Lattice simulation of QCD: status report

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Outline:

- Introduction
- Anomalous magnetic moment of muon
- Spectroscopy
- QCD critical end point
- Equation of state
- Transport properties of QCD
- "Melting" of heavy quarkonium at finite temperature
- Spatial confinement/deconfinement phase transition
- Conclusion

selected topics from Lattice 2024 and Lattice 2025

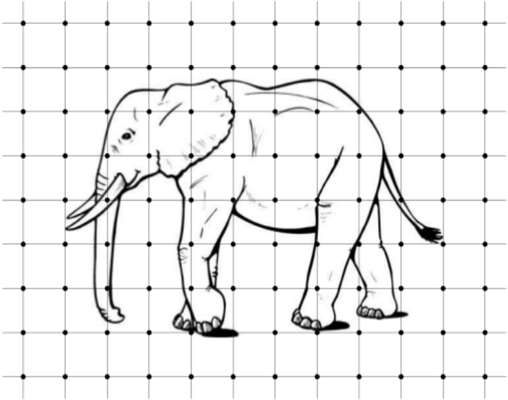
Theory of strong interactions (QCD)

- Degrees of freedom
  - Quarks q
  - Gluons A
- The QCD Lagrangian is well known

$$L = -\frac{1}{4}\sum_{a=1}^{8} F_a^{\mu\nu} F^a_{\mu\nu} + \sum_{f=u,d,s,\dots} \bar{q}_f (i\gamma^\mu \partial_\mu - m) q_f + g \sum_{f=1}^{N_f} \bar{q}_f \gamma^\mu \hat{A}_\mu q_f$$

- Non-linear equations of motion with $g \sim 1$
- The main problem: calculation of observables based on the QCD Lagrangian (Millennium problem)
- Theoretical approaches contain assumptions with systematic errors which are difficult to estimate

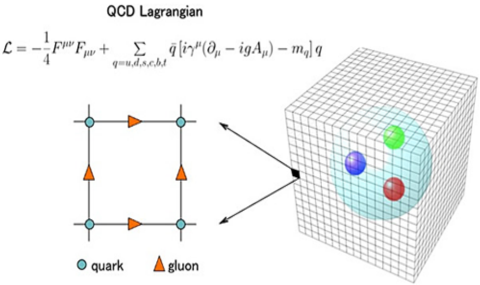
Lattice QCD



Lattice simulation

- Allows to study strongly interacting systems
- Based on the first principles of quantum field theory
- Powerful due to modern supercomputers and algorithms

Building lattice QCD



- Introduce regular cubic four dimensional lattice $N_s \times N_s \times N_s \times N_t = N_s^3 \times N_t$
- Lattice spacing–a
- Degrees of freedom
  - **Gluon fields:** 3x3 matrices $U \in SU(3)$, live on links
  - **Quarks fields:** column $q, \bar{q}$, live on sites

Lattice QCD

- We study QCD in thermodynamic equilibrium
- QCD partition function

$$Z = Sp[e^{-\hat{H}_{QCD}/T}] = \int DU \exp\left(-S_G(U)\right) \times \prod_{i=u,d,s\dots} \det\left(\hat{D}_i(U) + m_i\right)$$

- In continuum lattice partition function exactly reproduces QCD partition function
    - Gluon contribution: $S_G(U)\Big|_{a\to 0} = -\frac{1}{4}\sum_{a=1}^{8} F_a^{\mu\nu} F_{\mu\nu}^a$
    - Quark contribution:
      $$\bar{q}(\hat{D}(U) + m)q\Big|_{a\to 0} = \bar{q}(\gamma^\mu \partial_\mu + ig\gamma^\mu A_\mu + m)q$$
- Carry out continuum extrapolation $a \to 0$
- Uncertainties (discretization and finite volume effects) can be systematically reduced
- The first principles based approach. No assumptions!
- Parameters: coupling constant $g(a)$ and masses of quarks $m_q(a)$

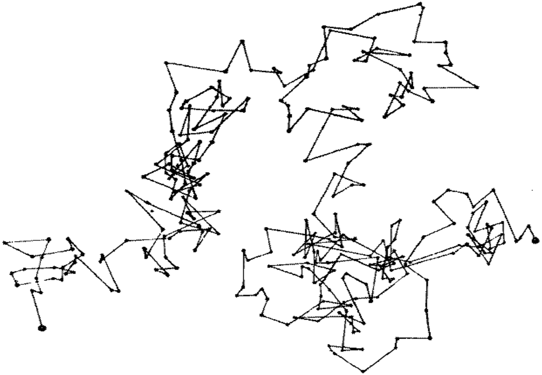
Lattice QCD

- We calculate partition function

  $$Z \sim \int DU e^{-S_G(U)} \prod_{i=u,d,s...} \det\left(\hat{D}_i(U) + m_i\right) = \int DU e^{-S_{eff}(U)}$$

  - 96×48^3
  - Variables: $96 \cdot 48^3 \cdot 4 \cdot 8 \sim 300 \cdot 10^6$
  - Matrices: $100 \cdot 10^6 \times 100 \cdot 10^6$
- Simulations of u, d, s, c, b quarks at physical masses and $a \sim 0.05$ fm
- Stochastic process: Hybrid Monte Carlo(HMC) generates $\{U_1\} \to \{U_2\} \to \{U_3\} \to ....$
- For sufficiently large n: $p(U) \sim e^{-S_{eff}(U)}$
- Calculation of observable $O(U)$
  $\langle O \rangle = \frac{1}{N} \sum_{i=1}^{N} O(U_i)$

Hybrid Monte Carlo algorithm



- HMC can be considered as Brownian motion of the system
- Accept/reject step at the end of the trajectory
  - if $S_{eff}(U_{n+1}) < S_{eff}(U_n)$ the U_{n+1} is accepted
  - otherwise U_{n+1} is accepted with $p \sim e^{-[S_{eff}(U_{n+1}) - S_{eff}(U_n)]}$
- Simulation of quantum system!
- For large number of the trajectories $p(U) \sim e^{-S_{eff}(U)}$

Muon g-2

- $\vec{\mu}_\mu = g_\mu \left(\frac{e}{2m_\mu}\right)\vec{s}$, $\quad a_\mu = \frac{g_\mu - 2}{2}$

- Experiment: $10^{10} a_\mu = 11659205.9(2.2)$ [FNAL'23]
- Theory: $10^{10} a_\mu = 11659181.0(4.3)$, [WP'20]

- Contributions to a_μ
  - QED: 11658471.8931(104)
  - EW: 15.36(10)
  - QCD: 693.7(4.3)

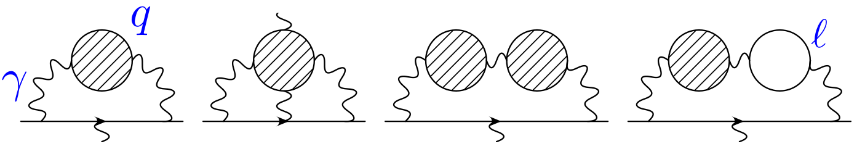


figure from Christine Davies, Lattice2024

Muon g-2: current status

- Tree-level Symanzik action
- $2 + 1 + 1$ staggered fermions
- Stout smearing
- $L \sim 6$ fm, $T \sim 9$ fm
- M_π and M_K are around physical point

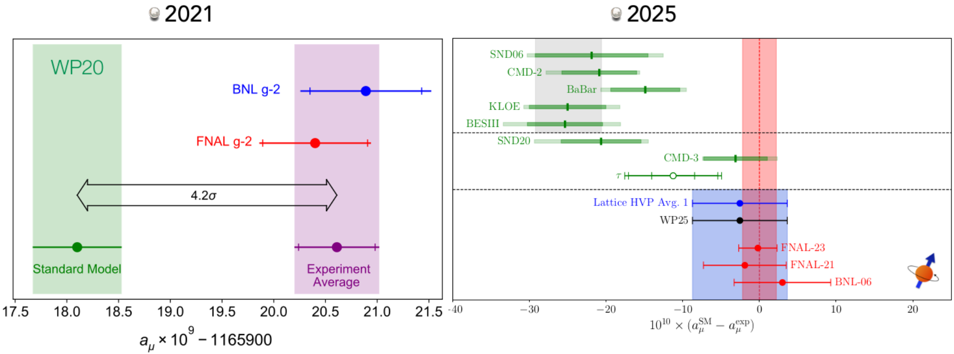
β	a/fm	$L \times T$	#conf
3.7000	0.1315	48×64	904
3.7500	0.1191	56×96	2072
3.7753	0.1116	56×84	1907
3.8400	0.0952	64×96	3139
3.9200	0.0787	80×128	4296
4.0126	0.0640	96×144	6980
4.1479	0.0483	128×192	4439

Ensembles for dynamical QED:

3.7000	0.1315	24×48	716
		48×64	300
3.7753	0.1116	28×56	887
3.8400	0.0952	32×64	4253

figure from from Finn M. Stokes, Lattice2025

Muon g-2: current status



Lattice results are in agreement with experiments!

figure from Aida El-Khadra, Lattice2025

Muon g-2: current status

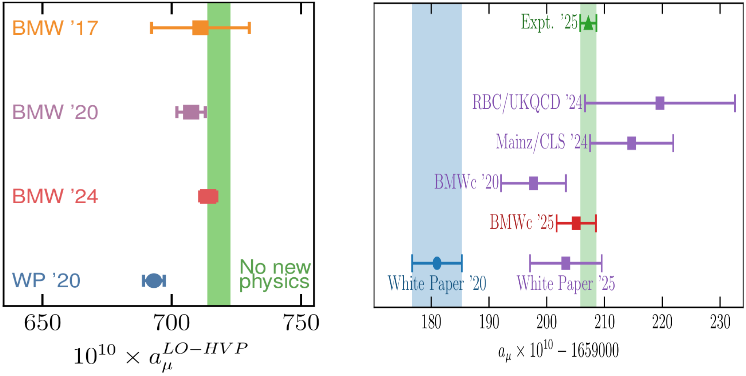


figure from Finn M. Stokes, Lattice2025

Spectroscopy: history

- Potential models
- "Relativistic potential models"
- QCD sum rules

  ...

Problems

- Systematic uncertainty
- Hadrons as a bound state of $q\bar{q}$, qqq
- Interaction potential?
- What about resonances?
  Higher states: $q\bar{q}g$, gg, $qq\bar{q}\bar{q}$,...?
- Satisfactory as the first approximation but not as an accurate study

Spectroscopy within lattice QCD has become accurate, reliable and engineering problem

Lattice spectroscopy

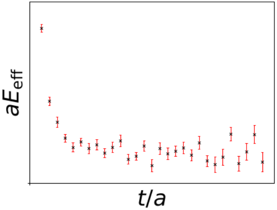
- For asymptotic states $(\pi, K, D, B, N...)$
  - Choose interpolator with correct quantum numbers

    for instance, for π meson $j_5 = \bar{d}\gamma_5 u$
  - Calculate the correlation function

    $C(\tau) = \int d^3x \langle 0|j_5(\tau, \vec{x}) j_5^+(0, \vec{0})|0\rangle$

    $C(\tau) = L^3 \sum_n e^{-E_n \tau} \langle 0|j_5(0, \vec{0})|n\rangle\langle n|j_5^+(0, \vec{0})|0\rangle$
  - Effective mass

    $aE_{eff} = \log \frac{C(\tau)}{C(\tau+1)}$
- For resonances: no such simple relation between finite-volume energy and resonance mass
- Finite-volume spectroscopy for $2 \to R \to 2$ resonances. Luscher method [R. Briceno'17]
- Finite-volume spectroscopy for $3 \to R \to 3$ resonances [M. Hansen'14]



Felix Erben, Lattice'24

Finite-volume spectroscopy

relevant interpolator basis

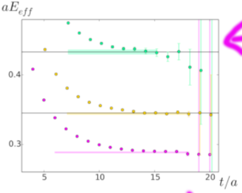
$$\rho(\vec{P},t) = \sum_{\vec{x}} e^{-i\vec{P}\cdot\vec{x}}(\bar{u}\gamma_i u - \bar{d}\gamma_i d)$$

$$(\pi\pi)(\vec{P},t) = \pi^+(\vec{p}_1,t)\pi^-(\vec{p}_2,t) - \pi^-(\vec{p}_1,t)\pi^+(\vec{p}_2,t)$$

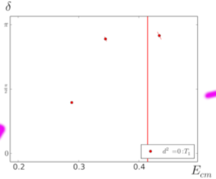
complete correlator matrix

$$\begin{pmatrix} \langle\rho(t)\rho^\dagger(t_0)\rangle & \langle\rho(t)(\pi\pi)_1^\dagger(t_0)\rangle & \langle\rho(t)(\pi\pi)_2^\dagger(t_0)\rangle & \cdots \\ \langle(\pi\pi)_1(t)\rho^\dagger(t_0)\rangle & \langle(\pi\pi)_1(t)(\pi\pi)_1^\dagger(t_0)\rangle & \langle(\pi\pi)_1(t)(\pi\pi)_2^\dagger(t_0)\rangle & \cdots \\ \langle(\pi\pi)_2(t)\rho^\dagger(t_0)\rangle & \langle(\pi\pi)_2(t)(\pi\pi)_1^\dagger(t_0)\rangle & \langle(\pi\pi)_2(t)(\pi\pi)_2^\dagger(t_0)\rangle & \cdots \\ \vdots & \vdots & \vdots & \ddots \end{pmatrix}$$

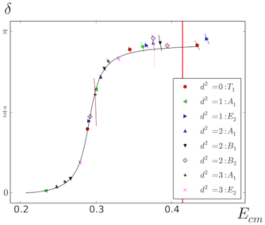
GEVP



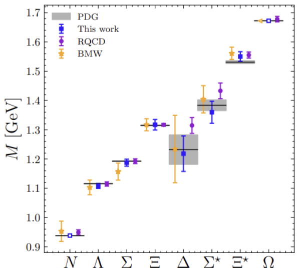
finite-volume formalism



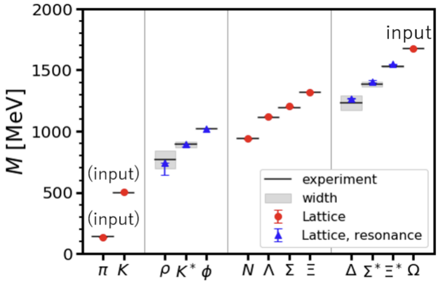
repeat for momenta, irreps



Spectroscopy: at physical quark masses

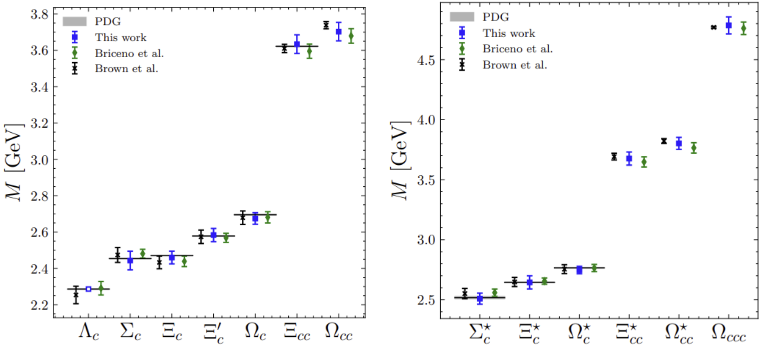


[C. Alexandrou'23]



[HAL QCD Collaboration'24]

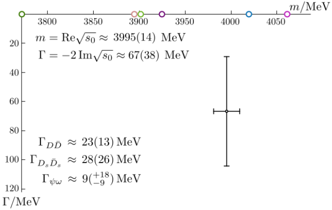
Spectroscopy: at physical quark masses

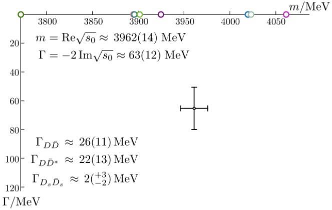


[C. Alexandrou'23]

Spectroscopy: state-of-the-art calculation

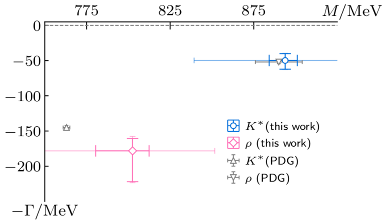
J^{PC}	hadron-hadron channels below $a_t E_{\mathsf{cm}} \approx 0.72$
0^{-+}	$\eta_c f_0$, $\chi_{c0}\eta$ $\{^1S_0\}$; $\psi\omega$, $D\bar{D}^*$, $D^*\bar{D}^*$ $\{^3P_0\}$; $\chi_{c2}\eta\{^5D_0\}$;
0^{++}	$\eta_c\eta$, $D\bar{D}$, $\eta_c\eta'$, $D_s\bar{D}_s$, $\psi\omega$, $D^*\bar{D}^*$, $\psi\phi$ $\{^1S_0\}$; $\chi_{c1}\eta$ $\{^3P_0\}$; $\psi\omega$, $D^*\bar{D}^*$, $\psi\phi$ $\{^5D_0\}$;
1^{-+}	$\eta_c\eta$, $\eta_c\eta'$, $\psi\omega$, $\psi\phi$ $\{^1P_1\}$; $\psi\omega$, $D\bar{D}^*$, $D^*\bar{D}^*$, $\psi\phi$ $\{^3P_1\}$; $\psi\omega$, $\psi\phi$ $\{^5P_1\}$; $\chi_{c1}\eta$ $\{^3S_1\}$; $\chi_{c1}\eta$ $\{^3D_1\}$; $\chi_{c2}\eta$ $\{^5D_1\}$;
1^{++}	$\psi\omega$, $D\bar{D}^*$ $\{^3S_1\}$; $\psi\omega$, $D\bar{D}^*$ $\{^3D_1\}$; $\psi\omega$, $D^*\bar{D}^*$ $\{^5D_1\}$; $\chi_{c0}\eta$, $\eta_c f_0$ $\{^1P_1\}$; $\chi_{c1}\eta$ $\{^3P_1\}$; $\chi_{c2}\eta$ $\{^5P_1\}$;
2^{-+}	$\psi\omega$, $D\bar{D}^*$, $D_s\bar{D}_s^*$, $\psi\phi$ $\{^3P_2\}$; $\chi_{c2}\eta$ $\{^5S_2\}$; $\chi_{c1}\eta$ $\{^3D_2\}$; $\chi_{c2}\eta$ $\{^5D_2\}$; $\eta_c f_0$ $\{^1D_2\}$
2^{++}	$\psi\omega$, $D^*\bar{D}^*$, $\psi\phi$, $D_s^*\bar{D}_s^*$ $\{^5S_2\}$; $\eta_c\eta$, $D\bar{D}$, $\eta_c\eta'$, $D_s\bar{D}_s$, $\psi\omega$, $D^*\bar{D}^*$, $\psi\phi$ $\{^1D_2\}$;
	$\psi\omega$, $D\bar{D}^*$, $D_s\bar{D}_s^*$, $D^*\bar{D}^*$, $\psi\phi$ $\{^3D_2\}$; $\chi_{c1}\eta$ $\{^3P_2\}$; $\chi_{c2}\eta$ $\{^5P_2\}$;
3^{-+}	$\psi\omega$, $\psi\phi$ $\{^5P_3\}$; $\chi_{c1}\eta$ $\{^3D_3\}$; $\chi_{c2}\eta$ $\{^5D_3\}$; $\eta_c\eta$, $\eta_c\eta'$, $\psi\omega$, $\psi\phi$ $\{^1F_3\}$; $\psi\omega$, $D\bar{D}^*$, $D^*\bar{D}^*$, $\psi\phi$ $\{^3F_3\}$; $\psi\omega$, $\psi\phi$ $\{^5F_3\}$;
3^{++}	$D\bar{D}^*$, $D_s\bar{D}_s^*$, $\psi\omega$, $\psi\phi$ $\{^3D_3\}$; $D^*\bar{D}^*$, $D_s^*\bar{D}_s^*$, $\psi\omega$, $\psi\phi$ $\{^5D_3\}$;
	$\eta_c f_0$ $\{^1F_3\}$; $\chi_{c2}\eta$ $\{^5P_3\}$; $\chi_{c0}\eta$ $\{^1F_3\}$; $\chi_{c1}\eta$ $\{^3F_3\}$; $\chi_{c2}\eta$ $\{^5F_3\}$;
4^{-+}	$\psi\omega$, $D\bar{D}^*$, $D^*\bar{D}^*$, $D_s\bar{D}_s^*$ $\{^3F_4\}$;
4^{++}	$\psi\omega$, $D^*\bar{D}^*$, $\psi\phi$, $D_s^*\bar{D}_s^*$ $\{^5D_4\}$; $\chi_{c1}\eta\{^3F_4\}$; $\chi_{c2}\eta\{^5F_4\}$;





[Hadron Spectrum Collaboration'24]

Spectroscopy: state-of-the-art calculation

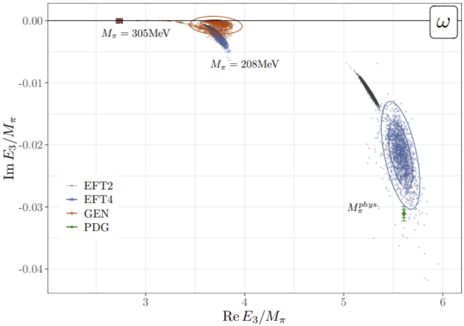


	this work [MeV]	PDG [MeV]
M_{K^*}	$893(2)_{\mathrm{stat}}(54)_{\mathrm{sys}}$	890(2)
$\Gamma_{K^*}/2$	$26(1)_{\mathrm{stat}}(6)_{\mathrm{sys}}$	25.6(1.2)
M_ρ	$796(5)_{\mathrm{stat}}(50)_{\mathrm{sys}}$	761 − 765
$\Gamma_\rho/2$	$96(5)_{\mathrm{stat}}(15)_{\mathrm{sys}}$	71 − 74

[Peter Boyle'24]

RBC/UKQCD 2 + 1 flavor, Mobius domain-wall, $a^{-1} = 1.7295(38)$ GeV, V = $48^3 \times 96$, Physical pion mass

Spectroscopy: state-of-the-art calculation

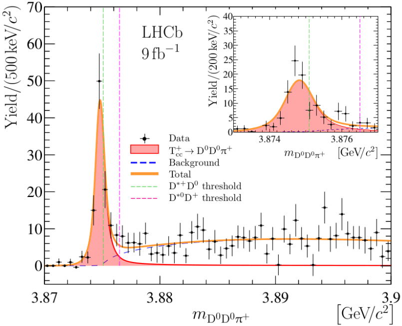


[Haobo Yan'24]

- Acount of $\pi\pi \to \pi\pi$ and $\pi\pi\pi \to \pi\pi\pi$
- Two pion masses and extrapolation to physical mass
- One lattice spacing: $a = 0.07746(18)$ fm
- $\sqrt{s}_\rho = 778.0(11.2) - i3.0(5)$ MeV
  PDG $m = 782.66(0.13)$, $\Gamma = 8.68(13)$

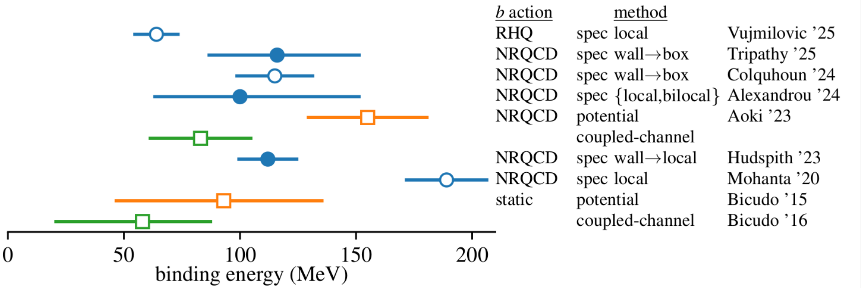
Spectroscopy: tetraquark $T_{cc\bar{u}\bar{d}}$

- $I(J^P) = 0(1^+)$ three body resonance
- $\delta = -273(61)$ KeV below $D^{*+}D^0$
- $\delta\Gamma = 440(165)$ KeV
- $DD^* - D^*D^*$ scattering: Virtual state
  [Hadron Spectrum Collaboration'25]
- Account of local $cc\bar{u}\bar{d}$ operators
  Resonance with pole at
  $\delta E = -5.2^{+0.7}_{-0.8} - i6.3^{+2.4}_{-4.8}$
  [S. Prelovsek et al'25'25]
- **More studies is required**



[LHCb Collaboration'22]

Spectroscopy: tetraquark $T_{bb\bar{u}\bar{d}}$



Jeremy R. Green, Lattice'25

- Open symbols: single lattice spacing or single pion mass
- Consensus: deeply bound state

Spectroscopy: Roper Resonance

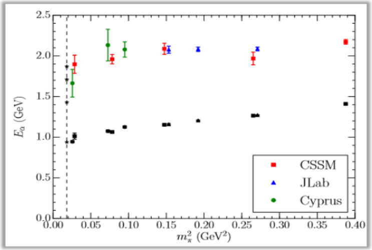
$N(1440)\ 1/2^+$ $I(J^P) = \frac{1}{2}(\frac{1}{2}^+)$

Re(pole position) = 1360 to 1380 (≈ 1370) MeV
−2Im(pole position) = 180 to 205 (≈ 190) MeV
Breit-Wigner mass = 1410 to 1470 (≈ 1440) MeV
Breit-Wigner full width = 250 to 450 (≈ 350) MeV

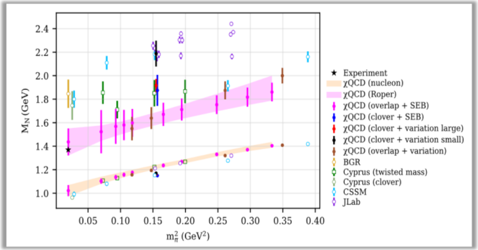
***N*(1440) DECAY MODES**	Fraction (Γ_i/Γ)	p (MeV/c)
$N\pi$	55–75 %	398
$N\eta$	<1 %	†
$N\pi\pi$	17–50 %	347
$\Delta(1232)\pi$, P-wave	6–27 %	147
$N\sigma$	11–23 %	–
$p\gamma$, helicity=1/2	0.035–0.048 %	414
$n\gamma$, helicity=1/2	0.02–0.04 %	413

Aim: understanding the first excited state of proton

Spectroscopy: Roper Resonance



D. Leinweber, NSTAR 2024
No presence of Roper state



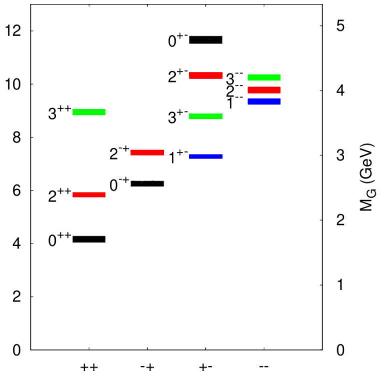
Importance of Chiral fermions:
χQCD (Sun et al). PRD 101, 054511 (2020)

N. Mathur, Lattice'24

- Not all groups see Roper Resonance
- Reliable lattice study requires
  - πN, σN, $\pi\Delta$ and $N\pi\pi$
  - Fermions that respect chiral properties
  - Large volume $>$ 5 fm
- **Necessary calculations can be performed in the next 5 years**

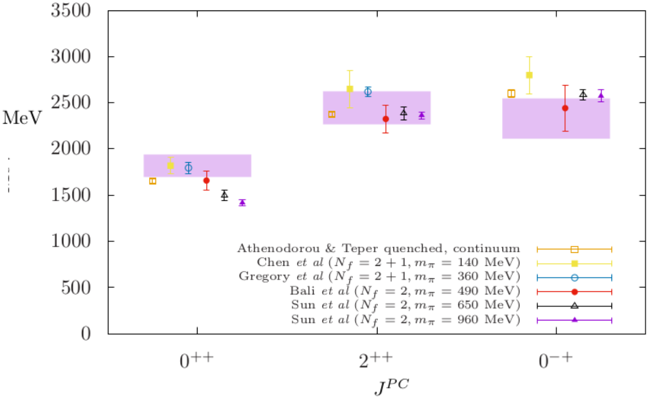
Spectroscopy: Glueballs

- Bound state of gluons with admixture of quark-antiquark states
- Masses are well studied only in gluodynamics ("real" glueballs): the lightest ~ 1700 MeV
- Poor knowledge of properties: no rigorous predictions on decay patterns and their branching ratios
- Difficult to identify. Information from lattice is required!
- Few candidates: $f_0(1370)$, $f_0(1500)$, $f_0(1710)$, $X(2370)$, ...



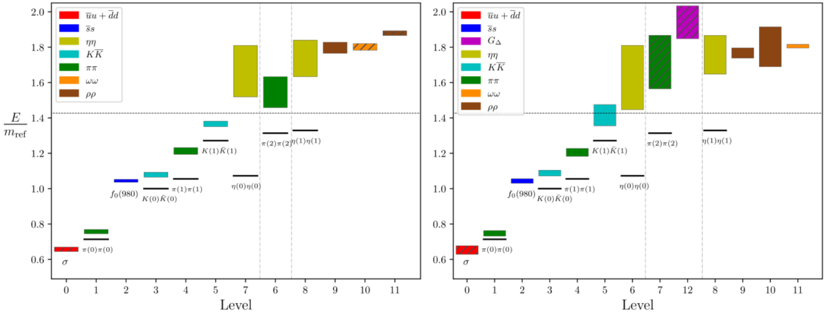
[Y. Chen et al.'05]

Spectroscopy: Glueballs



- Influence of quark loops on glueball spectrum [A. Athenodorou et al.'23]
- $N_f = 4$ and $m_\pi = 250$ MeV
- Only glueballs operators. No meson-meson operators!

Spectroscopy: Glueballs

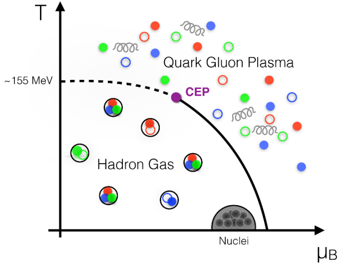


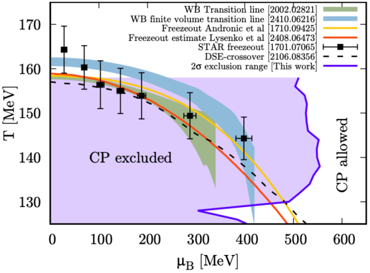
- Spectroscopy with and without scalas glueball [R. Brett(et al.'20]
  $N_f = 2+1$, $m_\pi = 390$ MeV
  scalar operators: 4 $\bar{q}q$ operators, 10 two-meson, 1 glueball
  without Luscher analysis
- Inclusion of scalar glueball does not influence the spectrum sigificanlty
- Mixing of 0^{-+} glueball and $q\bar{q}$ [X. Jiang et al.'23]
  $N_f = 2$, $m_\pi = 350$ MeV. Small mixing with quark sector $\sim 3.5\%$

A lot of work to be done to get better understanding

QCD critical end point

- HIC experiments are focused on CEP
- Crossover curve
  $\frac{T_c(\mu)}{T_c(0)} = 1 - 0.015(1)$ [H.T. Ding et al.'24]
- Contour of constant entropy:
  $(T_c^{CEP}, \mu_c^{CEP}) = (114(7), 602(62))$ MeV
  [H. Shah et al.'24]
- Lee-Yang zeros ($N_t = 6$):
  $(T_c^{CEP}, \mu_c^{CEP}) = (105^{+8}_{-18}, 422^{+80}_{-35})$ MeV
  [D. Clarke et al.'24]
- Taylor coefficients up to $O(\mu^8)$ ($N_t = 8$)
  $(T_c^{CEP}, \mu_c^{CEP}) \simeq (100, 580)$ MeV
  [G. Başar et al.'24]
- Phase diagram in space (T, μ_B, eB)
  $(T_c^{CEP}, \mu_c^{CEP}) = (100(25), 800(140)$ MeV
  [V.Braguta et al.'19]
- No CEP at $\mu_B < 450$ MeV at 2σ level
  [S. Borsányi et al.'25]
- The other point of view:
  transition is crossover [K. Fukushima'25]

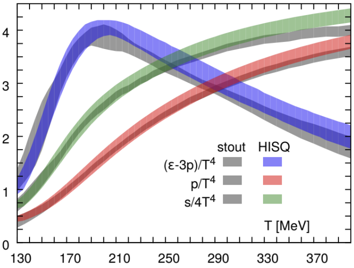


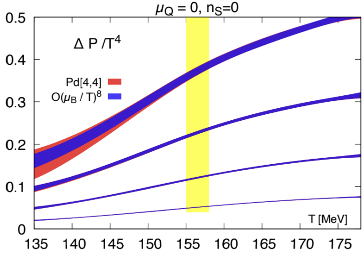


[S. Borsányi et al.'25]

QCD equation of state

- EoS at finite temperature
- EoS at $n_B \neq 0$ (Taylor expansion)
- EoS at finite magnetic field
- New trend:
  EoS of dense and manatized QGP
- EoS at ultrahigh temperature



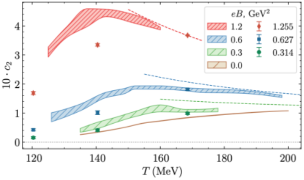


[F. Gross et al.'22]

EoS of dense and magnetized QCD

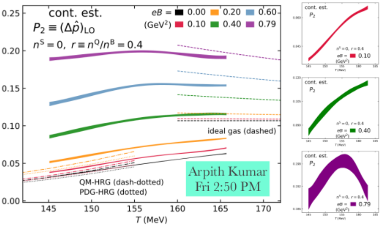
$$p(T,eB,\mu_B) = p(T,eB) + c_2(T,eB)\mu_B^2 + c_4(T,eB)\mu_B^2 + c_6(T,eB)\mu_B^6 + \ldots$$

Nf=2+1 QCD (Nt=8,10,12)
Analytical continuation, $\mu_S = 0$



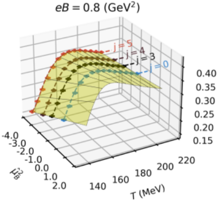
Astrakhantsev et al.,
Phys.Rev.D 109 (2024) 9, 094511

Nf=2+1 QCD (Nt=8 & 12)
Taylor expansion, strangness neutral



HTD, J.-B. Gu, A. Kumar, and S.-T. Li,
arXiv:2508.07532,2502.03152

Nf=2+1+1 QCD (Nt=8)
Imaginary μ_B with $\mu_S = 0$



S. Borsányi et al.,
arXiv:2502.01132, 2312.15118

QCD equation of state at ultrahigh temperature

Lattice set up

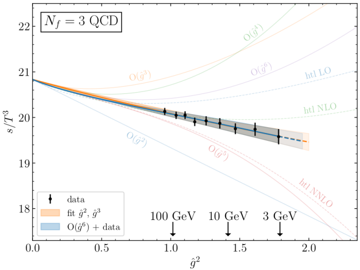
[M. Bresciani et al.'25]

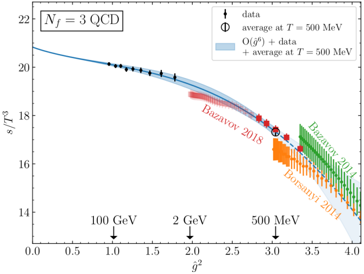
- $N_f = 3$ QCD
- Chiral limit
- 9 values of temperature in region $3 - 165$ GeV
- Reduced lattice artifacts: $O(a)$ - improved Wilson fermions
- Continuum limit extrapolation: $N_t = 4, 6, 8, 10$ $N_s = 144$

T	T (GeV)
T_0	164.6(5.6)
T_1	82.3(2.8)
T_2	51.4(1.7)
T_3	32.8(1.0)
T_4	20.63(63)
T_5	12.77(37)
T_6	8.03(22)
T_7	4.91(13)
T_8	3.040(78)

QCD equation of state at very high temperature

- Perturbative EoS:
  $\frac{s(T)}{T^3} = \frac{32\pi^2}{45}\left(\sum_{k=0} s_k \left(\frac{g}{2\pi}\right)^k\right)$
  $s_0 = 2.969, s_1 = 0, s_2 = -8.438,$
  $s_3 = 55.11, s_4 = -40.28 + 101.2\log g^2,$
  $s_5 = -1174,\ s_6 = 4791 - 1629\log g^2 + q_c$
  [K. Kajantie et al.'02]
- Enforcing exact SB limit
  $\frac{s(T)}{T^3} = \frac{32\pi^2}{45}\left(s_0 + s_2\left(\frac{g}{2\pi}\right)^2\right)$
  $s_0 = 2.954(15), s_2 = -3.6(7)$
- Fit with q_c and $s_7 \Rightarrow$
  Nonperturbative contribution from 3D Yang Mills is important - q_c
- PT is very poorly/slowly convergent





[M. Bresciani et al.'25]

Modelling heavy-ion collisions

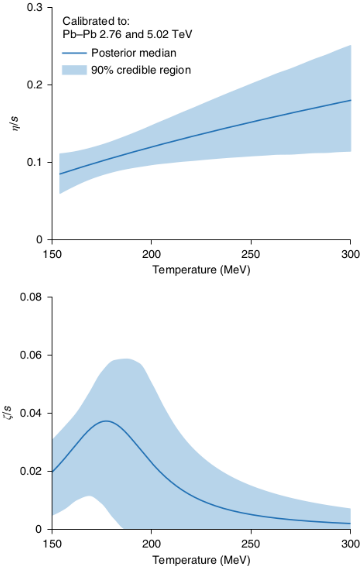
Heavy-ion collisions are modelled as a multi-stage evolution

- Initial conditions
- Pre-equilibrium stage
- Hydrodynamic evolution: relativistic viscous hydrodynamics (EoS and transport coefficients)
- Hadronization
- Hadronic transport

Transport coefficients:

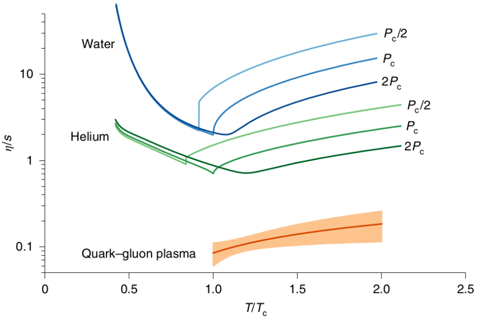
- Shear and bulk viscosities
- Diffusion coefficient
- Electromagnetic conductivity

  ...



[J. Bernhard et al.'19]

Shear viscosity of QGP



[J. Bernhard et al.'19]

- Perturbative QGP $\frac{\eta}{s} \sim 1$ [P. Arnold et al.'00]
- N=4 SYM $\frac{\eta}{s} = \frac{1}{4\pi} \sim 0.08$ [G. Policastro et al.'01]
- **QGP is the most ideal fluid!**

Transport properties on the lattice

- Transport coefficients $(\eta, \zeta, \sigma, D, ...)$ describe dissipative processes
- Related to Euclidean correlation functions due to linear response theory. Can be calculated in thermal equilibrium!
- For electromagnetic conductivity

$$C(\tau) = \int d^3x \langle j(\tau, \vec{x}) j(0, \vec{0}) \rangle$$

$$C(\tau) = \int \frac{\omega}{\pi} \rho(\omega) \frac{\cosh \omega (1/2T - \tau)}{\sinh(\omega/2T)}$$

$$\sigma = \lim_{\omega \to 0} \frac{\rho(\omega)}{\omega}$$

- On the lattice:
  - $O(10)$ points $C(\tau_i)$
  - The largest Euclidean time $\tau = \frac{1}{2T}$
  - Large perturbative contribution(shear and bulk viscosities)
  - Problems with Signal/Noise ratio

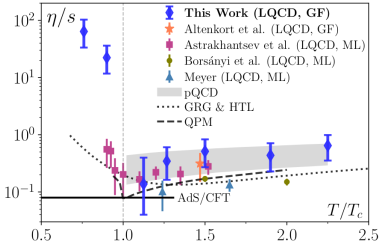
Shear and bulk viscosities in gluodynamics

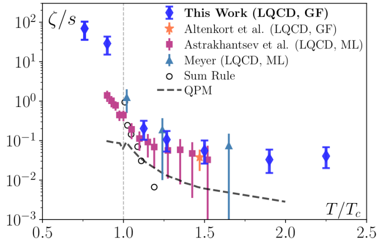
- Correlation function for viscosities $C(\tau) = \int d^3x \langle T_{12}(\tau, \vec{x}) T_{12}(0, \vec{0}) \rangle$
- Perturbative spectral function $\rho(\omega) \sim \omega^4$
- Huge perturbative contribution: 90% at $\tau = 1/2T$
- Rapid decrease of Signal/Noise ratio since $C(\tau) \sim 1/\tau^5$
- In gluodynamics problems can be solved due to multilevel algorithm
  [H. Meyer'02]
- No multilevel algorithm for fermions!

New study: [H.-T. Ding'26]

- Without multilevel algorithm
- Reached the accuracy 1% at $\tau = 1/2T$!

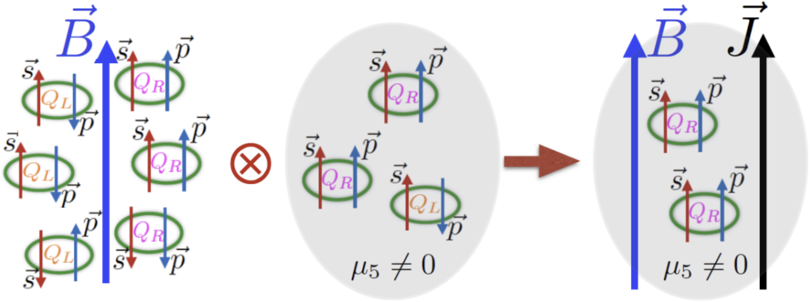
Ready to calculate viscosities with quarks!





[H.-T. Ding'26]

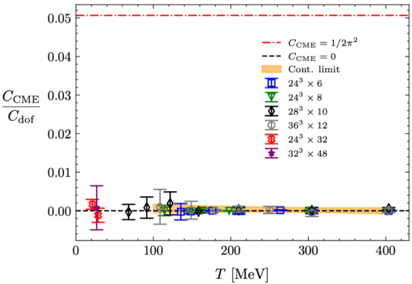
Chiral magnetic effect(CME)

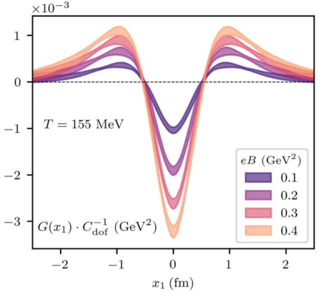


[D. Kharzeev'15]

- In HIC $\rho_5 = \rho_5(\mu_5)$ might be created
- CME: $\vec{J} = \frac{1}{2\pi^2}\mu_5 \times e\vec{B}$
- Related to chiral anomaly

CME in thermal equlibrium





[B. Brandt et al.'24, B. Brandt et al.'25] ,

- $\vec{J} = C_{CME} \times \mu_5 e\vec{B}$. Without interactions: $C_{CME} = \frac{1}{2\pi^2}$
- Homogeneous eB: $C_{CME} \simeq 0 \Rightarrow$ **no CME at thermal equilibrium**
- Inhomogeneous eB: spatially dependent current $\vec{J}$

Non-equilibrium CME

- QGP with $\vec{E}, \vec{B}$:

  $\frac{d\rho_5}{dt} = \frac{e^2}{4\pi^2}(\vec{E},\vec{B}) - \frac{\rho_5}{\tau}$

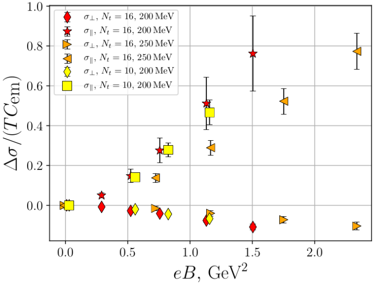
  $\rho_5 = \frac{e^2\tau}{4\pi^2}(\vec{E},\vec{B})$

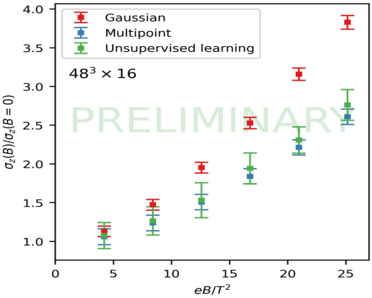
  $\rho_5 \sim \mu_5 B \Rightarrow \mu_5 \sim \frac{e^2\tau}{4\pi^2}\frac{(\vec{E},\vec{B})}{B}$

  $\vec{J} = \sigma\vec{E} + \frac{e^2}{2\pi^2}\vec{B}\times\mu_5$

  $\sigma_{\parallel}^{CME} \sim eB\tau$

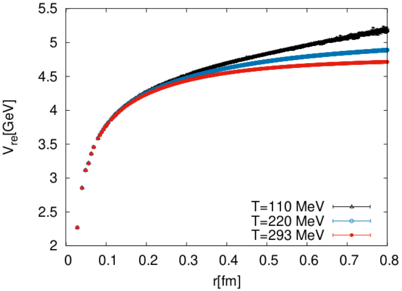
- **Manifestation of CME: rise of $\sigma_{\parallel}$ with B**
- Anomaly related quantum phenomenon (classically $\sigma_{\parallel}^{CME} = 0$)
- **Observed in experiment (Dirac semimetals)**
  [D. Kharzeev et al.'16] ,
- **Observed in lattice simulations**
  [N. Astrakhantsev'20, D. Bala, Lattice'25]

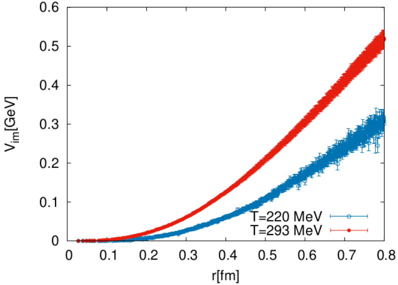




Static potential in hot QCD

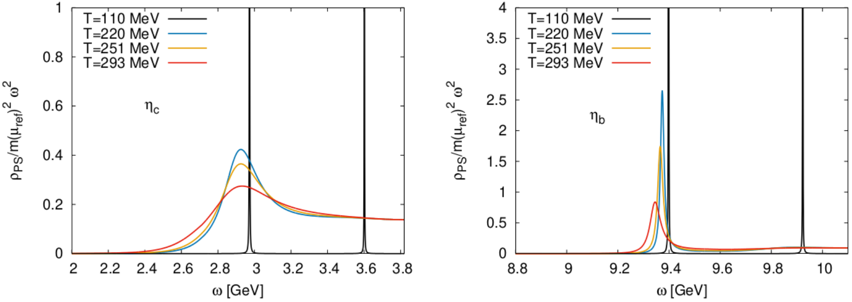
- Heavy quarkonium correlation function $C(t,\vec{r}) = \langle J_H(t,\vec{r}) J_H(0,\vec{0}) \rangle$
- $C(t,\vec{r})$ satisfies equation $\left(2M_Q - \frac{\vec{\nabla}^2}{M_Q} + V(\vec{r})\right) C(t,\vec{r}) = i\frac{\partial C(t,\vec{r})}{\partial t}$
- Perturbative potential
  $V(r) = -\alpha_s C_F \frac{e^{-m_D r}}{r} - i\alpha_s T C_F \varphi(m_D r)$
  [M. Laine'07]
- $ImV(r)$ desribes "melting" of heavy quarkonia at high temperatures
  $C(r,t) \sim e^{-ImV \cdot t}$
- $ReV(r)$ and $ImV(r)$ were calculated on lattice [H.-T. Ding'25]
- Determined the spectral density $\rho(\omega, T)$ from $C(t,\vec{r})$





[H.-T. Ding'25]

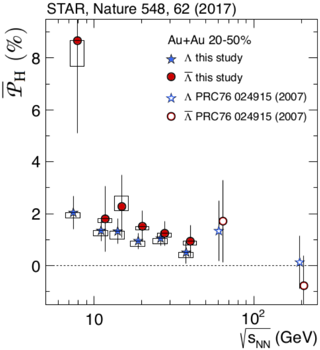
“Melting” of heavy quarkonium



[H.-T. Ding’25]

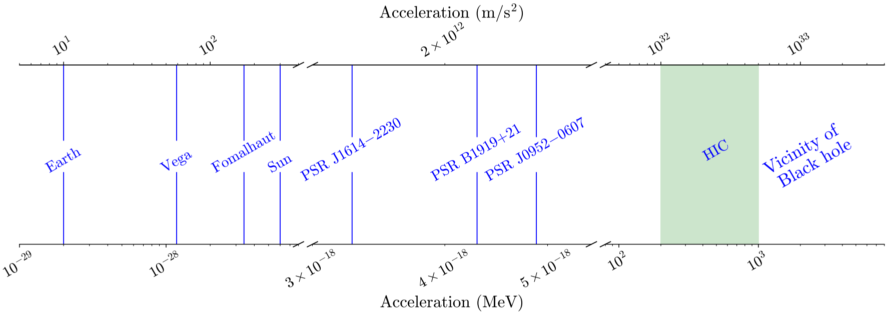
- Excitations of charmonium and bottomonium melt in deconfinement
- **Ground states of charmonium melt at $T \sim 300$ MeV**

The most vortical fluid ever observed



- Estimation of Ω from $\Lambda, \bar{\Lambda}$ polarization
- $\Omega \sim 10$ MeV or $\Omega \sim 10^{22}$ s^{-1}
  $v \sim c$ at distances $\sim 10 - 20$ fm
- **Relativistic rotation of QGP**

Acceleration in HIC



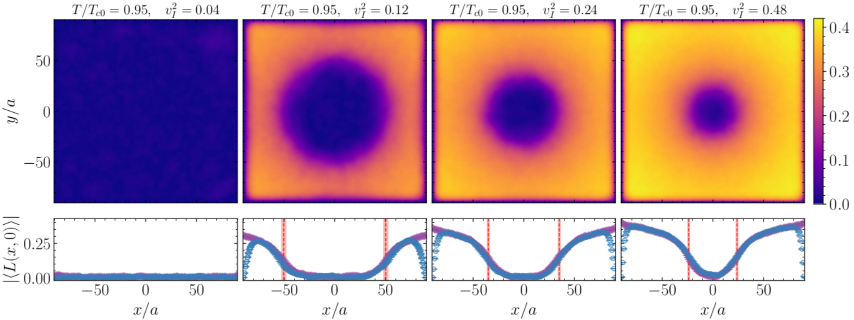
[V. Braguta'26]

- Characteristic accelerations in HIC: $\sim 0.1 - 1$ GeV

  [D. Kharzeev'05, G. Prokhorov'25]

- **One of the largest acceleration in universe!**

Spatial confinement/deconfinement transition



- Rotating gluodynamics Ω [V. Braguta'25 , V. Braguta'25,
- Accelerating gluodynamics [M. Chernodub'25, V. Braguta'26]
- **Observation of spatial confinement/deconfinement transition**

Conclusion:

- Lattice simulation is the most perspective approach for studying QCD
- A great number of new results were obtained within lattice simulation
- A lot of work to be done...

THANK YOU!