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Measuring the Hadronic Contribution to $(g-2)_\mu$

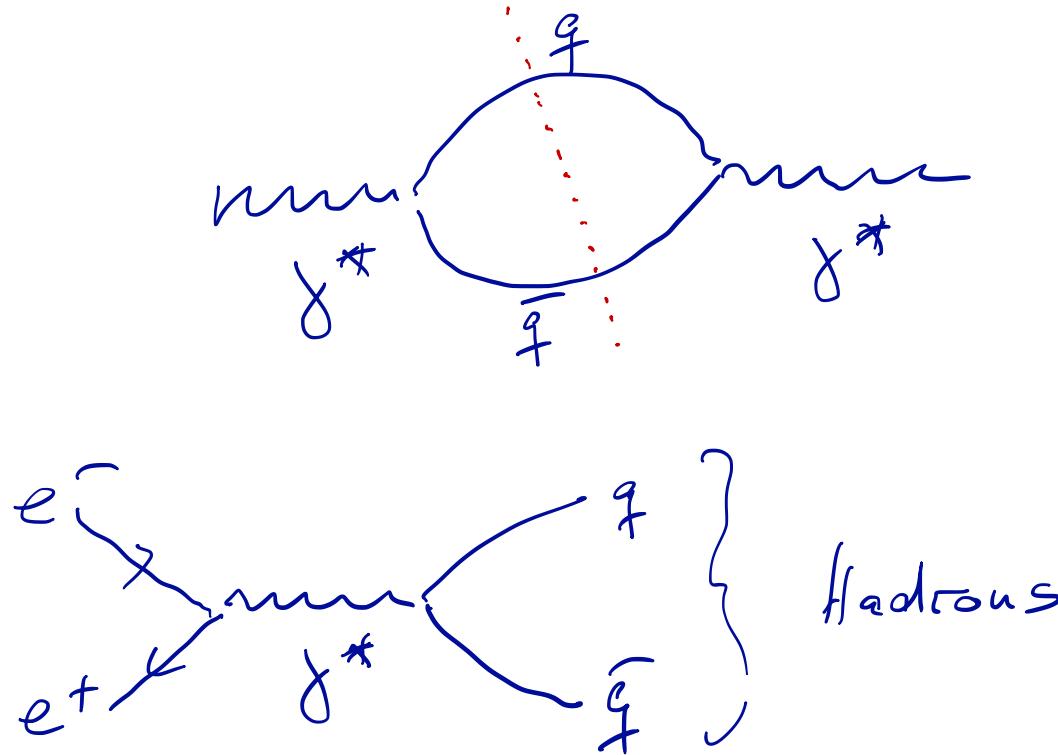
Part I: R_{had} Measurement

Part II: Meson Transition Form
Factors



International School on Muon Dipole
Moments and Hadronic Effects
BINP Novosibirsk, Sept 17 – 21, 2018

Reminder of Yesterday's Lecture

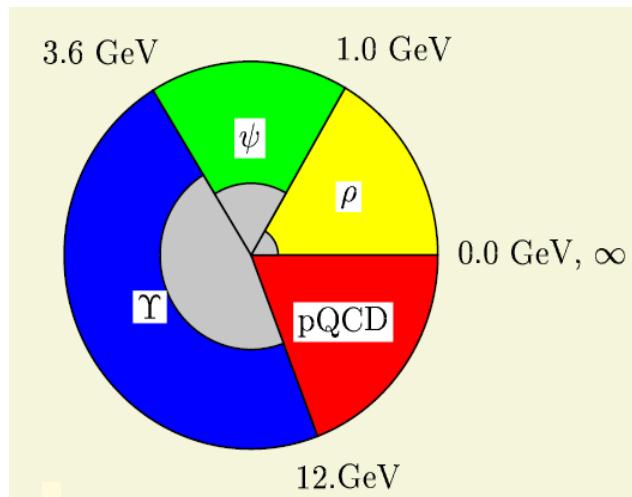


Reminder of Yesterday's Lecture

Running fine structure constant

$$\alpha_{\text{em}}(s) = \frac{\alpha(0)}{(1 - \Delta\alpha_{\text{em}}(s))}$$

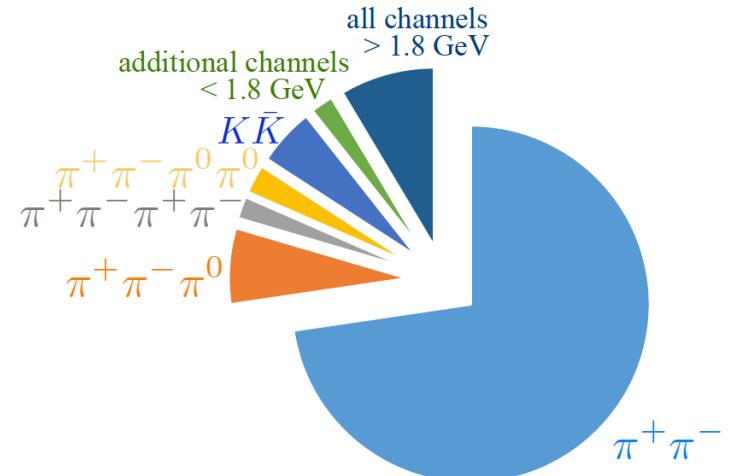
$$\Delta\alpha_{\text{had}}^{(5)}(s) = -\frac{\alpha s}{3\pi} \oint_{4m_\pi^2}^{E_{\text{cut}}^2} ds' \frac{R_\gamma^{\text{data}}(s')}{s'(s'-s)}$$



Anomalous magnet moment of the muon

$$a_\mu = (g-2)_\mu / 2$$

$$a_\mu^{\text{HVP}} = \frac{1}{4\pi^3} \int_{4m_\pi^2}^\infty ds K(s) \sigma_{\text{had}}$$



Outline

Part I: R_{had} Measurements

- R measurements via
Initial State Radiation



Part II: Measurements of Meson Transition Form Factors

- Hadronic Light-by-Light Contribution to $(g-2)_\mu$
- Meson Transition Form Factors at e^+e^- Colliders
- Production of $J^{PC}=1^{++}$ States in e^+e^- Annihilation

Summary (of both parts)



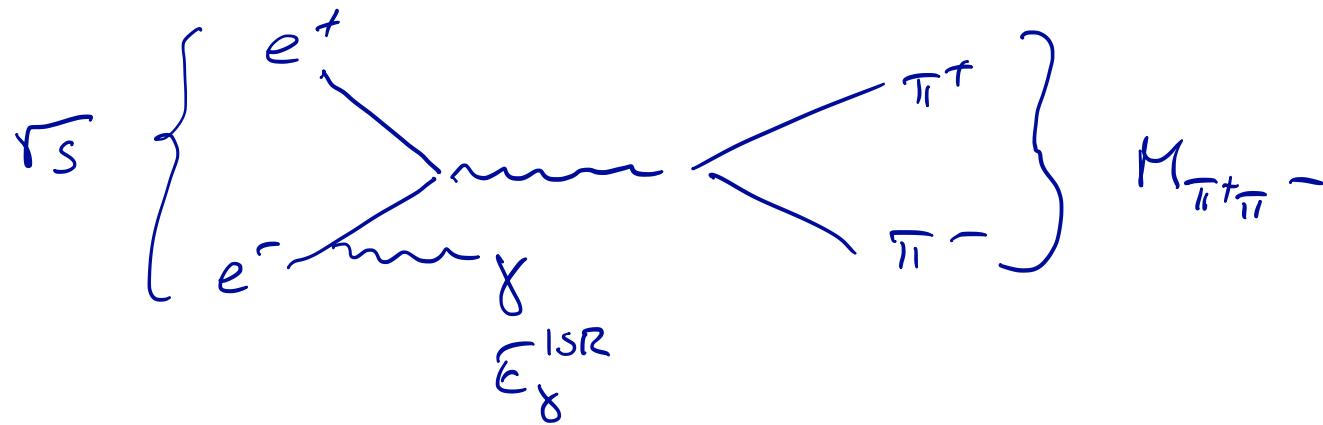
Hadronic Cross Section Measurements via Initial State Radiation

Initial State Radiation (ISR)

Complementary ansatz to traditional energy scan:

Consider events with **Initial State Radiation (ISR)**

Baier, Fadin (1968)
Baier, Khoze (1965)
Arbuzov (1998)
Benayoun et al. (1999)
Binner et al. (1999)

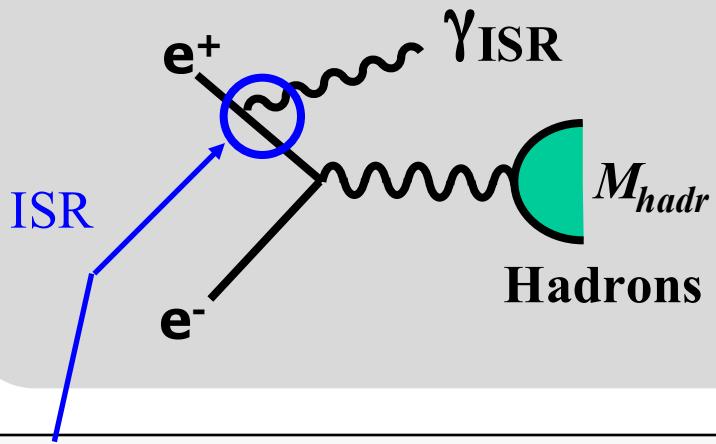


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'Radiative Return' to low-lying resonances
Normalization to theoretical radiator function

$$\frac{d\sigma_{\text{rad}}}{dM_{\text{Hadr}}^2} = \frac{d\sigma(e^+ e^- \rightarrow \text{Hadrons} + \gamma_{\text{ISR}})}{dM_{\text{Hadr}}^2}$$

for $M_{\text{Hadr}}^2 < s$

MC-Generator PHOKHARA = NLO

J. Kühn, H. Czyż, G. Rodrigo

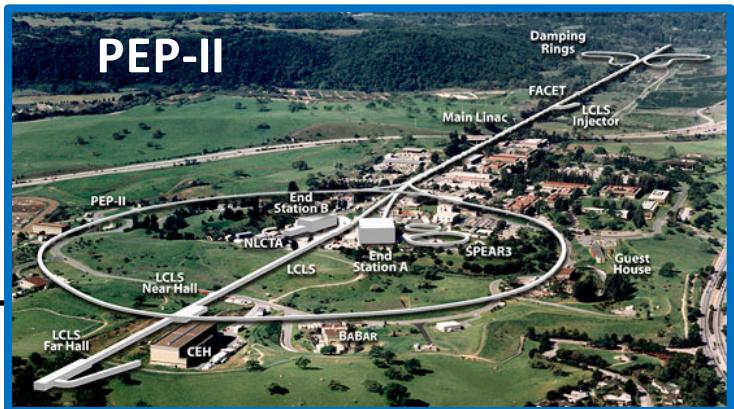
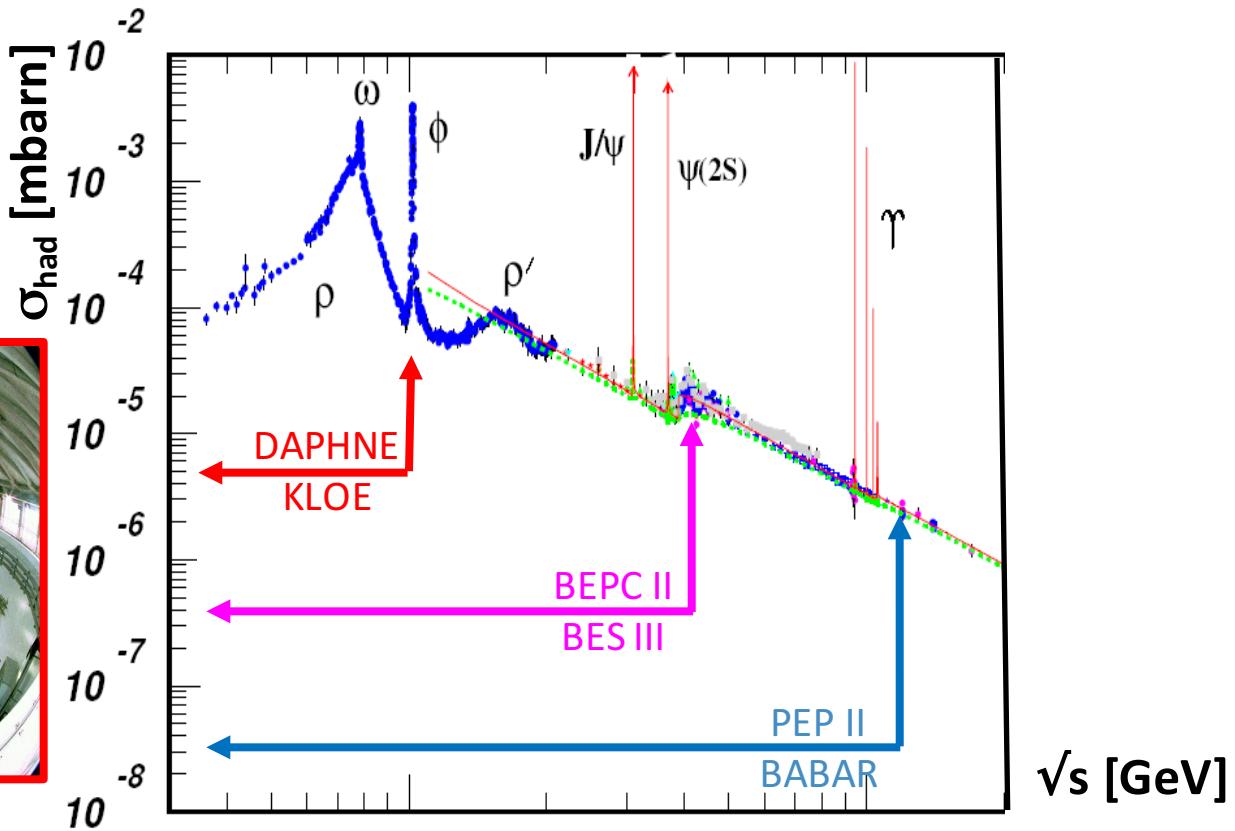
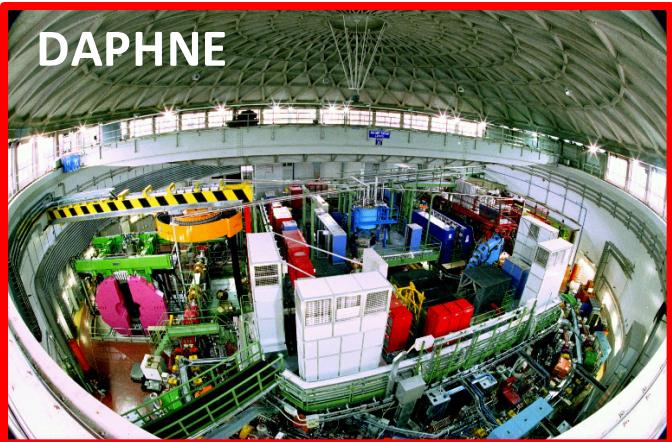
$$M_{\text{Hadr}}^2 \frac{d\sigma_{\text{rad}}}{dM_{\text{Hadr}}^2} = \sigma_{\text{Born}}(s) \times \mathbf{H}(s)$$

Radiator-Function

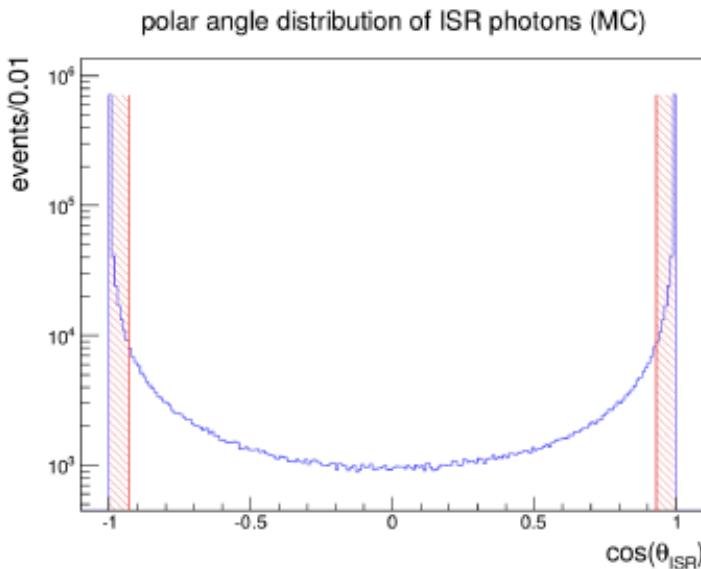
PHOKHARA 9.2: PRD94 (2016) 034033

All references at <http://ific.uv.es/~rodrigo/phokhara/>

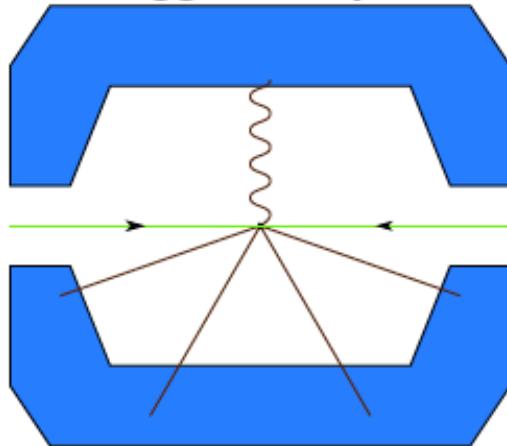
Initial State Radiation (ISR)



Initial State Radiation (ISR)



Tagged analysis



Tagged analysis:

ISR photon measured in Calorimeter

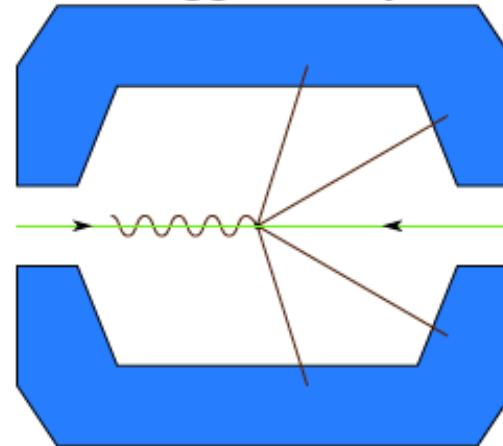
- Increased amount of Final State Radiation (FSR)

Untagged analysis:

No ISR detection; cut on missing momentum

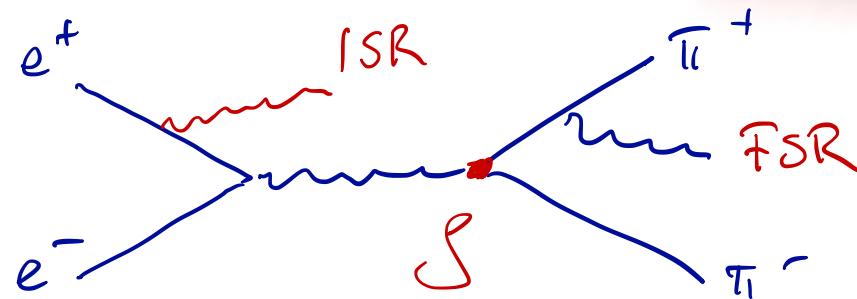
- Threshold mass region not accessible

Untagged analysis



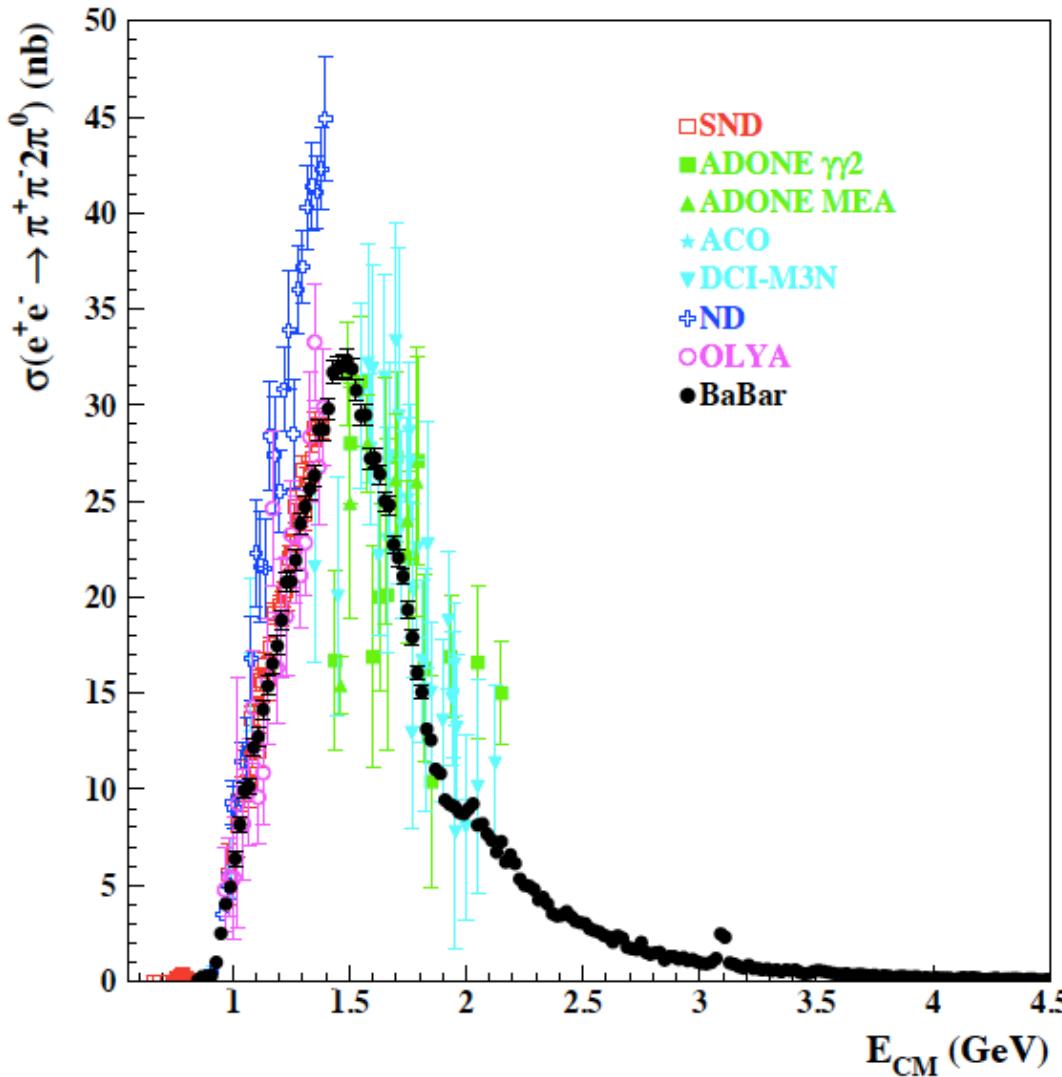
Final State Radiation (FSR)

Do you expect FSR radiation effects to be of similar size as ISR in case of hadronic events ?



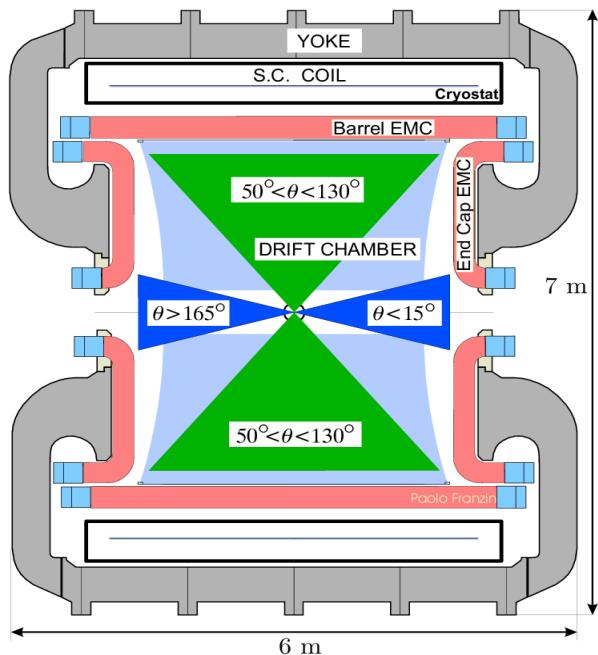
NEW

BABAR: $e^+e^- \rightarrow \pi^+\pi^- 2\pi^0\gamma_{ISR}$



- Tagged ISR analysis (2017)
- Huge improvement over existing data sets suffering from normalization issues

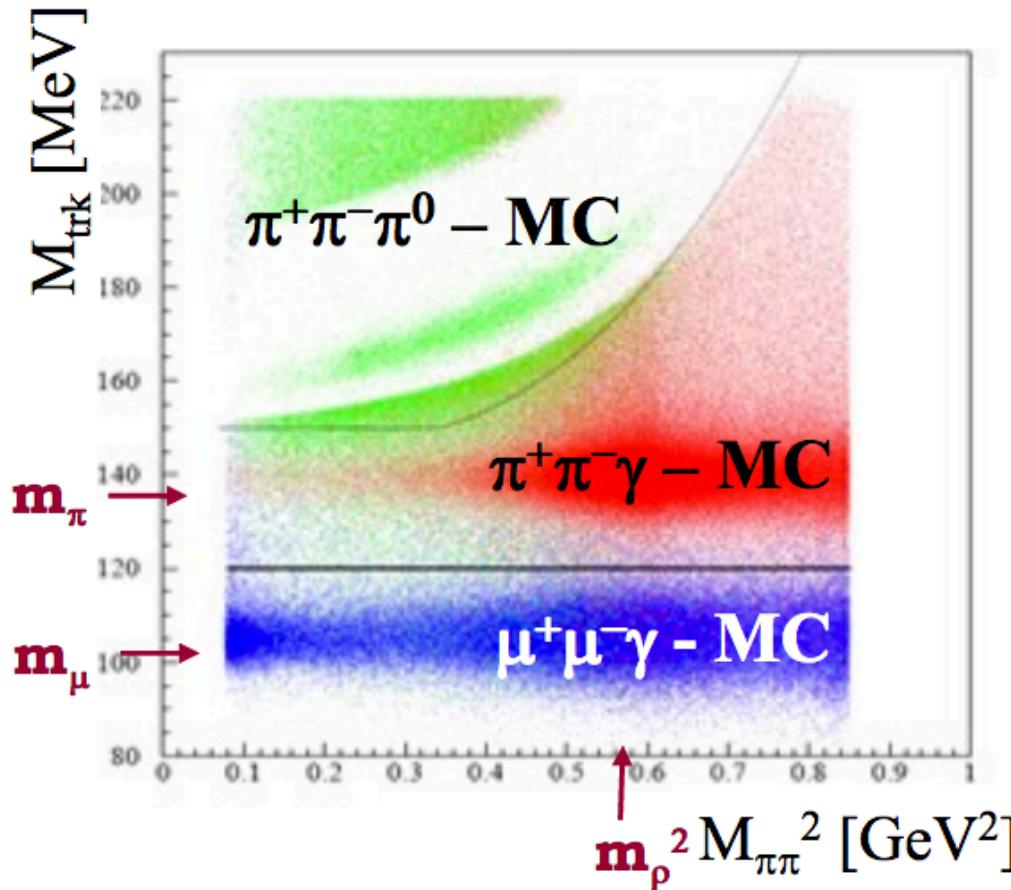
ISR@KLOE: $e^+e^- \rightarrow \pi^+\pi^-\gamma_{ISR}$



Publication	Mode	Normalization	Int. Luminosity*
Phys.Lett. B606 (2005) 12	untagged	Radiator	141 pb ⁻¹
Phys.Lett. B670 (2009) 285	untagged	Radiator	240 pb ⁻¹
Phys.Lett. B700 (2011) 102	tagged	Radiator	232 pb ⁻¹
Phys.Lett. B720 (2013) 336	untagged	$\mu^+\mu^-\gamma$	240 pb ⁻¹

ISR@KLOE: $e^+e^- \rightarrow \pi^+\pi^-\gamma_{ISR}$

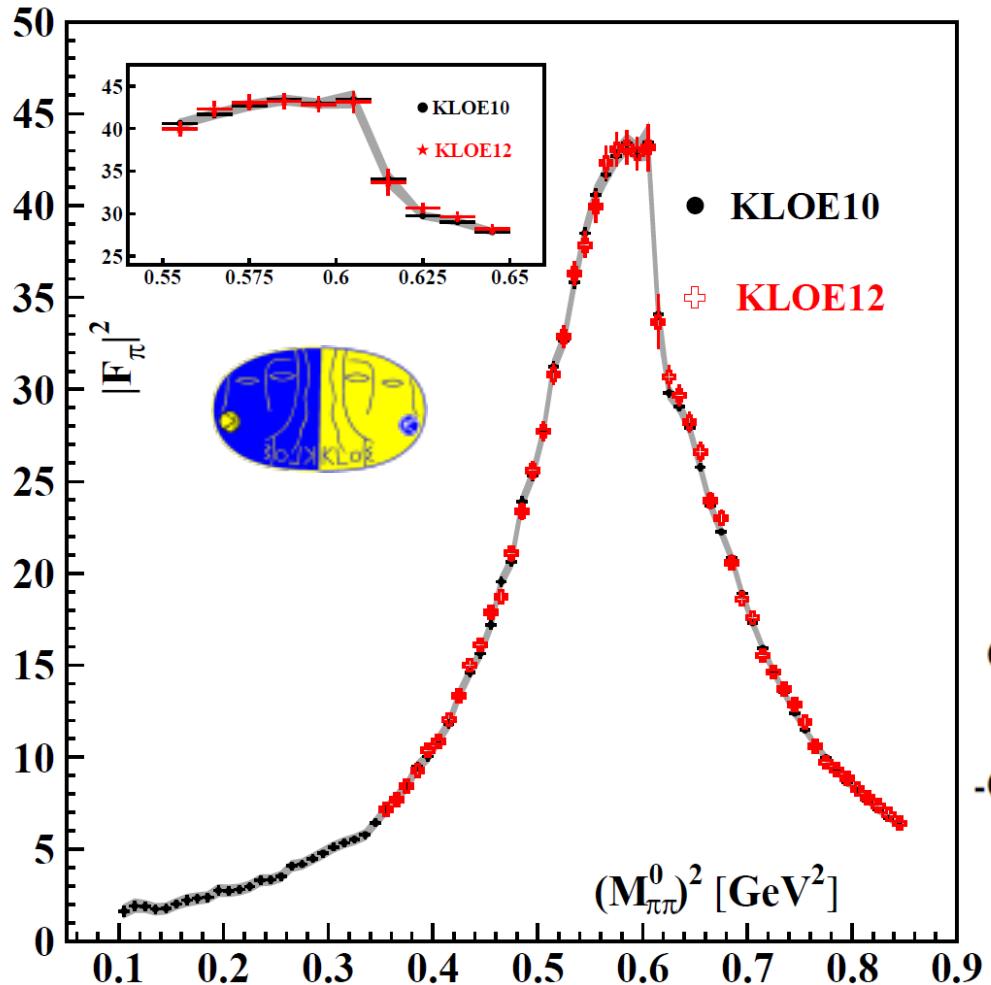
Track mass:
$$\left(M_\phi - \sqrt{\vec{p}_1^2 + M_{trk}^2} - \sqrt{\vec{p}_2^2 + M_{trk}^2} \right)^2 - (\vec{p}_1 + \vec{p}_2)^2 = q_\gamma^2 = 0$$



↑
Derive
this formula

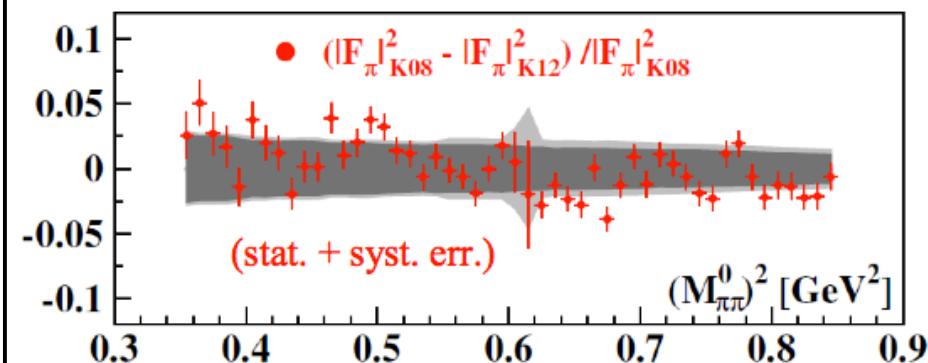


ISR@KLOE: Pion Form Factor

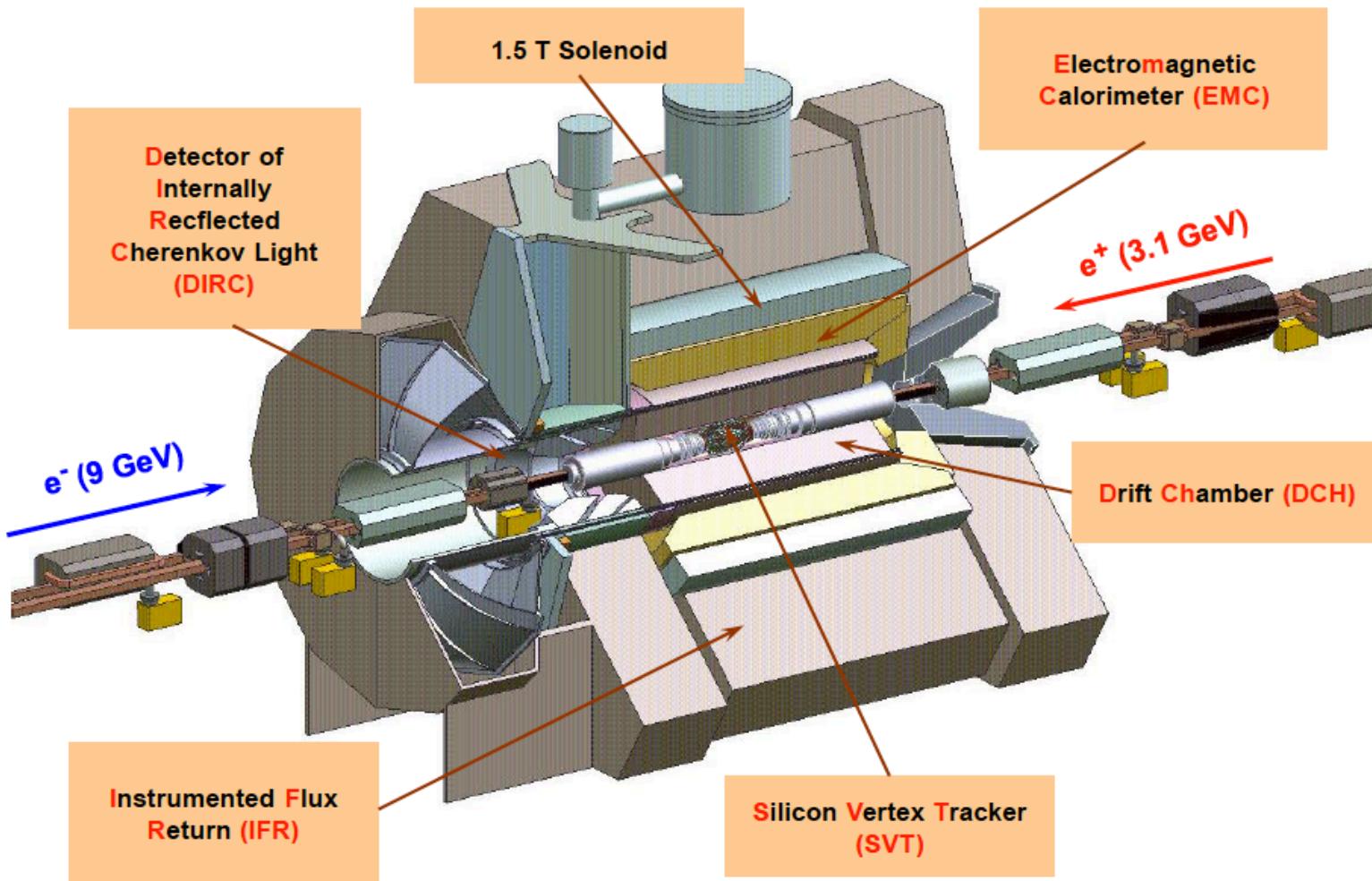


Features

- Below 1% systematic. uncertainties
- Coverage from threshold to 1 GeV
- Agreement btw. KLOE data sets
within systematic uncertainties
→ Cross check for Radiator and Luminosity

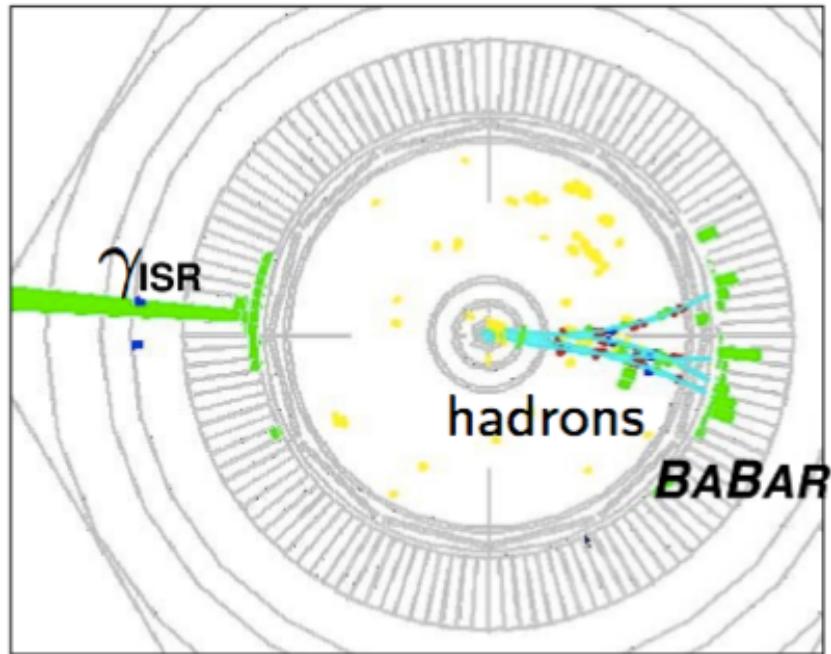


The BABAR-Experiment

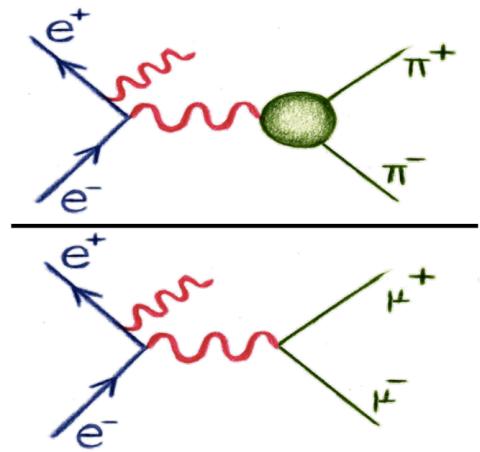


ISR Selection

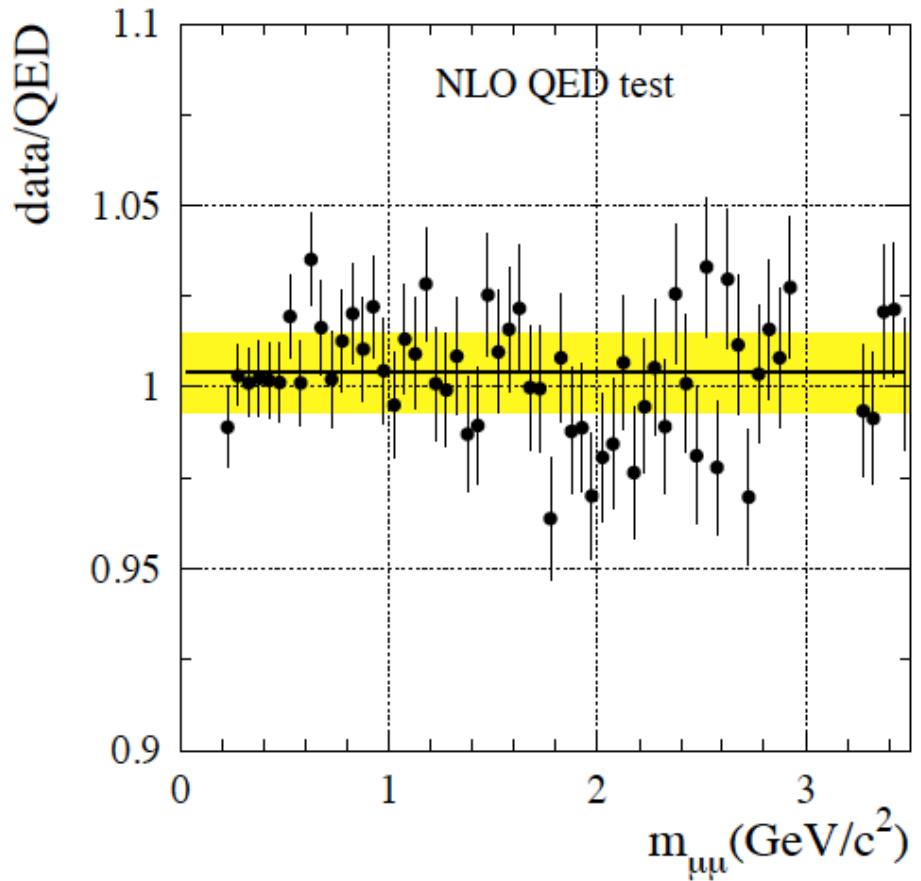
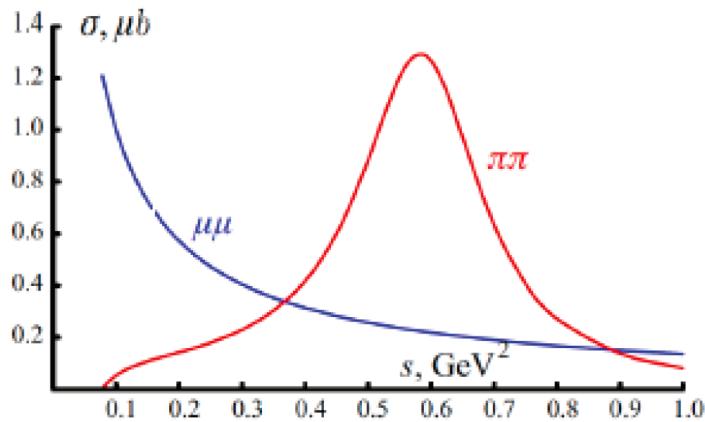
- **Detected high energy photon $E_\gamma > 3 \text{ GeV}$**
→ strong background subtraction
- **Event topology: back-to-back signature**
→ high geometrical acceptance
- **Kinematic fit including ISR**
→ very good energy resolution
- **Continuous measurement from threshold to $\sim 5 \text{ GeV}$**
→ provides common consistent systematic uncertainty



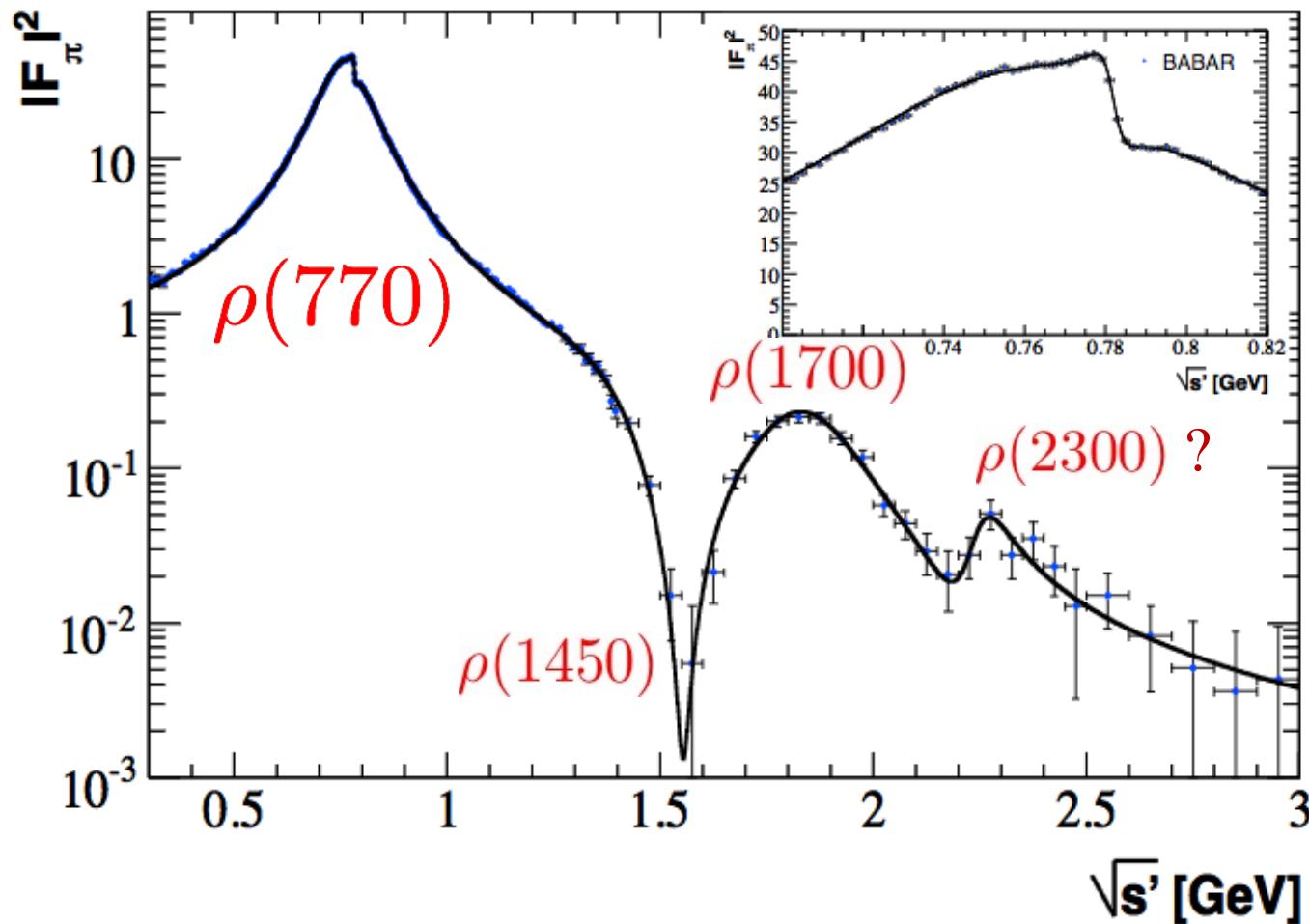
ISR@BABAR: Muon Normalization



$\sigma(e^+e^- \rightarrow \pi^+\pi^-)$ and $\sigma(e^+e^- \rightarrow \mu^+\mu^-)$



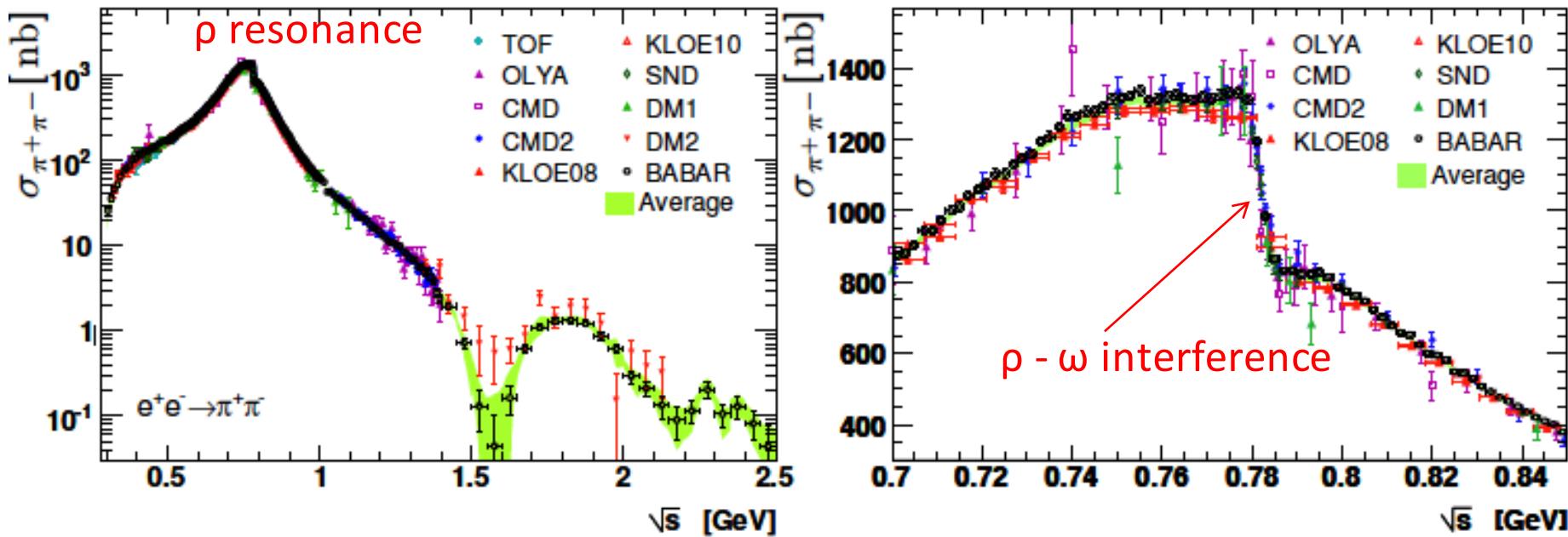
BABAR: Pion Form Factor



systematic. Uncertainty 0.5 – 1.4 %

BABAR, PRL 103, 231801 (2009)

Most relevant Channel: $e^+e^- \rightarrow \pi^+\pi^-$

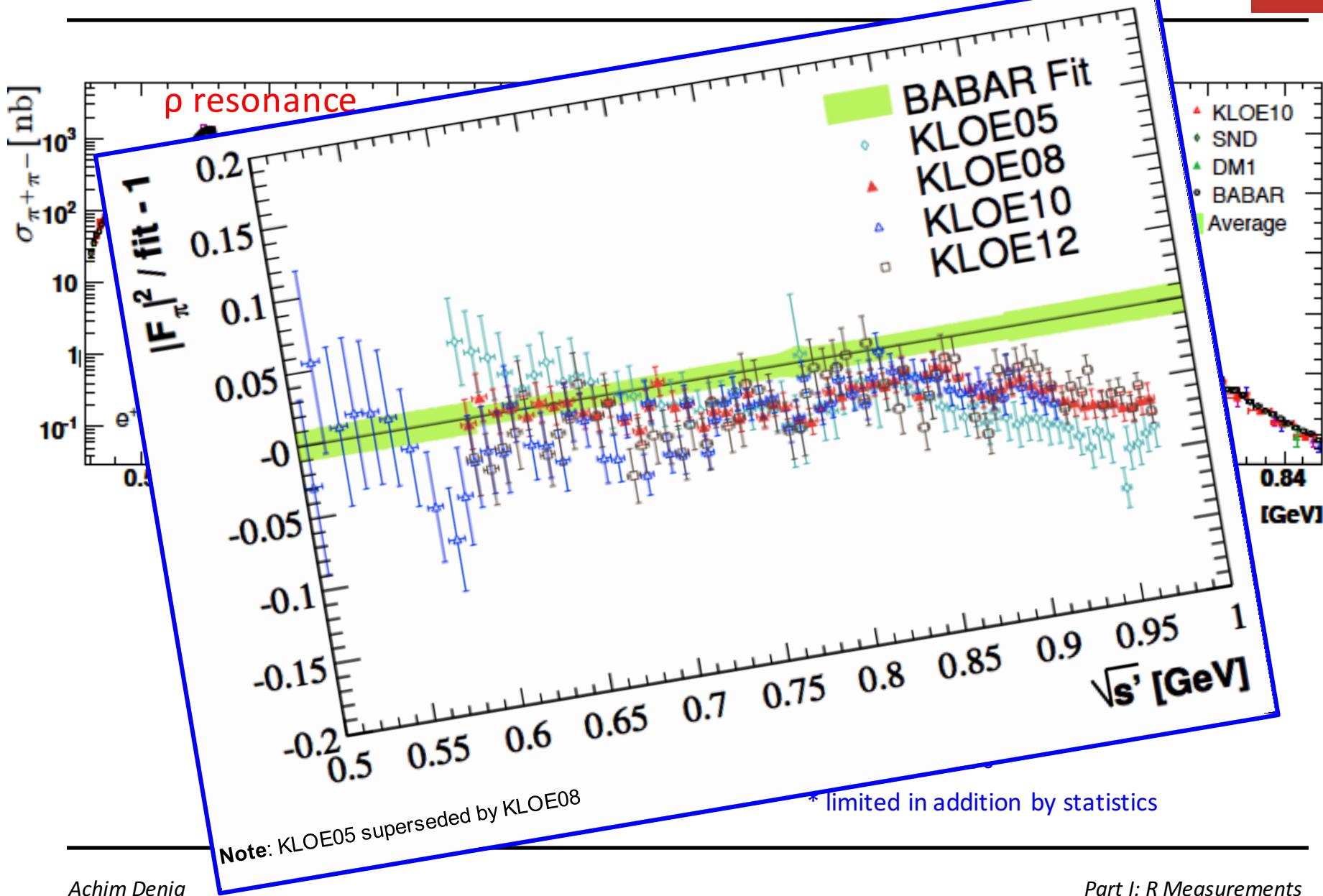


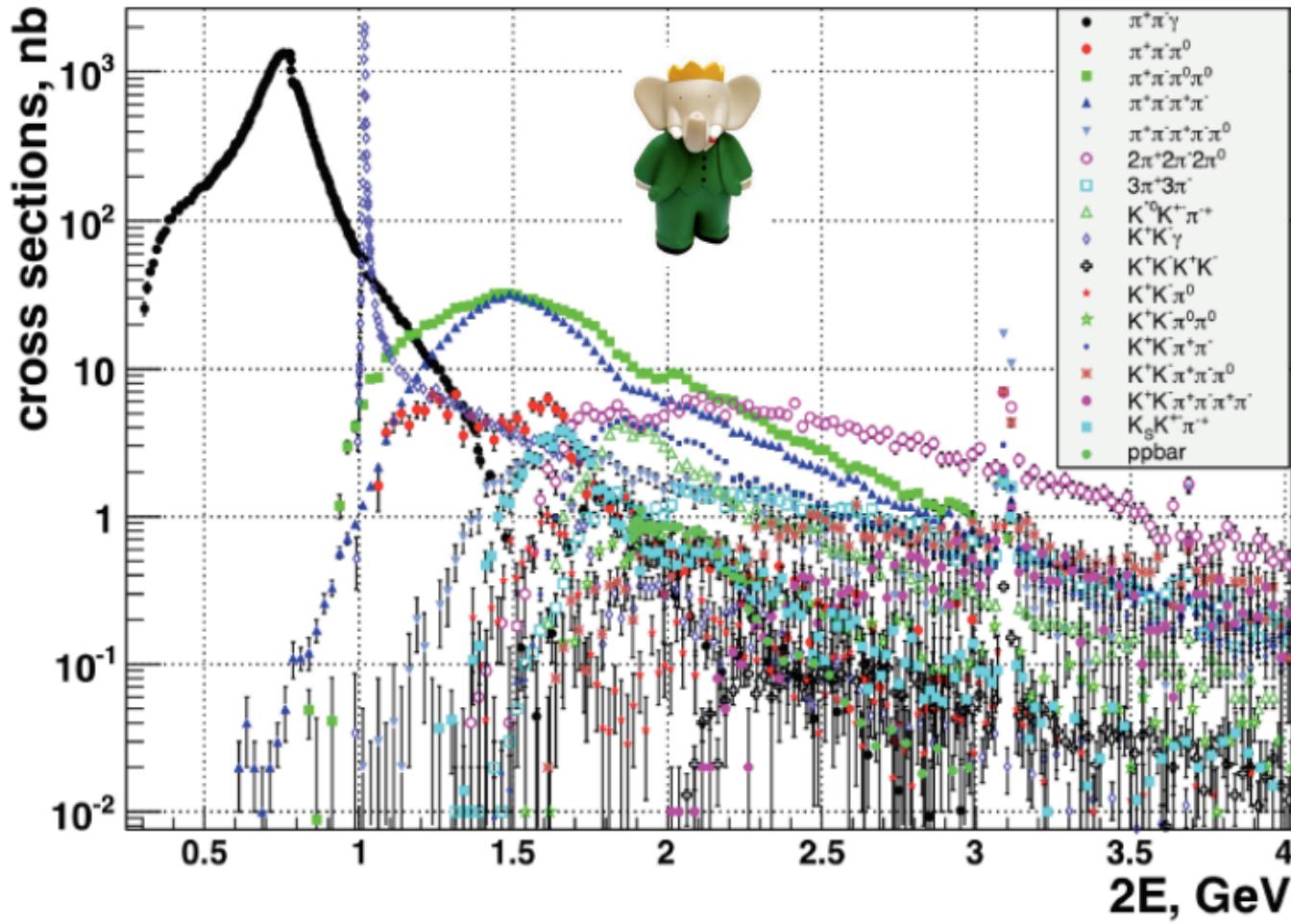
Systematic Uncertainties

- BABAR 0.5%
- KLOE 0.8%
- CMD2 0.8%*
- SND 1.5%*

* limited in addition by statistics

Most relevant Channel: $e^+e^- \rightarrow \pi^+\pi^-$





Precision:

2π : < 1%

3π : ~10%

4π : ~ 3%

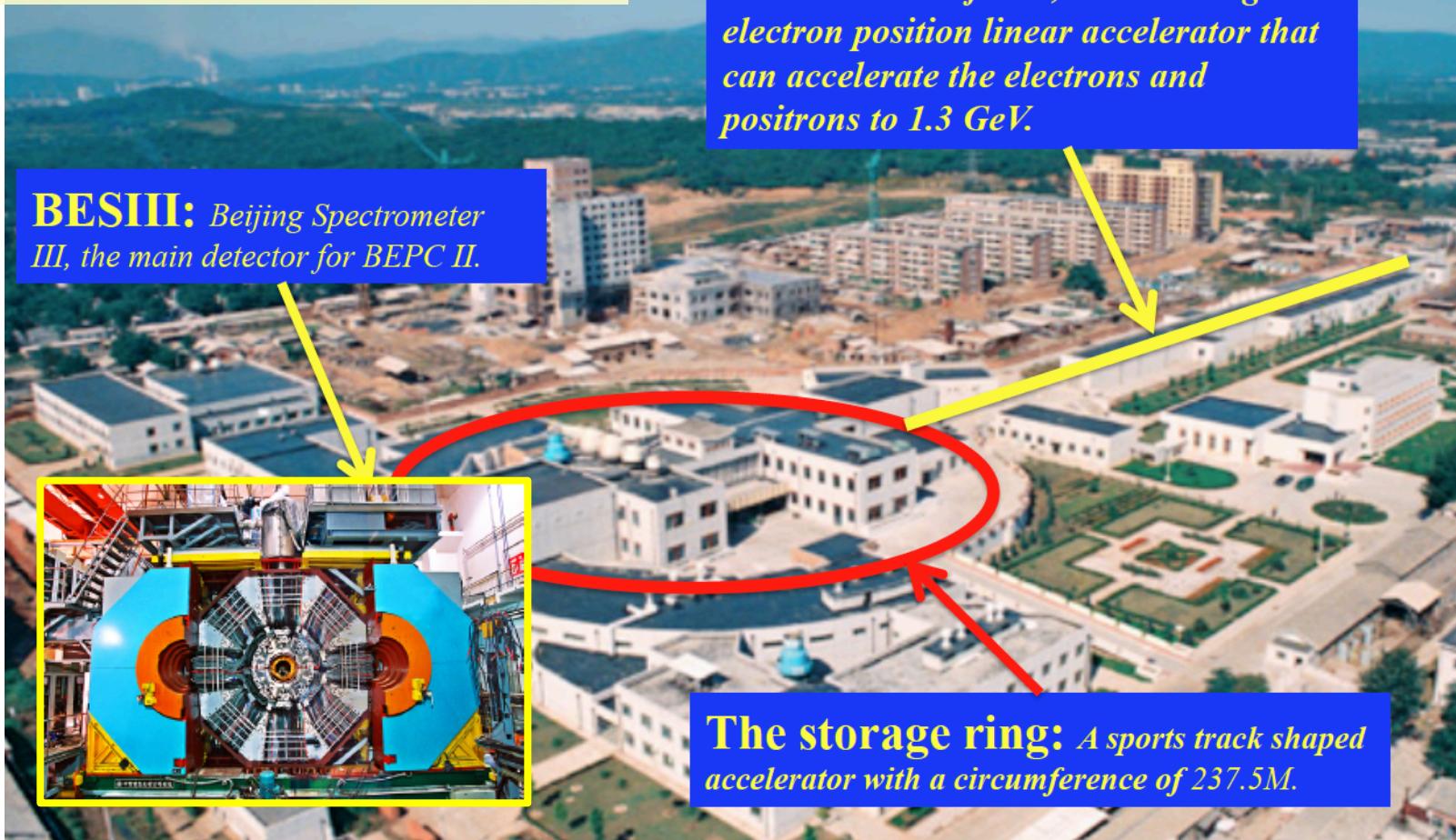
$\geq 5\pi$: 10% and higher

BESIII Experiment @ BEPCII

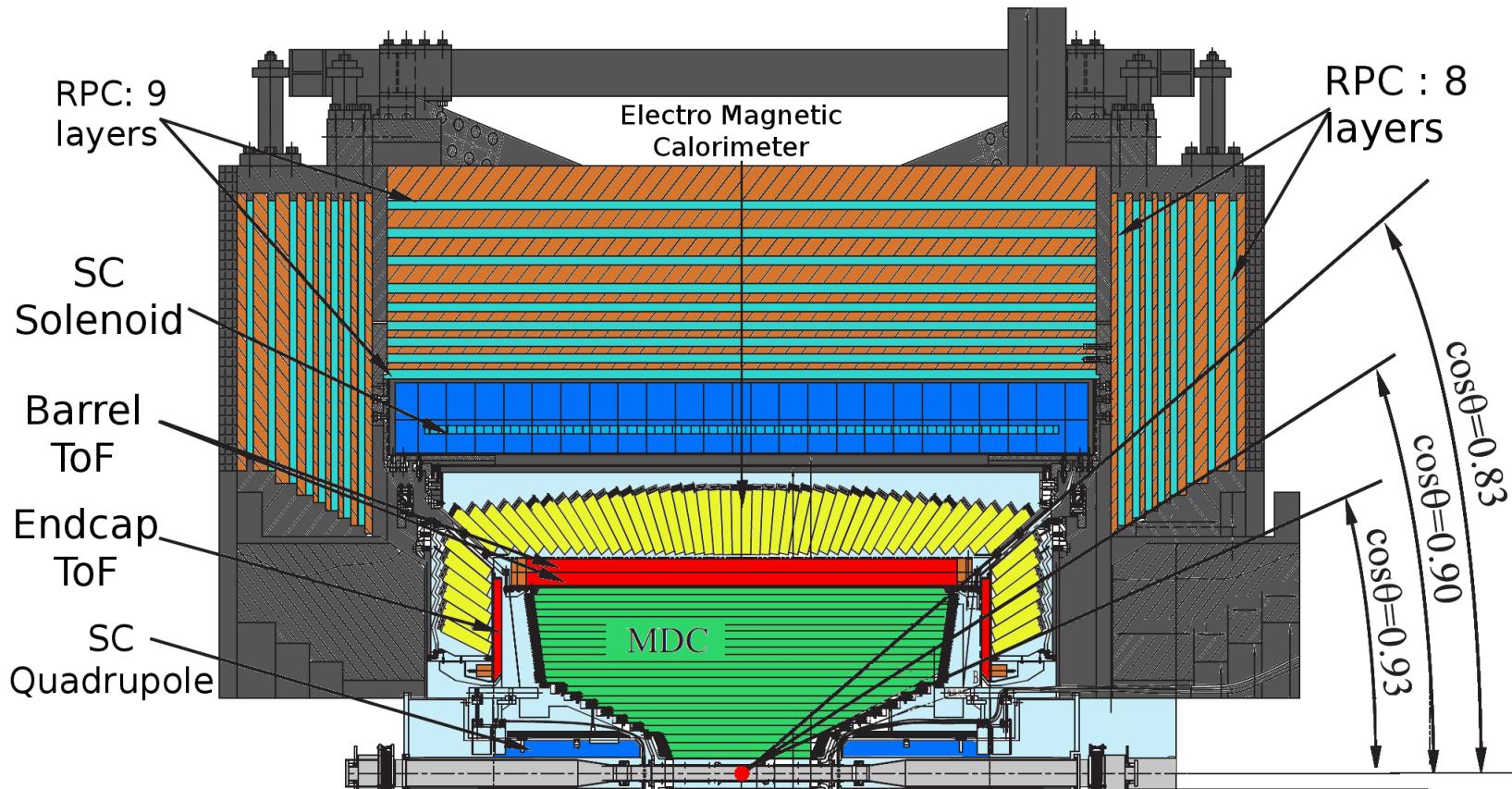
Electron-Positron Collider BEPCII

BEPCII Energy 2.0 – 4.6 GeV

Design Luminosity achieved $10^{33} \text{ cm}^{-2} \text{ s}^{-1}$

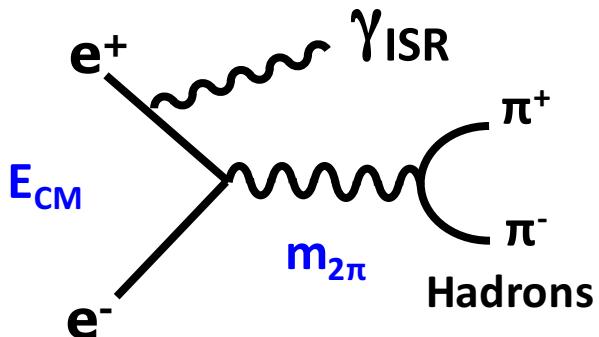


BESIII Experiment @ BEPCII



BESIII ISR Analysis: $e^+e^- \rightarrow \pi^+\pi^-\gamma_{ISR}$

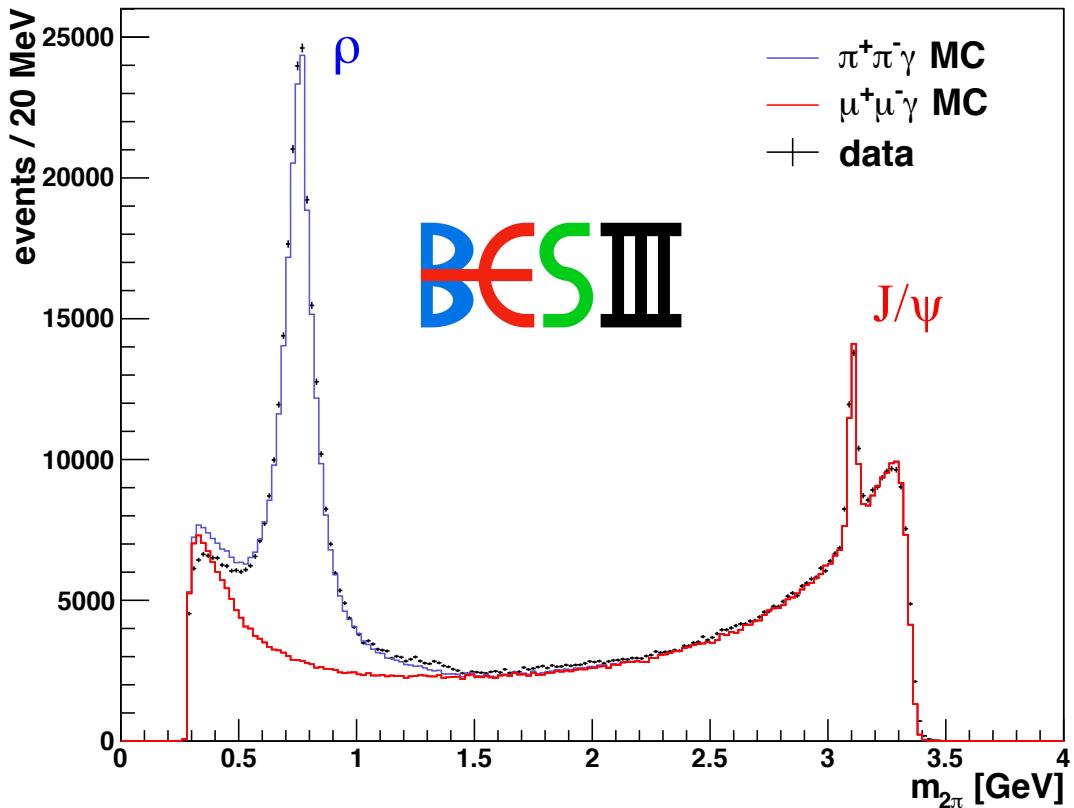
Initial State Radiation



Features:

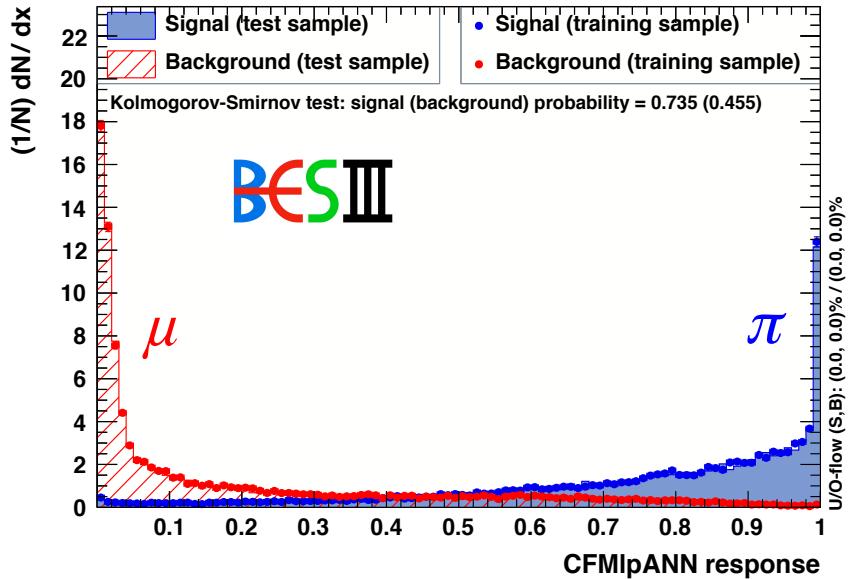
- $\psi(3770)$ data only (2.9 fb^{-1})
- no dedicated background subtraction
- tagged ISR photon

Event yield after acceptance cuts only



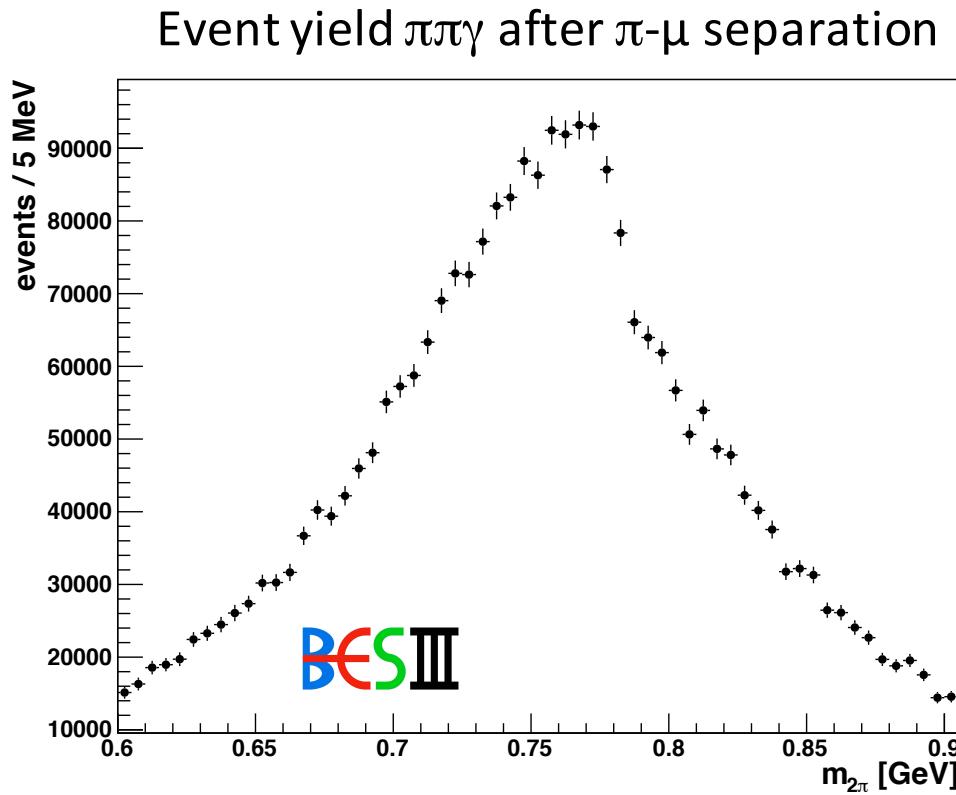
Pion Muon Separation needed
→ TMVA methods!

$e^+e^- \rightarrow \pi^+\pi^-\gamma_{ISR}$: π - μ Separation



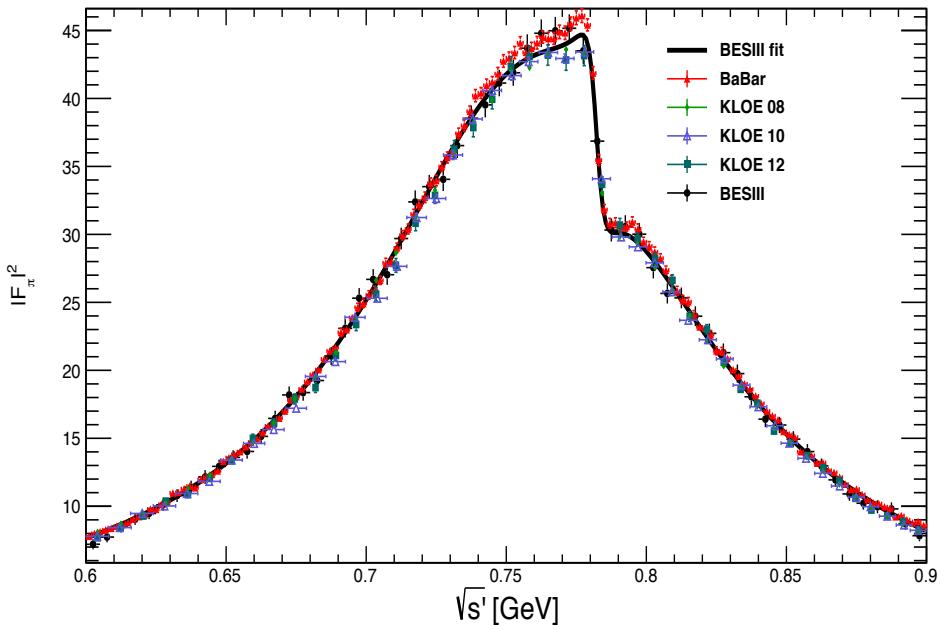
TMVA method (Neural Network):

- trained using $\mu\mu\gamma$ and $\pi\pi\pi\gamma$ MC events
- information based on track level
- efficiency matrix (p, Θ) for data, MC
- corrected for data - MC differences
- cross checked for different TMVA methods

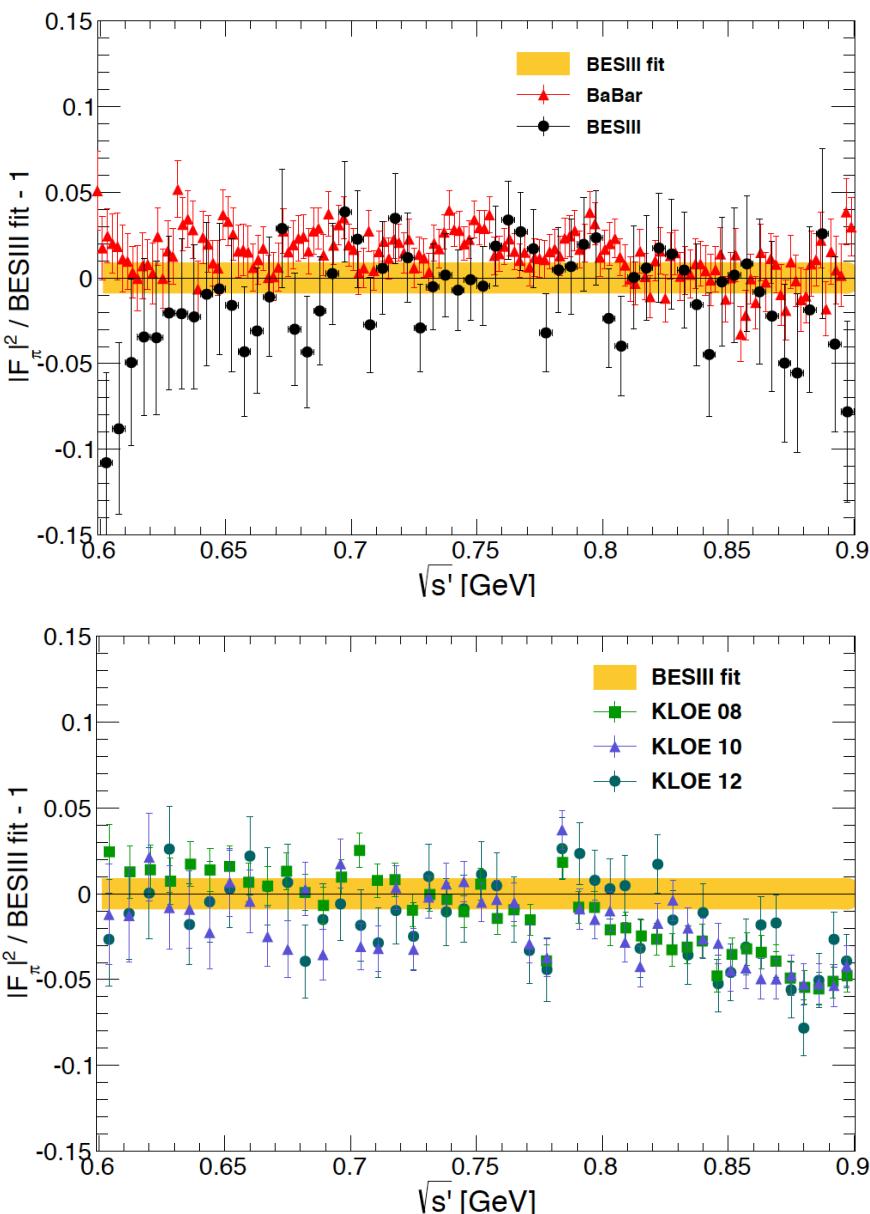


Comparison with existing Data

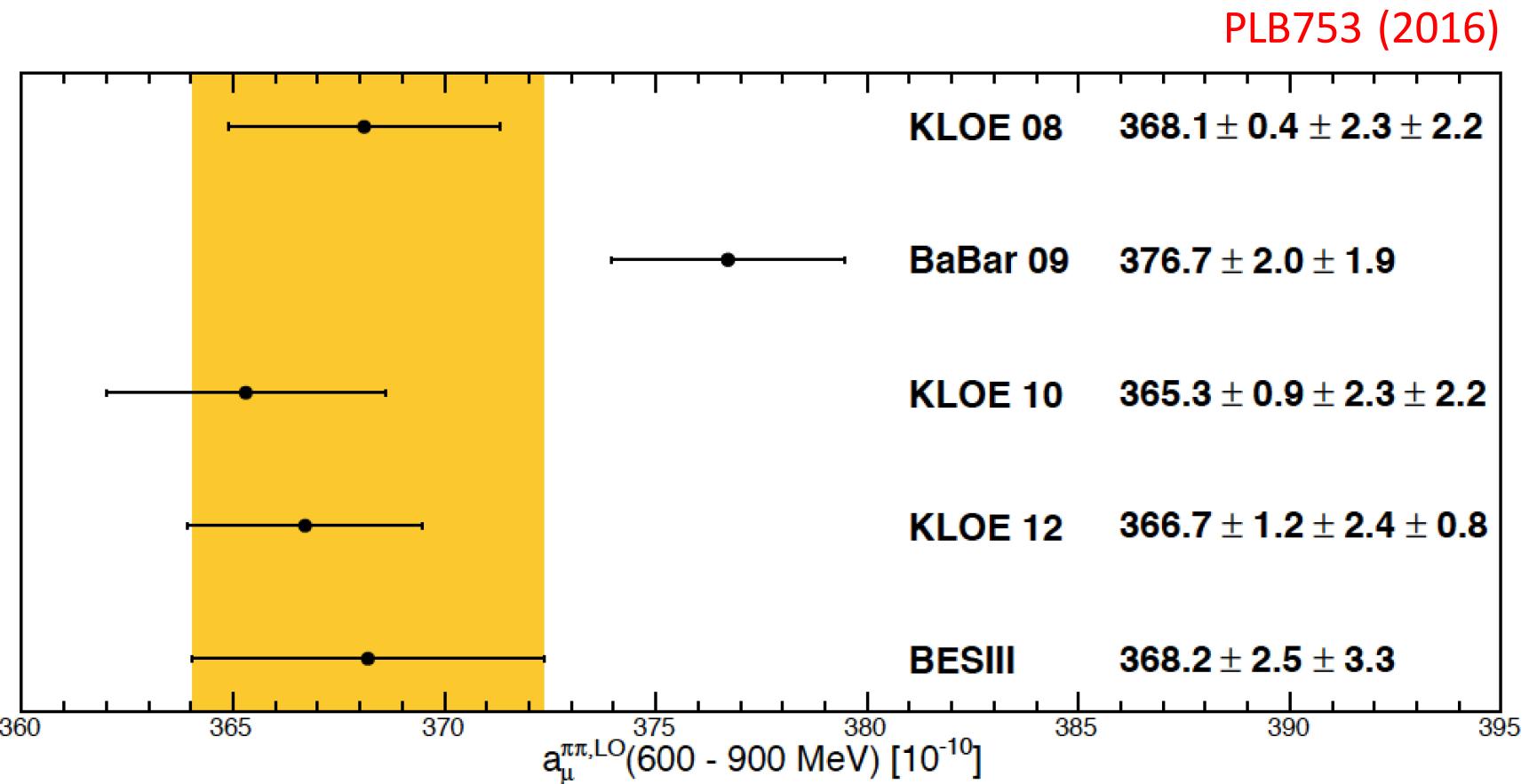
Pion Form Factor F_π



0.9 % accuracy
normalization to luminosity / radiator function



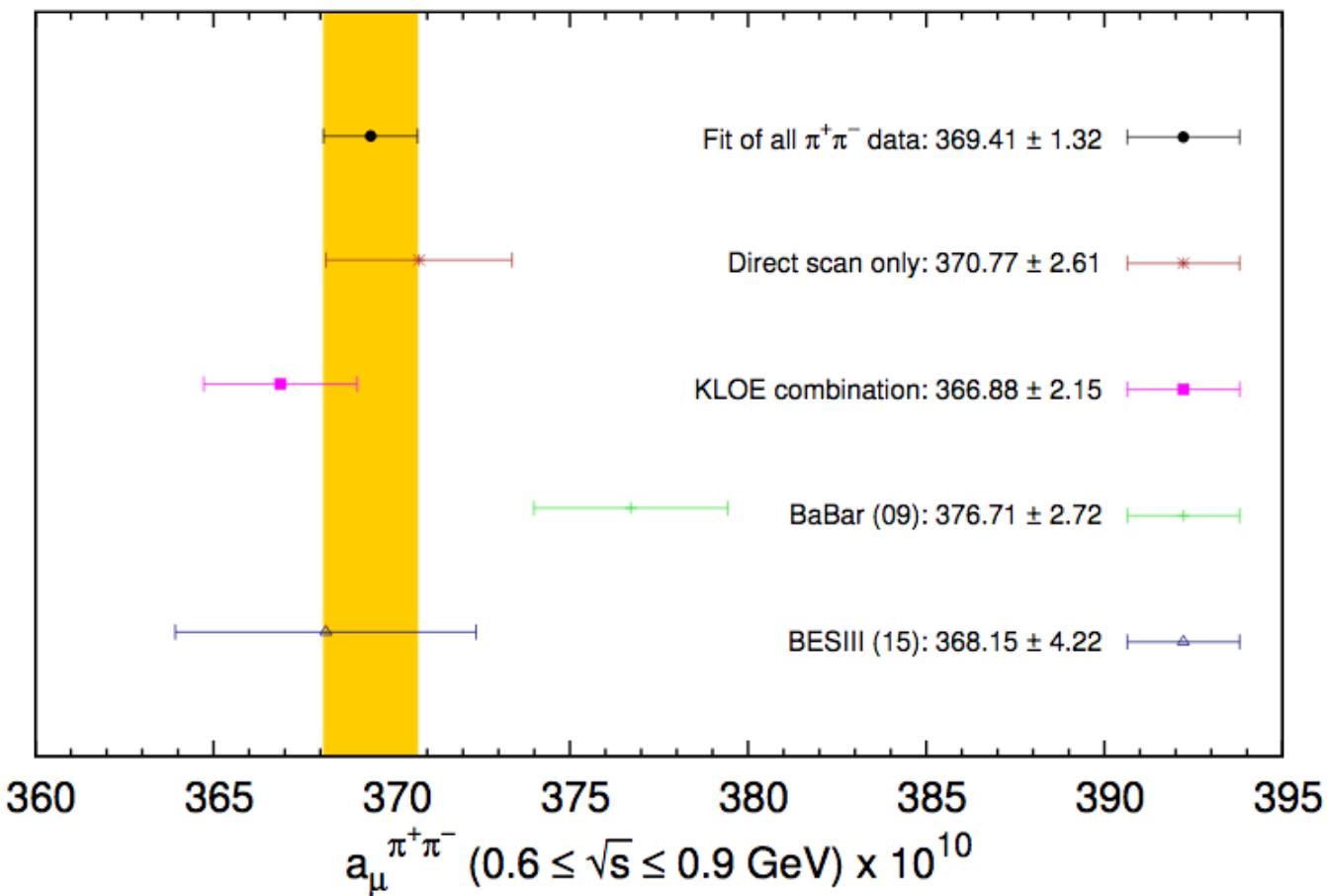
Impact on Hadronic Vacuum Polarization



Good agreement found with KLOE !

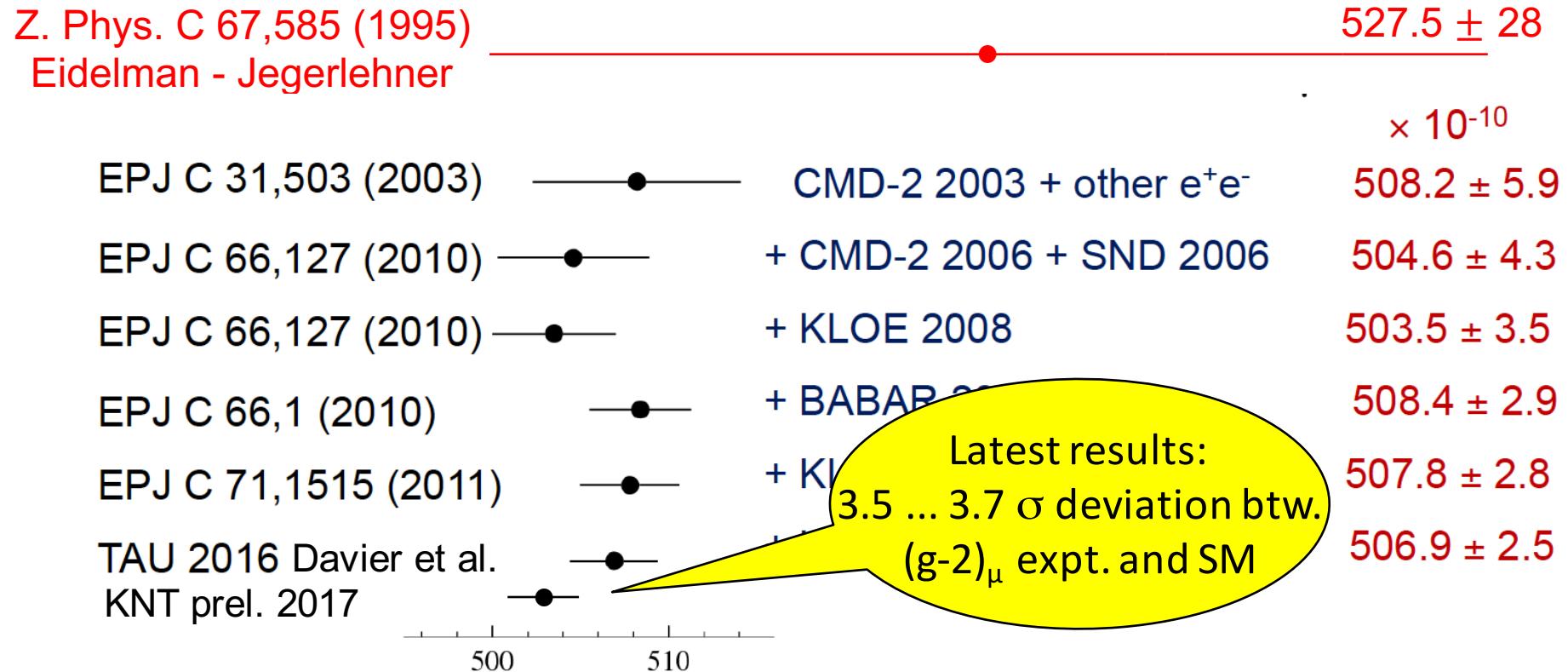
BES III confirms the $(g-2)_{\mu}$ deviation at 3 ... 4 sigma level !!!

Impact on Hadronic Vacuum Polarization



Mandatory to understand KLOE-BABAR-BES III discrepancy !!!
New data (energy scan!) from VEPP-2000 eagerly awaited !!!

Improvement in 2π Contribution to $(g-2)_\mu$



Complementarity ISR Experiments

*** top, ** good, * fair

	KLOE	BABAR	BES III
Energy range covered	*	***	***
Available Statistics	***	***	**
Resolution DC	***	**	**
Threshold for untagged method	***	*	**
FSR model uncertainty	*	***	**



Hadronic Light-by-Light Contribution to $(g-2)_\mu$

Standard Model Prediction of $(g-2)_\mu$

Hadronic contribution **non-perturbative**, the limiting contribution

$$a_\mu^{SM} = a_\mu^{\text{QED}} + a_\mu^{\text{weak}} + a_\mu^{\text{had}} = (11\,659\,180.2 \pm 4.9) \cdot 10^{-10}$$

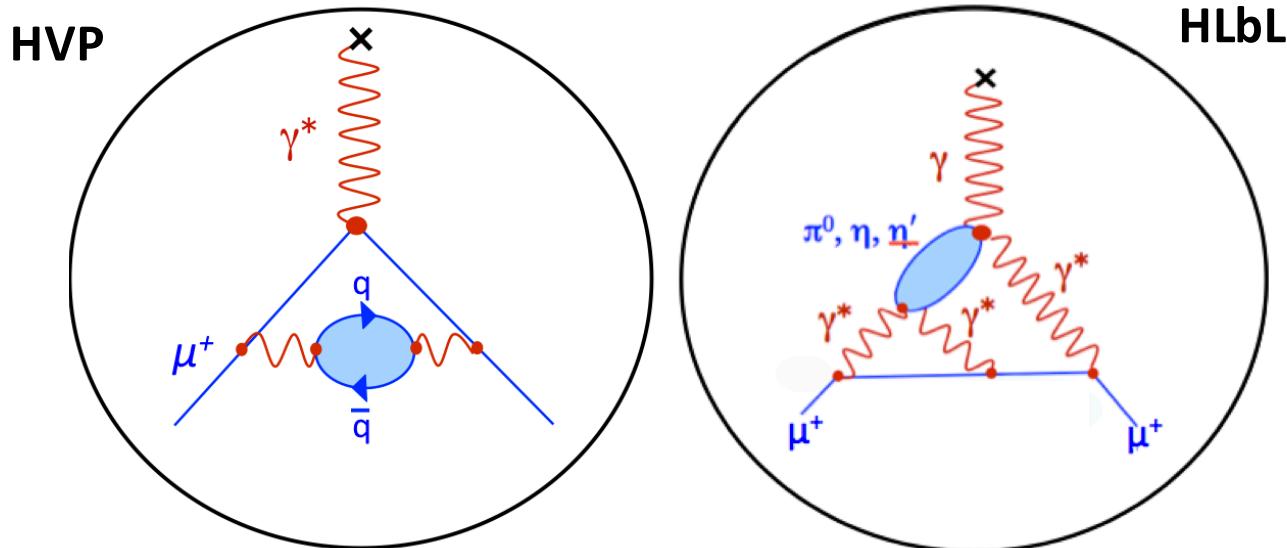
Teubner et al. '11

Keshavarzi et al. 2018

→ **HVP**: Hadronic Vacuum Polarization $(693.27 \pm 2.46) \cdot 10^{-10}$

NLO $(-9.8 \pm 0.1) \cdot 10^{-10}$; NNLO $(1.2 \pm 0.01) \cdot 10^{-10}$)

→ **HLbL**: Hadronic Light-by-Light $(9.8 \pm 2.6) \cdot 10^{-10}$



Hadronic Light-by-Light ($g-2$) $_{\mu}$

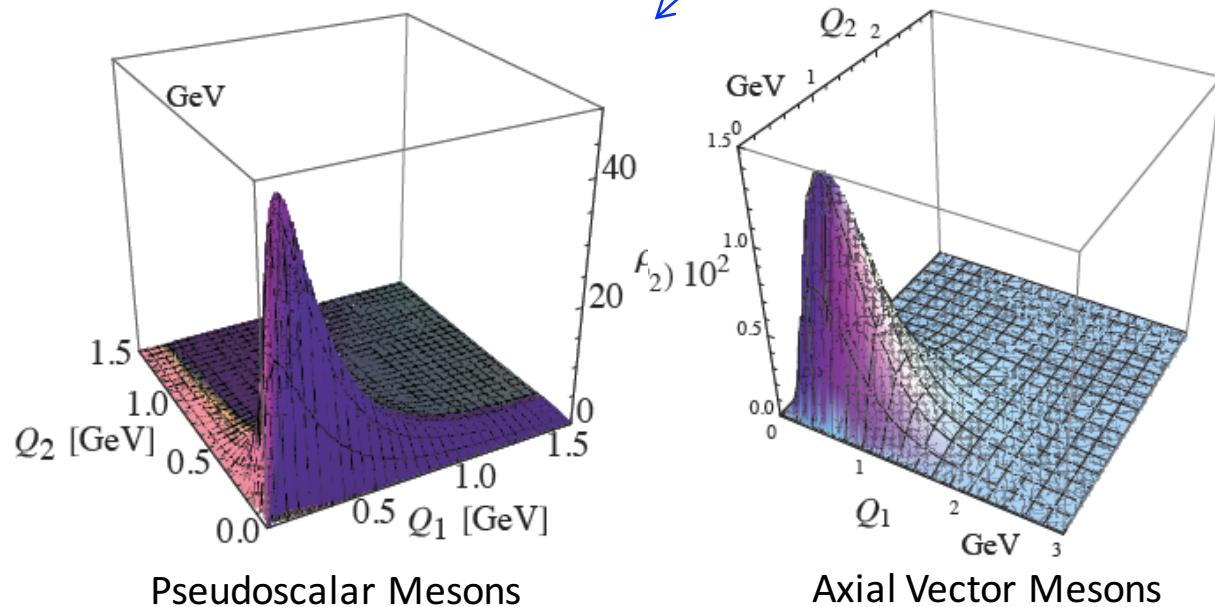
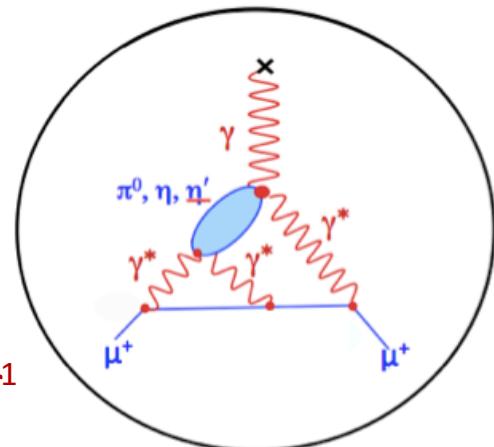
Leading contributions are pole contribution from π^0, η, η'

2D integral representation

$$a_{\mu}^{\text{HLbL};\pi^0} = \int_0^\infty dQ_1 \int_0^\infty dQ_2 \sum_i w_i(Q_1, Q_2) f_i(Q_1, Q_2)$$

weighting function form factor

[Knecht, Nyffeler 2002]



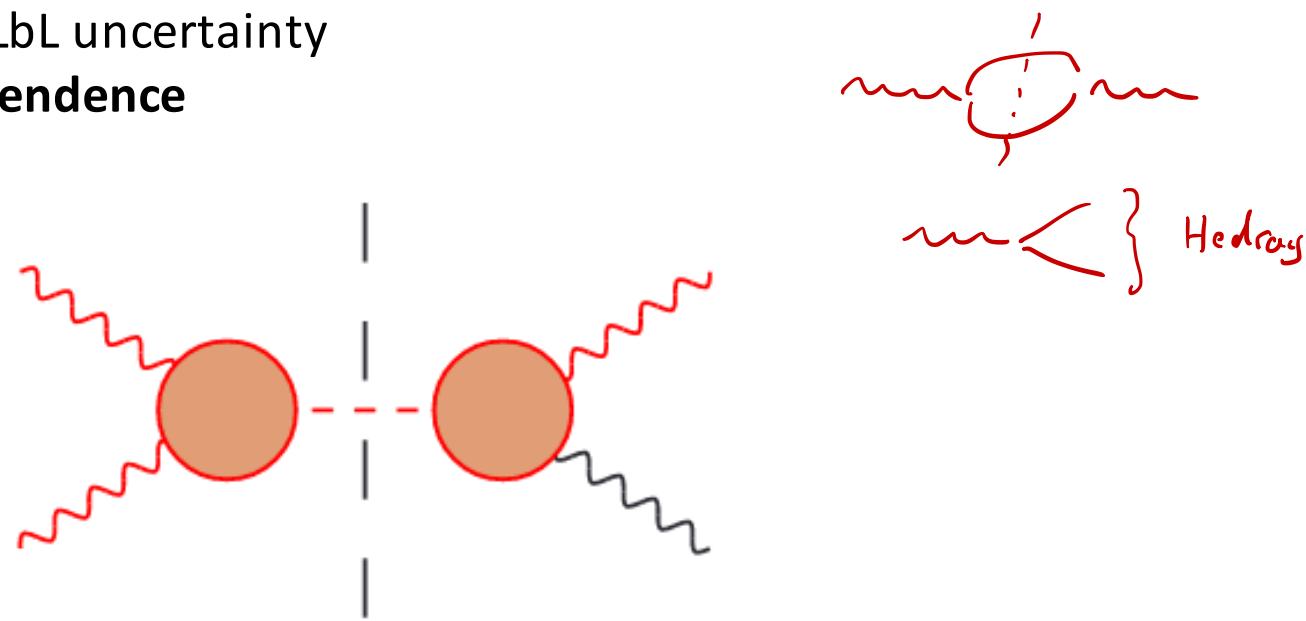
→ Need doubly virtual form factors of π^0, η, η' at low Q^2

Data-Driven Approaches $F(Q^2, 0)$

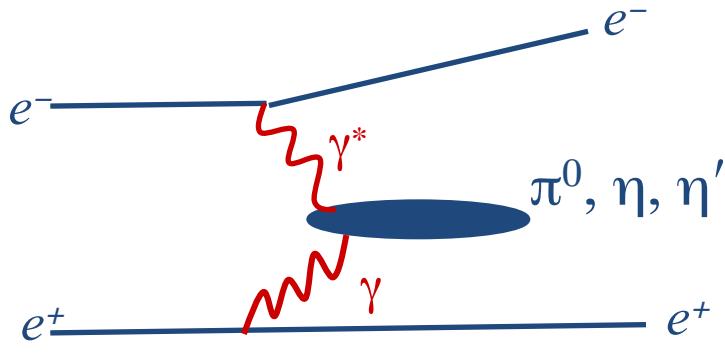
Dispersive Approaches for a_μ^{HLbL}

[Colangelo, Hoferichter, Kubis, ...2014, 2015]
 [Pauk, Vanderhaeghen, 2015, 2016]

- Describe dominating contributions with dispersion relations
- Remaining contributions calculated classically (model-dependent)
- Relate to measurable quantities
- Relate singly-virtual TFFs with double-virtual TFFs
- Goal: 10 ... 20% of HLbL uncertainty
- **Reduced model-dependence**



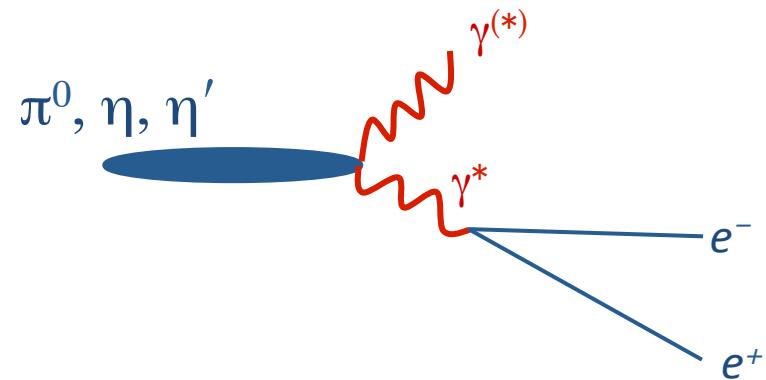
Transition Form Factors



Spacelike Measurement

at BES III: $Q^2 = 0.3 - 3.1 \text{ GeV}^2$

no previous precision data



Timelike Measurement

$$Q^2 < M_{\text{Meson}}^2$$



Meson Transition Form Factors at e+e- Colliders

Spacelike FFs $\gamma \gamma^* \rightarrow P$

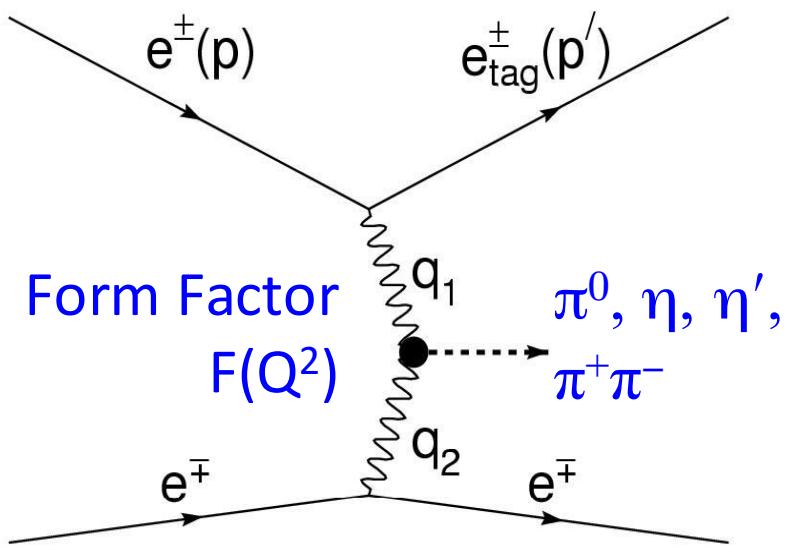
Single Tag Method

Selection criteria

- 1 electron (positron) detected
 - 1 positron (electron) along beam axis
 - Meson fully reconstructed
- **cut on angle of missing momentum**

Momentum transfer

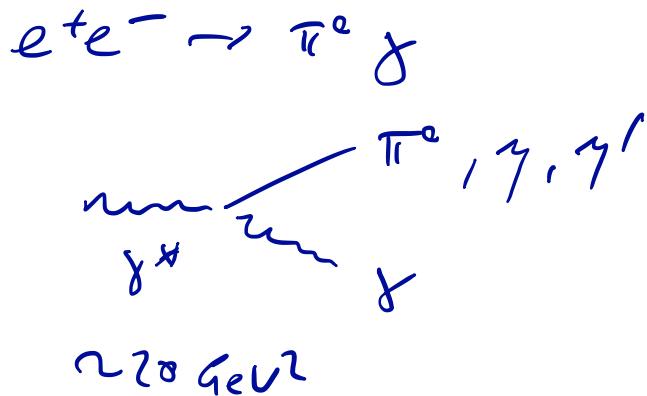
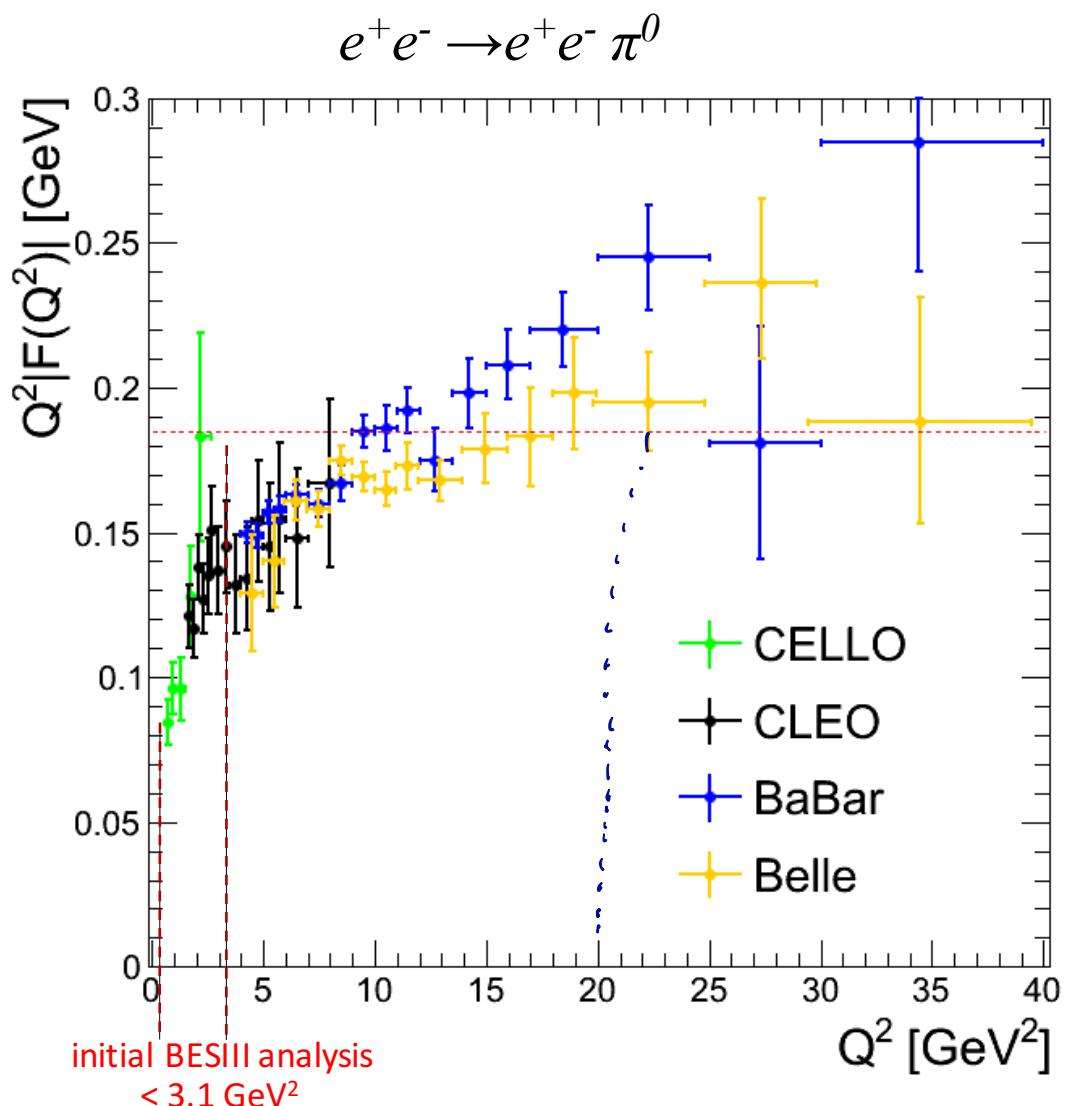
- tagged: $Q^2 = -q_1^2 = -(p - p')^2$
→ Highly virtual photon
- untagged: $q^2 = -q_2^2 \sim 0 \text{ GeV}^2$
→ Quasi-real photon



$$Q^2 = 4 \cdot E \cdot E' \cdot \sin^2(\theta/2)$$

EKHARA event generator
Czyż, Ivashyn

Existing Data on SL Transition FFs



Features:

- recent high- Q^2 data from BABAR and BELLE $Q^2 > 4 \text{ GeV}^2$
- above 1.5 GeV^2 data from CLEO
- below 1.5 GeV^2 data from CELLO

Meson Distribution Amplitudes

Meson Structure:

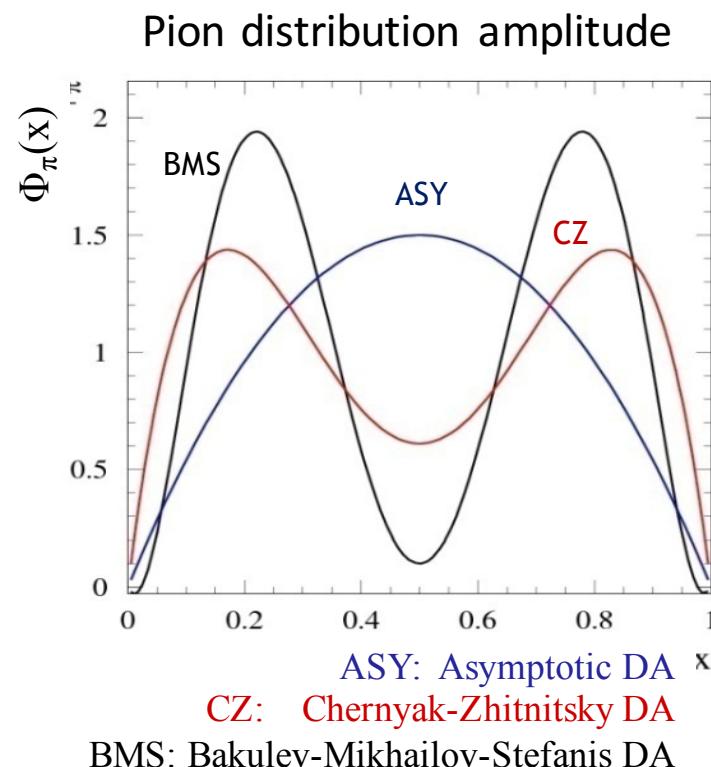
Transition form factors F give access to the meson distribution amplitudes (DA) $\Phi(x, Q^2)$ important for many QCD processes

(x : fraction of the meson momentum carried by one of the quarks)

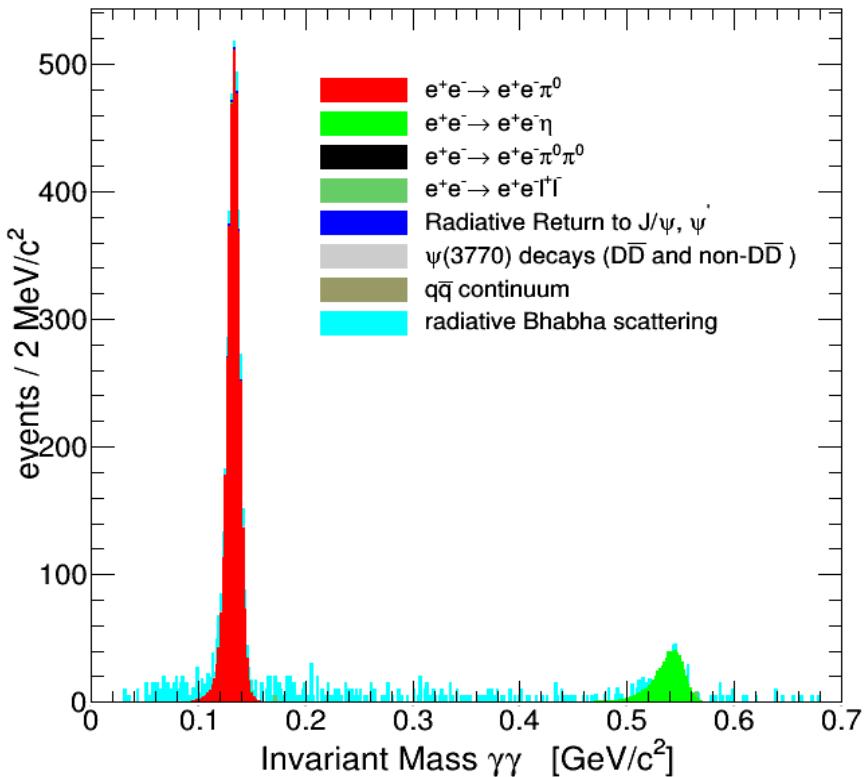
$$F(Q^2) = \int T(x, Q^2) \Phi(x, Q^2) dx$$

hard scattering amplitude
for $\gamma\gamma \rightarrow qq$

Nonperturbative pion
distribution amplitude DA
for $\gamma\gamma \rightarrow \pi/\eta^{(\prime)}$



BES III Analysis: $e^+e^- \rightarrow e^+e^-\pi^0$



Event Selection:

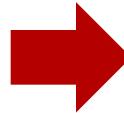
- exactly one lepton candidate
 - at least two, max four photons
 - Helicity angle $\cos \Theta_H > 0.8$
 - Kinematic cuts to reject ISR background
- **cut on angle of missing momentum**

Strategy:

Count
 π^0 yield in
bins of Q^2

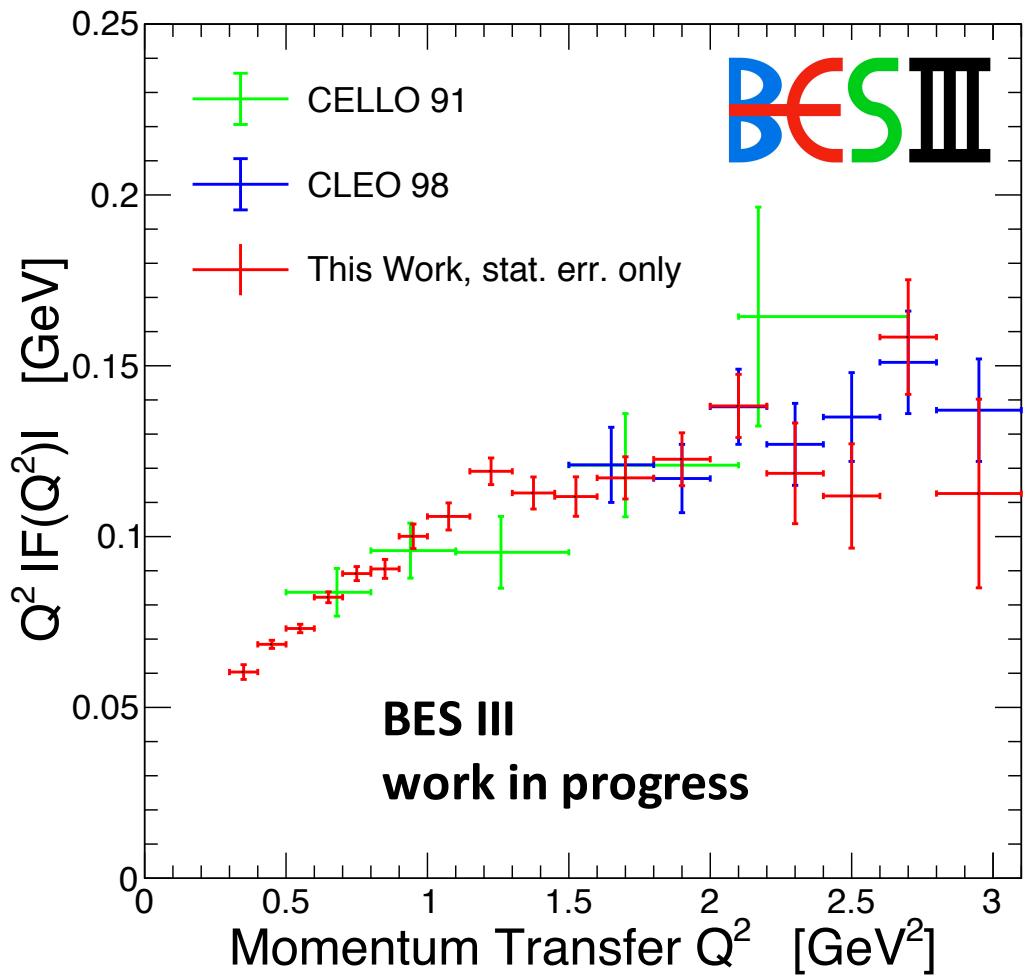


$d\sigma/dQ^2$



Form factor
 $F(Q^2)$

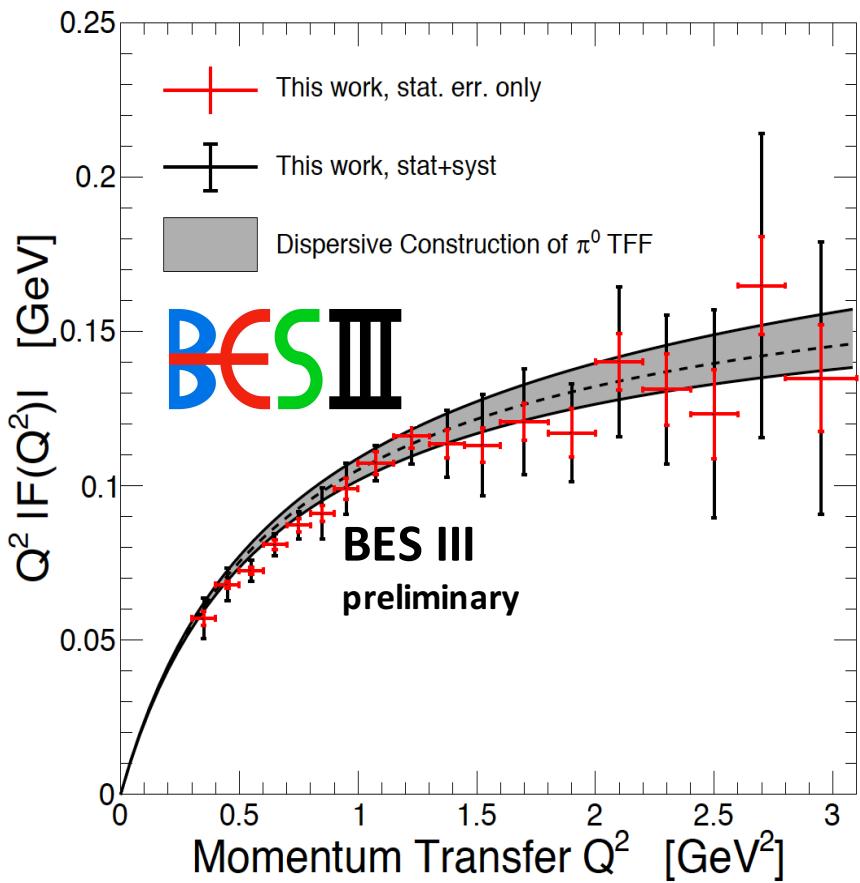
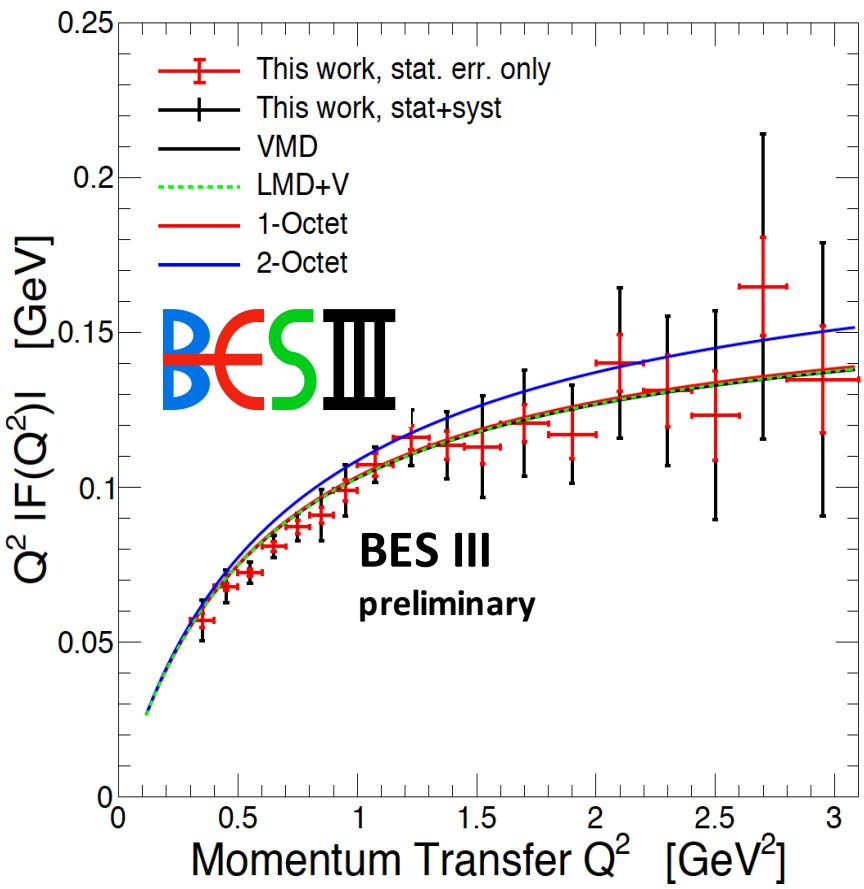
BES III Analysis: $\gamma\gamma^* \rightarrow \pi^0$



- $L_{int}: 2.92 \text{ fb}^{-1}$
 - Extract TFF for $0.3 \leq Q^2[\text{GeV}^2] \leq 3.1$
- Unprecedented Accuracy $Q^2 < 1.5 \text{ GeV}^2$**
- Challenges Hadronic Models**

BES III Analysis: $\gamma \gamma^* \rightarrow \pi^0$

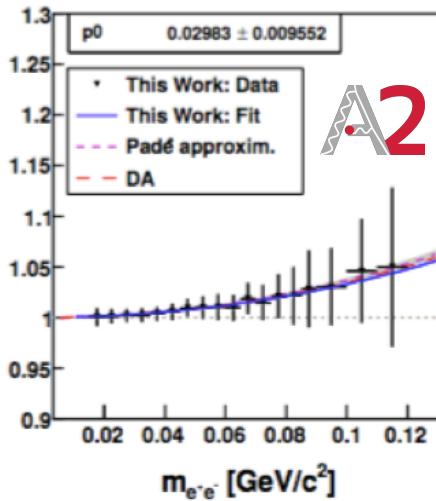
→ Challenges Hadronic Models



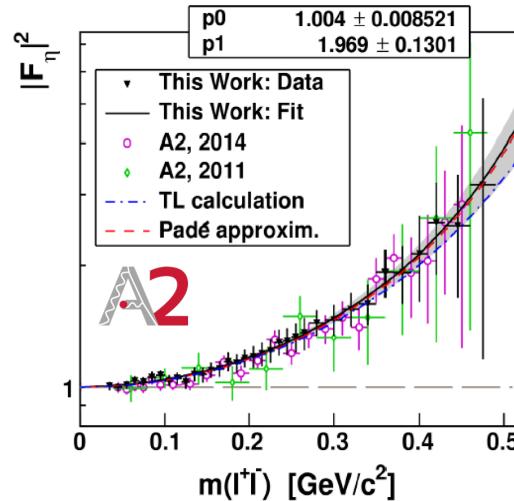
Further TFF Measurements

- Preliminary results for $\gamma\gamma^* \rightarrow \pi^+\pi^-$ from BES III expected soon
- Experiments (BABAR, BES III, BELLE II) embark on first measurements with **two virtual photons (double-tag)**
- **Timelike TFFs from meson decays** at various meson facilities worldwide (MAMI, COSY, JLAB, BES III, Frascati, BELLE II, CERN-SPS, ...)

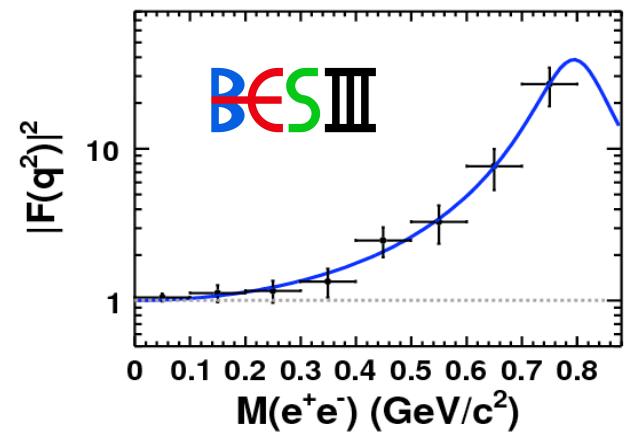
$$\pi^0 \rightarrow \gamma\gamma^*$$



$$\eta \rightarrow \gamma\gamma^*$$



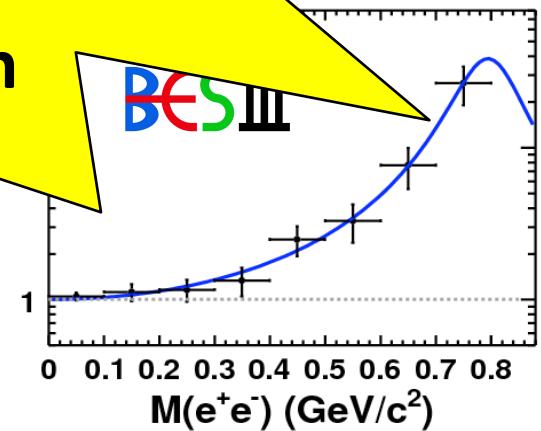
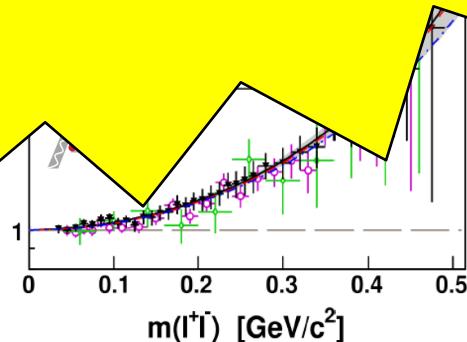
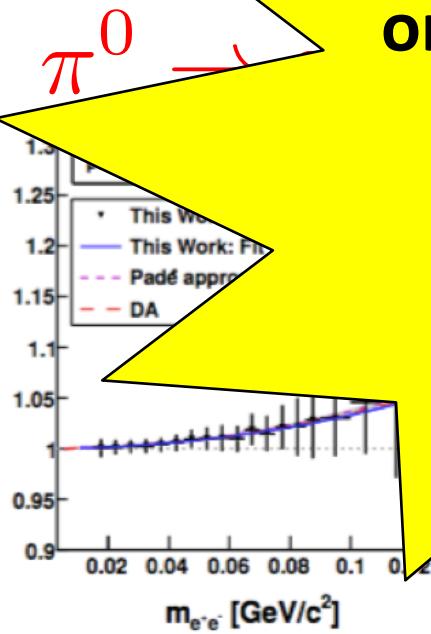
$$\eta' \rightarrow \gamma\gamma^*$$



Further TFF Measurements

- Preliminary results for $\gamma\gamma^* \rightarrow \pi^+ \pi^-$ from BES expected soon
- Experiments (BABAR, BES III, BELLE) with two virtual photons measureable
- Time-like form factors (MAMI, E615) measured

**Major experimental progress
on all fields in understanding
of transition form factors
for HLBL contribution**





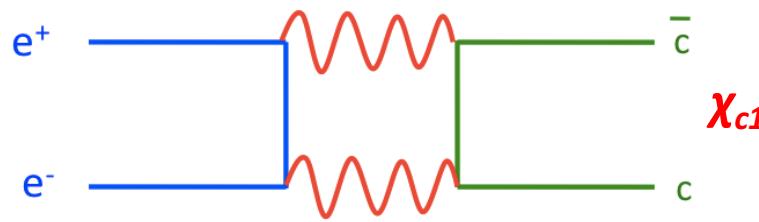
Production of

$J^{PC}=1^{++}$

State in e^+e^- Annihilation

Production $e^+e^- \rightarrow \chi_{c1}$

$$e^+ e^- \rightarrow 1^{++}$$

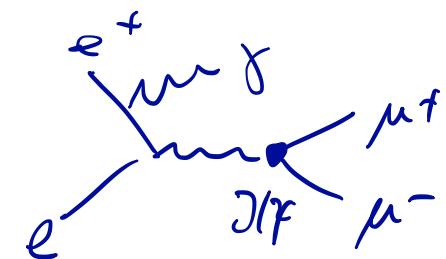


$$\begin{array}{c} \nearrow \\ \nearrow \end{array} \quad \begin{array}{c} S, C, \phi \\ \text{mn.} \end{array}$$

- **χ_{c1} Parameters:** Mass $\chi_{c1} = 3.5107$ GeV
Width $\chi_{c1} = 0.86$ MeV
Main decay channel (35% BR): $\chi_{c1} \rightarrow \gamma J/\Psi$

→ Signal process: $e^+e^- \rightarrow \gamma J/\psi \rightarrow \gamma \mu^+\mu^-$

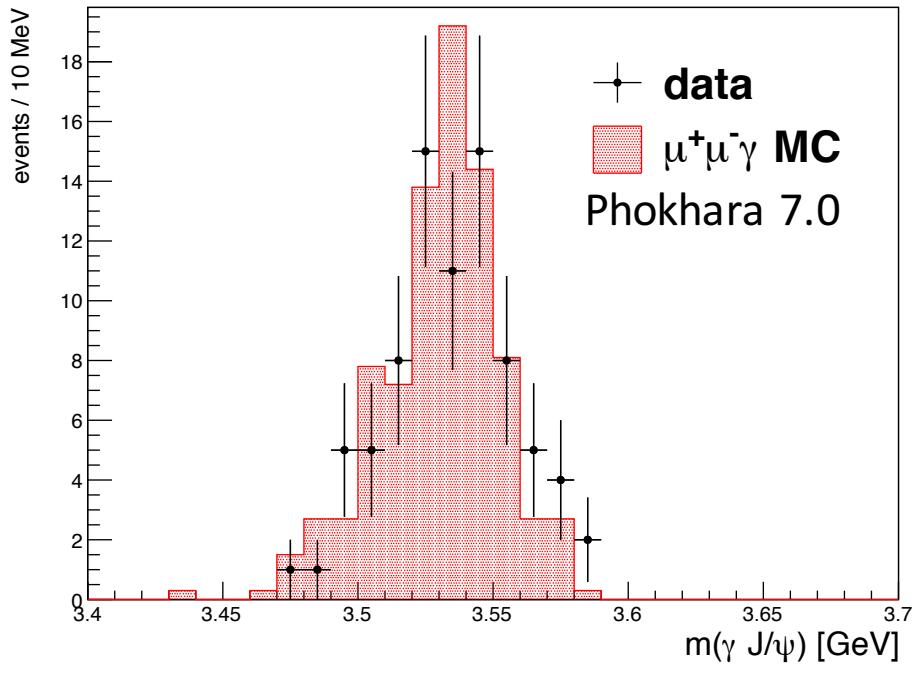
Irreducible background process: ISR production of J/ψ



ISR Background $e^+e^- \rightarrow \mu^+\mu^-\gamma_{ISR}$

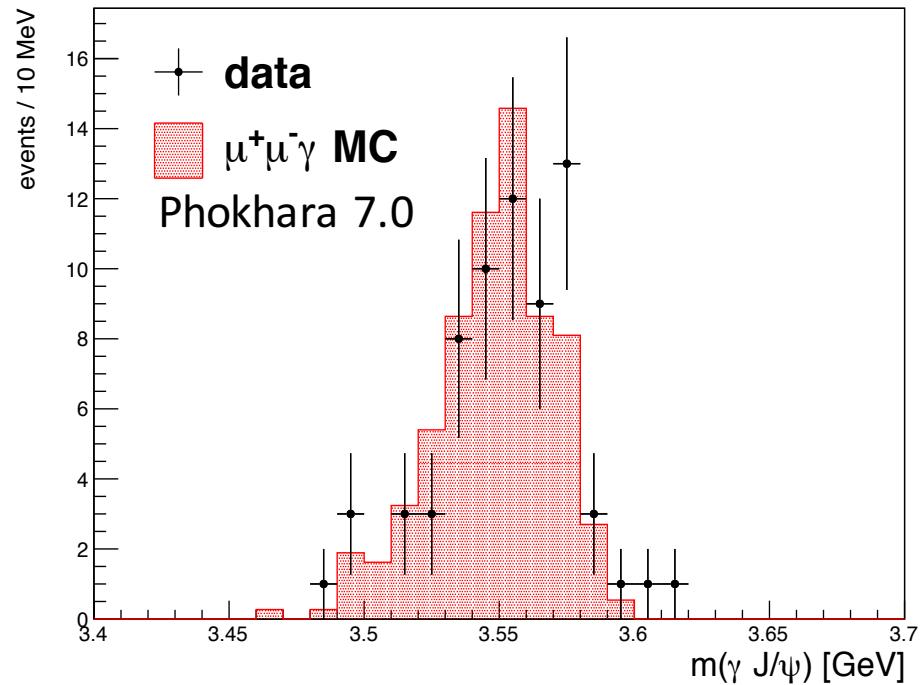
31 MeV above signal

$E_{cms} = 3.542$ GeV



50 MeV above signal

$E_{cms} = 3.561$ GeV

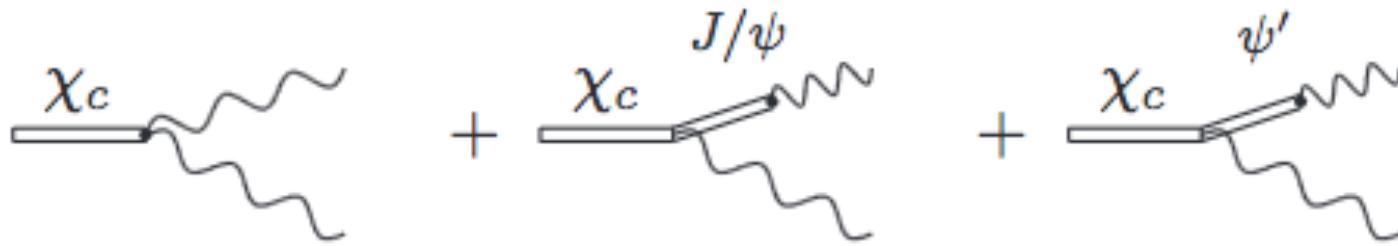


- excellent agreement, no indication of additional background
- effective cross section background 20.6 pb

Electronic Width of χ_{c1}

Czyz, Kühn, Trasc, arXiv:1605.06803

Interference effects lead to a value for Γ_{ee} for χ_{c1} of 0.41 eV



J. Kaplan, H.Kühn, PLB78 (1978) 252

Vector Meson Dominance (without ψ') predicts Γ_{ee} for χ_{c1} of 0.46 eV

N. Kivel and M. Vanderhaeghen, JHEP 02, 032 (2016)

Soft Collinear Theory Γ_{ee} for χ_{c1} is 0.09 eV

used for our
beam time
proposal

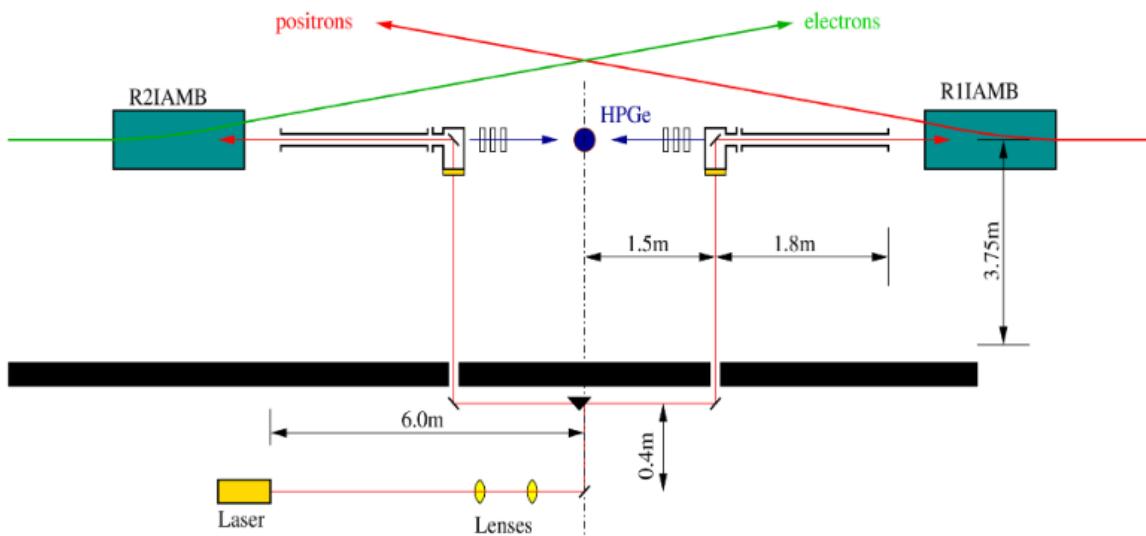
A.D., F.-K. Guo, C. Hanhart, A. Nefediev, PLB 736, 221 (2016)

Vector Meson Dominance (without ψ') predicts Γ_{ee} for χ_{c1} of 0.1 eV

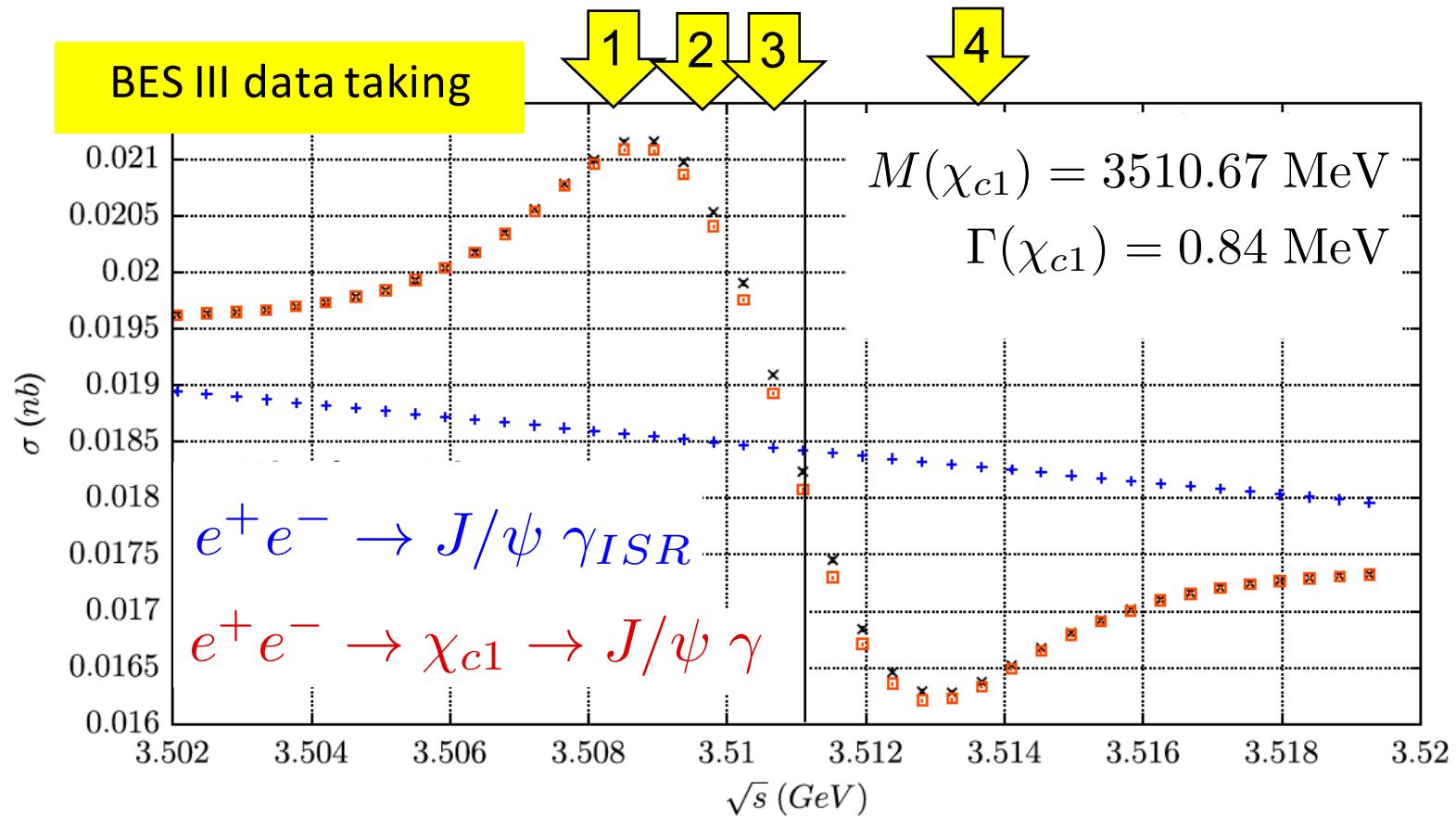
Data Taking Plan at BES III $e^+e^- \rightarrow \chi_{c1}$

3 weeks of data taking ($> 300 \text{ pb}^{-1}$)

- Potential to observe 5 sigma effect (assuming no phase!) Beam energy spread, ISR effects included in calculation
- Beam Energy Measurement System Compton Backscattering developed by Novosibirsk group $<< 10^{-4}$ accuracy



Signal Cross Section $e^+e^- \rightarrow \chi_{c1}$

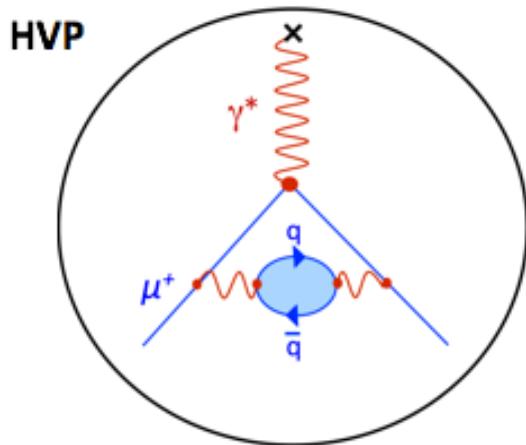


Effect of the resonance $\sim 10\%$



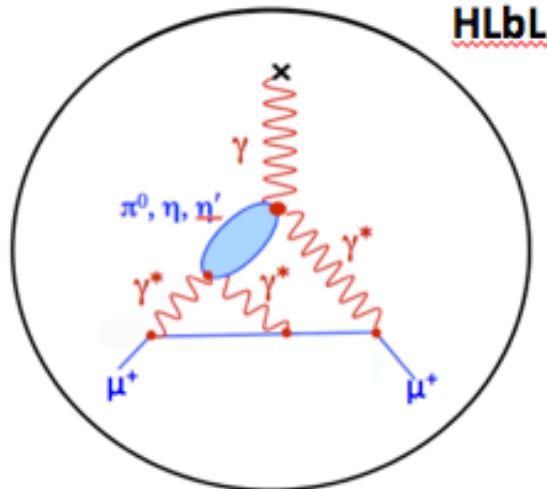
Conclusions and Outlook

Conclusions HVP



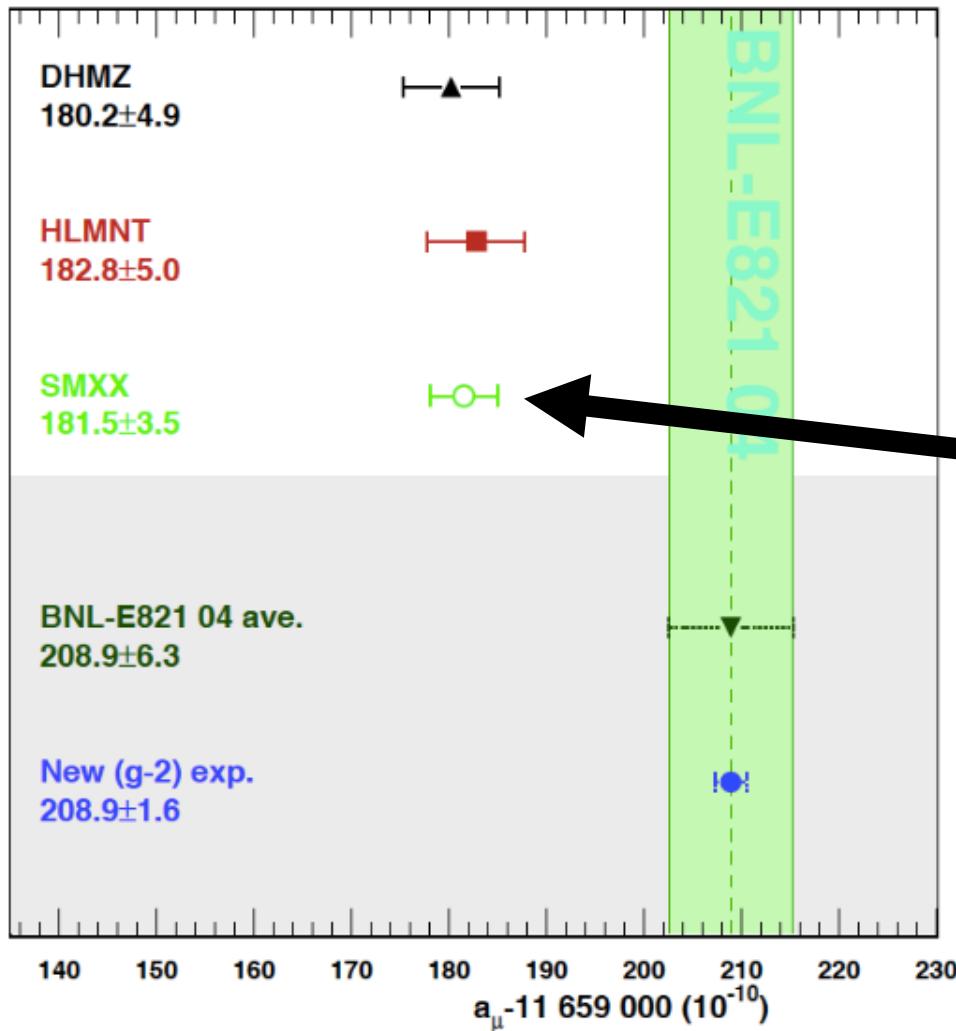
- Dispersion relation to relate hadronic cross section measurements to HVP contribution to $g-2$ of the muon
 - Energy scan experiments at Novosibirsk < 2 GeV and Beijing < 5 GeV
 - Initial State Radiation (ISR) at e^+e^- particle factories:
KLOE, BABAR, BES III, BELLE (II)
 - Extremely impressive progress in the field
-

Conclusions *HLbL*



- Up to some years considered as sub-leading hadronic contribution
→ not any more !
 - Hadronic models (VMD-like) used for calculation → uncertainty ?
 - Dispersion relations are now worked out: data-driven approach !
→ requires measurements of meson transition form factors $P \rightarrow \gamma\gamma$
 - Extremely impressive progress in the field
-

Conclusions



Expectation for the final reduction of SM uncertainty already now achieved

Whitepaper (2013)
arxiv:1311.2198



g-2

A phantastic field to work on