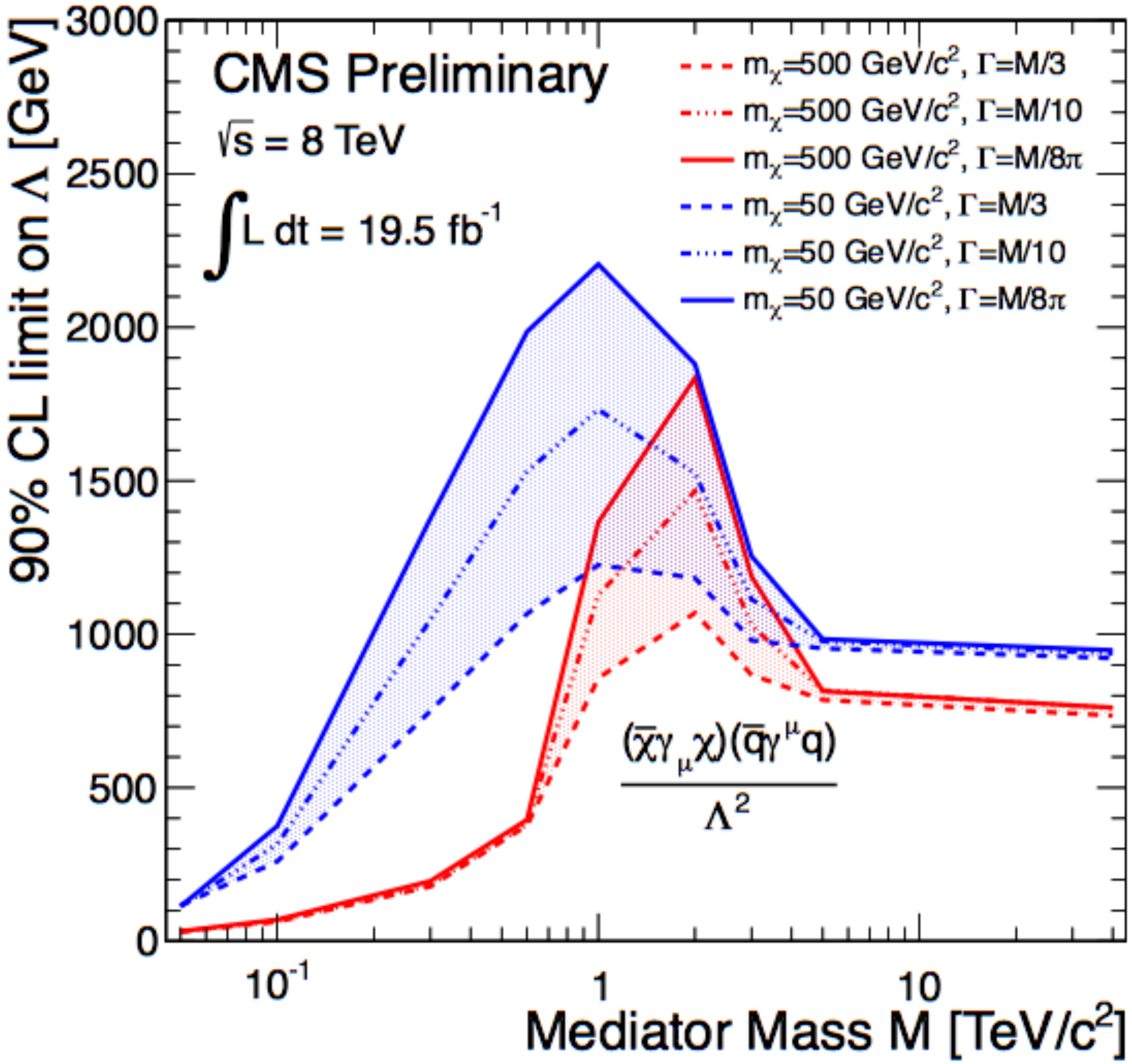
$\langle Q^2 \rangle^{1/2} \gtrsim 500\,\mathrm{GeV}$ @ 8TeV LHC

# Beyond effective operators

=> results can be cast as a limit on the contact interaction scale $\Lambda$

LHC limits are stringent for contact operators, but can go away completely for light mediators!

=> accessible UV content can be caught in "*simplified models*" with content **SM + DM + mediator**

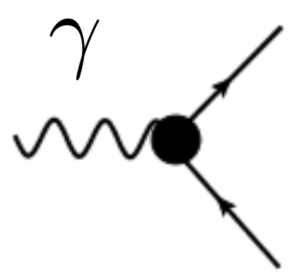



e.g. arXiv:1507.0096

# What are the force carriers/mediators?

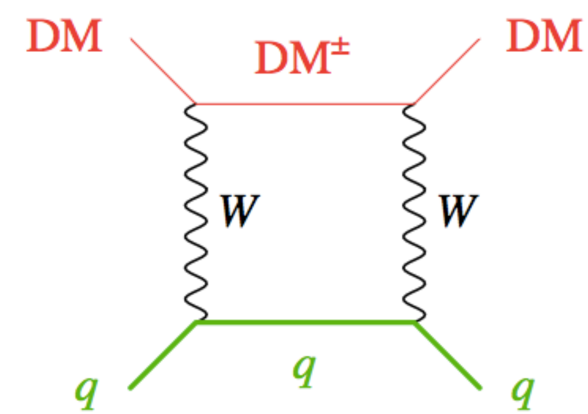
## Photon:
milli-charged DM; neutral DM interacting via EM form factors

… Kouvaris (2013);
Ho, Scherrer (2013);
Weiner, Yavin (2013)…




## EW-bosons:
DM in electroweak multiplets

Cirelli, Fornengo, Strumia (2007)




## Higgs boson:
Inert Higgs, Higgs portal models, SUSY

Deshpande, Ma (1978);
Silveira, Zee (1985);
McDonald (1993);
Burgess, Pospelov, ter Veldhuis (2000)
…

## New physics mediators
squarks, SUSY gauginos, dark photons…
(whatever you can think of)

# Original expectations

Those were the good ole days

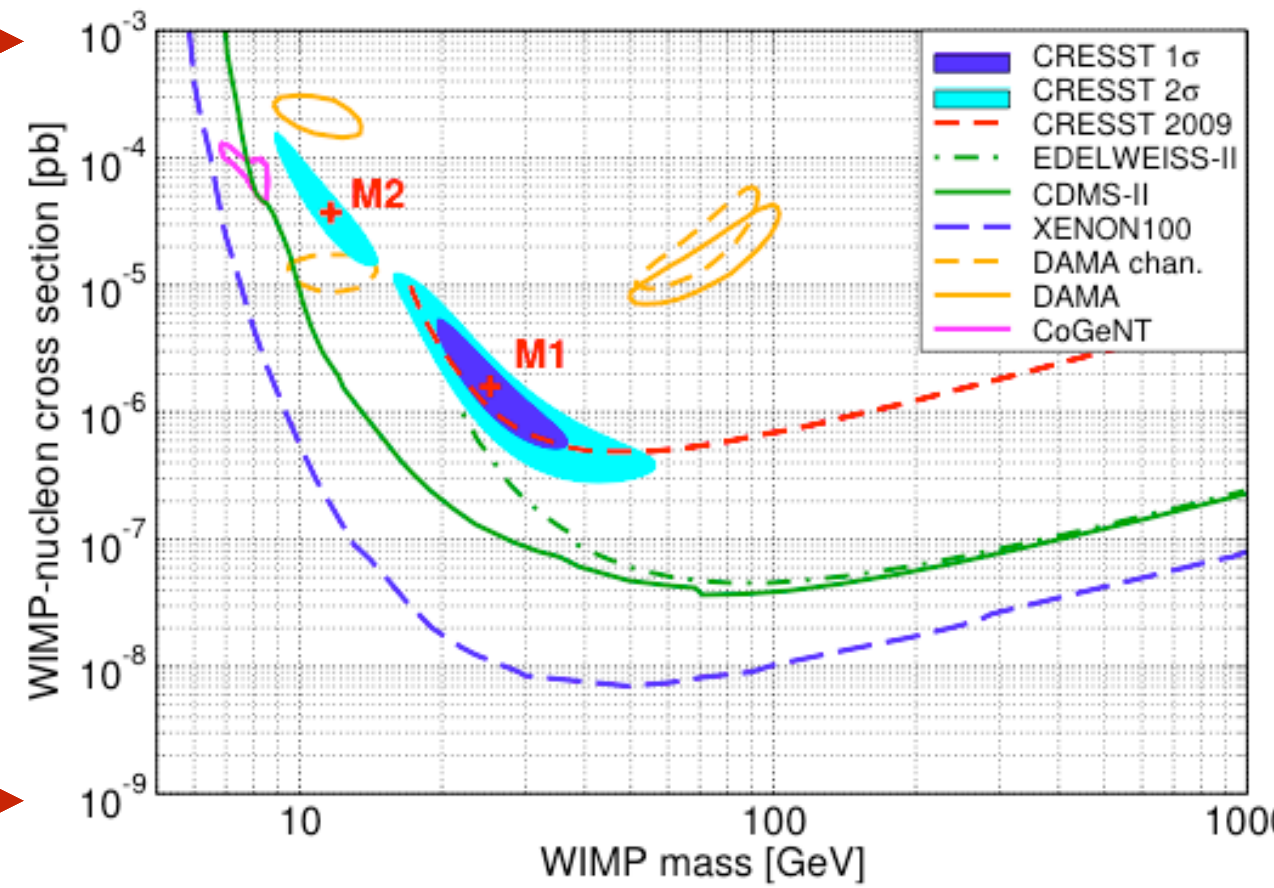
The “weakly” interacting 100 GeV WIMPs are long excluded

Z-mediated cross section

$\sigma_n \sim 10^{-3}\,\mathrm{pb}$

*Higgs-mediated* interactions are being probed right now!

$\sigma_n \sim 10^{-(9-10)}\,\mathrm{pb}$




NB: direct detection may never completely exclude neutralino:

*pure* neutralino (wino, bino, higgsino) has suppressed higgs couplings $h^\dagger \tilde{h} \tilde{w} \;\; h^\dagger \tilde{h} \tilde{b}$
pure wino/bino does not couple to Z
*cancelations* in couplings to Z and Higgs

# Light Dark Sectors
# (with relevance for the intensity frontier)

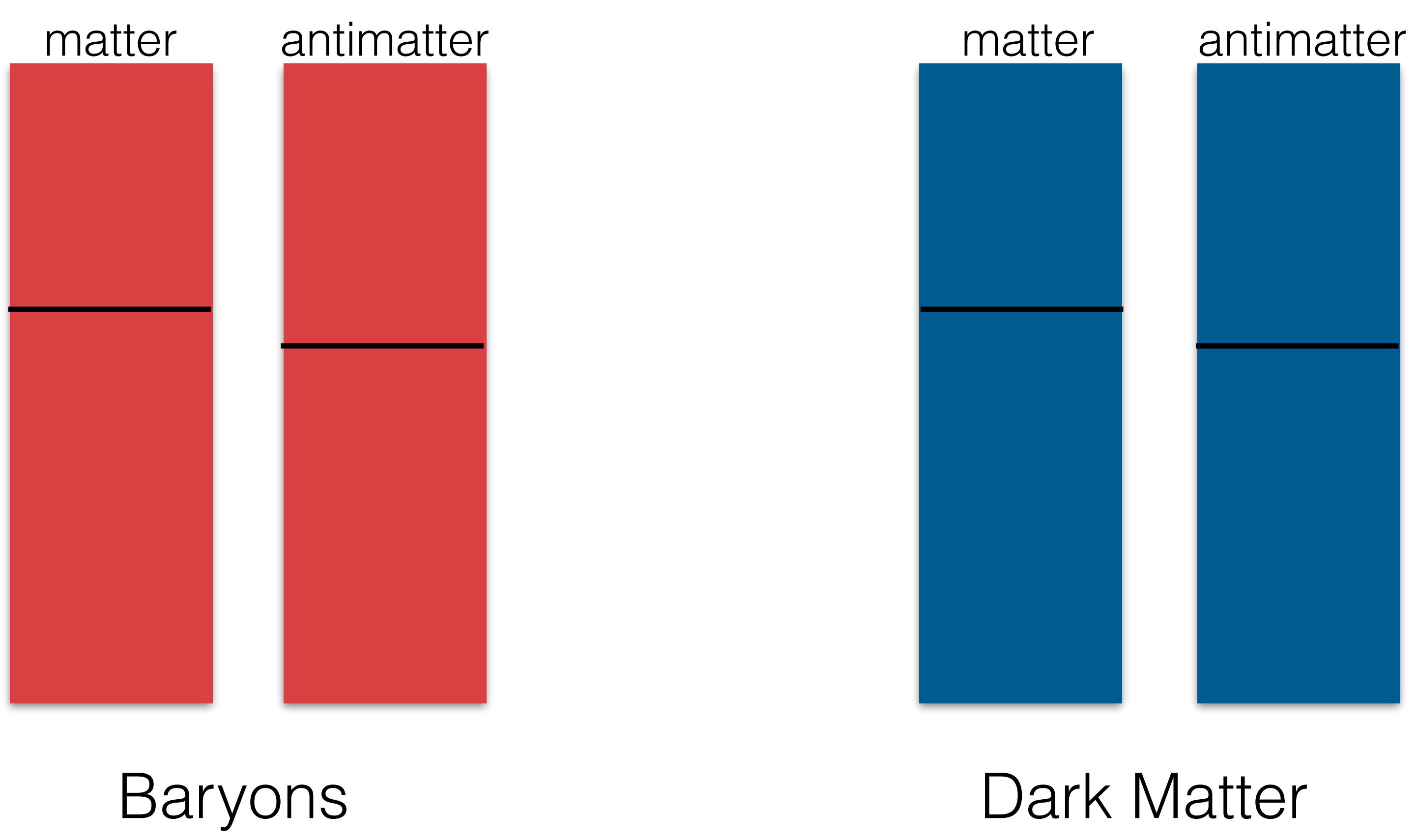
# An alternative to the WIMP miracle

$$\Omega_{dm} \simeq 5\Omega_b$$

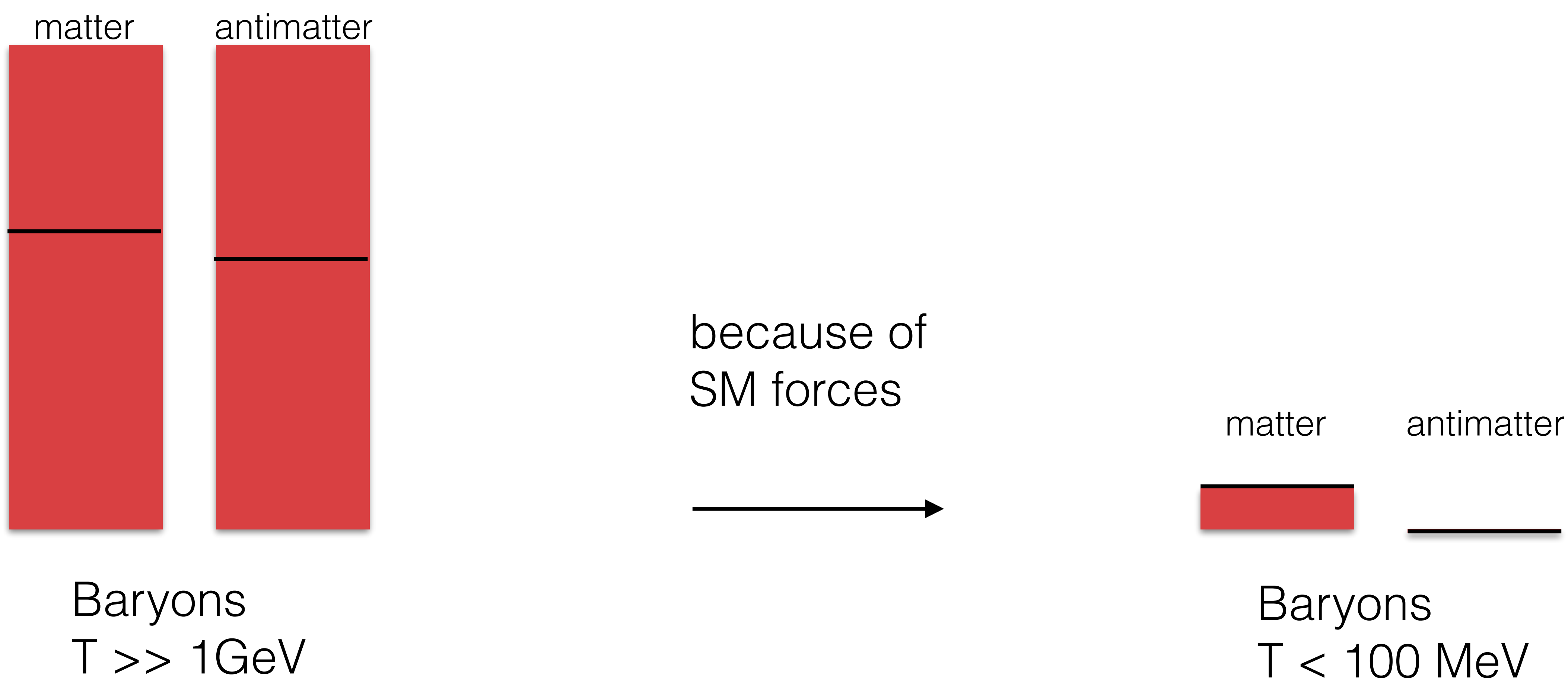
—

Why?

Maybe because dark matter carries a chemical potential
(i.e. a matter-antimatter asymmetry)

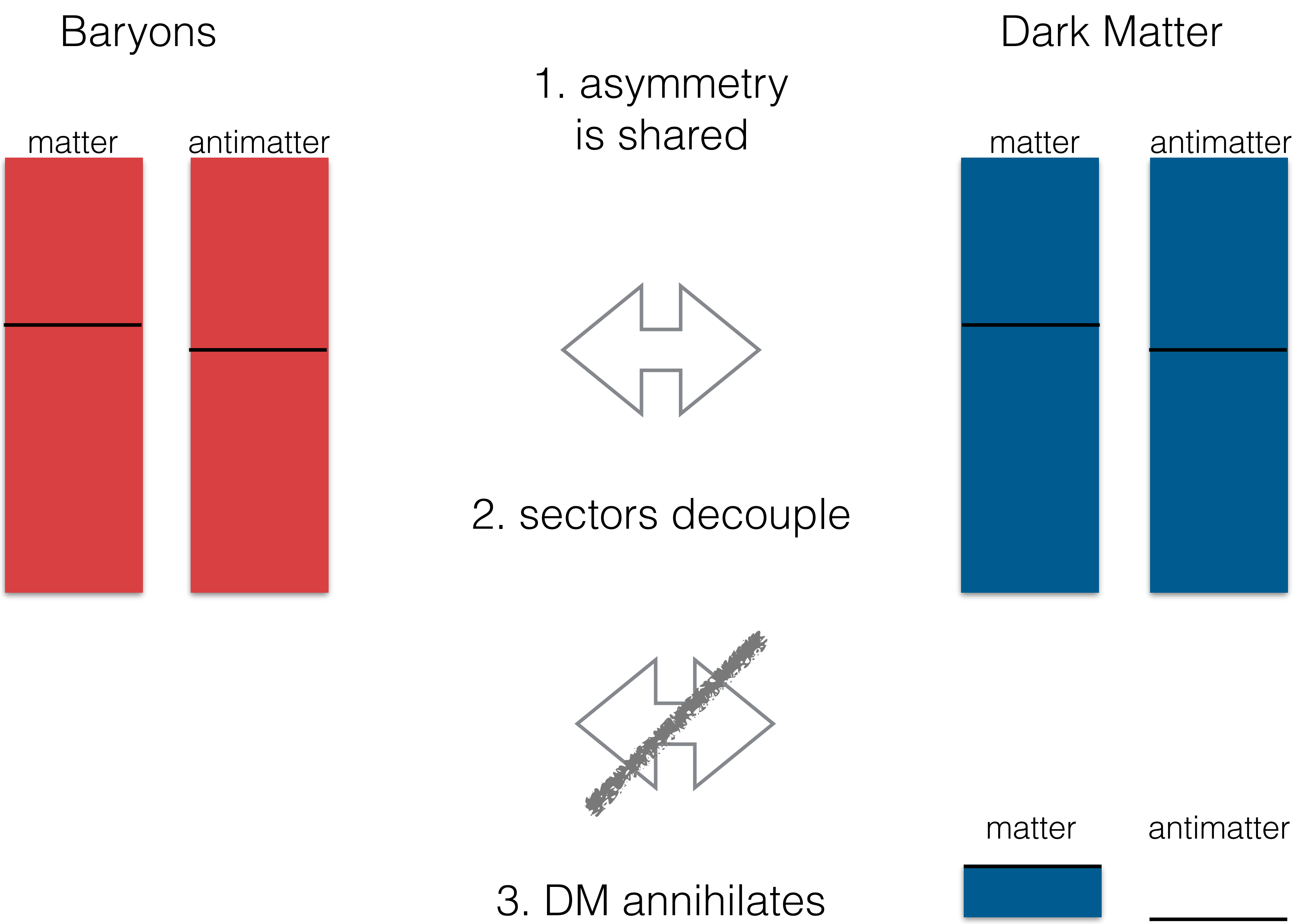


# An alternative to the WIMP miracle




Without a chemical potential of baryons, we would be living in a Universe with $n_b/n_\gamma = 10^{-18}$ and not $n_b/n_\gamma = 10^{-10}$

# An alternative to the WIMP miracle

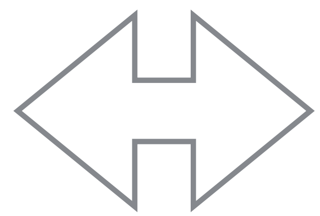


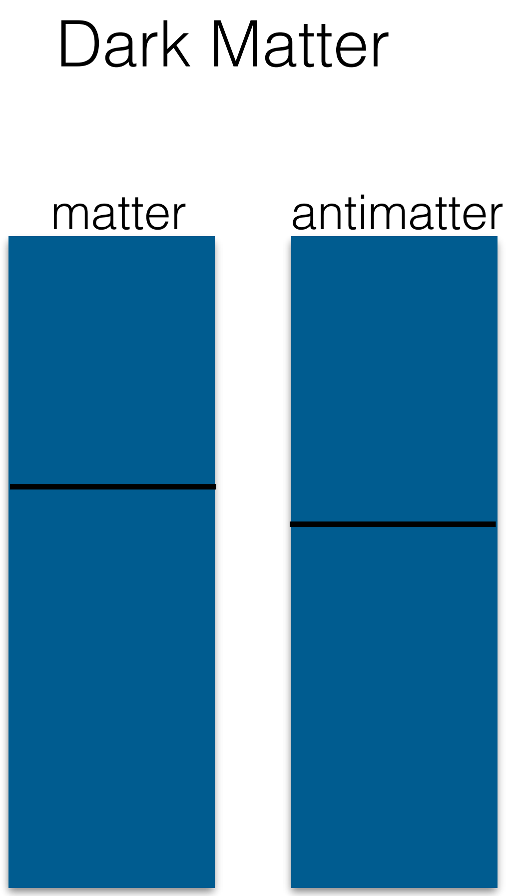

=> abundance is set by the chemical potential

# An alternative to the WIMP miracle - asymmetric DM

$$n_\chi - n_{\bar{\chi}} \sim n_B - n_{\bar{B}}$$

1. asymmetry is shared






Use sphalerons, or
higher dimensional operators

$$\mathcal{O}_{B-L} = u^c d^c d^c,\ q\ell d^c,\ \ell\ell e^c,\ \ell H_u$$

$$\mathcal{O}_D = X^n$$

$$W = \frac{\mathcal{O}_D \mathcal{O}_{B-L}}{M^{m+n-3}}$$

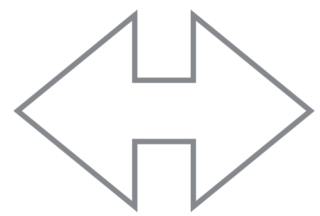
# An alternative to the WIMP miracle - asymmetric DM

Dark Matter

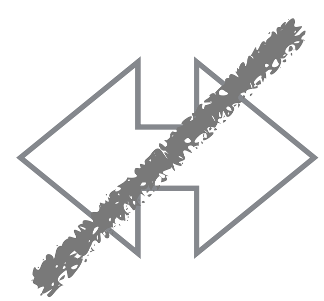
$$n_\chi - n_{\bar\chi} \sim n_B - n_{\bar B}$$

1. asymmetry is shared

matter antimatter



2. sectors decouple



$$\rho_{\rm DM}/\rho_B \simeq 5$$

$$m_\chi \sim 5 m_B \simeq 5\,{\rm GeV}$$

3. DM annihilates

matter antimatter

# An alternative to the WIMP miracle - asymmetric DM

Relic density determined by chemical potential.

Annihilation must be efficient enough to remove the symmetric abundance:

EW scale mediators don't work, required couplings are excluded by LHC.

$$\sigma_{ann} v = \frac{g_d^2 g_f^2 m_X^2}{\pi m_M^4} \simeq 10^{-26}\ \mathrm{cm}^3/\mathrm{s} \left(\frac{g_d g_f}{0.25}\right)^2 \left(\frac{10\mathrm{GeV}}{m_X}\right)^2 \left(\frac{200\ \mathrm{GeV}}{m_M}\right)^4$$

Relic abundance typically relies on new, light mediators that enhance annihilation cross section.

Note: Indirect detection prospects / astro-constraints depend on the residual abundance of anti-DM after freeze out! The symmetric component does not annihilate.

Detection prospects are built on the interaction that annihilates the symmetric component (model dependent.)

# New physics "portals"

Limited options to couple new physics directly in a renormalizable way

**Vector portal**

$$\mathcal{L} \supset \epsilon V_\mu J^\mu_{\rm SM}$$

Most studied case is when $J_{SM}$ is the electromagnetic current. It originates from "kinetic mixing" with $U(1)_Y$ hypercharge field strength

$$(\kappa/2) V_{\mu\nu} F_Y^{\mu\nu}$$

=> minimal extension of the SM by an additional U(1) gauge group

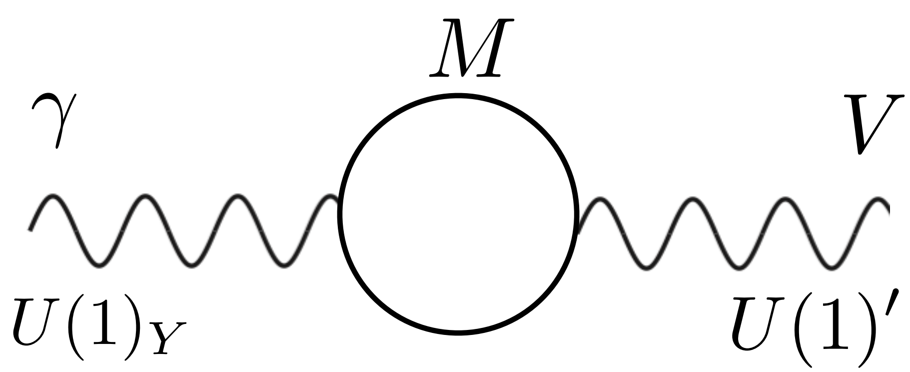
$$\mathrm{SU}(3)_c \times \mathrm{SU}(2)_L \times \mathrm{U}(1)_Y \times U(1)'$$

Standard Model x "dark sector" with vector particle $V^\mu$

# New physics “portals”

Limited options to couple new physics directly in a renormalizable way

**Vector portal**




$$\longrightarrow \quad -\frac{\kappa'}{2} F^Y_{\mu\nu} V^{\mu\nu}$$

Assume there are particles charged both under $U(1)_Y$ and $U(1)'$ of *arbitrarily heavy* mass M

$$\kappa \sim \frac{g_Y g'}{16\pi^2} \times \log\left(\frac{\Lambda_{UV}}{M}\right)$$

“non-decoupling” [Holdom ’85]

=> kinetic mixing can be a low-energy messenger from high scale

# New physics "portals"

Limited options to couple new physics directly in a renormalizable way

**Vector portal**

$$-\frac{\kappa'}{2} F^{Y}_{\mu\nu} V^{\mu\nu} \xrightarrow[\text{energies}]{\text{at low}} -\frac{\kappa}{2} F_{\mu\nu} V^{\mu\nu}$$

hypercharge field strength

photon field strength

V or A' is called "Dark Photon", "Hidden Photon", "U-Boson", …

Note: must be massive as otherwise kinetic mixing can just be rotated away => 2 Options

1. "hard photon mass" (Stuckelberg)

$$\mathcal{L} \supset -\frac{1}{2} m_V V_\mu^2$$

2. Higgsed U(1)'

$$\mathcal{L} \supset -\frac{1}{2} m_V V_\mu^2 + e' m_V h' V_\mu^2 + \frac{1}{2} e'^2 \, h'^2 V_\mu^2$$

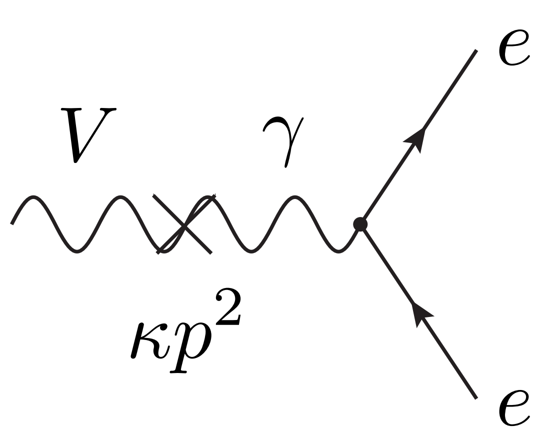
+ h' self-interactions

# Dark photon searches

Two equivalent ways to think about $-\frac{\kappa}{2}F_{\mu\nu}V^{\mu\nu}$

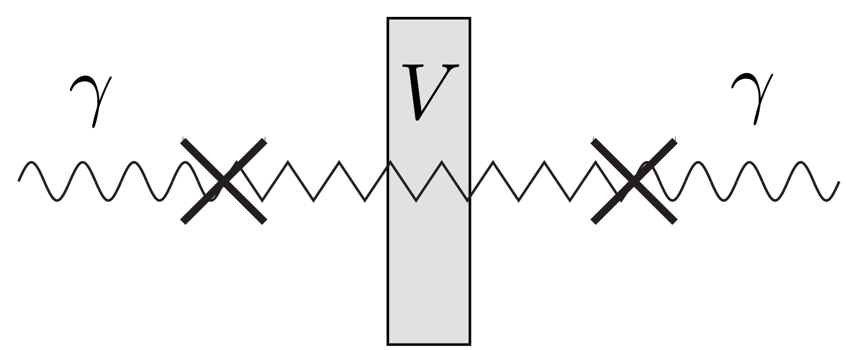
**A.** Keep the mixing as a perturbation:

"Light-shining-through-wall" (LSW) experiments




$eA_\mu J^\mu_{EM}$

…suggests…




Photon-Dark Photon mixing manifest

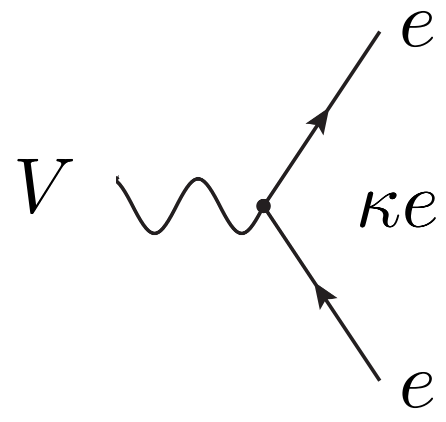
probability $\propto \kappa^4$

sensitivity when $m_V \sim \omega_\gamma$

# Dark photon searches

Two equivalent ways to think about $$-\frac{\kappa}{2} F_{\mu\nu} V^{\mu\nu}$$

**B.** Diagonalize kinetic term:

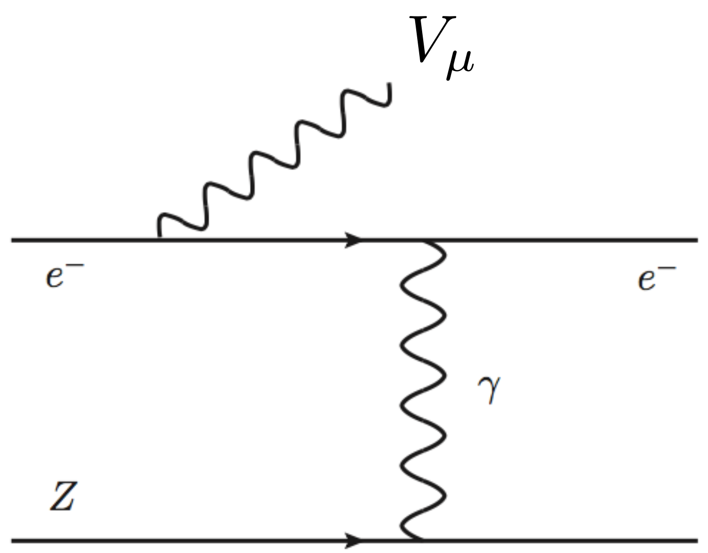



...suggests...

$$e A'_\mu J^\mu_{EM} - \kappa e V'_\mu J^\mu_{EM}$$

Ordinary matter has millicharge under new force

Direct production in experiment:




"Intensity Frontier"

# Dark photon searches

Primary production mechanisms

- Bremsstrahlung on nucleus with charge Z in fixed target configurations (mass reach till beam energy )

  $e^- \; \mathrm{Z} \to e^- \; \mathrm{Z}A'$ or $\mathrm{p\,Z} \to \mathrm{p\,Z}A'$

- Annihilation in electron-positron colliders (or positron beams on fixed targets)

  $e^+e^- \to \gamma A'$

- Meson decay through coupling to quarks

  Dalitz decays, $\pi^0/\eta/\eta' \to \gamma A'$ , rare meson decays, e.g. $K \to \pi A'$

- Drell-Yan in hadron colliders / proton fixed target expts.

  $q\bar{q} \to A' \to (\ell^+\ell^- \text{ or } h^+h^-)$

# Dark photon searches

Primary detection mechanism

A: bump hunts

in visible mass $A' \to \ell^+\ell^-$ or $A' \to h^+h^-$

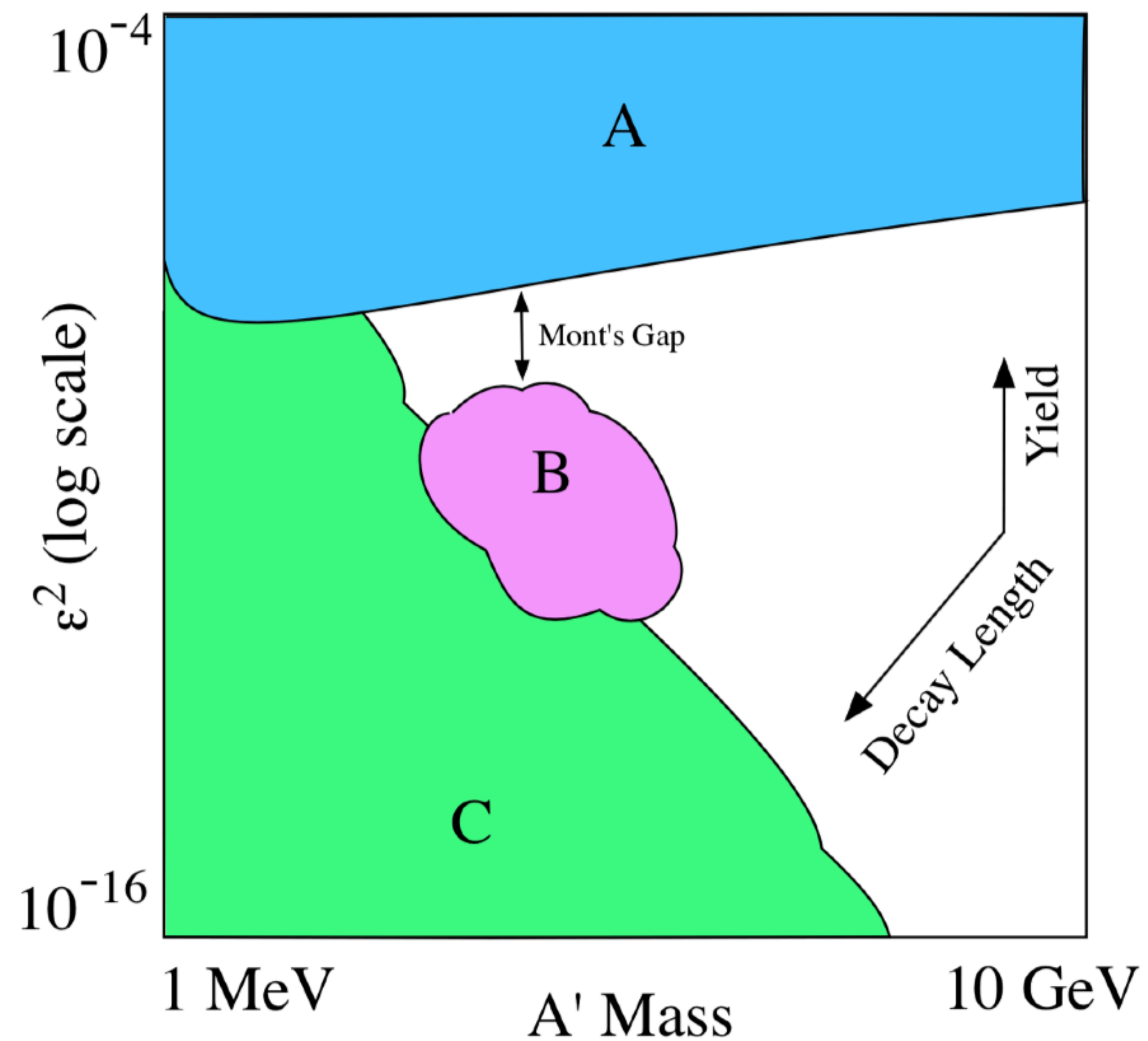
or invisible mass $e^+e^- \to \gamma A'$

B: displaced vertex detection, short decay lengths

$A' \to \ell^+\ell^-$ $\qquad \Gamma_{e^+e^-} = \frac{1}{3}\varepsilon^2\alpha m_{A'}$

C: displaced vertex searches, long decay lengths

decay length ~ $(\varepsilon^2 m_{A'})^{-1}$



# Dark photon searches

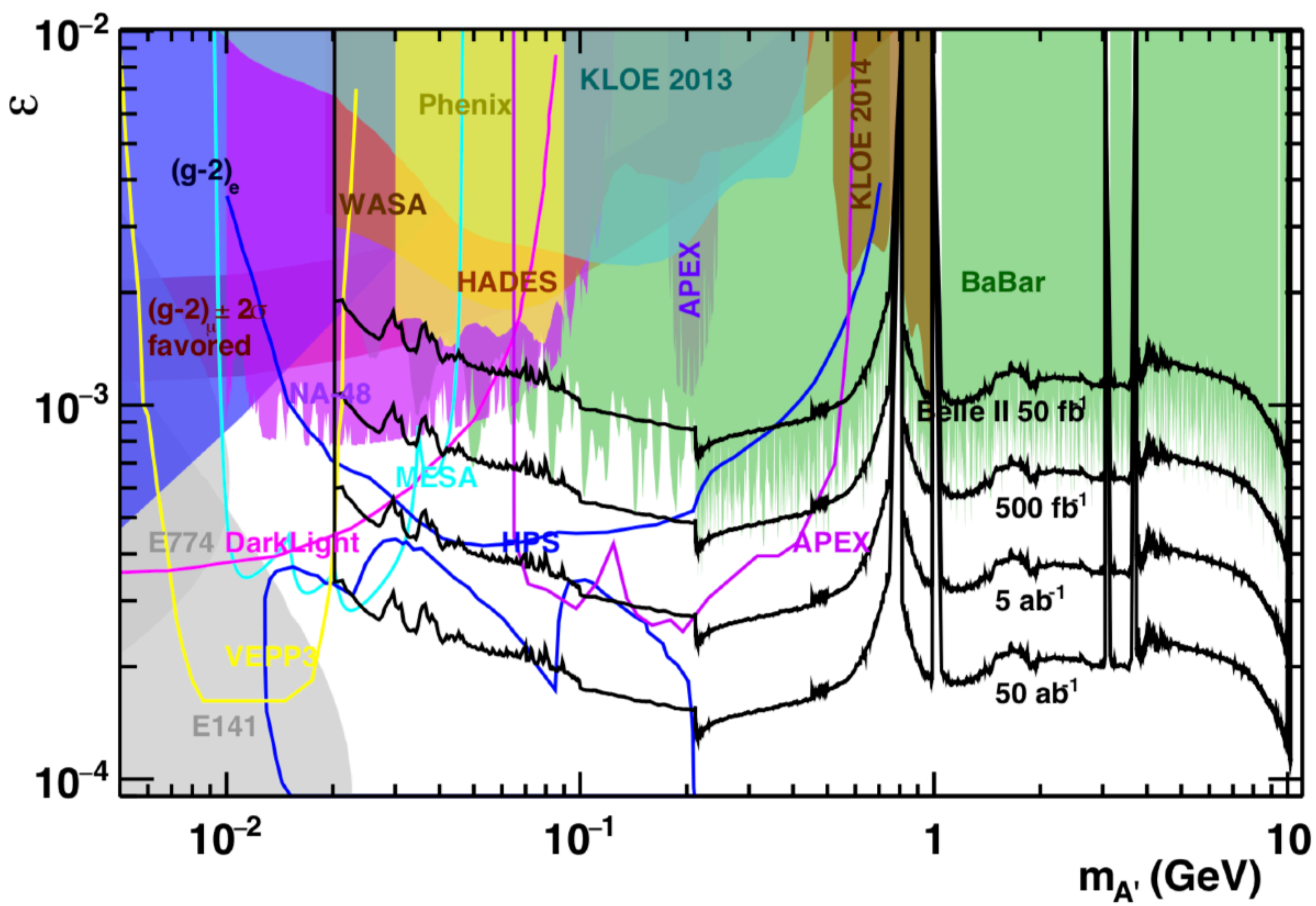
A very active experimental field

$Ze^- \to Ze^-V \to Ze^-e^+e^-$

beams on fixed target

precision tests

$(g-2)_e$




flavour factories

e-beam dumps

$e^+e^- \to \gamma V \to \gamma l^+l^-$

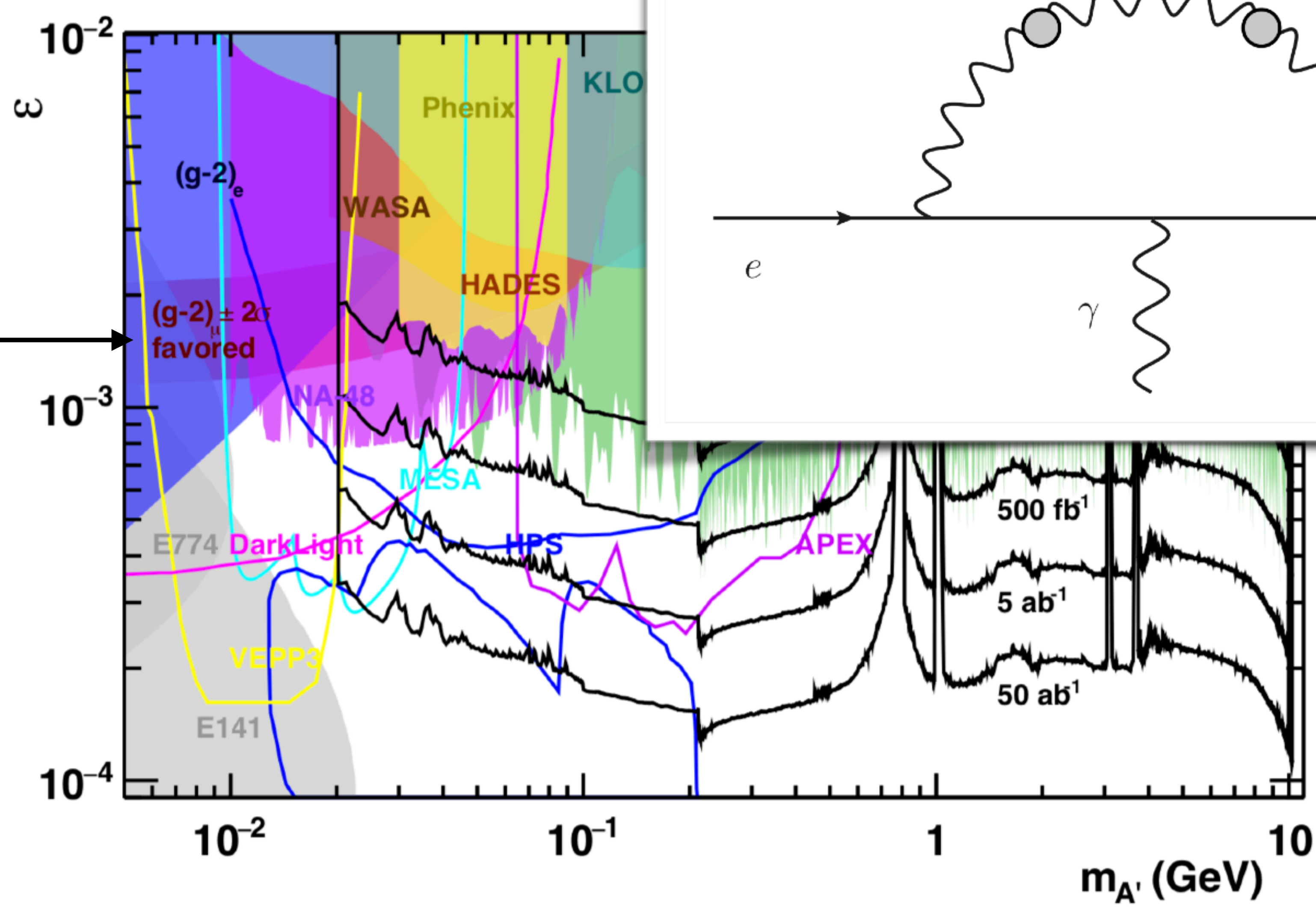
$\pi^0 \to \gamma V \to \gamma e^+e^-$

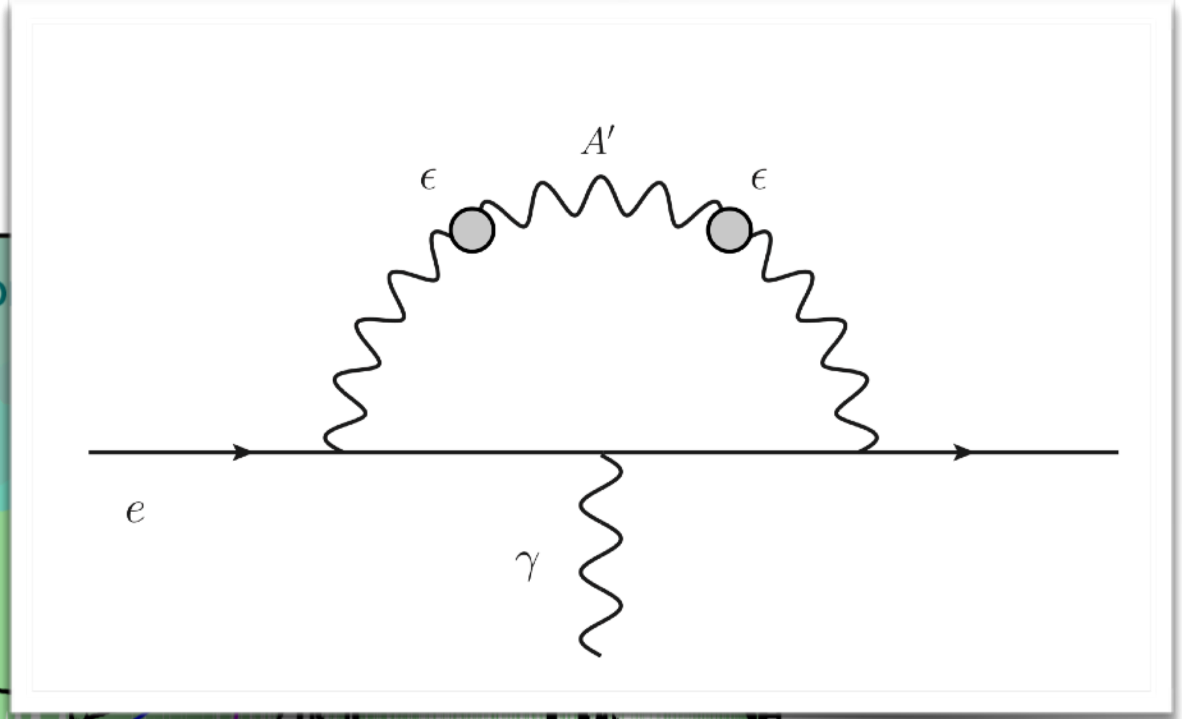
$Ze^- \to Ze^-V \to Ze^-e^+e^-$

# Dark photon searches

A very active experimental field

Note:




Experimental activity in part motivated by its contribution to g-2

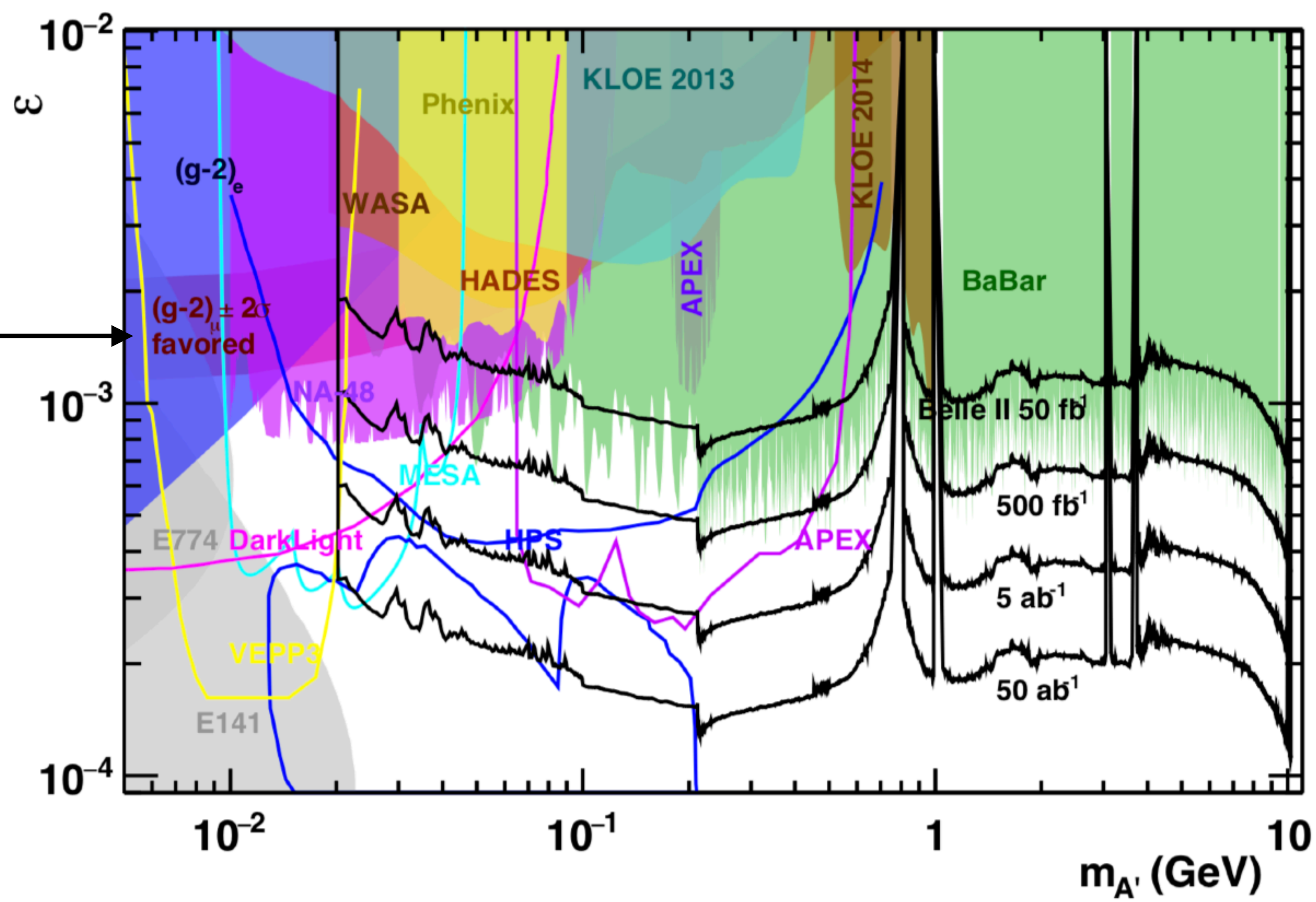
$$a_l^V = \frac{\alpha\kappa^2}{2\pi} \times \begin{cases} 1 \quad \text{for} \quad m_l \gg m_V, \\ 2m_l^2/(3m_V^2) \quad \text{for} \quad m_l \ll m_V. \end{cases}$$

Pospelov 2008

# Dark photon searches

A very active experimental field

Note:




Experimental activity in part motivated by its contribution to g-2

Other “dark photon” models can still shift muon (g-2), while expt. allowed, e.g. $L_\mu - L_\tau$

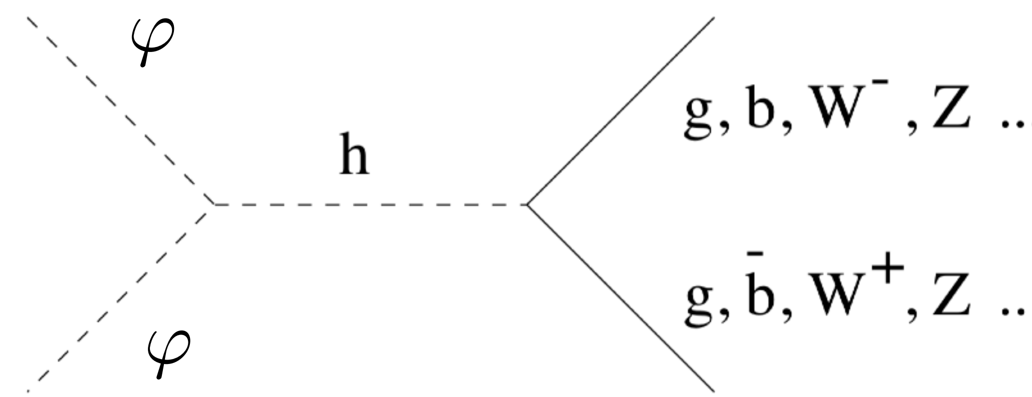
Gninenko, Krasnikov 2002

# New physics “portals”

Limited options to couple new physics directly in a renormalizable way

**Higgs portal**

$$\mathcal{L}_{\rm int} = (A\varphi + \lambda\varphi^2)H^\dagger H$$




when A=0, there is an additional $Z_2$ symmetry and $\varphi$ is stable (Dark Matter candidate)

After EWSB, $H^0 = (v+h)/\sqrt{2}$. Mass-mixing with the SM Higgs is induced $\theta = \dfrac{Av}{m_h^2 - m_\varphi^2}$

=> SM-Higgs like fermion coupling of $\varphi$, i.e. $\theta\,(m_f/v)$

=> Higgs can decay invisibly (LHC limits the invisible decay with of the Higgs Boson to < 10%)

# New physics “portals”

Limited options to couple new physics directly in a renormalizable way

**Higgs portal**

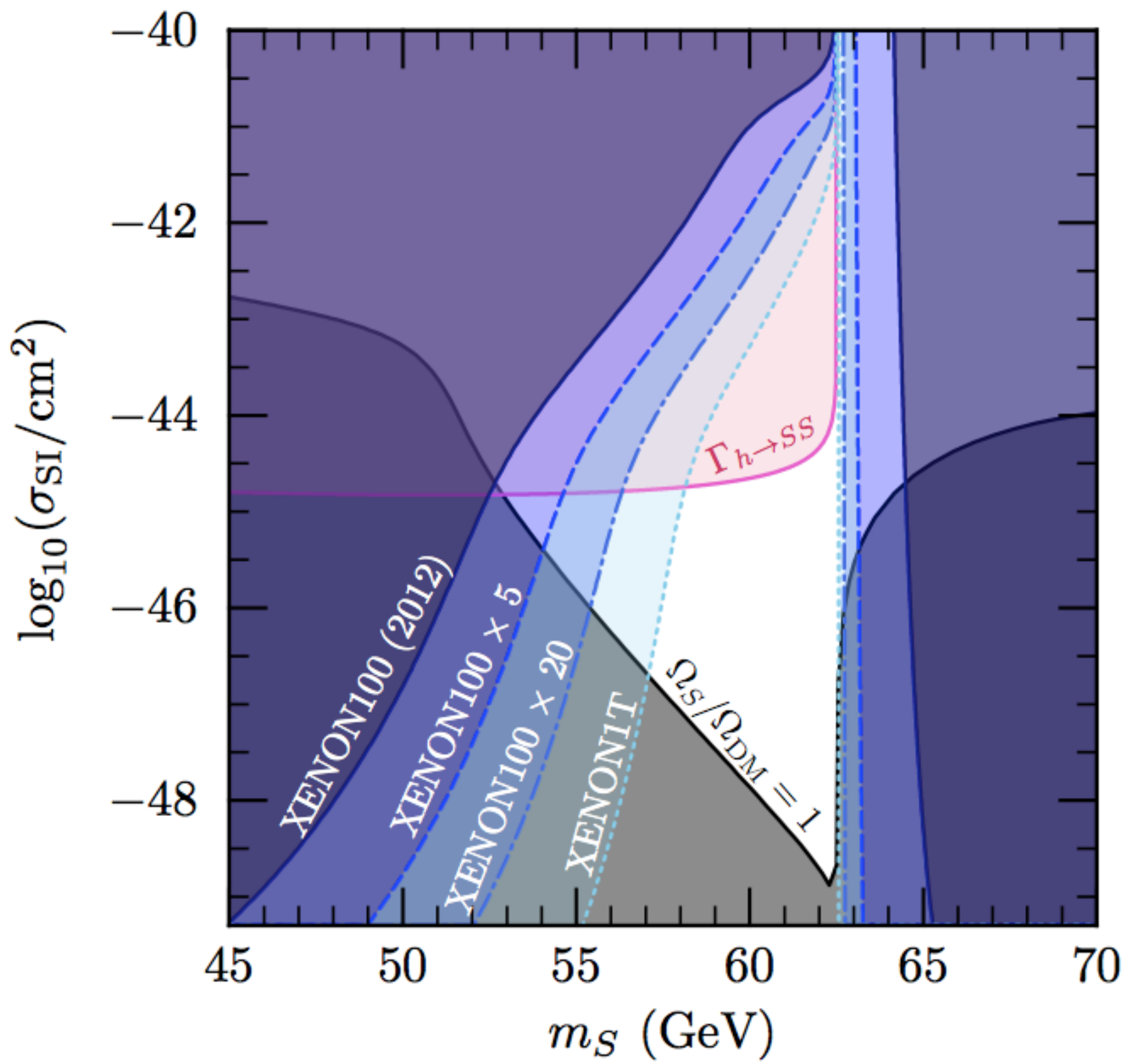
$$\mathcal{L}_{\rm int} = (A\varphi + \lambda\varphi^2)H^\dagger H$$

=> relic abundance via annihilation through H

=> direct detection via $\langle n|m_q\bar{q}q|n\rangle$ H-nucleon coupling

LHC results slain DM models with $m_{\rm DM} \lesssim 60\,\mathrm{GeV}$

Higgs decays invisibly into DM




e.g. Cline et al. (2013)

# New physics “portals”

Limited options to couple new physics directly in a renormalizable way

**Higgs portal**

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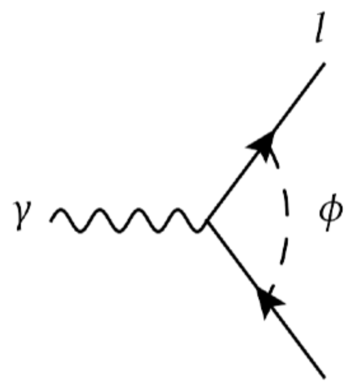
Interesting connection to muon (g-2):




A SM-like light Higgs boson ($m_h << m_\mu$) with v=246 GeV creates a shift

$$\Delta a_\mu = \frac{3}{16\pi^2} \times \left(\frac{m_\mu}{v}\right)^2 \simeq 3.5 \times 10^{-9}$$

=> for the Higgs portal model multiply by $\theta^2$

# New physics “portals”

Limited options to couple new physics directly in a renormalizable way

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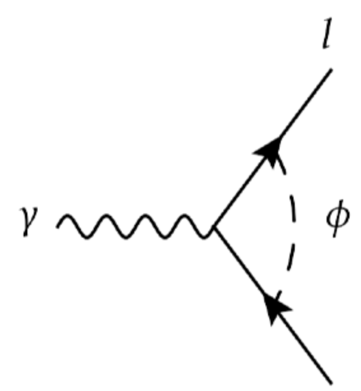


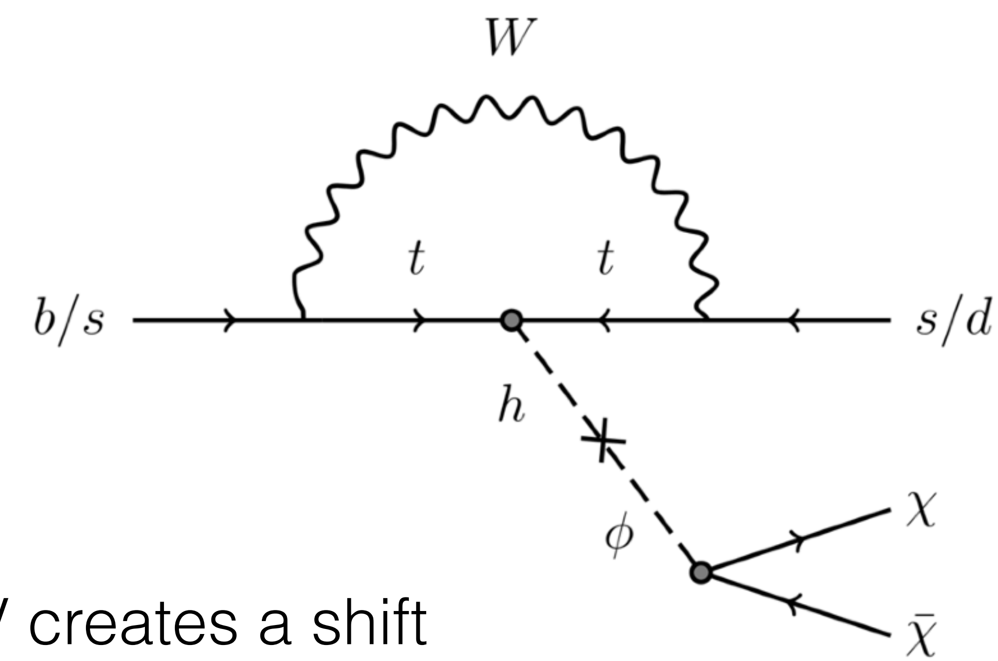


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$$\Delta a_\mu = \frac{3}{16\pi^2} \times \left(\frac{m_\mu}{v}\right)^2 \simeq 3.5 \times 10^{-9}$$

=> for the Higgs portal model multiply by $\theta^2$

which is at the right order of magnitude shift to explain (g-2), but $\theta = O(1)$ excluded e.g. from $B \to K\varphi$

# New physics “portals”

Limited options to couple new physics directly in a renormalizable way

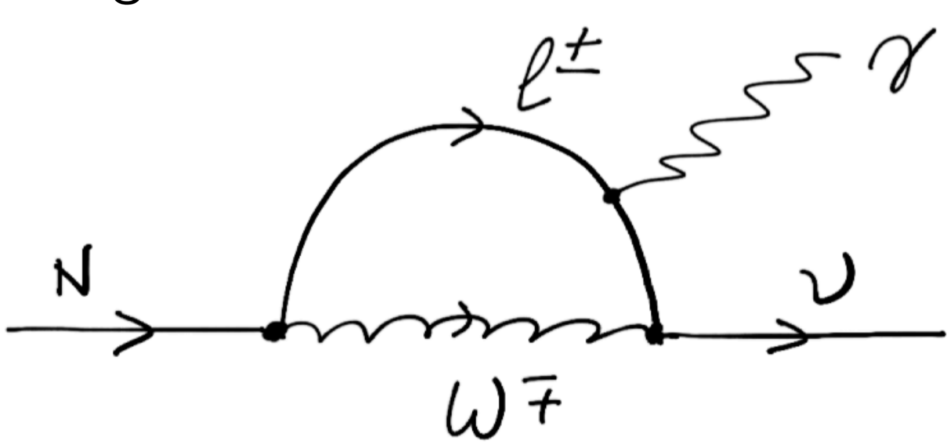
**Neutrino portal**

$$\mathcal{L} \supset y\bar{L}HN + \text{h.c.}$$

N is a SM singlet (a.k.a. right handed neutrino or sterile neutrino).

Given that neutrino have mass, there is an ample chance that this portal is indeed realized in nature. N can have a Majorana mass and give rise to Seesaw mechanism.

N is a dark matter candidate - one which will decay through the active-sterile neutrinos mixing



# Light Dark Matter - a call for mediators

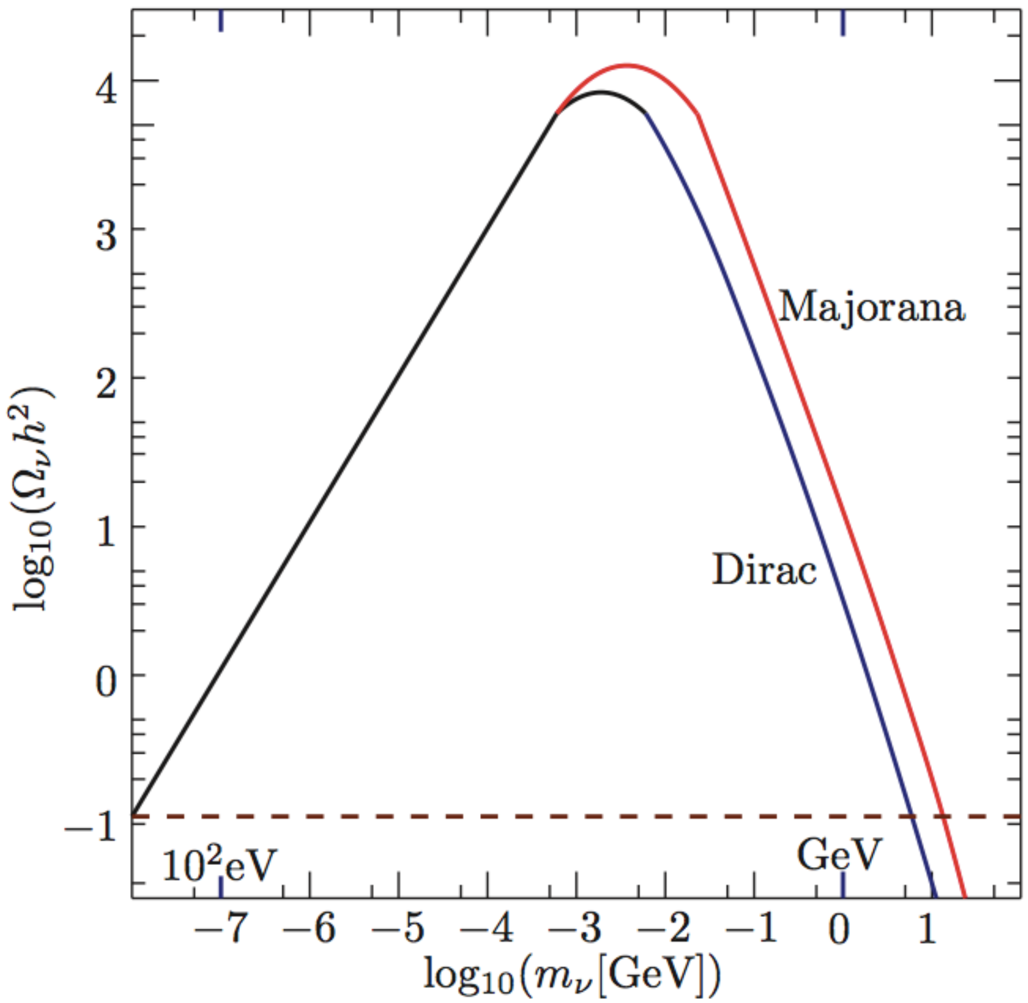
Relic Abundance issues

Lee-Weinberg bound:

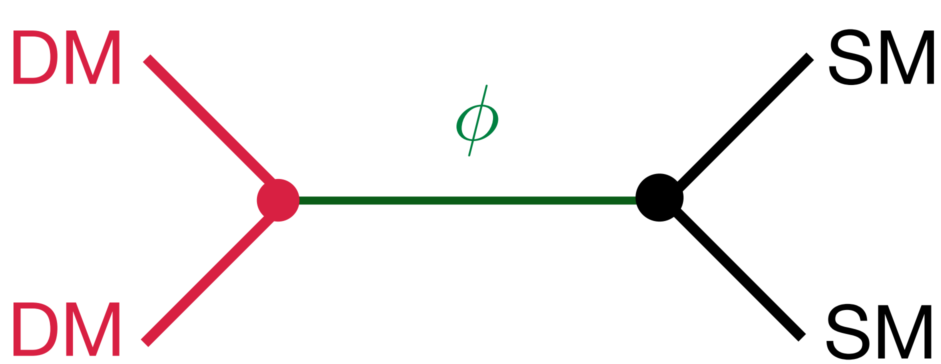
Annihilation of a heavy neutrino through SM mediators excludes masses below ~ few GeV

$$\langle \sigma v \rangle \sim G_F^2 m_\nu^2 / 2\pi$$

*A way out are new, light mediators*



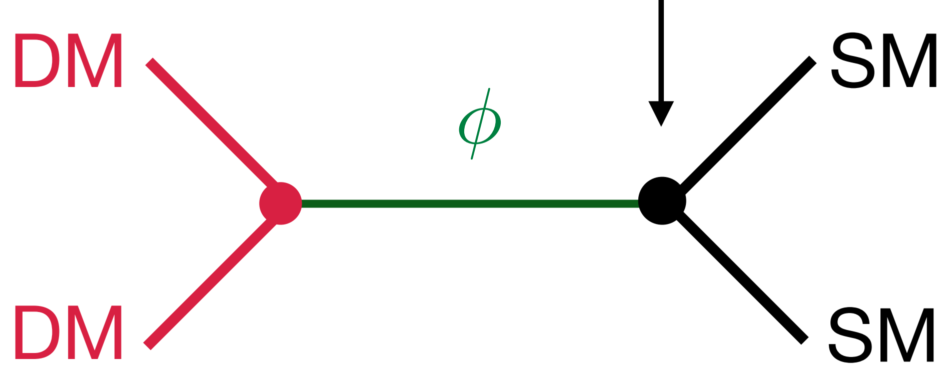

$$G_F \to g^2 / m_\phi^2$$



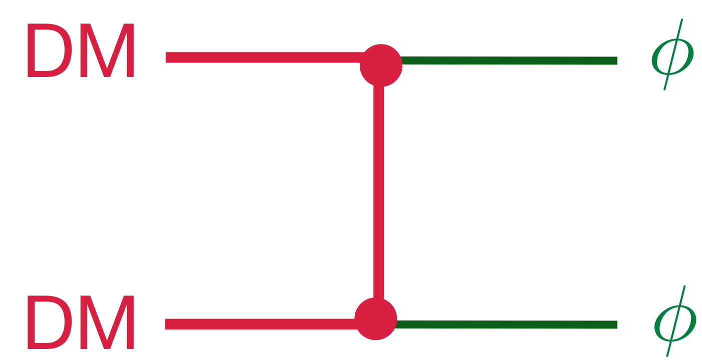

=> $\phi$ can be a dark photon or a light Higgs

# Relic abundance with light mediators

here a minimum coupling
to SM required for
efficient annihilation



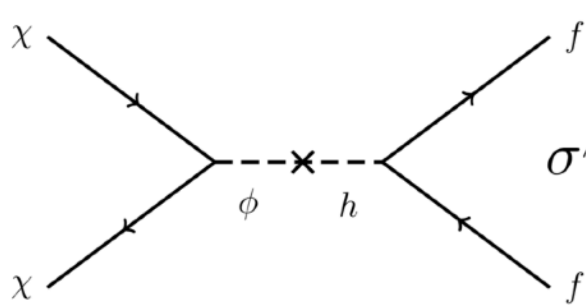

vs.




$m_{\rm DM} < m_\phi$

$m_{\rm DM} > m_\phi$

e.g.




$\sigma v \propto g_\chi^2 g_f^2$

Here, even a minuscule coupling of $\phi$ to the SM can be allowed and the model can essentially be secluded.

# Astrophysical and Cosmological Implications of Light Dark Sectors

# Astrophysical and cosmological implications of light dark sectors

At low energy, high intensity colliders we can probe new physics up to the GeV-scale and below - but by how much??

New light degrees of freedom can interfere in….

- **Object-based astronomy**
  1. Photons and neutrinos from sources are affected during their propagation
  => photon-axion conversion, neutrino oscillations
  2. Decay products of particles from distant sources
  => gamma/X-rays
  3. Emission of light, weakly interacting particles leads to energy loss in stars

- **Cosmology**
  Cosmic Microwave Background, Structure formation, primordial nucleosynthesis, modifications of Newtonian gravity…

# **Astrophysical** and Cosmological Implications of Light Dark Sectors

# Stars as laboratories for new physics

A star in a nutshell:

Virial theorem:

$$\langle E_{\rm kin}\rangle = -\frac{1}{2}\langle E_{\rm grav}\rangle$$

(imagine, the star forms from an initially dispersed cloud)

$$\frac{3}{2}T = \frac{1}{2}\frac{GM_\odot m_p}{R_\odot}$$

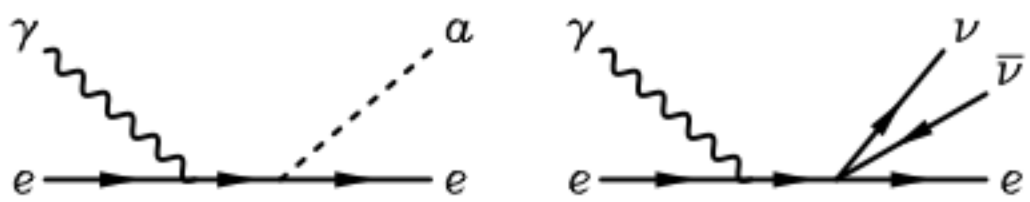
=> $T = O(\mathrm{keV})$ core temperature of solar mass star

=> Particles with mass < $O(\mathrm{keV})$ are kinematically accessible and can be produced in stars

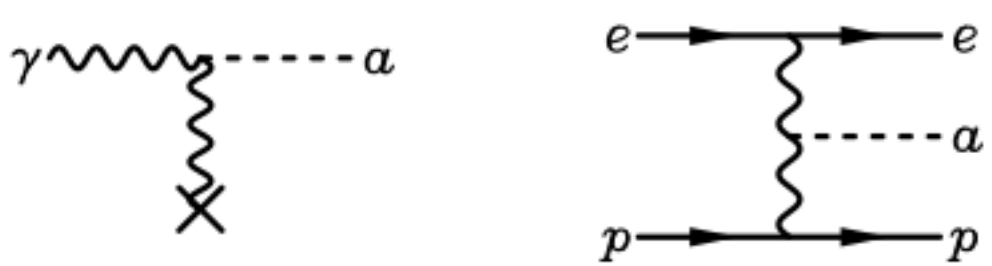
NB: in core-collapse supernovae O(10) MeV-temperatures are reached and MeV-scale particles can be produced

# Stars as laboratories for new physics

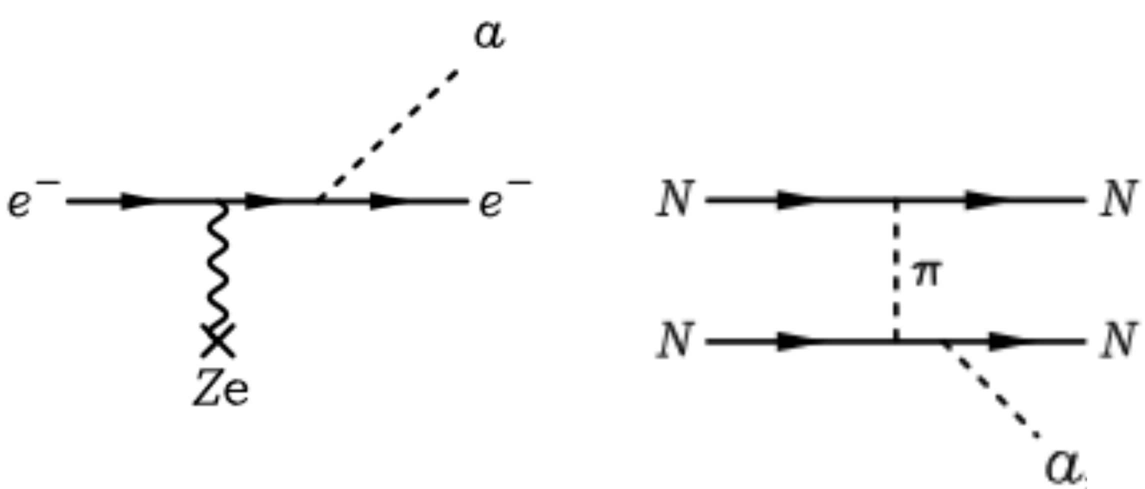
Typical production mechanisms




Compton-process




EM-field assisted




Bremsstrahlung

Free-bound transitions, bound-bound transitions, pair annihilation,…

# Stars as laboratories for new physics

**If interaction is "strong"**, particles can be trapped, just like photons

=> such particles are not necessarily harmless, as they contribute to radiative energy transfer

=> mean free path must be shorter than for photons, and therefore likely challenged by laboratory experiments

**If interaction is "weak",** particles can escape, just like neutrinos

=> if their interaction-rate is much weaker than neutrinos, then typically harmless

Impact on stars often maximised when new particle's mean free path is of order the geometric dimension of the system.

# Stars as laboratories for new physics

Reaction of a star to energy loss, i.e., $\langle E_{\rm kin} + E_{\rm grav} \rangle$ decreases.

**1. Stars supported by radiation pressure (active stars):**

Virial theorem: $\langle E_{\rm kin} \rangle = -\frac{1}{2} \langle E_{\rm grav} \rangle$

=> Gravitational potential energy becomes more negative (tighter bound)

=> average kinetic energy increases, star becomes hotter, negative heat capacity

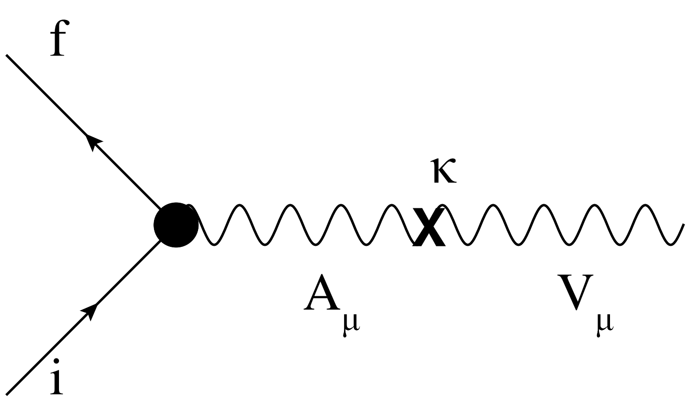
**2. Stars supported by degeneracy pressure (white dwarfs, neutron stars):**

possess positive heat capacity, the star actually cools by the energy loss

# Stars as laboratories for new physics

Example: stellar production of dark photon

$$\frac{\kappa}{2} F_{\mu\nu} V^{\mu\nu} + e J^{\mu}_{\rm em} A_{\mu} \xrightarrow{\text{on-shell V}} \mathcal{L}_{\rm int} = -\kappa m_V^2 A_\mu V^\mu + e J^{\mu}_{\rm em} A_\mu .$$



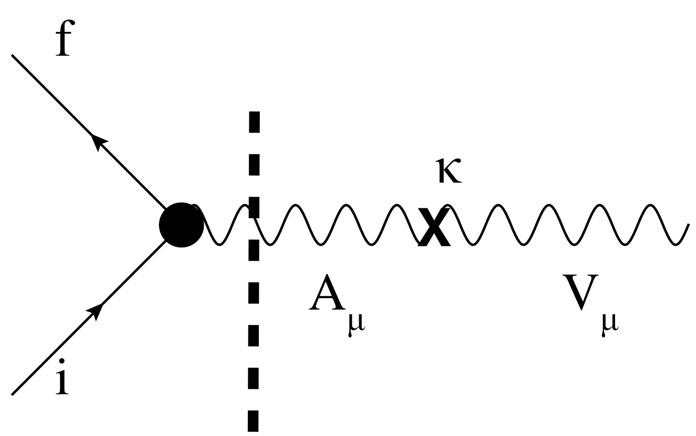

$$\mathcal{M}_{i \to f + V_{T(L)}} = \kappa m_V^2 \, [e J_{\rm em \, \mu}]_{fi} \, \langle A^\mu, A^\nu \rangle \, \epsilon_\nu^{T(L)}$$

in-medium photon propagator

# Stars as laboratories for new physics

Example: stellar production of dark photon

$$\frac{\kappa}{2} F_{\mu\nu} V^{\mu\nu} + e J^{\mu}_{\rm em} A_{\mu} \xrightarrow{\text{on-shell V}} \mathcal{L}_{\rm int} = -\kappa m_V^2 A_\mu V^\mu + e J^{\mu}_{\rm em} A_\mu .$$




$$\mathcal{M}_{i \to f + V_{T,L}} = - \frac{\kappa m_V^2}{m_V^2 - \Pi_{T,L}} [e J^{\mu}_{\rm em}]_{fi} \epsilon_\mu^{T,L}$$

transverse resonance

$$m_V^2 = \mathrm{Re}\,\Pi_T = \omega_p^2$$

$\omega_p$ = plasma frequency

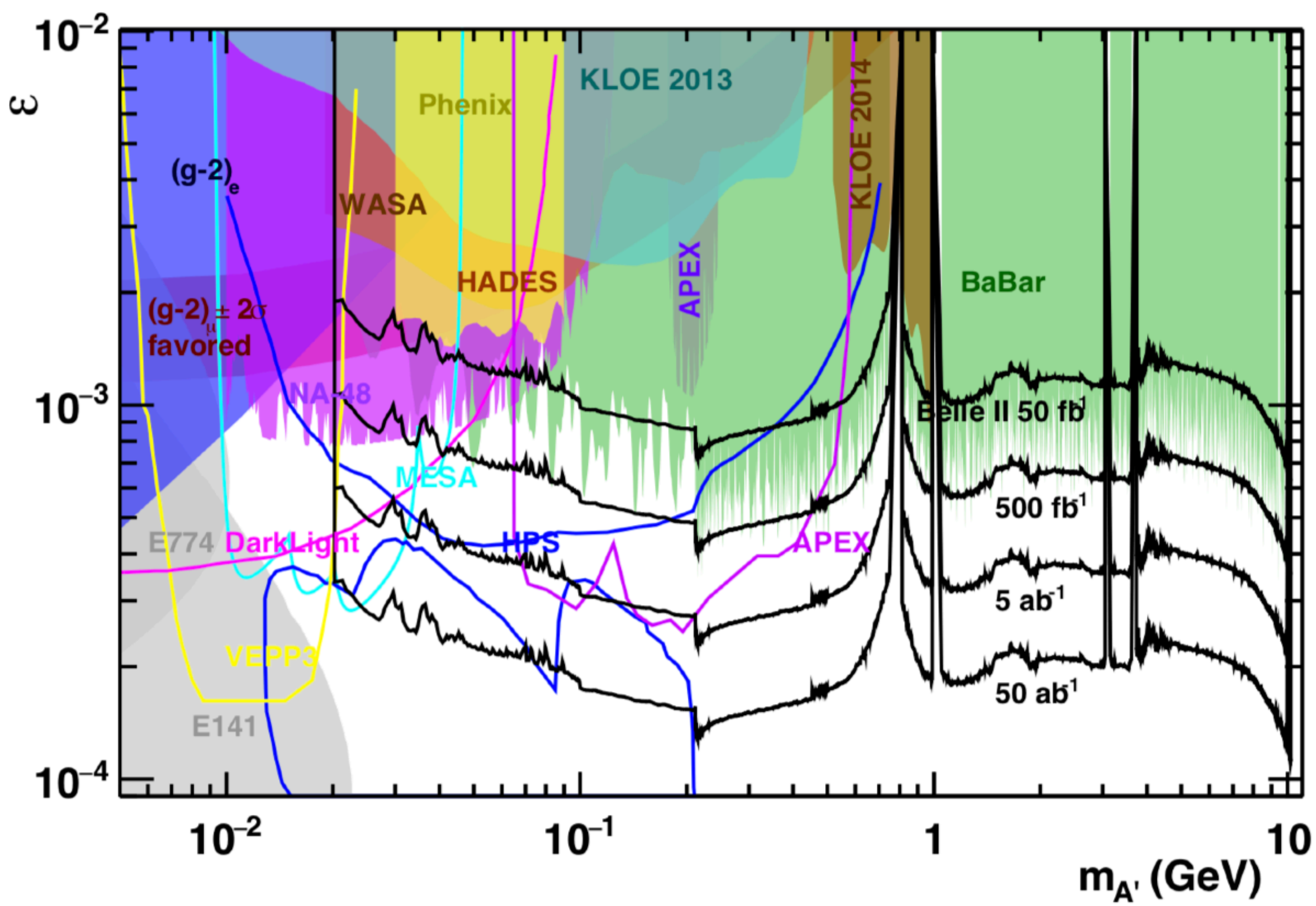
longitudinal resonance

$$m_V^2 = \mathrm{Re}\,\Pi_L = \omega_p^2 m_V^2 / \omega^2$$

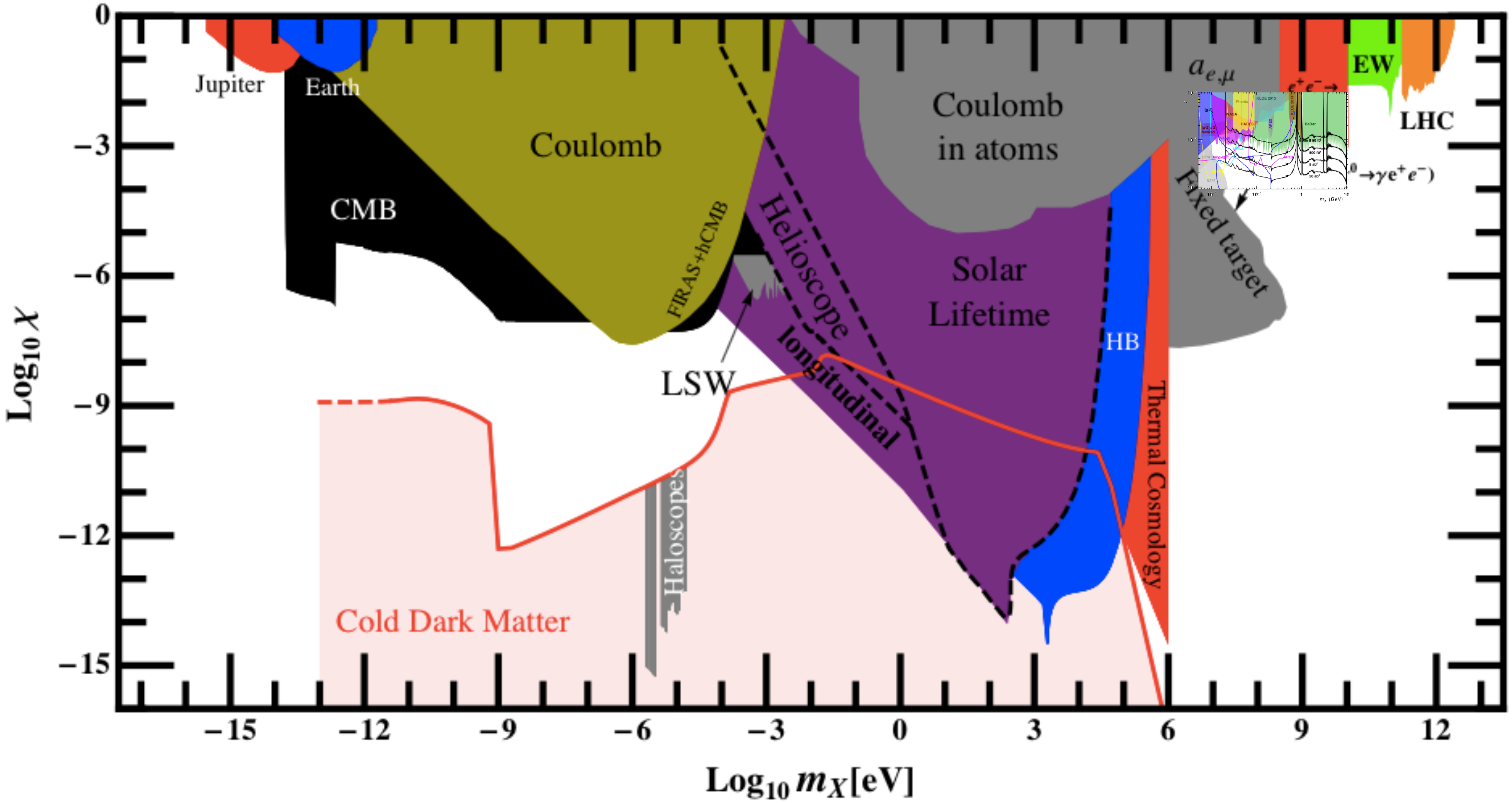
$$\Leftrightarrow \omega^2 = \omega_p^2$$

Energy loss heats up the sun => greater neutrino flux than observed!

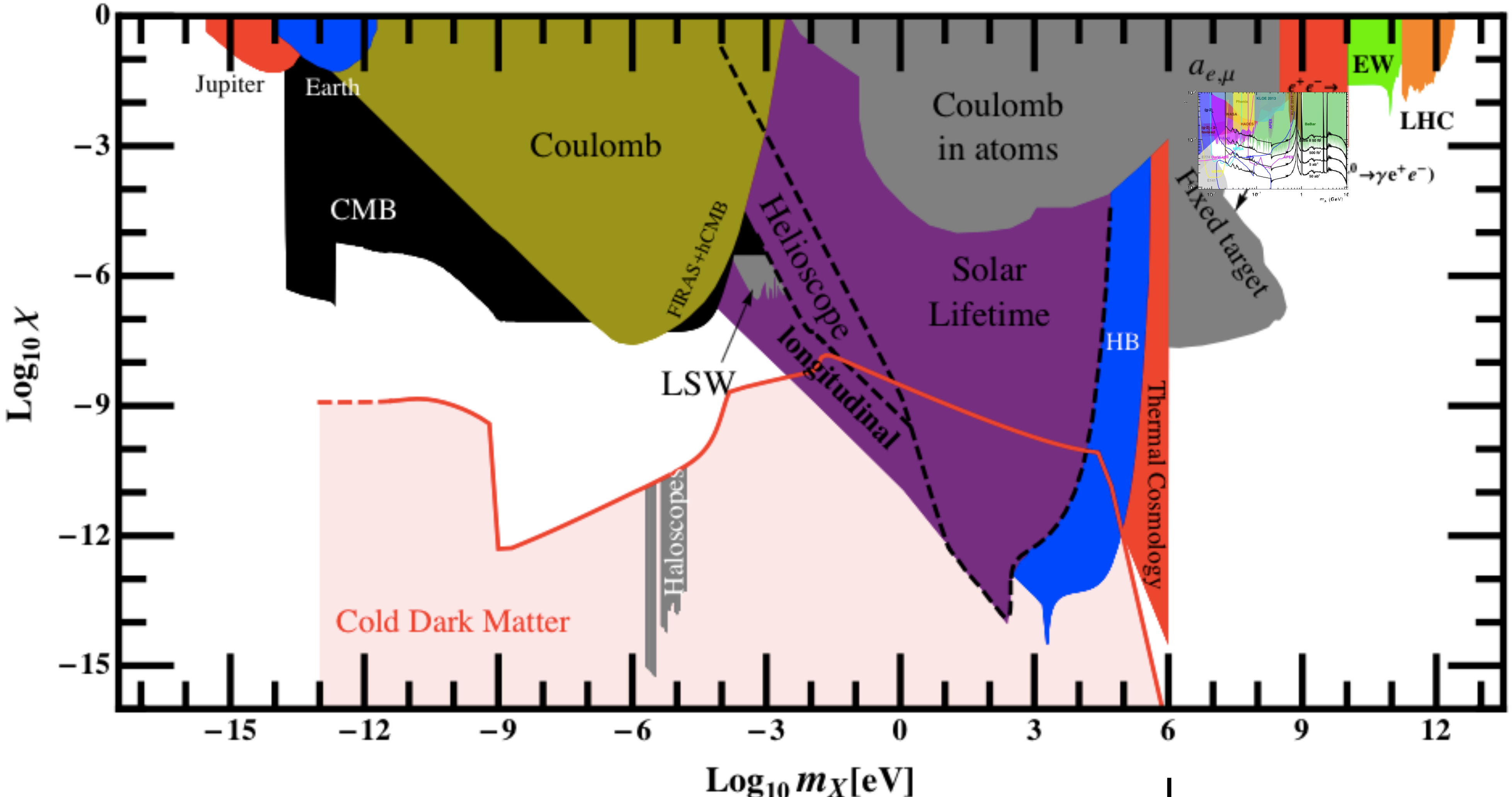
# Dark photon searches



# Dark photon searches



# Dark photon searches



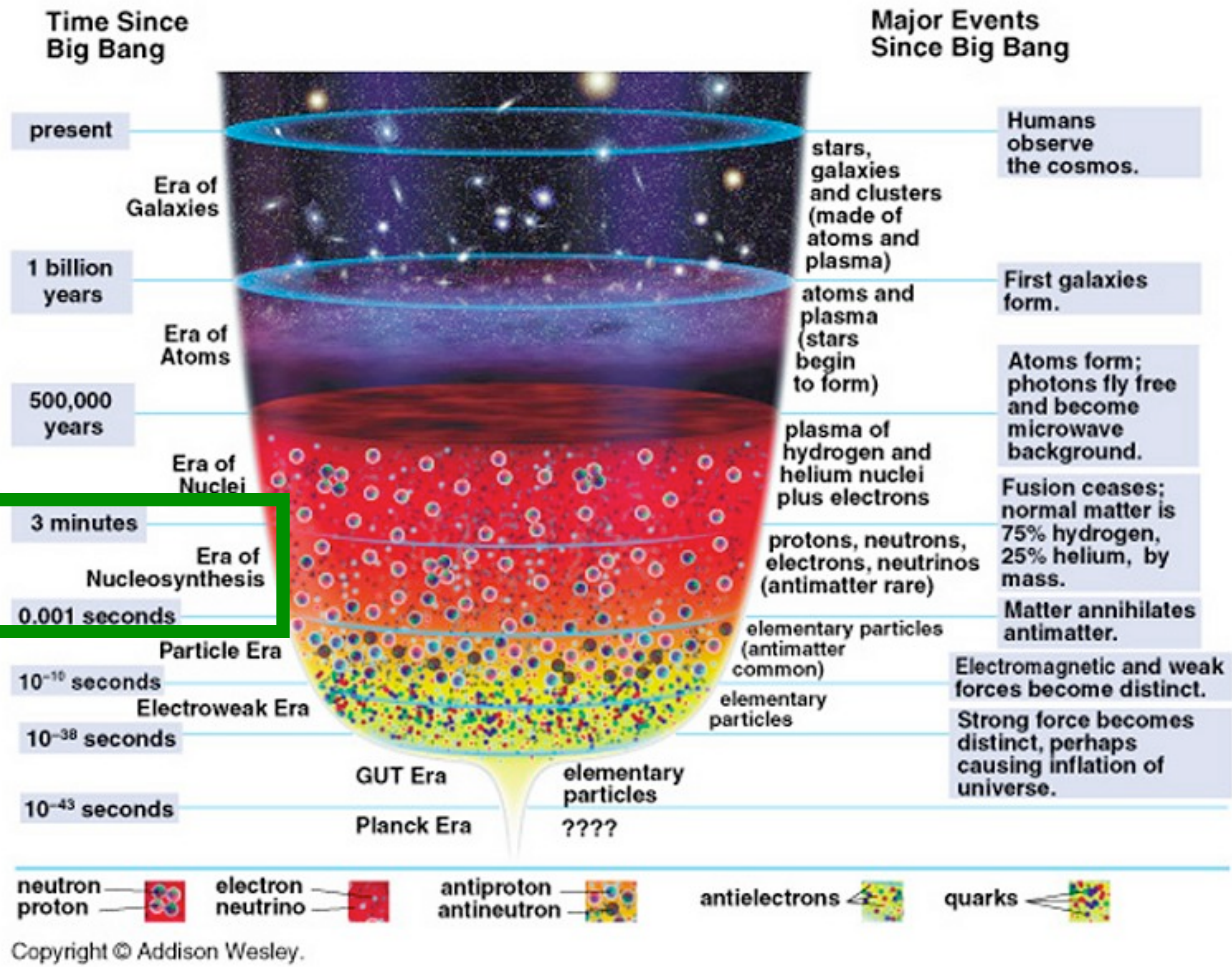

Dark Photon becomes a dark matter candidate

Decays to e+ e- possible

# Astrophysical and **Cosmological** Implications of Light Dark Sectors

# The origin of chemistry

Primordial Nucleosynthesis



# The origin of chemistry

| | | | | | | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| $\boxed{\text{hydrogen 1 H 1.0079}}$ | | | | | | | | | | | | | | | | | | $\boxed{\text{helium 2 He 4.0026}}$ |
| $\boxed{\text{lithium 3 Li 6.941}}$ | beryllium 4 Be 9.0122 | | | | | | | | | | | | boron 5 B 10.811 | carbon 6 C 12.011 | nitrogen 7 N 14.007 | oxygen 8 O 15.999 | fluorine 9 F 18.998 | neon 10 Ne 20.180 |
| sodium 11 Na 22.990 | magnesium 12 Mg 24.305 | | | | | | | | | | | | aluminium 13 Al 26.982 | silicon 14 Si 28.086 | phosphorus 15 P 30.974 | sulfur 16 S 32.065 | chlorine 17 Cl 35.453 | argon 18 Ar 39.948 |
| potassium 19 K 39.098 | calcium 20 Ca 40.078 | | scandium 21 Sc 44.956 | titanium 22 Ti 47.867 | vanadium 23 V 50.942 | chromium 24 Cr 51.996 | manganese 25 Mn 54.938 | iron 26 Fe 55.845 | cobalt 27 Co 58.933 | nickel 28 Ni 58.693 | copper 29 Cu 63.546 | zinc 30 Zn 65.39 | gallium 31 Ga 69.723 | germanium 32 Ge 72.61 | arsenic 33 As 74.922 | selenium 34 Se 78.96 | bromine 35 Br 79.904 | krypton 36 Kr 83.80 |
| rubidium 37 Rb 85.468 | strontium 38 Sr 87.62 | | yttrium 39 Y 88.906 | zirconium 40 Zr 91.224 | niobium 41 Nb 92.906 | molybdenum 42 Mo 95.94 | technetium 43 Tc [98] | ruthenium 44 Ru 101.07 | rhodium 45 Rh 102.91 | palladium 46 Pd 106.42 | silver 47 Ag 107.87 | cadmium 48 Cd 112.41 | indium 49 In 114.82 | tin 50 Sn 118.71 | antimony 51 Sb 121.76 | tellurium 52 Te 127.60 | iodine 53 I 126.90 | xenon 54 Xe 131.29 |
| caesium 55 Cs 132.91 | barium 56 Ba 137.33 | 57-70 * | lutetium 71 Lu 174.97 | hafnium 72 Hf 178.49 | tantalum 73 Ta 180.95 | tungsten 74 W 183.84 | rhenium 75 Re 186.21 | osmium 76 Os 190.23 | iridium 77 Ir 192.22 | platinum 78 Pt 195.08 | gold 79 Au 196.97 | mercury 80 Hg 200.59 | thallium 81 Tl 204.38 | lead 82 Pb 207.2 | bismuth 83 Bi 208.98 | polonium 84 Po [209] | astatine 85 At [210] | radon 86 Rn [222] |
| francium 87 Fr [223] | radium 88 Ra [226] | 89-102 * * | lawrencium 103 Lr [262] | rutherfordium 104 Rf [261] | dubnium 105 Db [262] | seaborgium 106 Sg [266] | bohrium 107 Bh [264] | hassium 108 Hs [269] | meitnerium 109 Mt [268] | ununnilium 110 Uun [271] | unununium 111 Uuu [272] | ununbium 112 Uub [277] | | ununquadium 114 Uuq [289] | | | | |

| | | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| * Lanthanide series | lanthanum 57 La 138.91 | cerium 58 Ce 140.12 | praseodymium 59 Pr 140.91 | neodymium 60 Nd 144.24 | promethium 61 Pm [145] | samarium 62 Sm 150.36 | europium 63 Eu 151.96 | gadolinium 64 Gd 157.25 | terbium 65 Tb 158.93 | dysprosium 66 Dy 162.50 | holmium 67 Ho 164.93 | erbium 68 Er 167.26 | thulium 69 Tm 168.93 | ytterbium 70 Yb 173.04 |
| * * Actinide series | actinium 89 Ac [227] | thorium 90 Th 232.04 | protactinium 91 Pa 231.04 | uranium 92 U 238.03 | neptunium 93 Np [237] | plutonium 94 Pu [244] | americium 95 Am [243] | curium 96 Cm [247] | berkelium 97 Bk [247] | californium 98 Cf [251] | einsteinium 99 Es [252] | fermium 100 Fm [257] | mendelevium 101 Md [258] | nobelium 102 No [259] |

# Big Bang Nucleosynthesis (BBN) - A pillar of modern cosmology

- recent progress primarily clarifies state of the Universe at z = few (Galaxy surveys, SN,…) and exposes relevant physics at recombination $z = 1000$

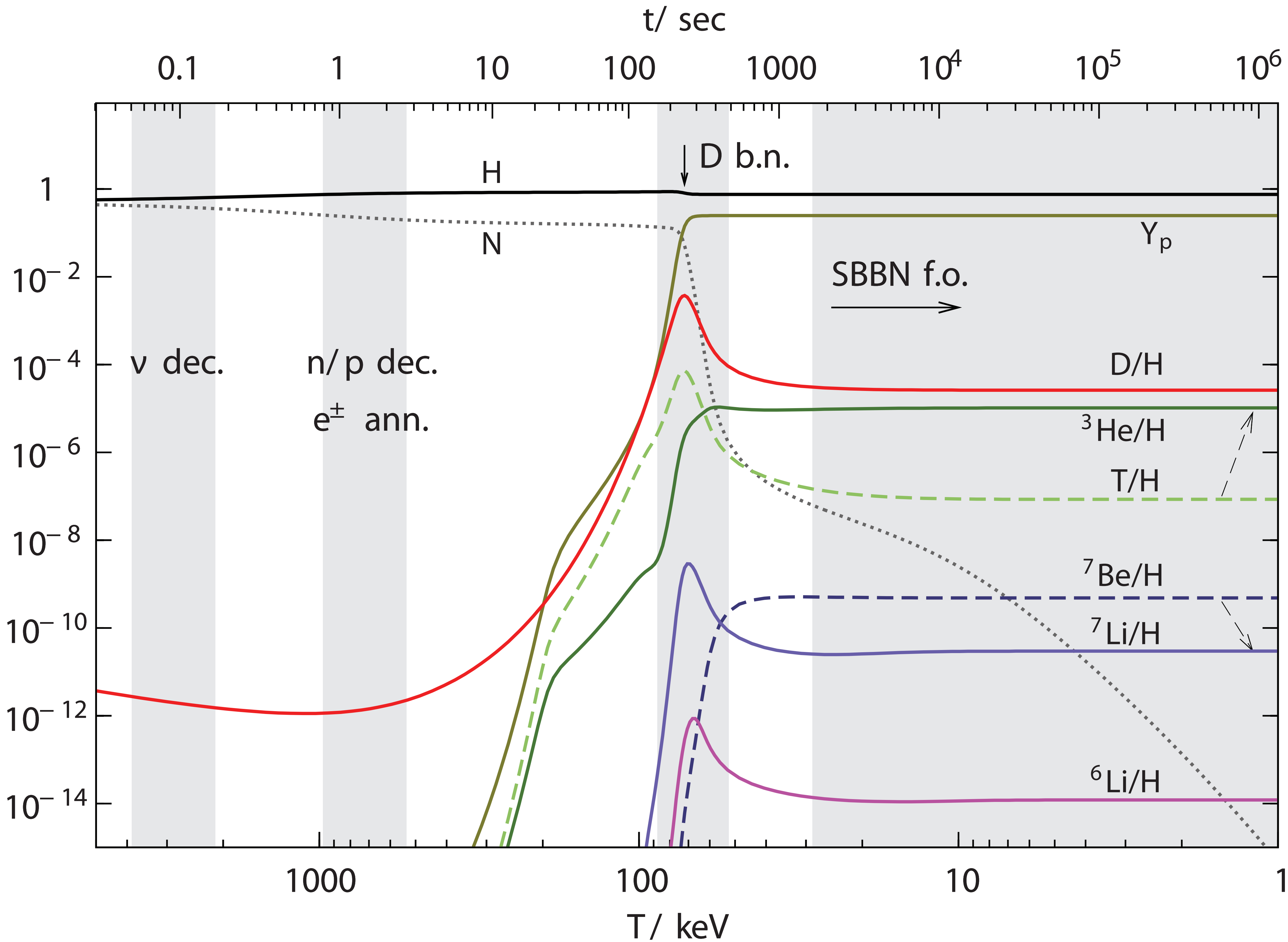
- light element formation happens at $z = 10^9$ ; direct window into the early Universe at t=1sec

- qualitative agreement between $z = 0 \div 10^3$ and $z = 10^9$ tells us that early Universe was governed by the same physical laws and contained similar particle content

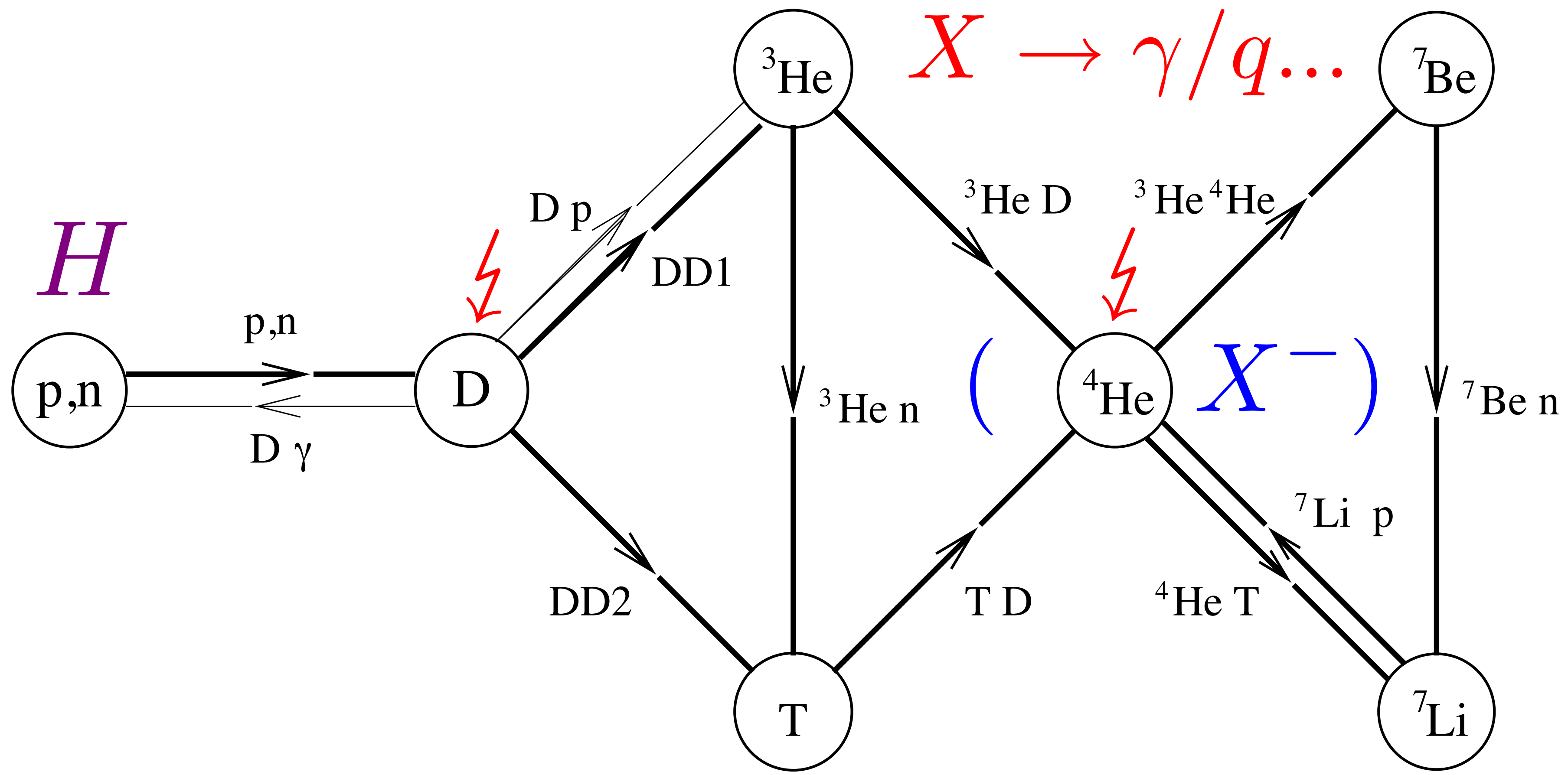
  => invaluable piece that helps to establish the standard cosmological model

- BBN can react sensitively on departures from General Relativity and the Standard Model of particle physics => a toolbox to test new physics

# The Universe at a redshift of a billion



# Beyond SBBN



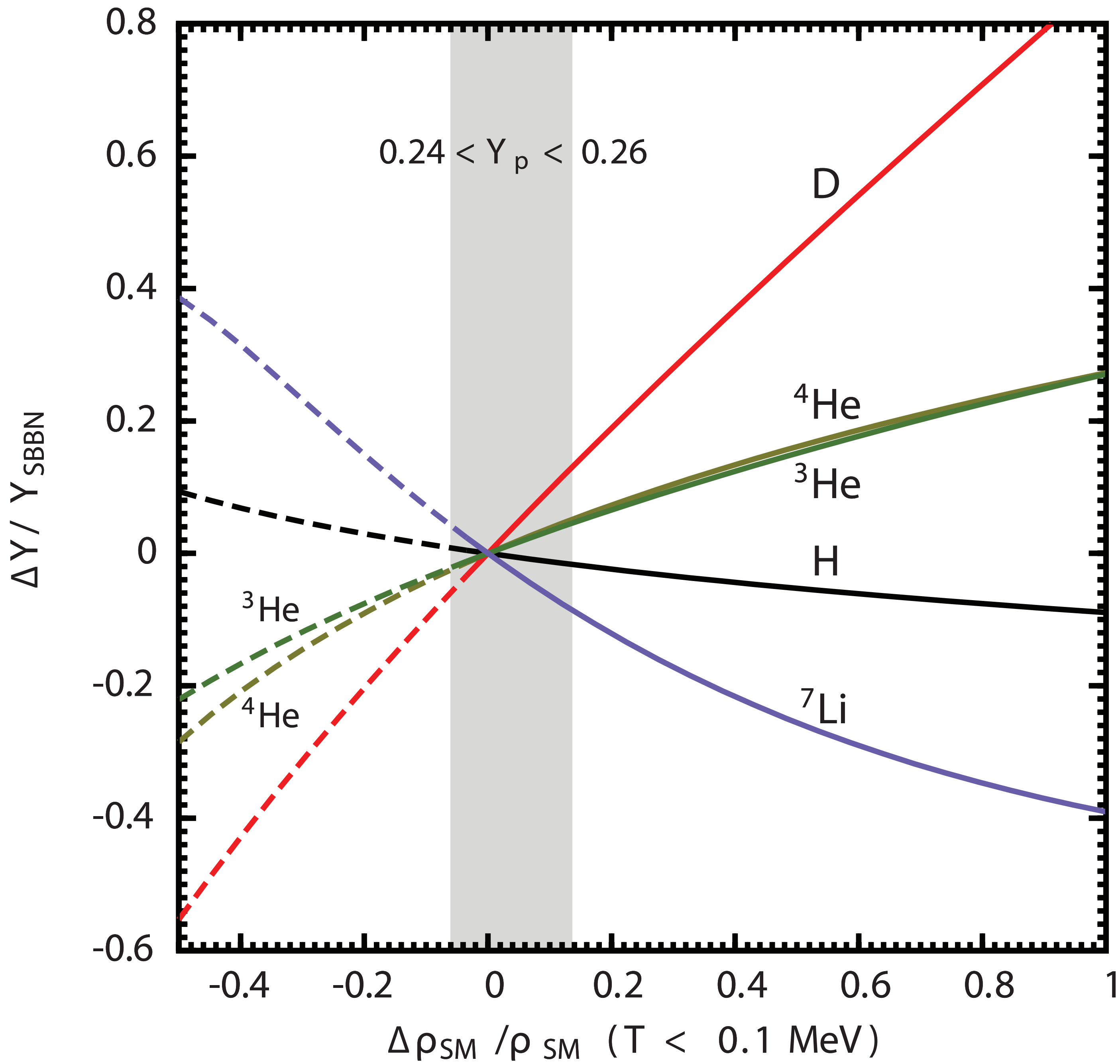

Change in timing

non-equilibrium BBN

catalyzed BBN

# Change in timing

$$H_{\rm SBBN} \to H = H_{\rm SBBN}\sqrt{1+\rho_{dr}/\rho_{\rm SM}}$$





$$N_{\rm eff} = 3.15 \pm 0.23$$

Planck

Extra radiation energy increases He4 abundance

# Concrete example

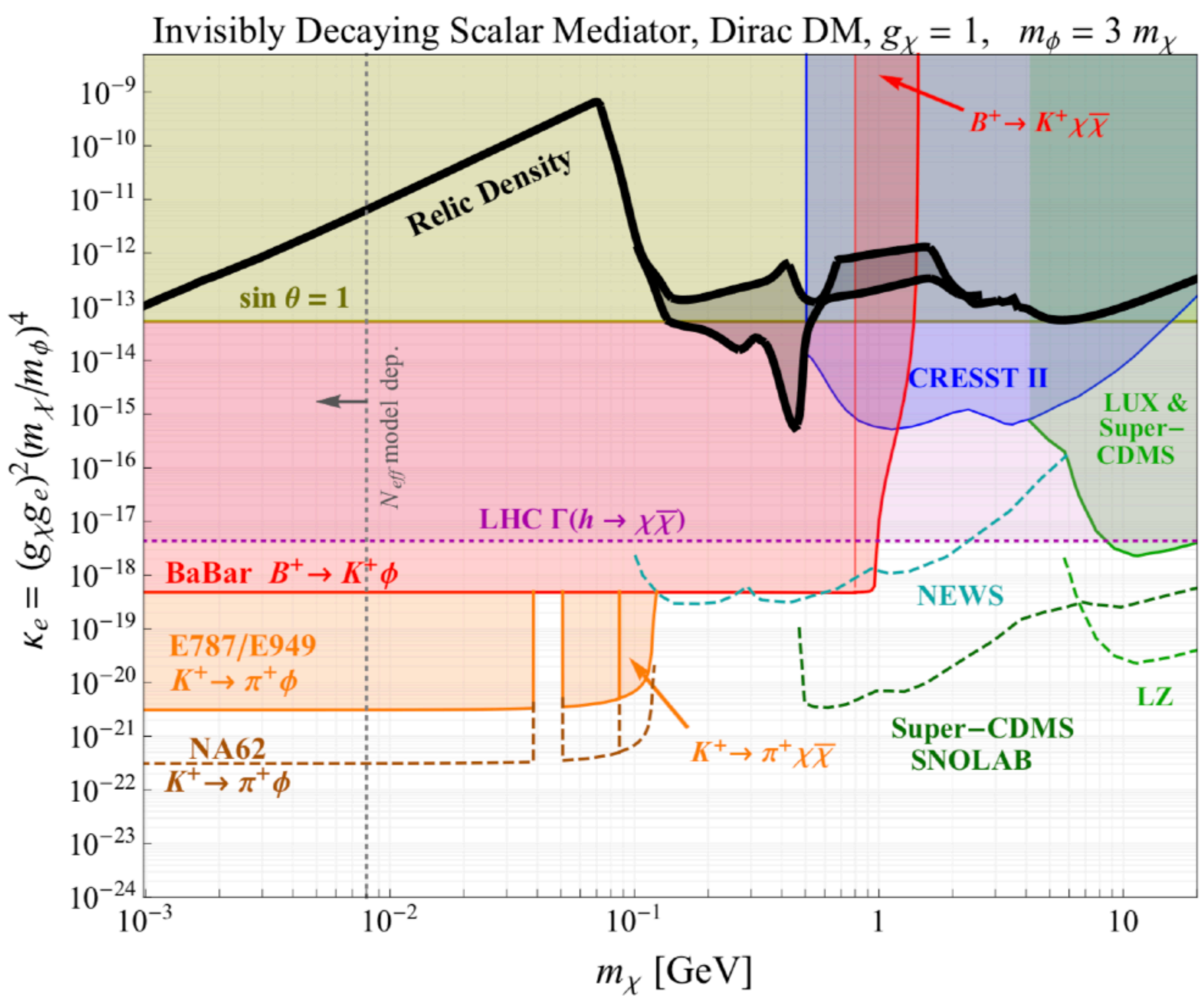
Mediator interaction with SM through Higgs portal

$$\mathcal{L}_{\Phi,H} = (A_{\Phi H}\Phi + \lambda_{\Phi H}\Phi^2)H^\dagger H \quad => \quad \mathcal{L}_{\phi,\mathrm{SM}} = \phi \sin\theta \sum_f \frac{m_f}{v}\bar{f}f \;\;,\;\; g_f \equiv \frac{m_f}{v}\sin\theta$$

DM interaction with the mediator through Yukawa coupling $\mathcal{L}_{\phi,\mathrm{DM}} = g_\chi \phi \bar{\chi}\chi$

=> parameter space of light DM

(here decay $\phi \to \bar{\chi}\chi$ kinematically allowed)



# Concrete example

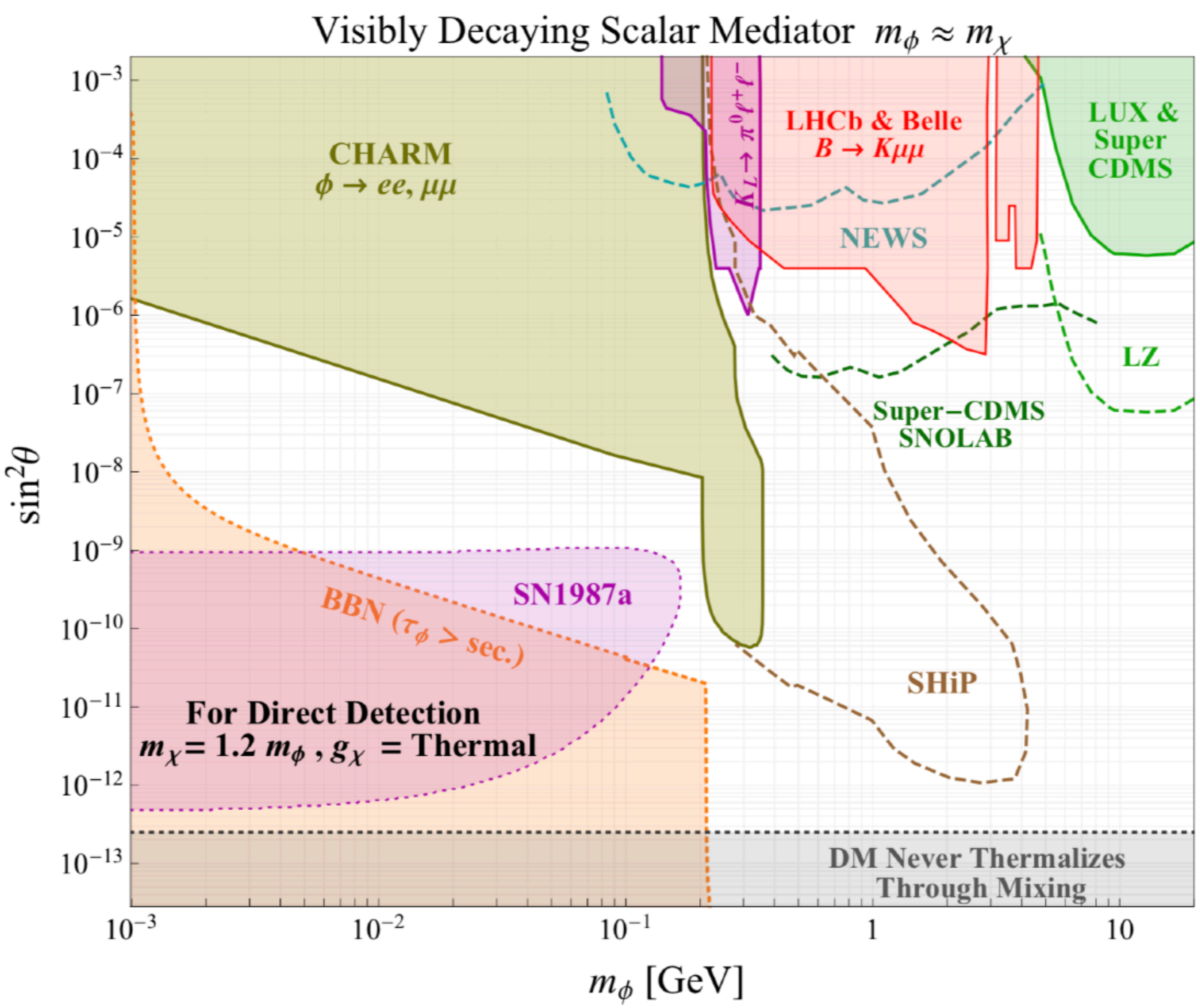
Mediator interaction with SM through Higgs portal

$$\mathcal{L}_{\Phi,H} = (A_{\Phi H}\Phi + \lambda_{\Phi H}\Phi^2)H^\dagger H \quad \Rightarrow \quad \mathcal{L}_{\phi,\mathrm{SM}} = \phi \sin\theta \sum_f \frac{m_f}{v}\bar{f}f \ , \quad g_f \equiv \frac{m_f}{v}\sin\theta$$

DM interaction with the mediator through Yukawa coupling $\mathcal{L}_{\phi,\mathrm{DM}} = g_\chi \phi\bar{\chi}\chi$

=> parameter space of the mediator

(here decay $\phi \to \bar{\chi}\chi$ kinematically allowed)



# Summary

- Organisational principle of new physics in terms of effective operators in direct detection and at the collider; approach has its limitation, when mediator goes on-shell

- With the Higgs discovery, many models fell on July 4, 2012. A minimal extension of the dark sector by a new mediator opens up many new possibilities of phenomenology (e.g. intensity frontier)

- Alternatives to the WIMP miracle exist, e.g. asymmetric DM. Light DM typically requires introduction of new particles/force carriers to achieve correct relic abundance

- Astrophysics and Cosmology severely limit the existence of dark sector states below keV-MeV mass range. Above that “blind spot” for both astro/cosmo and direct detection that can be tested at the intensity frontier.

Thank you — and keep up the good work!