Achim Denig

Institute for Nuclear Physics
PRISMA Cluster of Excellence
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Measuring the Hadronic Contribution to $(g-2)_\mu$

Part I: R_{had} Measurements

SFB 1044

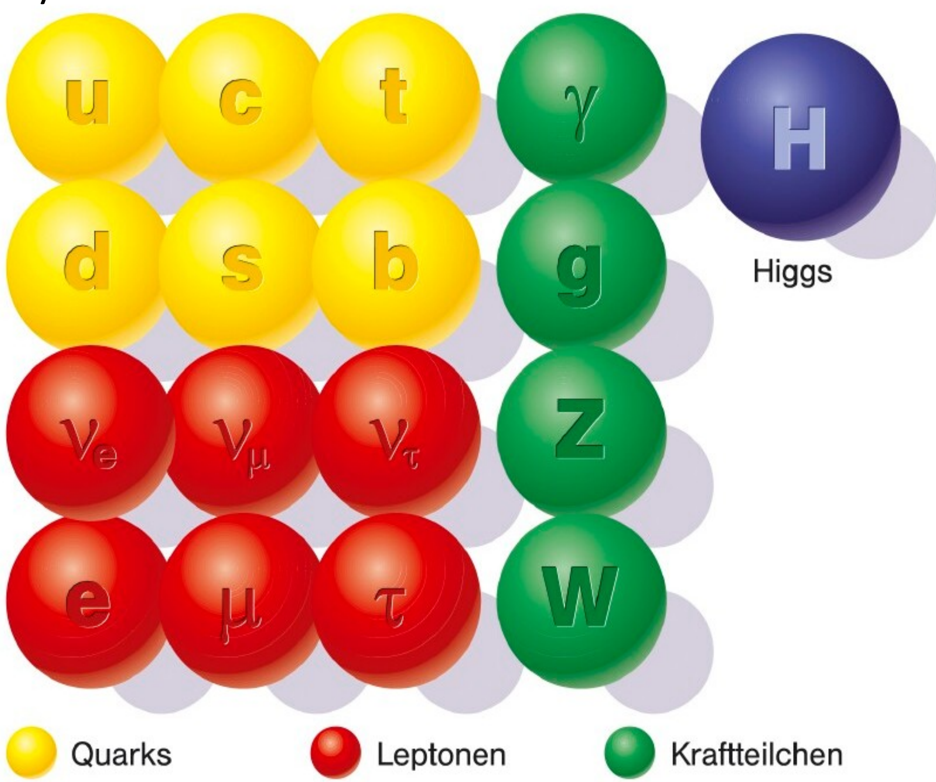
Cluster of Excellence Precision Physics, Fundamental Interactions and Structure of Matter PRISMA

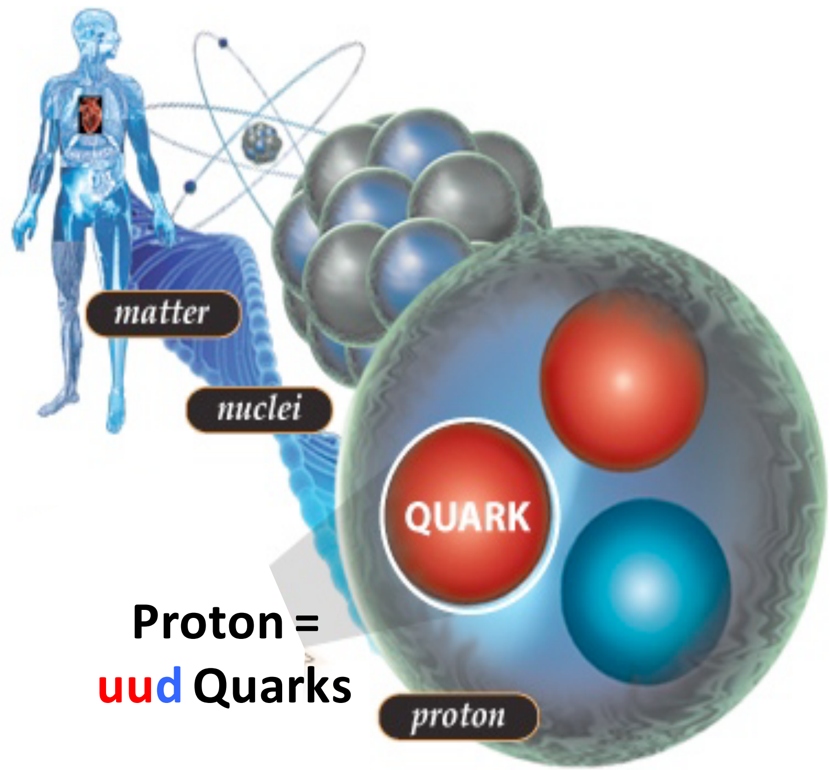
International School on Muon Dipole Moments and Hadronic Effects
BINP Novosibirsk, Sept 17 - 21, 2018

Standard Model of Particle Physics

Fundamental fermions
= building blocks of matter

Family 1 2 3



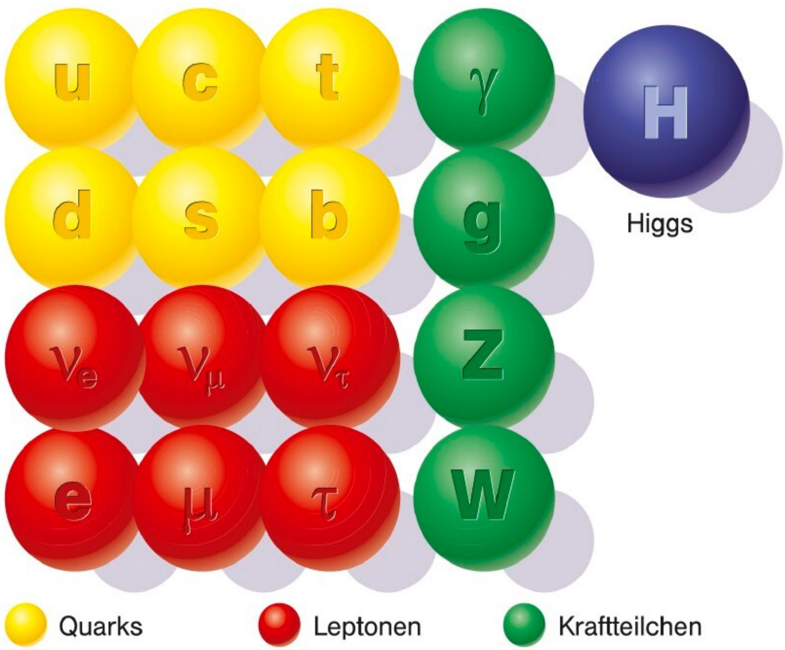


Proton =
uud Quarks

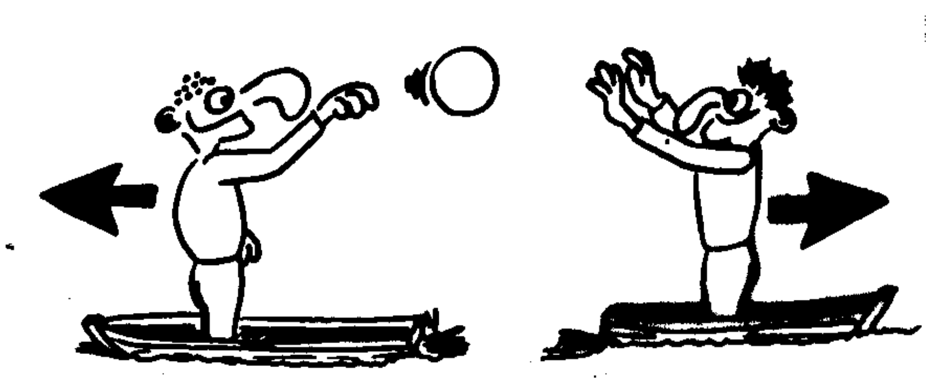
Standard Model of Particle Physics

Describes 3 of the 4 fundamental interactions, i.e. forces:

- electromagnetic force
- weak nuclear force
- strong force



$$\Delta E \cdot \Delta t \approx h$$

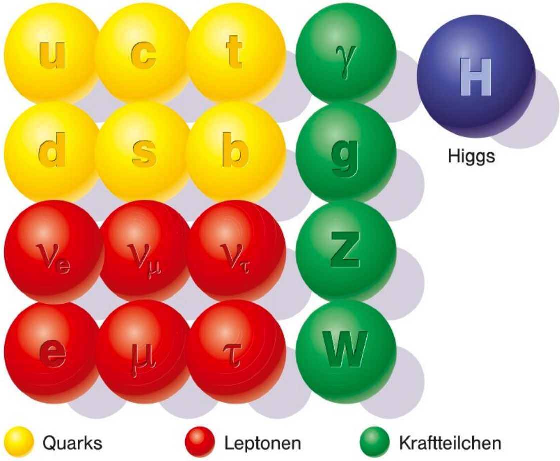


Standard Model of Particle Physics

Describes 3 of the 4 fundamental interactions, i.e. forces:

- electromagnetic force
- weak nuclear force
- ***strong nuclear force***

QCD ***Quantum Chromo Dynamics***



Nobel Prize 2004

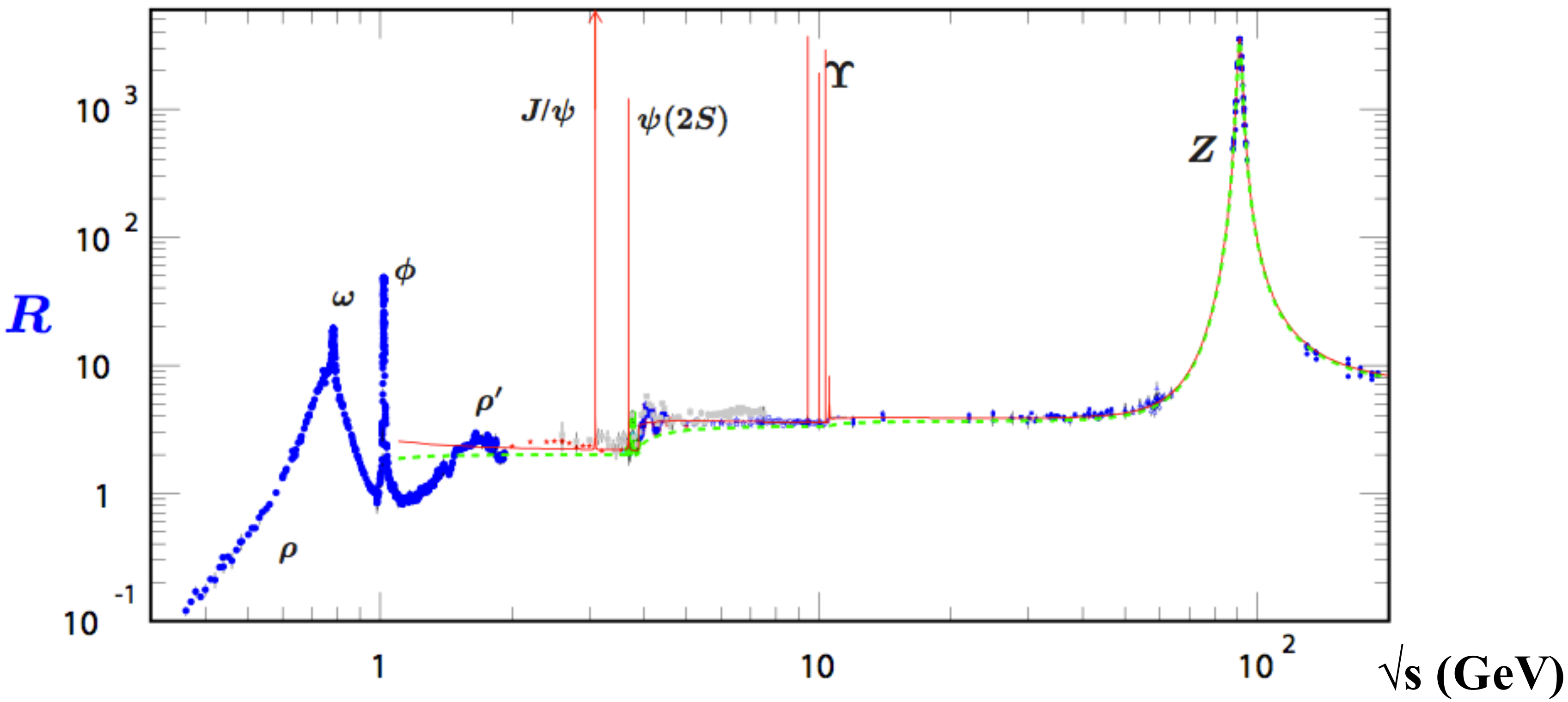
*"for the discovery of **asymptotic freedom** in the theory of the **strong interaction**"*

The R Ratio

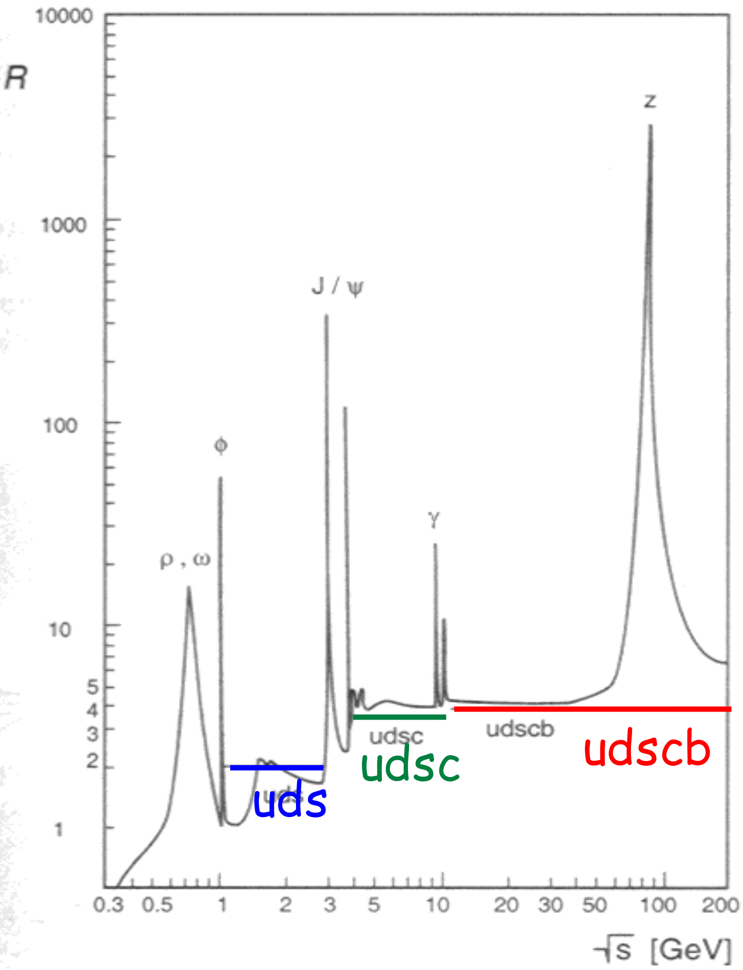
$$R = \frac{\sigma^{(0)}(e^+e^- \to \text{hadrons})}{\sigma^{(0)}(e^+e^- \to \mu^+\mu^-)}$$

R lead to formulation of Standard Model

- **Number of colours $N_c = 3$**
- **Number of quark flavours N_f**



The R Ratio



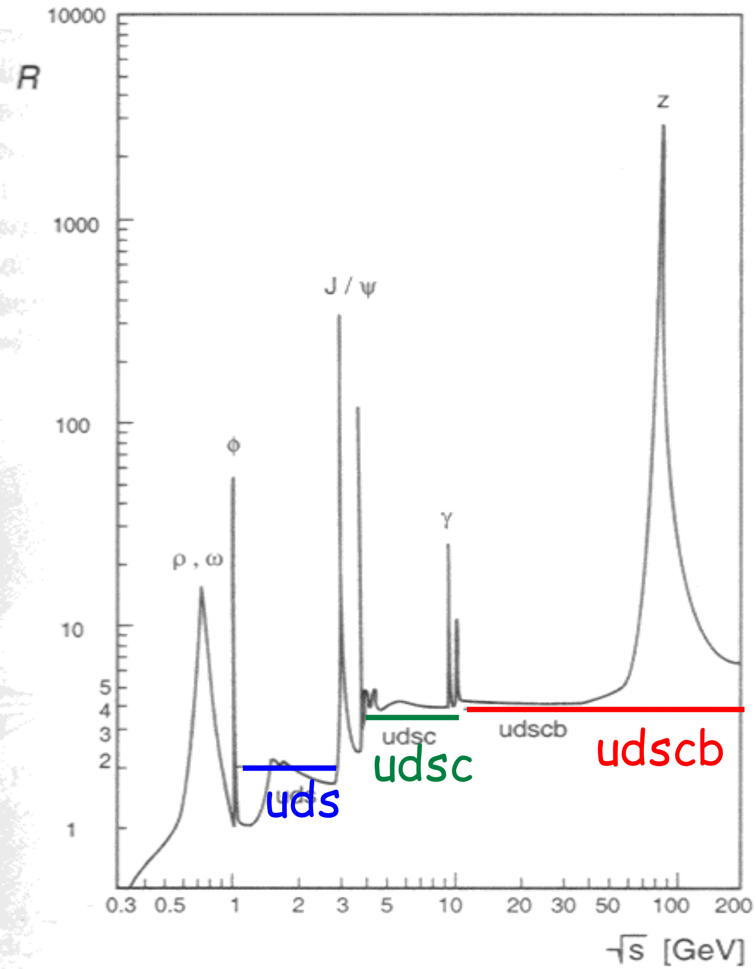
$$R = \frac{\sigma(e^+e^- \to \text{Hadrons})}{\sigma(e^+e^- \to \mu^+\mu^-)}$$

$$= \frac{\Sigma_f\, \sigma(e^+e^- \to f\bar{f})}{\sigma(e^+e^- \to \mu^+\mu^-)} = \Sigma_f\ z_f^2$$

$$= [\,(2/3)^2 + (-1/3)^2 + (-1/3)^2 + (2/3)^2 + (-1/3)^2]$$

u **d** **s** **c** **b**

The R Ratio



$$R = \frac{\sigma(e^+e^- \to \text{Hadrons})}{\sigma(e^+e^- \to \mu^+\mu^-)}$$

Number Colours

R G B

$$= \frac{\Sigma_f\, \sigma(e^+e^- \to f\bar{f})}{\sigma(e^+e^- \to \mu^+\mu^-)} = 3\ \Sigma_f\ z_f^2$$

$$= 3\ [\,(2/3)^2 + (-1/3)^2 + (-1/3)^2 + (2/3)^2 + (-1/3)^2]$$

u d s c b

18/9

30/9

33/9

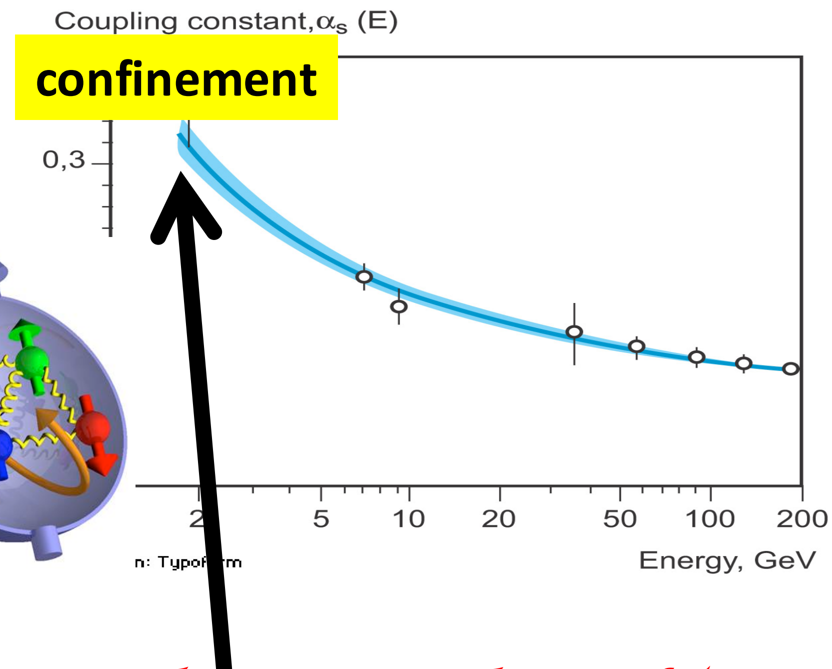
Good agreement between theory and experiment → Proof of concept of colour charge of quarks in QCD!

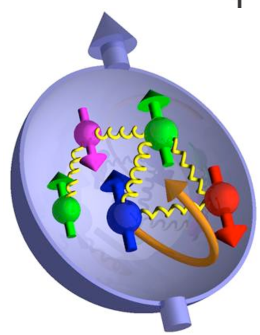
Standard Model of Particle Physics

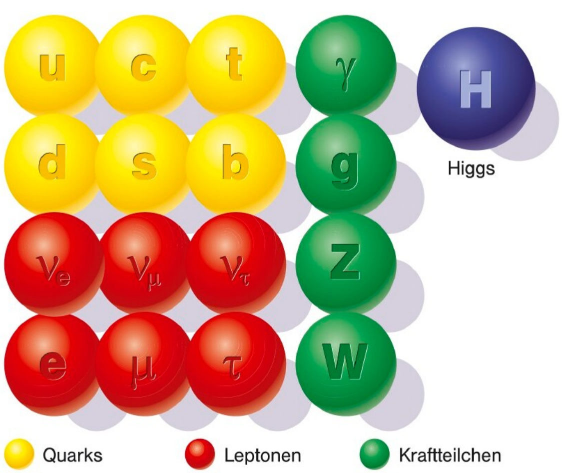
Describes 3 of the 4 fundamental interactions, i.e. forces:

- electromagnetic force
- weak nuclear force
- ***strong nuclear force***

QCD ***Quantum Chromo Dynamics***





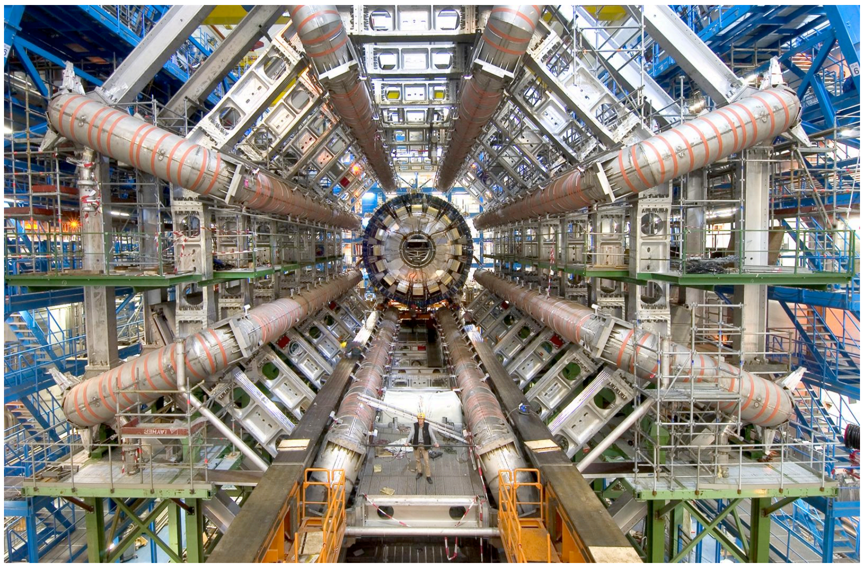


Low Energy Frontier

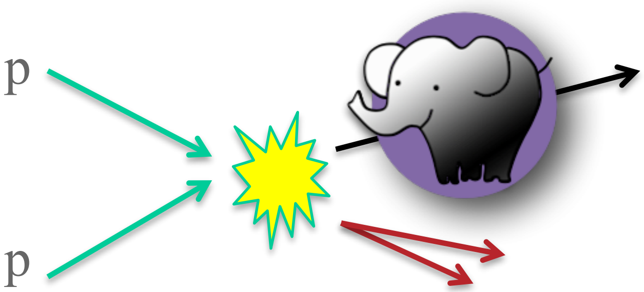
The World of Hadrons as bound objects of quarks

Search for New Physics

High Energy Frontier

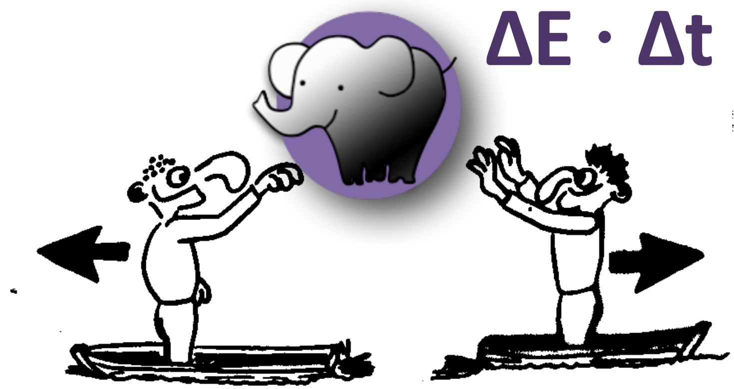


$$E = M \cdot c^2$$



Precision Frontier

$$\Delta E \cdot \Delta t \approx h$$

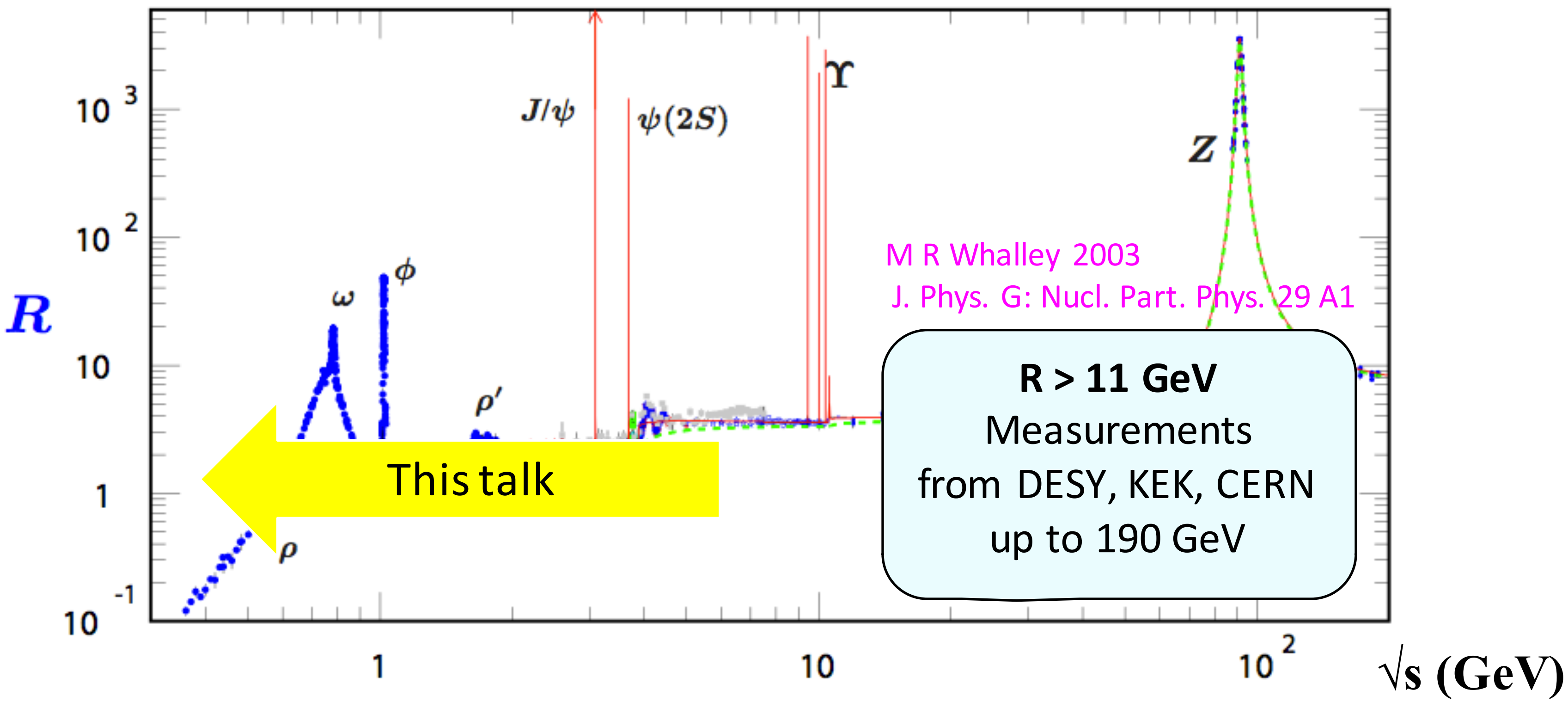


The R Ratio

$$R = \frac{\sigma^{(0)}(e^+e^- \to \text{hadrons})}{\sigma^{(0)}(e^+e^- \to \mu^+\mu^-)}$$

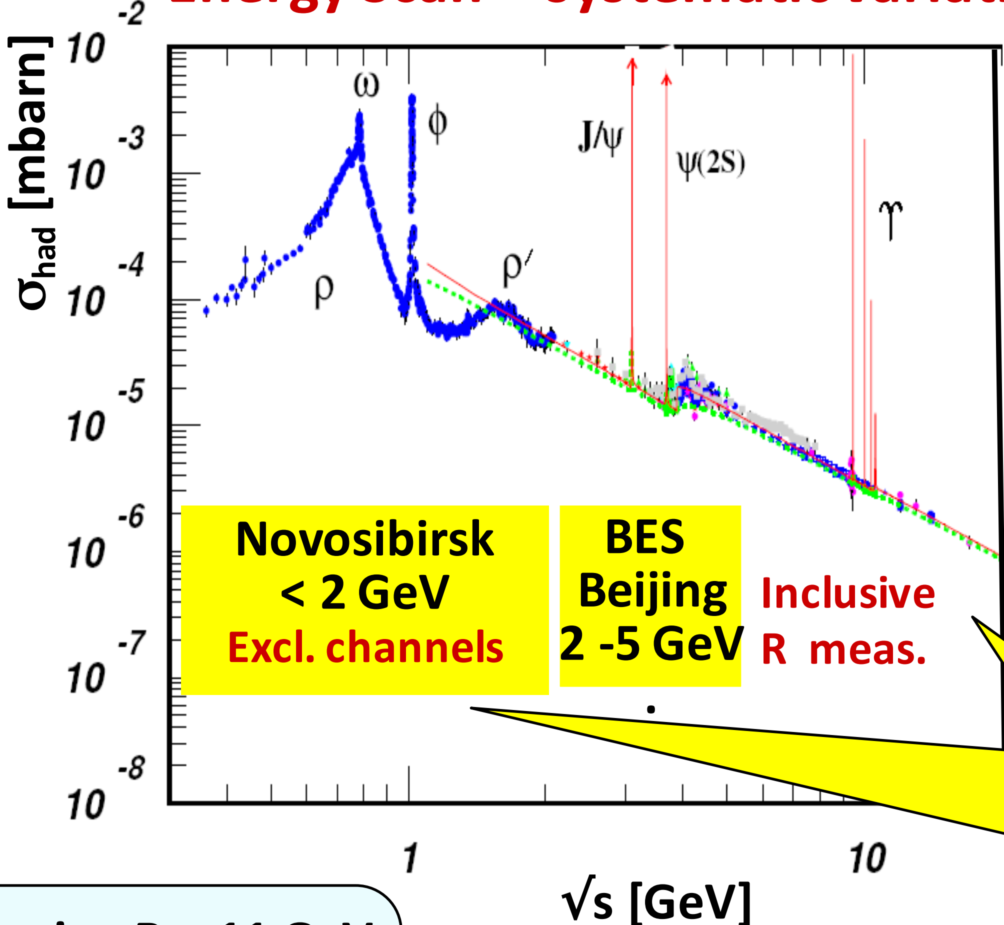
R helps to overcome hadronic uncertainties

- **running EM fine structure constant**
- **anomalous magnetic moment of muon**



*Hadronic Cross Section via **Energy Scan***

Energy Scan = systematic variation of collider's √s



2^{nd}, 3^{rd} Generation Expts.
→ new millenium
VEPP-2M, VEPP2000
Novosibirsk
BEPC, BEPC II Beijing

1^{st} Generation R < 11 GeV
→ 60's, 70's, 80's, 90's
measurements from Orsay, Frascati, SLAC, Novosibirsk

Outline (today)

- **R measurements** and the Standard Model of Particle Physics
- The „running“ fine structure „constant“
- The anomalous magnetic moment of the muon
- R measurements via energy scan at Novosbirsk
- R measurements via Intial State Radiation
- Summary (today)

Tomorrow: Meson transition form factors

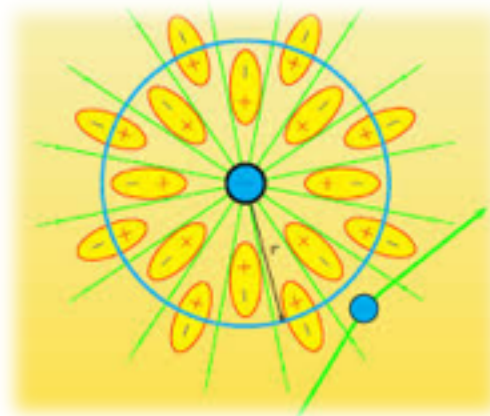
Motivation for R_{had}: The Fine Structure constant $\alpha_{EM}(M_Z^2)$

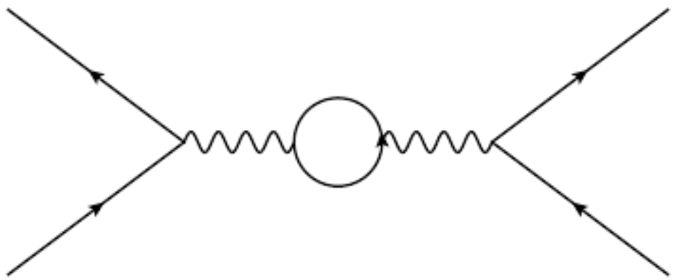
Hadronic Cross Section Data and $\alpha_{em}(M_Z^2)$

Running of α_{em} (s) with s due to vacuum polarization corrections

- Leptonic Vacuum Polarization calculable within QED
- Hadronic Vacuum Polarization not accessible in pQCD

  → **Dispersion relation**





$$\alpha_{\text{em}}(s) \quad = \quad \frac{\alpha(0)}{(1 - \Delta\alpha_{\text{em}}(s))}$$

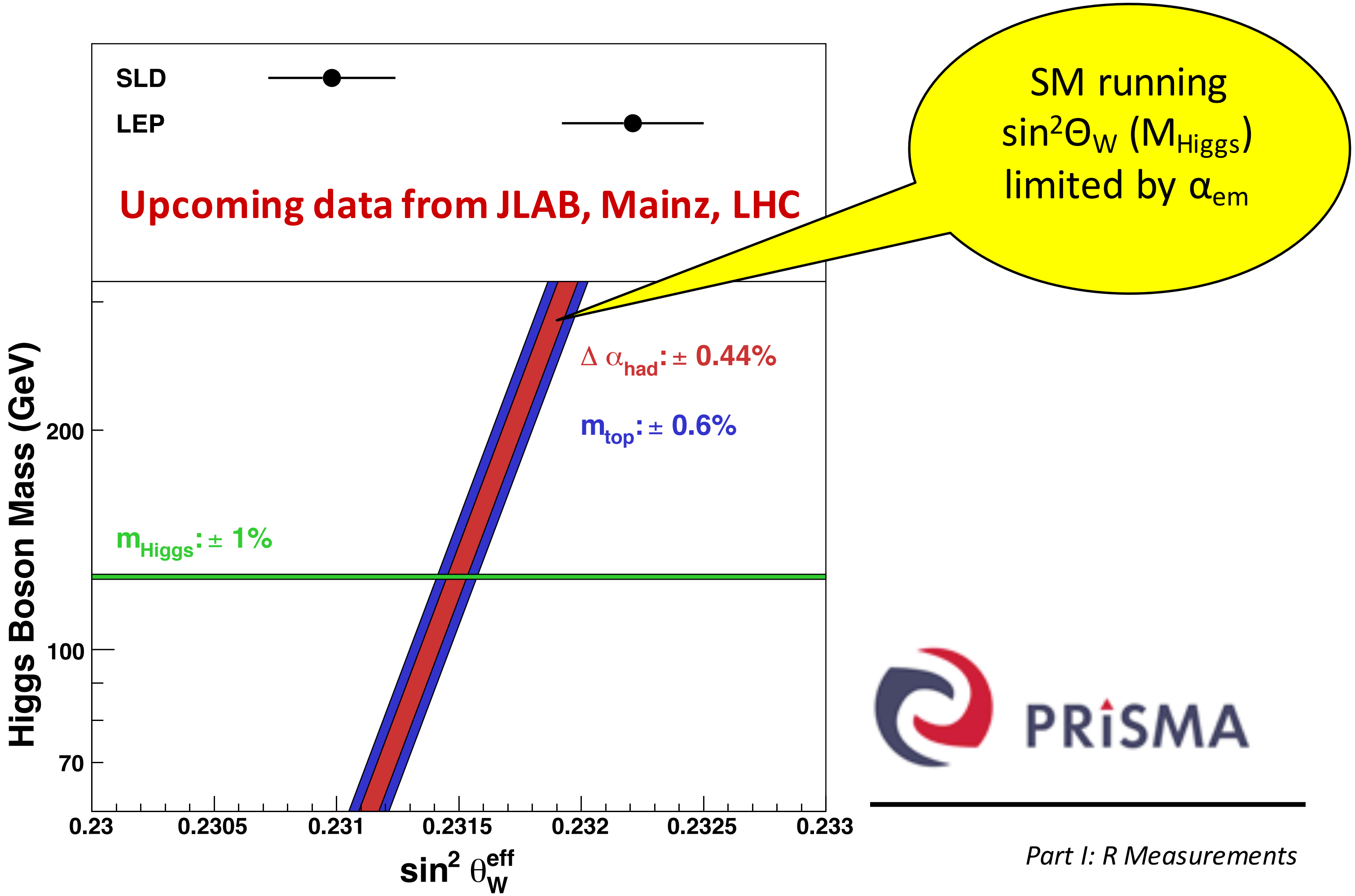
$$\alpha^{-1}(M_Z^2) = 128.947 \pm 0.012$$

Davier, et al.(2017)

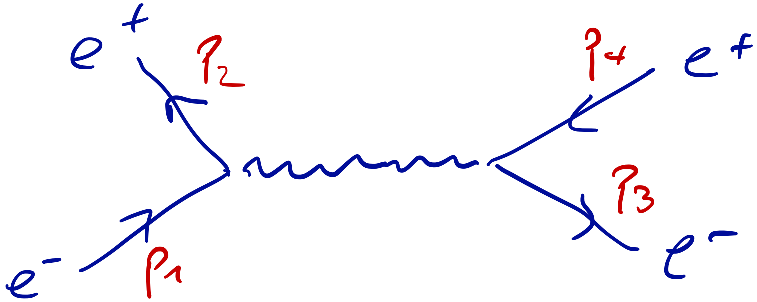
QED	$\Delta\alpha_{\text{lep}}(M_Z^2) \;=\; 314.97686 \cdot 10^{-4}$
strong	$\Delta\alpha_{\text{had}}(M_Z^2) = (275.3 \pm 0.9) \cdot 10^{-4}$

$\alpha_{em}(M_Z^2)$ limiting electroweak precision fits

→ Test overall consistency of the electroweak Standard Model
→ Since the discovery of the Higgs boson more timely than ever



Hadronic Cross Section and $\alpha_{em}(M_Z^2)$



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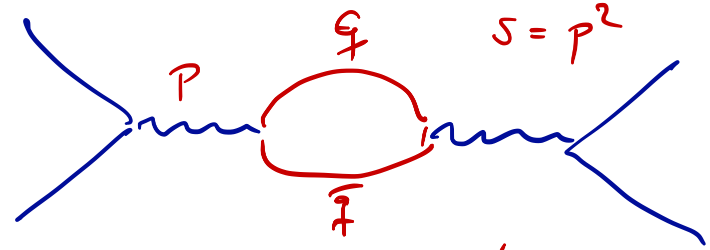


$p = p_1 + p_2$

$s = p^2$

HVP contribution

$$\Pi(p^2) = \Pi(s)$$

$$\frac{ie^2}{s}\,\bar{v}(2)\,\gamma^\mu\, u(1)\;\bar{v}(4)\,\gamma_\mu\, u(3)\left(1 + \frac{\Pi(s)}{s}\right)$$

$$\underbrace{\left(1 + \frac{\Pi(s)}{s}\right)}_{\displaystyle \frac{1}{1 - \frac{\Pi(s)}{s}}}$$

$$\Rightarrow \alpha(s) = \frac{\alpha(0)}{1 + \Delta\alpha(s)} \qquad (*)$$

$$\Delta\alpha(s) \equiv -\,\mathrm{Re}\left(\frac{\Pi(s)}{s}\right)$$

Hadronic Cross Section and $\alpha_{em}(M_Z^2)$

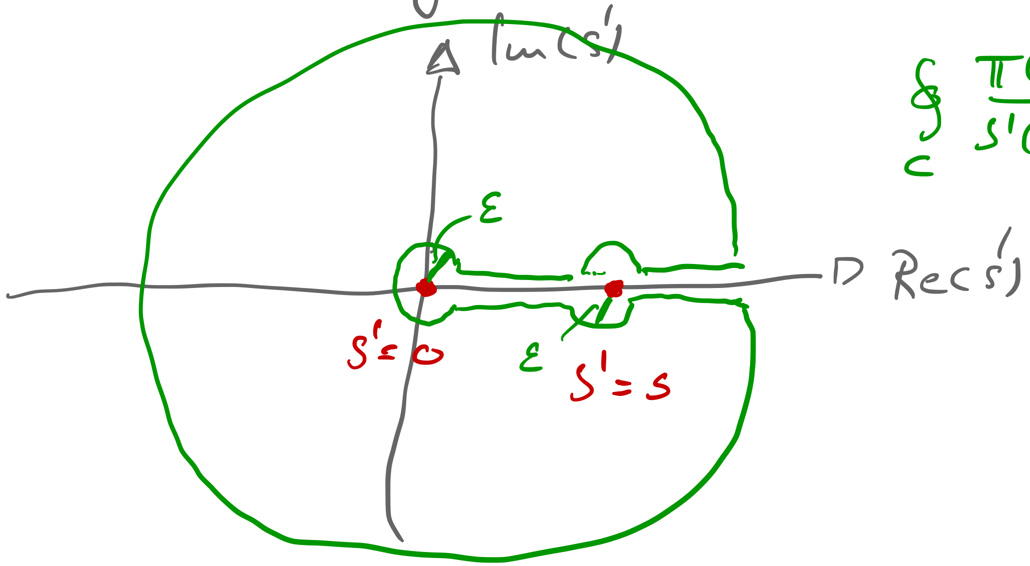
Dispersion relations : follow analytical structure functions

$$\Pi_{ren}(s) = \frac{s}{\pi} \int_{s_{min}}^{\infty} ds' \frac{\text{Im}\,\Pi(s')}{s'(s'-s-i\varepsilon)}$$

↑ $\Pi(s) - \Pi(0)$

↑ $s_{min} > 0$ due to non-massless quarks

Analytical continuation of s in the complex plane



$$\oint_C \frac{\Pi(s')}{s'(s'-s)} = 0$$

Hadronic Cross Section and $\alpha_{em}(M_Z^2)$

$$\Delta\alpha(s) \overset{(*)}{=} -\mathrm{Re}\left(\frac{\Pi(s)}{s}\right) = -\frac{s}{\pi}\,\mathrm{Re}\int ds' \,\frac{\mathrm{Im}\,\Pi(s')}{s'^2\,(s'-s-i\varepsilon)}$$

notice : $$\frac{1}{s'-s-i\varepsilon} = P\,\frac{1}{s'-s} + i\,\delta(s'-s)$$

↑ principle value

$f(x)$ with a singularity at $x = x_0$, $a < x_0 < b$

$$P \hat{=} \lim_{\varepsilon \to 0^+} \int_a^{x_0-\varepsilon} f(x)\,dx + \int_{x_0+\varepsilon}^{b} f(x)\,dx$$

$$\Delta\alpha(s) = -\frac{s}{\pi}\,P\int_{s_{min}}^{\infty} \frac{ds'}{s'^2}\,\frac{\mathrm{Im}\,\Pi(s')}{s'-s} \qquad (**)$$

Hadronic Cross Section and $\alpha_{em}(M_Z^2)$

Optical Theorem: $\quad \mathrm{Im}\,\Pi(s') = \frac{s'^2}{4\pi\alpha}\,\sigma_{tot}^{had}$ (***)

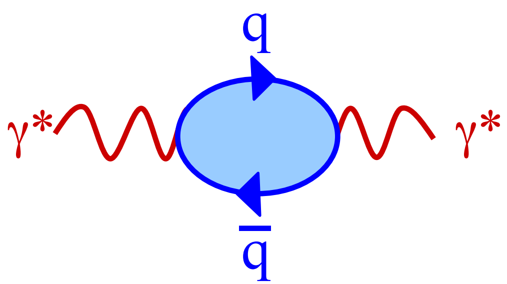
follows from causality

Finally: $\quad \Delta\alpha = -\frac{s}{\pi}\,\frac{1}{4\pi\alpha}\,P\int_{4m_\pi^2}^{\infty} ds'\,\frac{\sigma_{tot}^{had}}{s'-s}$

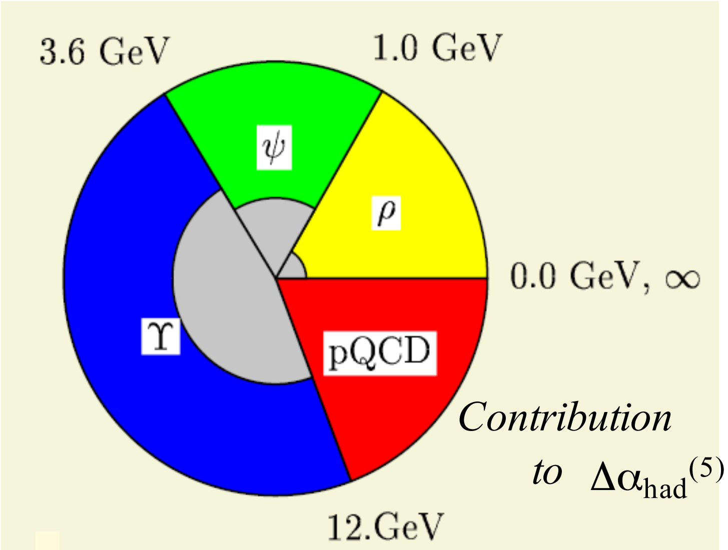
$$R^{had}(s) = \frac{\sigma(e^+e^- \to hadrons)}{\sigma(e^+e^- \to \mu^+\mu^-)} = \frac{3s}{4\pi\alpha^2}\,\sigma_{tot}^{had}$$

$$\Delta\alpha = -\frac{\alpha s}{3\pi}\int_{4m_\pi^2}^{\infty}\frac{ds'}{s'}\,\frac{R_{had}}{s'-s}$$

Running Fine Structure Constant



$$\Delta\alpha_{\rm had}^{(5)}(s) = -\frac{\alpha s}{3\pi}\left(\mathcal{P}\!\!\int_{4m_\pi^2}^{E^2_{\rm cut}} ds' \frac{R_\gamma^{\rm data}(s')}{s'(s'-s)} + \mathcal{P}\!\!\int_{E^2_{\rm cut}}^{\infty} ds' \frac{R_\gamma^{\rm pQCD}(s')}{s'(s'-s)} \right)$$

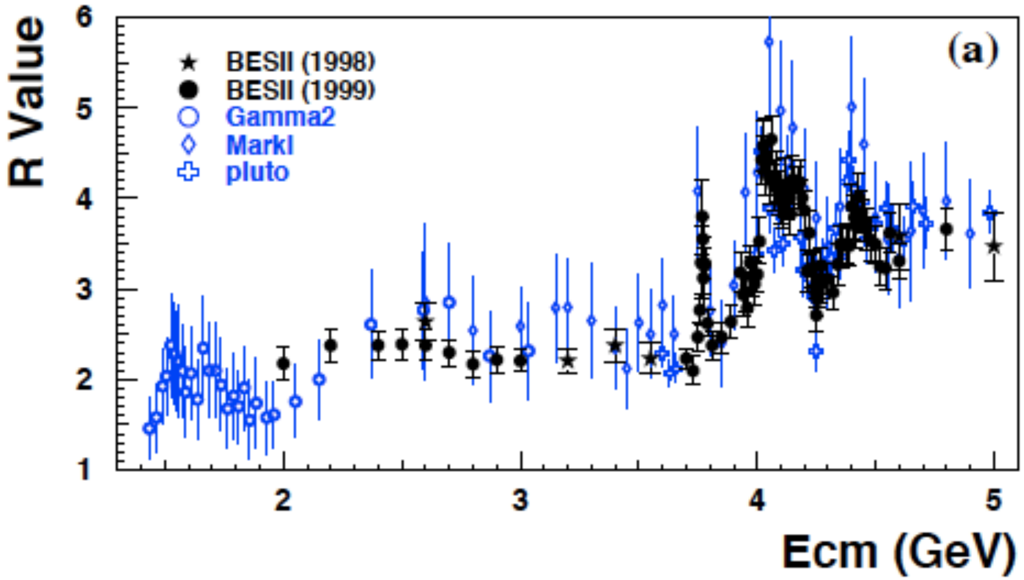


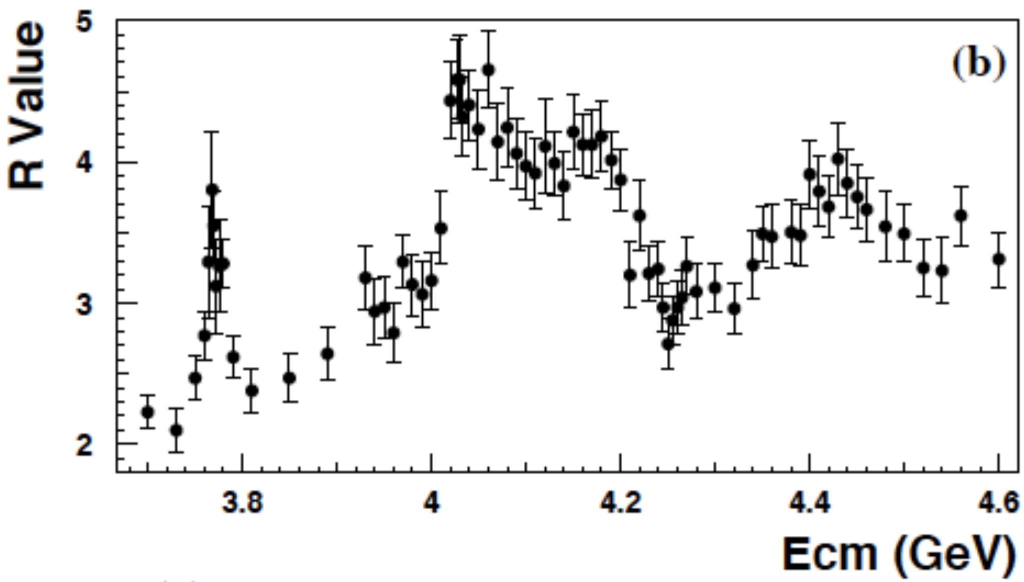
[F. Jegerlehner ArXiv:0807.4206]

- $\Delta\alpha_{\rm had}^{(5)}$ is the hadronic vacuum polarization contribution of the five lightest quarks u,d,s,c,b
- Low and intermediate energy regions important in dispersion integral

  ⇒ **from which energy range on use pQCD?**

BEPC (1997 – 2004) : $\sigma_{incl}(e^+e^- \rightarrow Hadrons)$





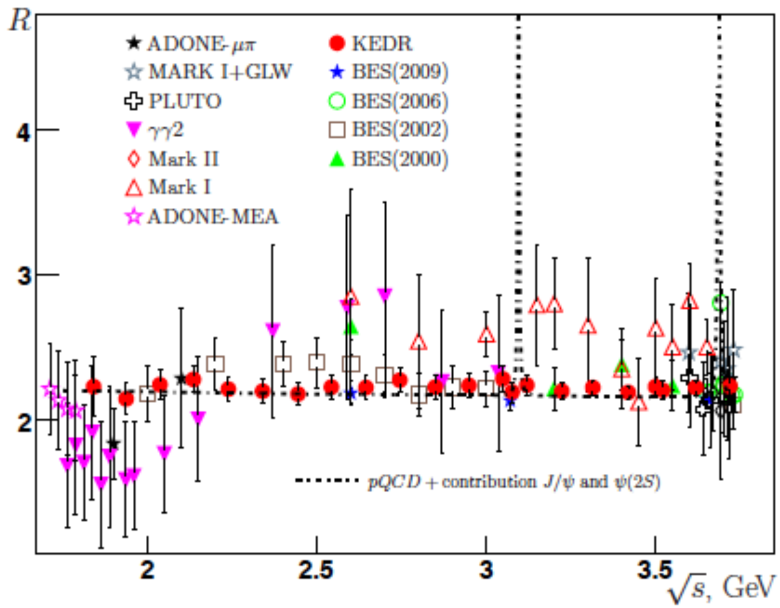
arXiv:hep-ex/9908046
arXiv:hep-ex/0102003

Results

- 3-5 % statistical accuracy per scan point
- Systematic uncertainty: ~5 …8%

- **Major improvement of R**
- **Important QCD test**
- **Of utmost importance for $\alpha_{em}(m_Z^2)$**

KEDR: $\sigma_{incl}(e^+e^- \rightarrow Hadrons)$



S. Eidelman
@ g-2 workshop
Mainz, 06/2018

1.84-3.05 GeV $R = 2.225 \pm 0.020 \pm 0.047$ ($R_{\mathrm{pQCD}} = 2.18 \pm 0.02$)

V.V. Anashin et al., Phys. Lett. B770 (2017) 174

3.05-3.72 GeV $R_{\mathrm{uds}} = 2.204 \pm 0.013 \pm 0.030$ ($R_{\mathrm{pQCD}} = 2.16 \pm 0.01$)

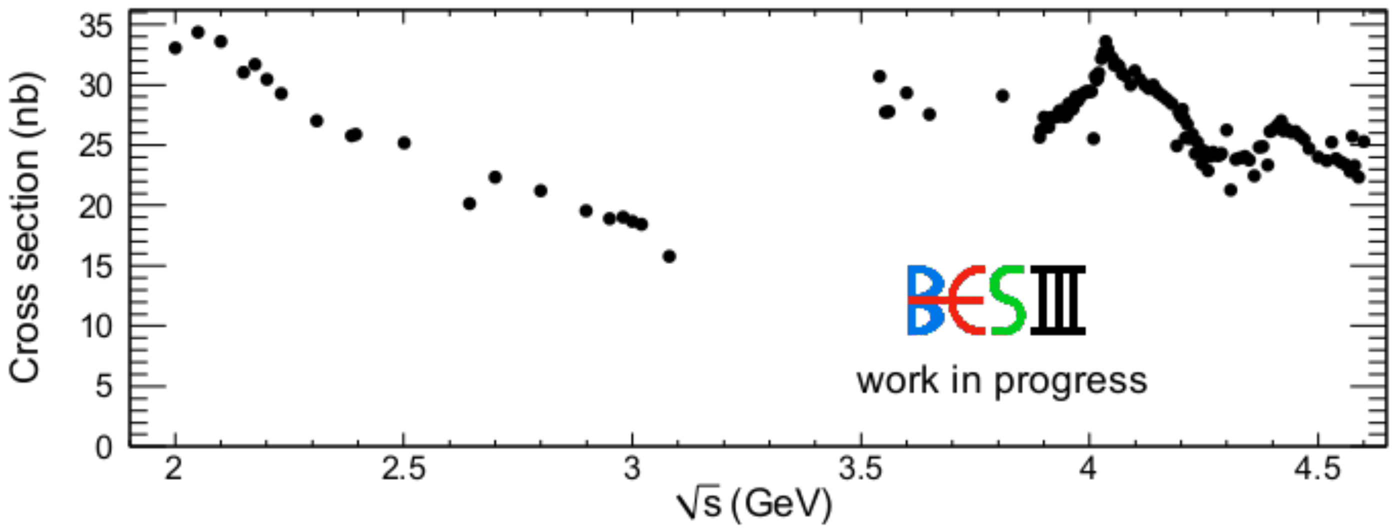
V.V. Anashin et al., Phys. Lett. B753 (2016) 533; arXiv:1805.06235

Total (syst. error) 3.9% (2.4%) at low, 2.6% (1.9%) at high $\sqrt{s}$

R measurement from 5 to 7 GeV in progress

BES III @ BEPC II: High Statistics R Scan

Online spectrum of R scan recently finished



$R_{incl} = \sigma_{had}/\sigma_{\mu\mu}$ ratio with targeted 3% systematic accuracy (statistical error <<1%)

125 scan points with >10^5 hadronic events each
→ statistical error negligible

How good are Monte-Carlo generators to simulate hadronic events ?

Motivation for R_{had}: Anomalous Magentic Moment of the Muon $(g-2)_\mu$

Muon Anomaly $(g-2)_\mu$

Magnetic Moment: $\vec{m} = \mu_B \, g \, \vec{S}$ *μ_B: Bohr magneton, g: gyromagnetic factor ~ 2*

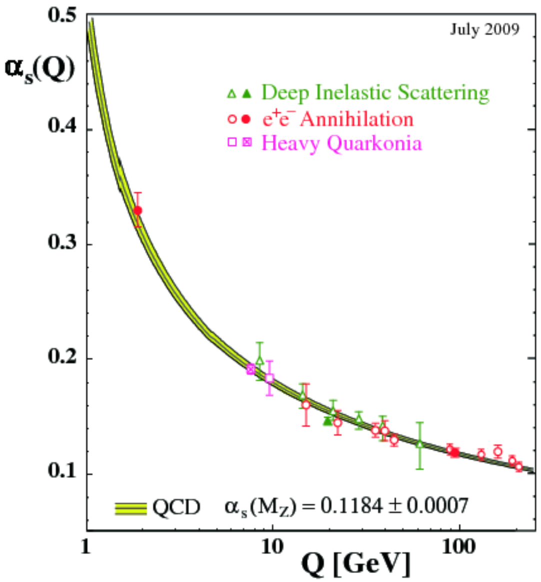
Muon Anomaly: $a_\mu = (g-2)_\mu / 2 = \alpha_{em} / 2\pi + \ldots = 0.001161\ldots$

- **Standard Model (SM) prediction a_μ^{SM}:**

  - QED: $a_\mu^{QED} = (11\,658\,471.8971 \pm 0.007) \cdot 10^{-10}$
  - weak: $a_\mu^{weak} = (15.36 \pm 0.10) \cdot 10^{-10}$
  - hadronic: $a_\mu^{hadr} = (694.49 \pm 3.55) \cdot 10^{-10}$

  $a_\mu^{SM} = (\,11\,659\,182.05 \pm 3.56\,) \cdot 10^{-10}$ Keshavarzi et al. 2018

  SM prediction entirely limited by **hadronic contribution**!



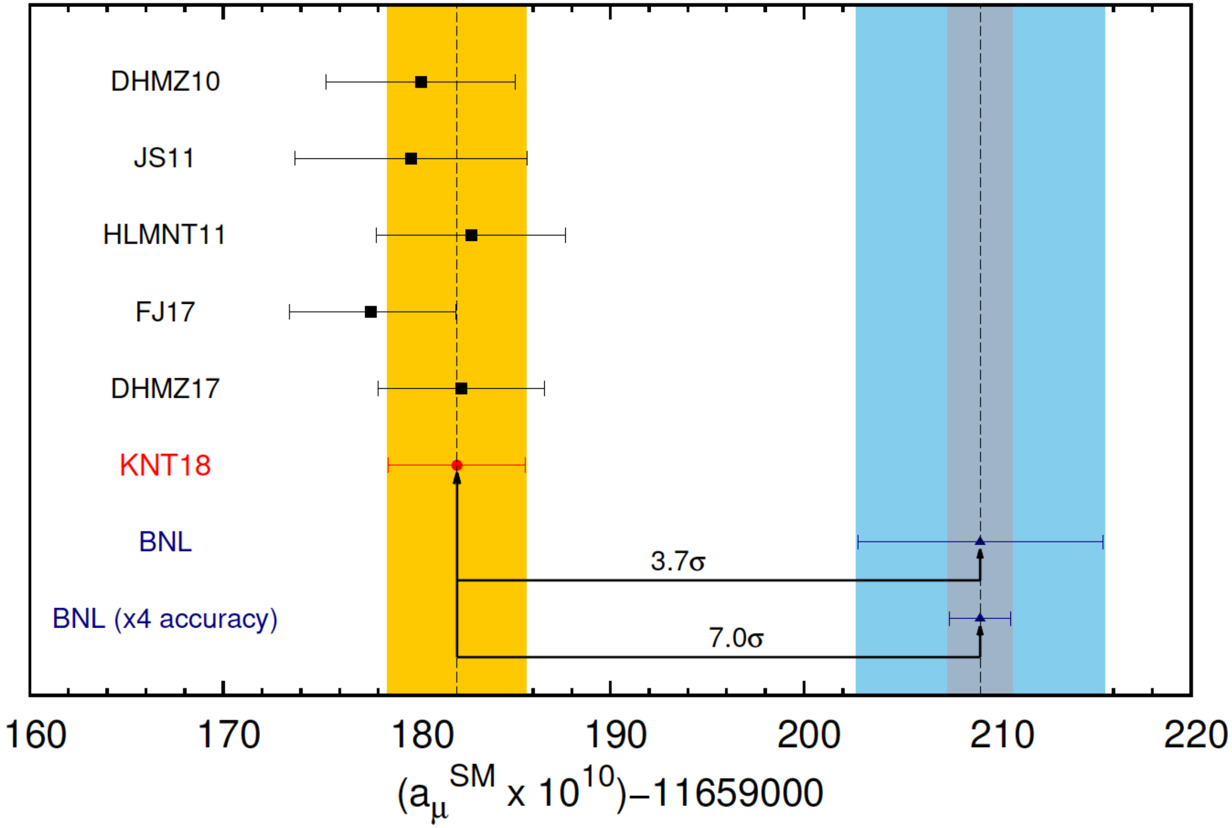
- **Direct measurement BNL-E821 a_μ^{exp}:**

  $a_\mu^{exp} = (\,11\,659\,208.9 \pm 6.3\,) \cdot 10^{-10}$

Massimo Passera
Yannis Semertzidis

Muon Magnetic Moment: $(g\text{-}2)_\mu$

Keshavarzi et al. 2018



One of the most significant tests of the SM

$\Delta a_\mu = a_\mu^{exp} - a_\mu^{SM} = (27.05 \pm 7.28) \cdot 10^{-10}$ (3.7σ) !!!

Error or New Physics ???

Pradler

Standard Model Prediction of $(g\text{-}2)_\mu$

JG|U

Hadronic contribution **non-perturbative,** the **limiting** contribution

$$a_\mu^{SM} = a_\mu^{QED} + a_\mu^{weak} + a_\mu^{had} = (\,11\,659\,180.2 \pm 4.9\,) \cdot 10^{-10}$$

Teubner et al. '11

→ **HVP**: Hadronic Vacuum Polarization (693.27 ± **2.46**) · 10^{-10}

NLO (-9.82 ± **0.04**) · 10^{-10} ; NNLO (1.24 ± **0.01**) · 10^{-10} Keshavarzi et al. 2018

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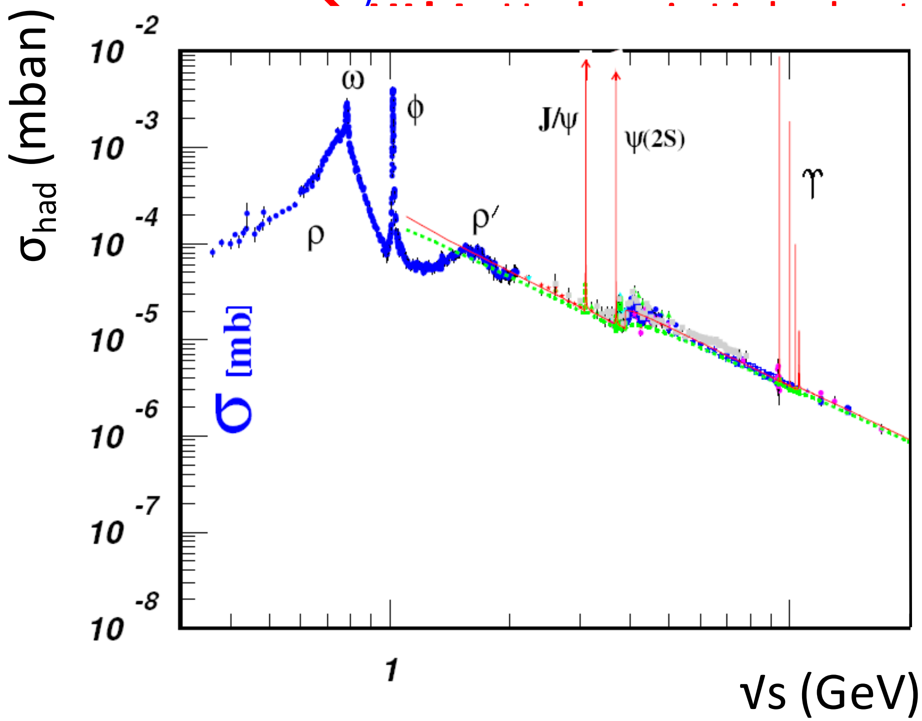

$$a_\mu^{HVP} = \frac{1}{4\pi^3}\int_{4m_\pi^2}^{\infty} ds\, K(s)\, \sigma_{had}$$

$\sigma_{had} = \sigma(\, e^+e^- \to \; hadrons \,)$

~ 1/s → Data at low energies needed!

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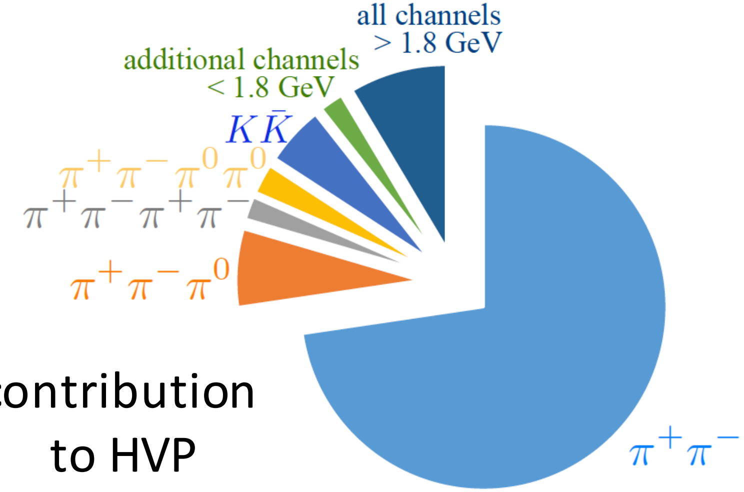
Hadronic Vacuum Polarization (HVP)

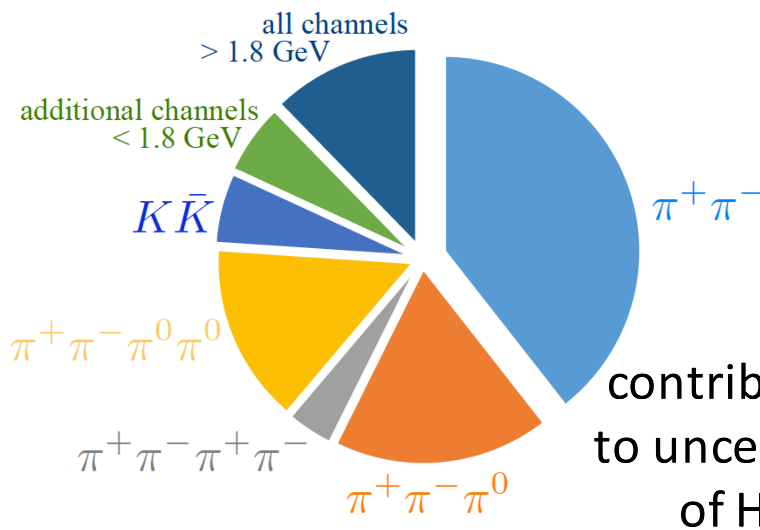
$$a_\mu^{HVP} = \frac{1}{4\pi^3} \int_{4m_\pi^2}^{\infty} ds\, K(s)\, \sigma_{had}$$

$$\sigma_{had} = \sigma(\, e^+e^- \rightarrow \; hadrons \,)$$

~ 1/s → Data at low energies needed!



contribution to HVP



contribution to uncertainty of HVP

Most relevant channels:

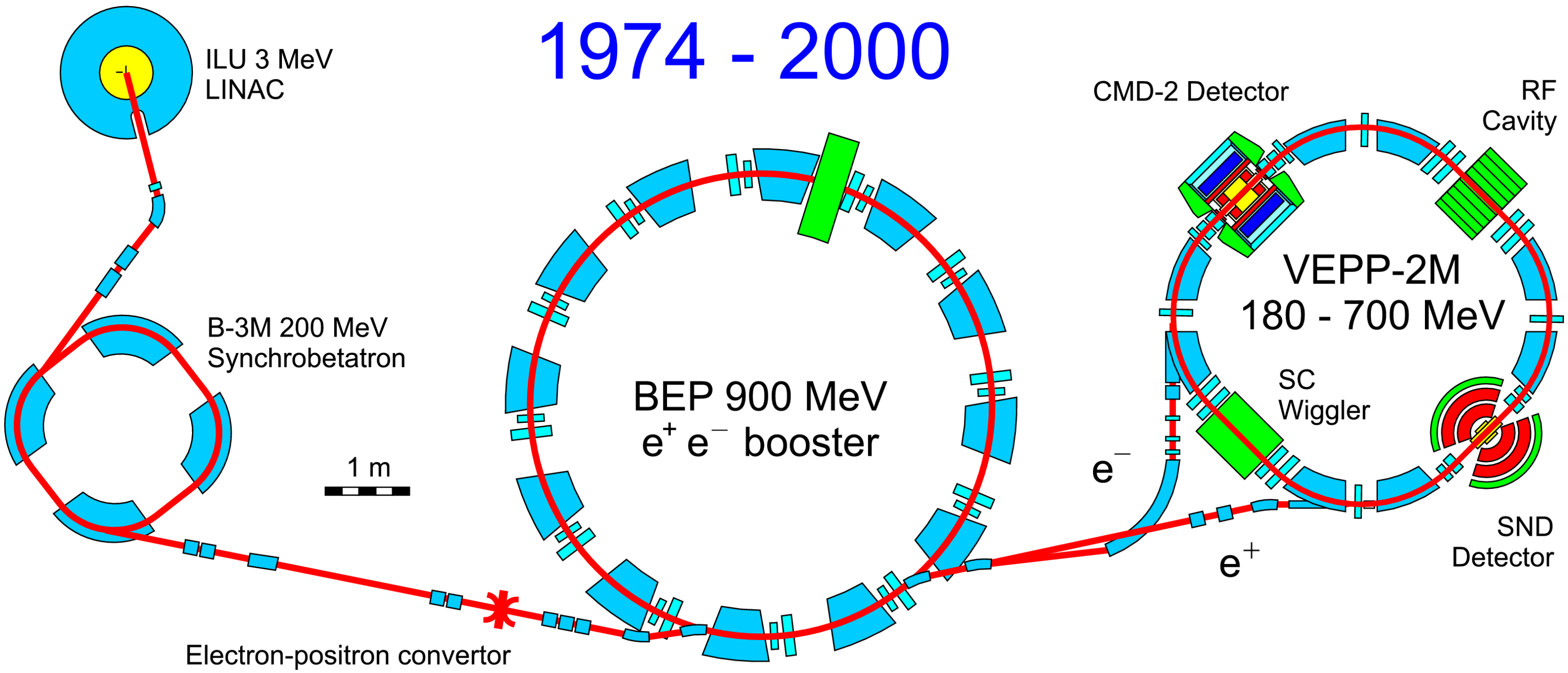
$$e^+e^- \rightarrow \pi^+\pi^- \quad e^+e^- \rightarrow \pi^+\pi^-\pi^0 \quad e^+e^- \rightarrow 4\pi \quad e^+e^- \rightarrow K\bar{K}$$

Why there is no decay

$$e^+e^- \to \rho \to \pi^0\pi^0$$

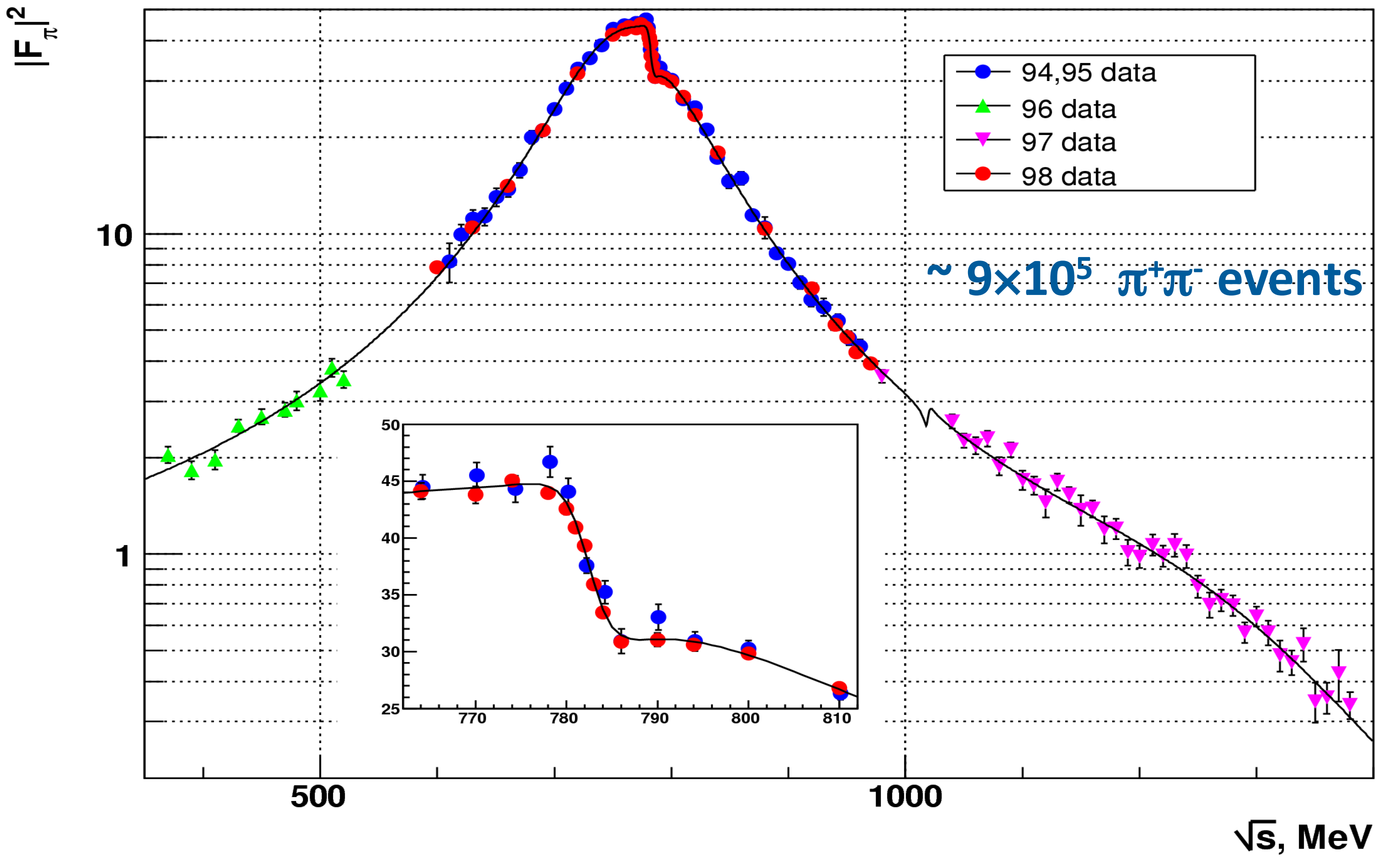
Hadronic Cross Section Measurements via Energy Scan from Novosibirsk

Overview Novosibirsk Results



- VEPP-2M collider: 0.36-1.4 GeV in c.m., **$L \approx 2 * 10^{30}$ cm^{-2}s^{-1}** at 1 GeV
- Detectors **CMD-2** (solenoidal B field) and **SND** (w/o B field)
- Integrated Luminosity collected by CMD-2 and SND: 70 pb^{-1}
  to be compared to 6 pb^{-1} in Orsay and Frascati at higher energies

CMD-2 Measurement $\sigma(e^+e^- \rightarrow \pi^+\pi^-)$



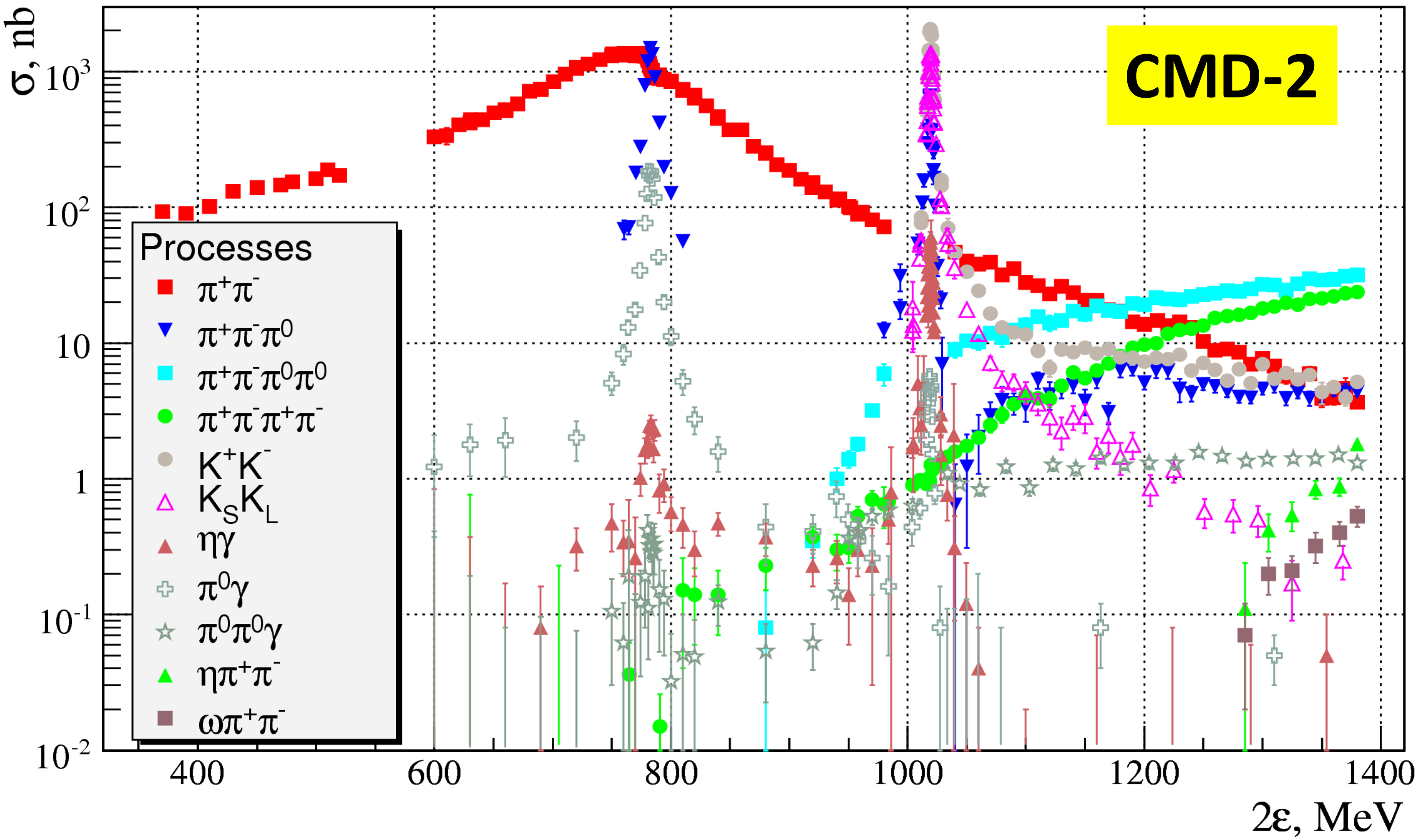
systematic uncertainty: 0.7% | 0.6% (95)/ 0.8% (98) | 1.2-4.2%

- 4 running campaigns, overall fair agreement
- Radiative corrections with 0.3% uncertainty

Phys.Lett.B578(2003)285
JETP.Lett.82(2005)743
JETP.Lett.84(2006)413
PLB648(2007)78

Overview CMD-2 Results

- CMD-2: $\pi^+\pi^-$ <1%, higher multiplicities few % accuracy
- SND measurement of $\pi^+\pi^-$ with 1.2% accuracy



VEPP – 2000 Collider (since 2010)

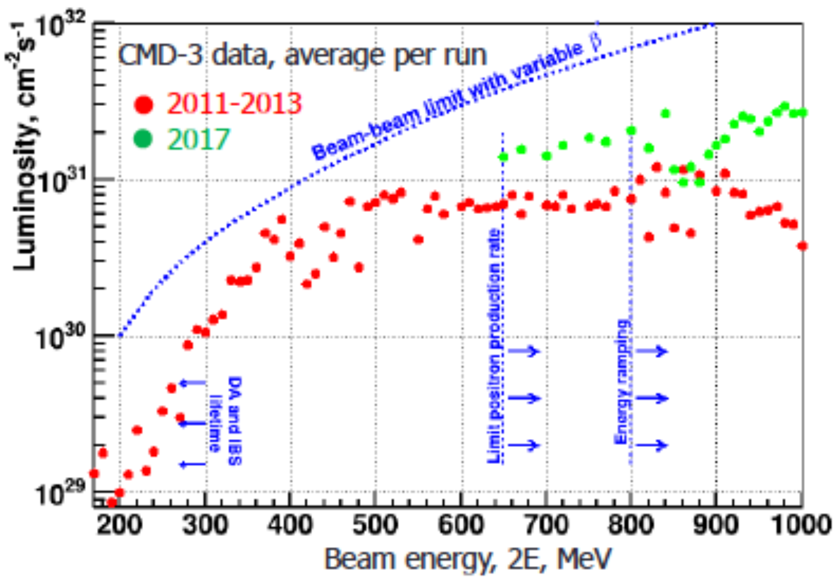
CMS energy range is 0.32-2.0 GeV

unique optics –"round beams"

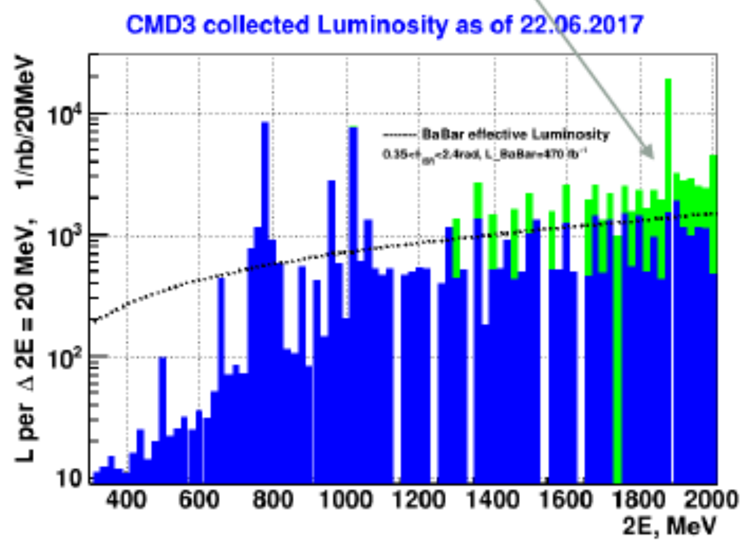
Design luminosity is $L=10^{32}/cm^2 s @ s=2$GeV

Experiments with two detectors, CMD-3 and SND

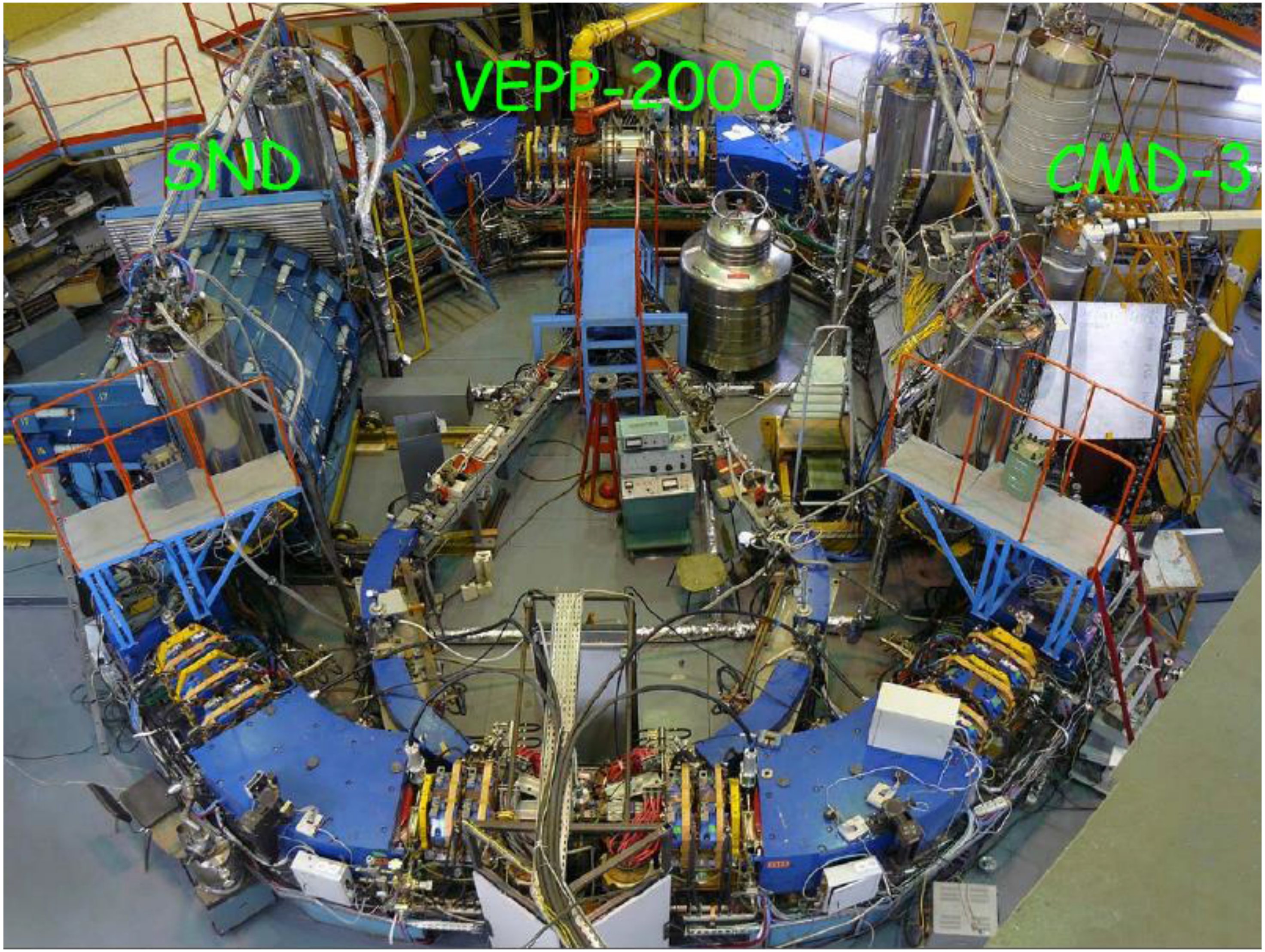
2017 data taking



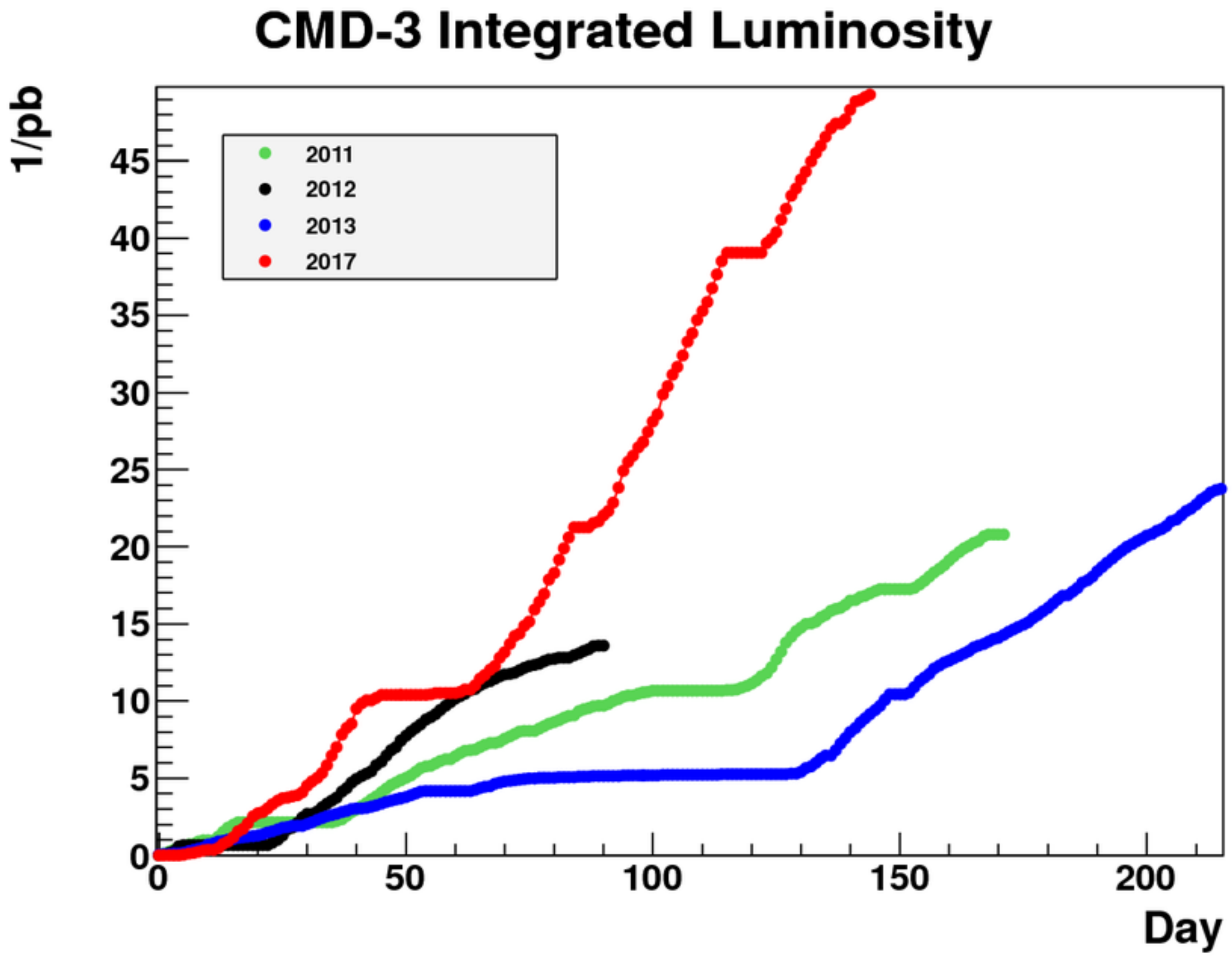
CMD-3
26/01-20/06/2017



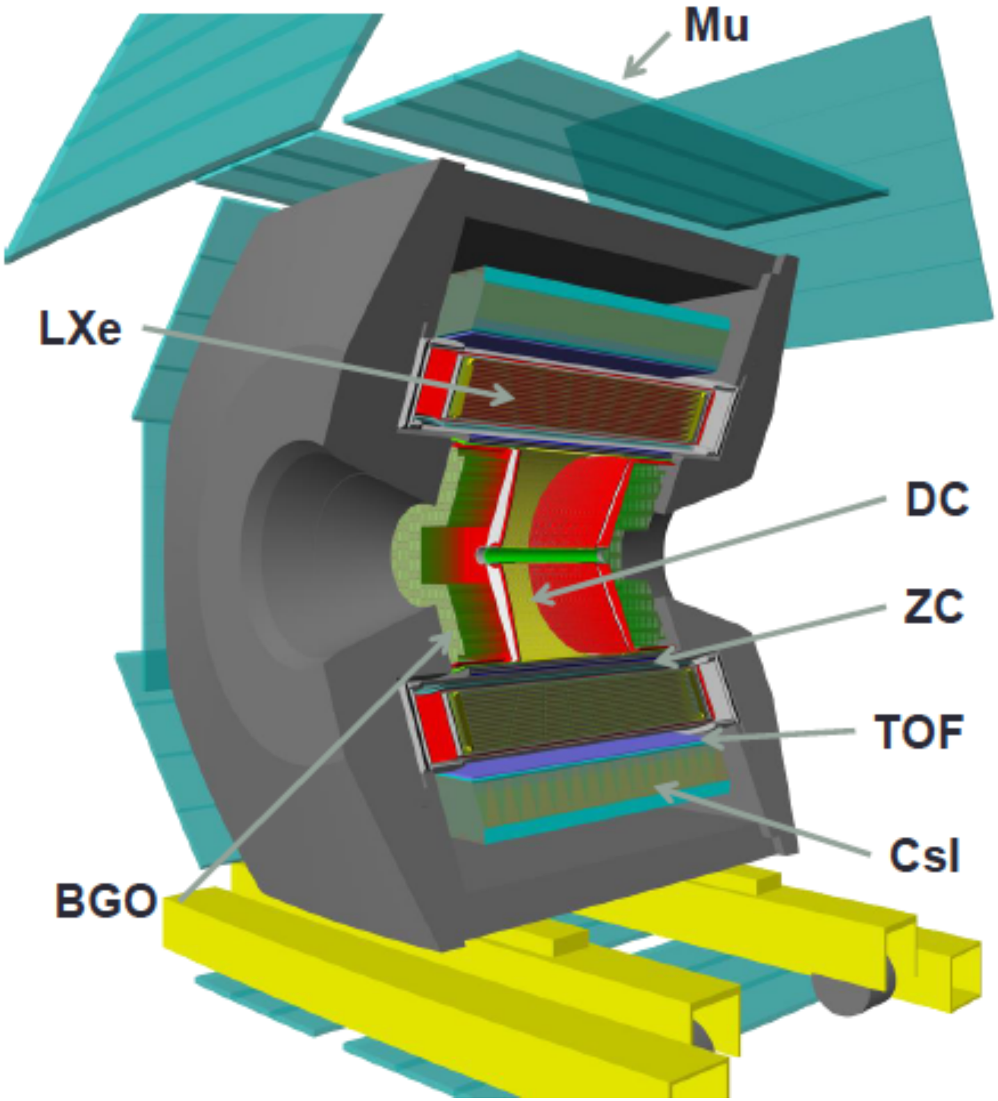
VEPP – 2000 Collider



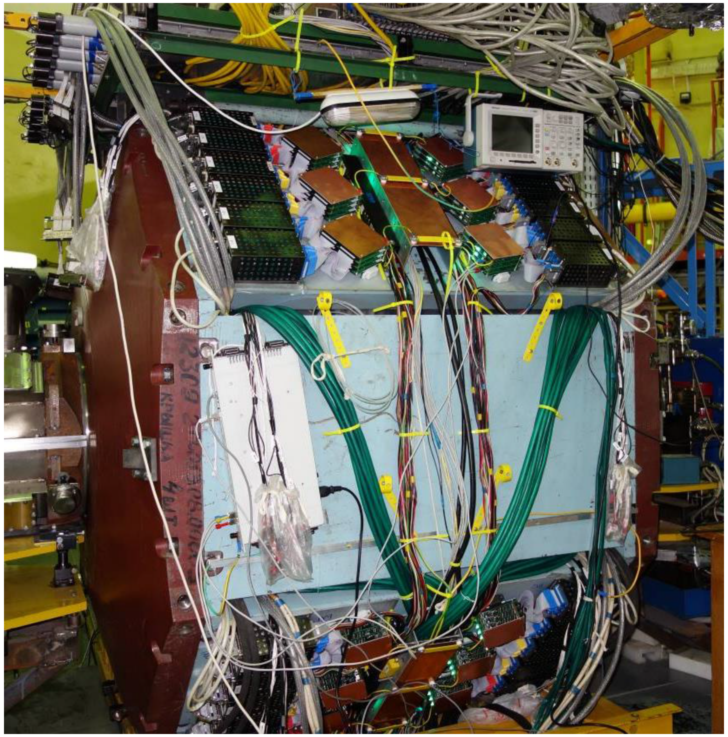
VEPP – 2000 Collider (since 2010)



CMD-3 Detector



Very compact multipurpose detector with high resolution tracking and detection of EM particles



Overview CMD-3 Results

- 2 charged

$$e^+e^- \to \pi^+\pi^-, K^+K^-, K_SK_L, p\bar{p}$$

- 2 charged + γ's

$$e^+e^- \to \pi^+\pi^-\pi^0, \pi^+\pi^-\eta, K^+K^-\pi^0, K^+K^-\eta, K_SK_L\pi^0, \pi^+\pi^-\pi^0\eta,$$
$$\pi^+\pi^-\pi^0\pi^0, \pi^+\pi^-\pi^0\pi^0\pi^0, \pi^+\pi^-\pi^0\pi^0\pi^0\pi^0$$

- 4 charged

$$e^+e^- \to \pi^+\pi^-\pi^+\pi^-, K^+K^-\pi^+\pi^-, K_SK^*$$

- 4 charged + γ's

$$e^+e^- \to \pi^+\pi^-\pi^+\pi^-\pi^0, \pi^+\pi^-\eta, \pi^+\pi^-\omega, \pi^+\pi^-\pi^+\pi^-\pi^0\pi^0, K^+K^-\eta, K^+K^-\omega$$

- 6 charged

$$e^+e^- \to \pi^+\pi^-\pi^+\pi^-\pi^+\pi^-$$

- γ's only

$$e^+e^- \to \pi^0\gamma, \eta\gamma, \pi^0\pi^0\gamma, \pi^0\eta\gamma, \pi^0\pi^0\pi^0\gamma, \pi^0\pi^0\eta\gamma$$

- other

$$e^+e^- \to n\bar{n}, \pi^0e^+e^-, \eta e^+e^-$$

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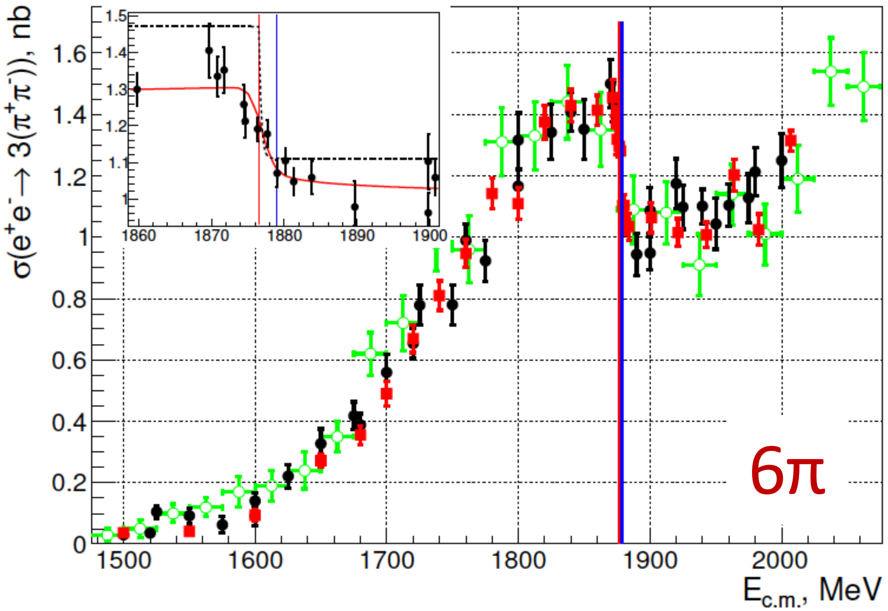
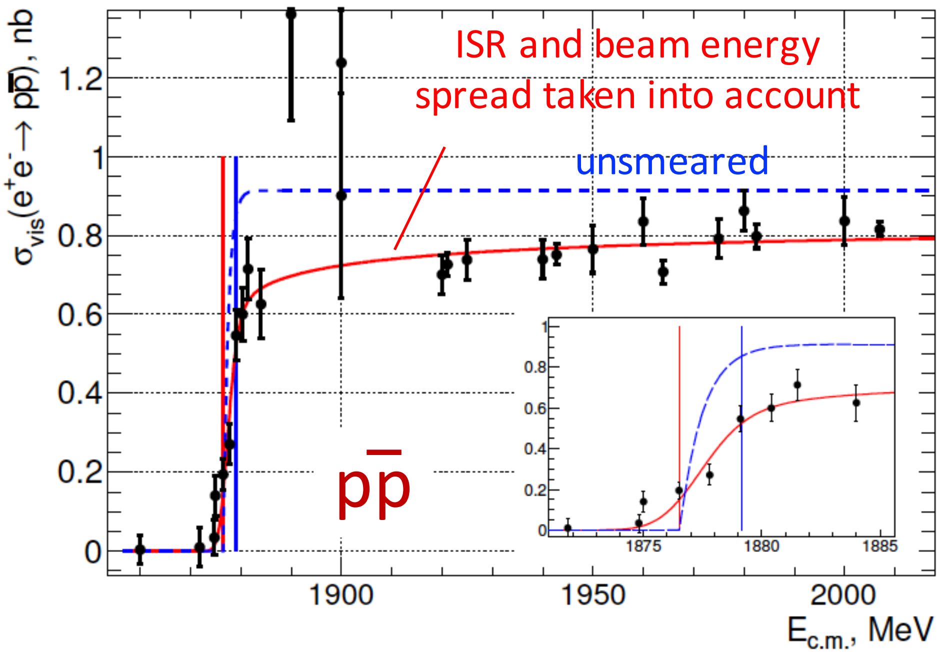
CMD-3: Structure at the Nucleon Threshold

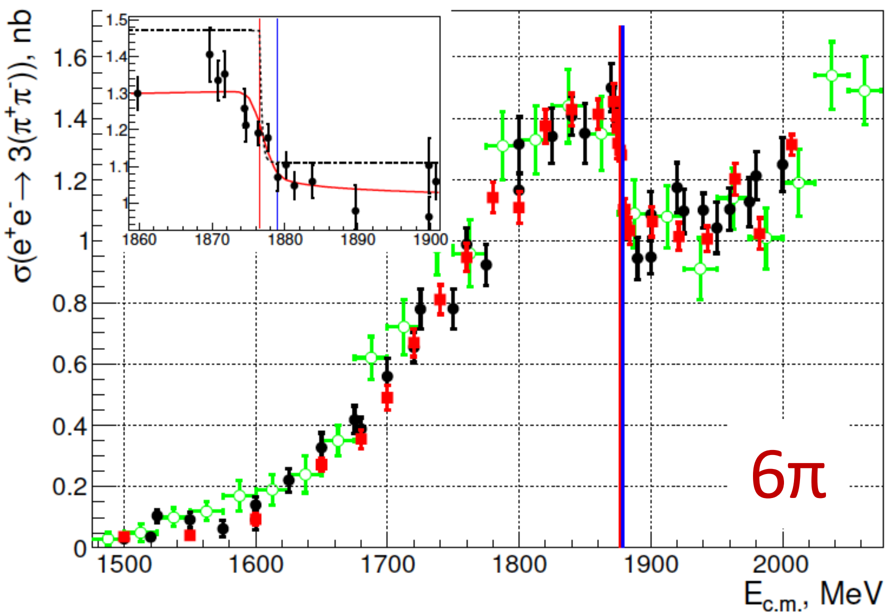
Structures seen in various cross section spectra at $N\bar{N}$ threshold

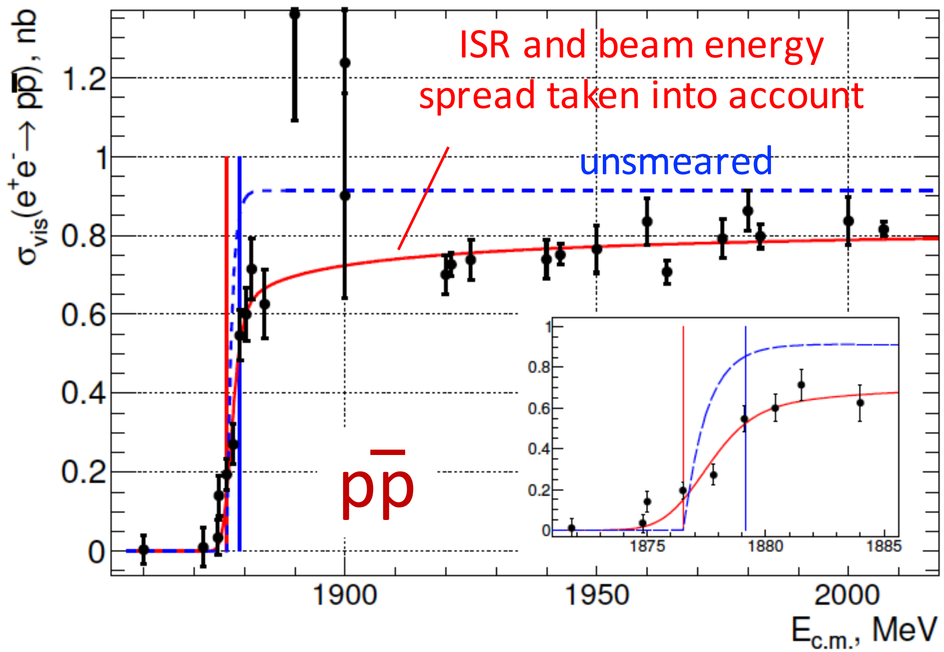




CMD-3: Structure at the Nucleon Threshold

Structures seen in various cross section spectra at $N\bar{N}$ threshold



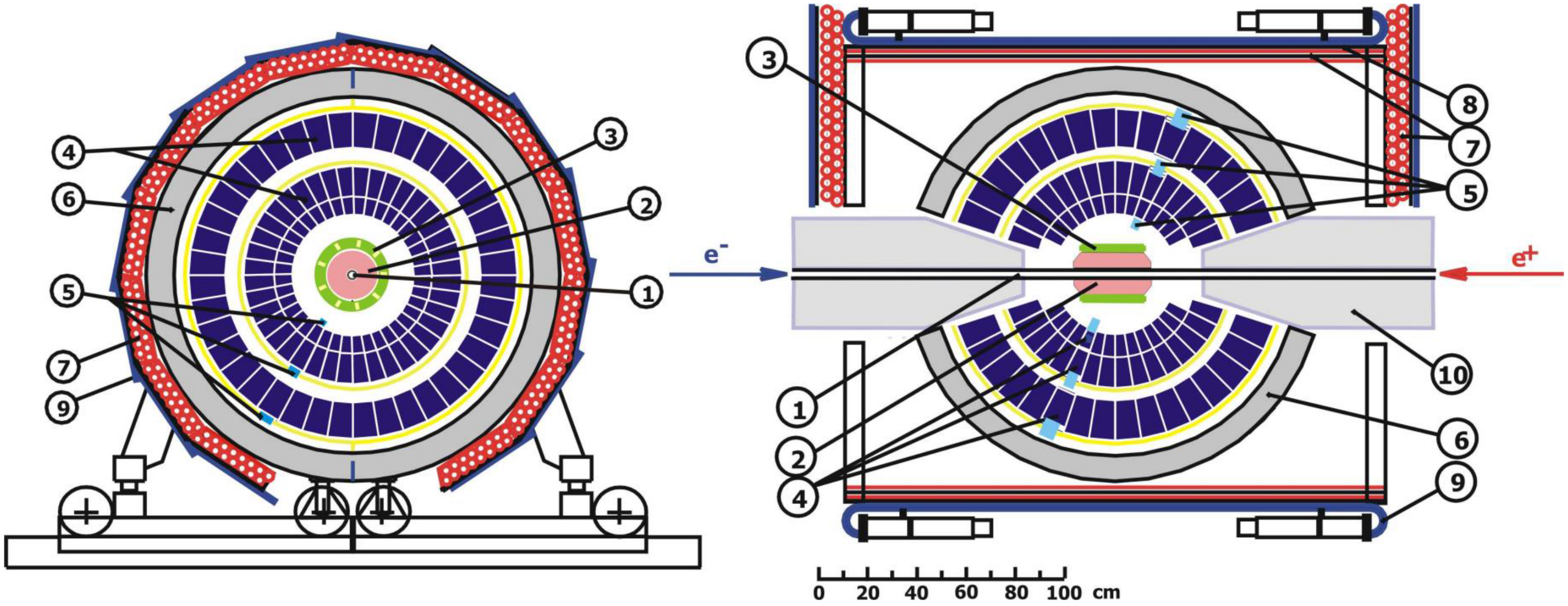


Fit: $\sigma_{\mathrm{Born}}(\mathrm{E}_{\mathrm{c.m.}}) = \mathrm{A} + \mathrm{B}\left[1 - \exp\left(-\frac{(\mathrm{E}_{\mathrm{c.m.}} - \mathrm{E}_{\mathrm{thr}})}{\sigma_{\mathrm{thr}}}\right)\right]$ **~ 1 MeV width**

Interpretation:
- Narrow structure due to rescattering effects $p\bar{p}$, $n\bar{n}$?
- Subnucleon threshold ?

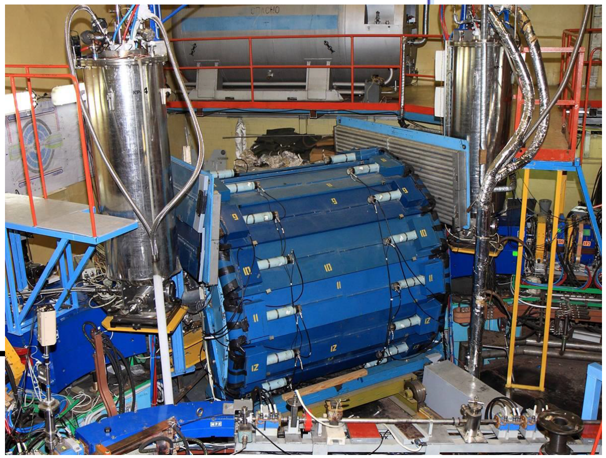
arxiv: 1808.00145

SND Detector



1 – **beam pipe, 2 – tracking system, 3 – aerogel Cherenkov counters , 4 – NaI(Tl) crystals, 5 – phototriodes, 6 – iron muon absorber, 7–9 – muon detector, 10 – focusing solenoids.**

Very compact multipurpose detector with high resolution detection of EM particles



Overview SND Results

1. $e^+e^- \rightarrow \pi^0\pi^0\gamma$, Phys.Rev.D, (2013)
2. $e^+e^- \rightarrow nn$, Phys.Rev.D,(2014)
3. $e^+e^- \rightarrow NN+6\pi$, JETP Lett.,(2014)
4. $e^+e^- \rightarrow \eta\gamma$, Phys.Rev.D,(2014)
5. $e^+e^- \rightarrow \eta'$, Phys.Lett.B,(2015)
6. $e^+e^- \rightarrow \eta\pi^+\pi^-$, Phys.Rev.D,(2015)
7. $e^+e^- \rightarrow \pi^+\pi^-\pi^0$, JETP,(2015)
8. $e^+e^- \rightarrow \eta$ JETP Lett.,(2015)
9. $e^+e^- \rightarrow \omega\eta\pi^0$, Phys.Rev.D,(2016)
10. $e^+e^- \rightarrow \omega\eta$, Phys.Rev.D,(2016)
11. $e^+e^- \rightarrow \pi^0\gamma$, Phys.Rev.D,(2016)
12. $e^+e^- \rightarrow \pi^0\pi^0\gamma$, Phys.Rev.D, (2016)
13. $e^+e^- \rightarrow K^+K^-$ Phys. Rev. D, (2016)

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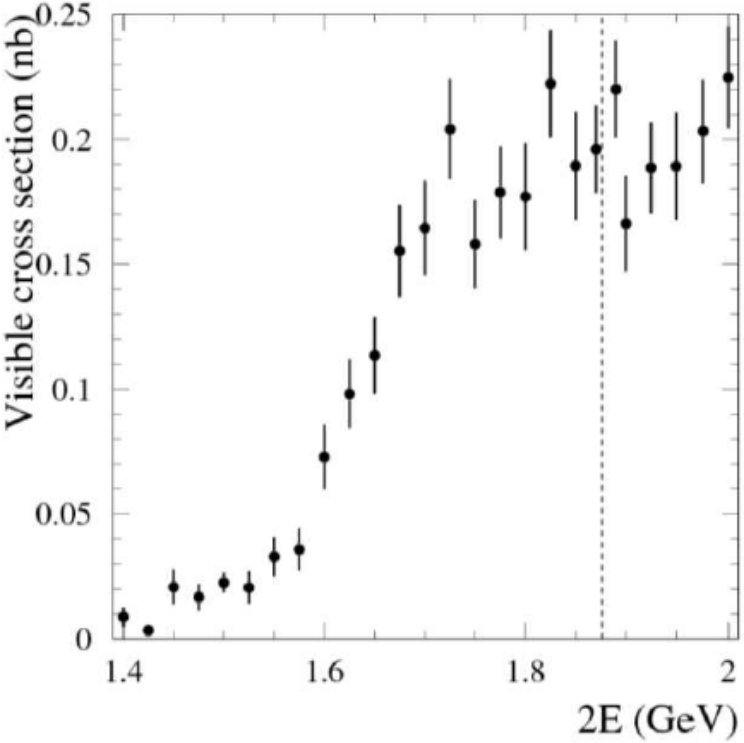
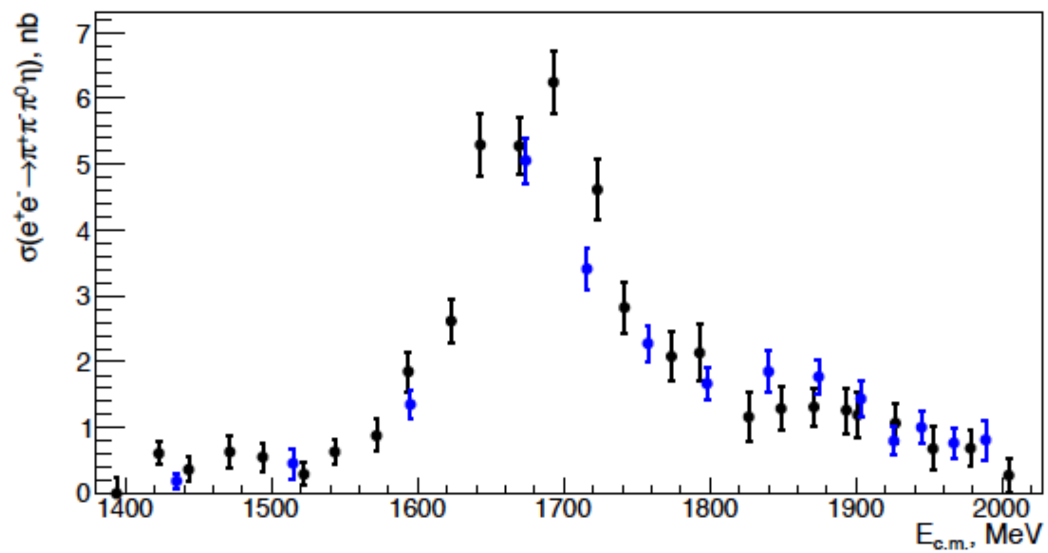
SND: High Multiplicity Channels

$e^+e^- \to \pi^+\pi^-4\pi^0$



$e^+e^- \to \pi^+\pi^-\pi^0\eta$

$\omega\eta,\ \phi\eta,\ a_0(980)\rho$



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Very rare and complicated decays attacked by VEPP-2000 experiments