Швингеровские процессы. Новые идеи и результаты

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Outline of the talk

- Pair creation. A bit history and generalities
- Probability. Effective action approach
- Probability. Worldline instantons
- Nonperturbative decay of the particles. Proton decay in electric field
- Pair creation in holographic SYM and QCD
- Entanglement in the pair production
- Astrophysics and condmat
- Comparison with the false vacuum decay

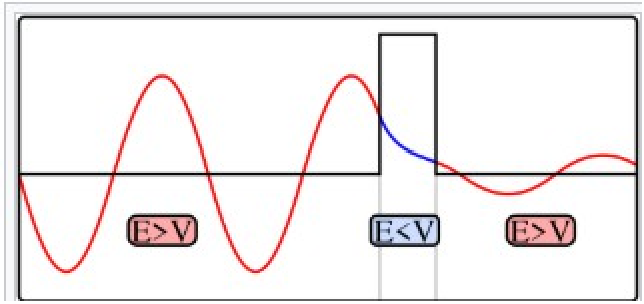
Some history

- Sauter 1932 vacuum is unstable
- Schwinger 1952 — effective action in the QFT electric field - probability evaluated
- Alvarez, Affleck, Manton -1982 worldline instanton approach
- Nikishov-Ritus 1988 «Stable» particles in external field are unstable. Proton is the example
- Holographic QCD — calculation of probability at strong coupling ~ since 2000

Tunneling in QM and QFT

Quantum mechanics. Tunneling through the barrier .

Energy splitting in two-wall potential



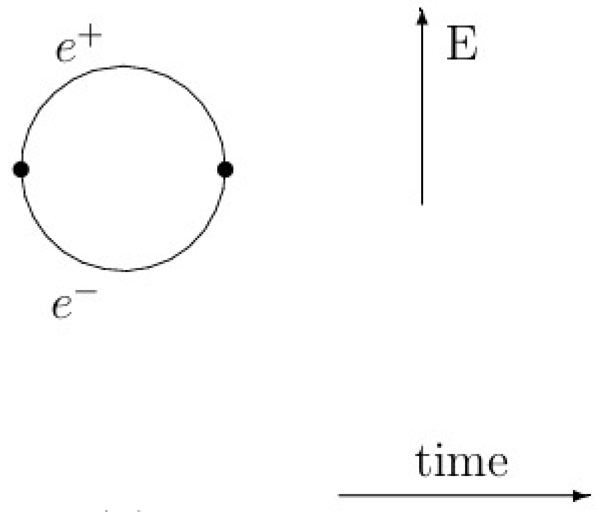
Quantum field theory. Tunneling of the whole system is impossible since infinite number degrees of freedom. Therefore the local tunneling takes place.

Two well-known tunneling phenomena in QFT.

Schwinger process in the external field. Different objects can be created Particles, strings, membranes, Universe etc

False vacuum decay

Simple physics

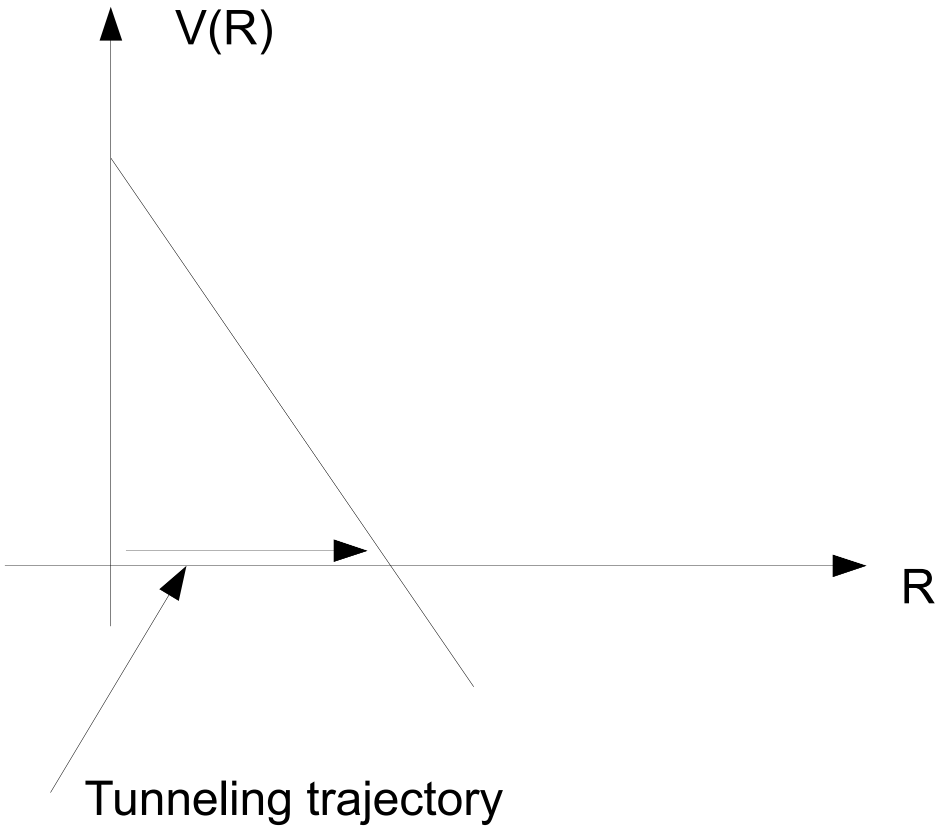


$$S_{eff} = mL - gEA$$

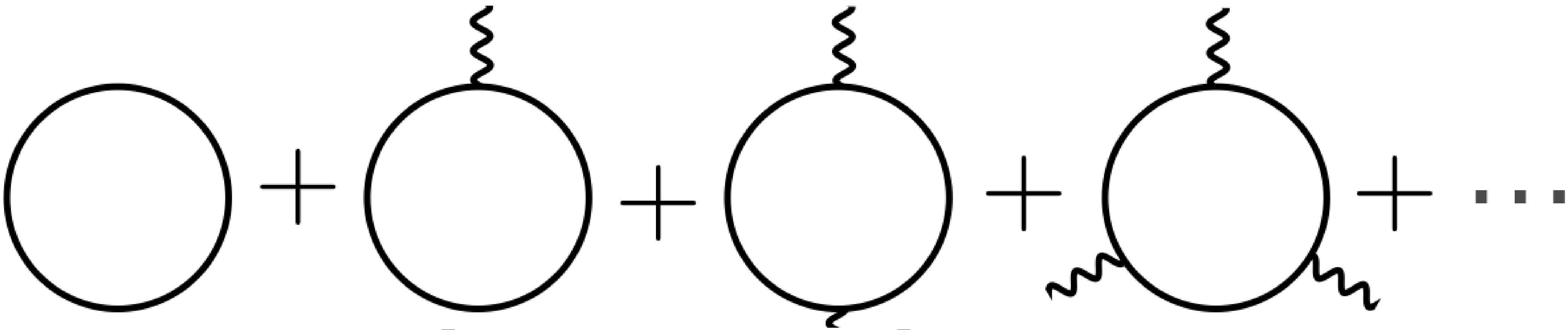
$$S_{eff} = 2\pi mR - \pi gER^2$$

$$R_{min} = m/(gE).$$

The energy required for pair production 2m
It is stolen from a electric field



Effective action in external field



$$\langle 0_{\rm out} \,|\, 0_{\rm in} \rangle = \exp\left[\frac{i}{\hbar} \left\{ {\rm Re}(\Gamma) + i\,{\rm Im}(\Gamma) \right\} \right]$$

$$P_{\rm production} = 1 - |\langle 0_{\rm out} \,|\, 0_{\rm in} \rangle|^2 = 1 - \exp\left[-\frac{2}{\hbar}\,{\rm Im}\,\Gamma \right] \approx \frac{2}{\hbar}\,{\rm Im}\,\Gamma$$

- Resummation of the whole series in the electric field is necessary.
- According to unitarity the imaginary part of the effective action yields the probability rate for the pair creation

Probability of the pair creation

$$\omega = \frac{(eE)^2}{4\pi^2} \sum_{n=1}^{\infty} \frac{1}{n^2} \exp\left(-\frac{n\pi m^2}{eE}\right)$$

fermions

$$Im\mathcal{L}^{(1)}_{\text{scalar}} \sim \frac{e^2E^2}{16\pi^3} \sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{n^2} \exp\left[-\frac{m^2\pi n}{eE}\right].$$

scalars

$$Im\mathcal{L}^{(1)}_{\text{spinor}} \sim \frac{e^2EB}{8\pi^2} \sum_{n=1}^{\infty} \frac{1}{n} \coth\left(\frac{B}{E}n\pi\right) \exp\left[-\frac{m^2\pi n}{eE}\right]$$

Parallel electric and magnetic fields

Probability is nonperturbative in the coupling constant !

Worldline instantons for pair production

Feynman's first quantized picture in terms of sum over trajectories

$$S = \int d\tau m\sqrt{\dot{x}^2} + i\oint A$$

$$m\frac{d}{d\tau}\left(\frac{\dot{x}_\mu}{\sqrt{\dot{x}^2}}\right) = iF_{\mu\nu}\dot{x}_\nu$$

Worldline instanton — solution to the Euclidean equations of motion

$$S = 2\pi r_0 M - q\pi r_0^2 E.$$

Electric field in Minkowski= Magnetic field In Euclid

$$r_0 = \frac{M}{qE}, \quad S = \frac{\pi M^2}{qE}.$$

Larmour circles=worldline instantons

$$\Gamma \sim e^{-S}.$$

Affleck, Manton, Alvarez 81',82'

Pair production in time-varying field

$$P \sim \exp\left[-\pi\, E_c\, g(\gamma)\right] \qquad \gamma \equiv \frac{m\omega}{eE}\ .$$

Keldysh approach for atom ionization 1964

Brezin-Itzykson 77
Popov-Marinov 78

$$E\, g(\gamma) = \frac{2}{\pi}\int_{-1}^{1}\sqrt{\frac{1-u^2}{1+\gamma^2 u^2}}\, du$$

$$= \frac{4}{\pi}\frac{\sqrt{1+\gamma^2}}{\gamma^2}\left[\mathbf{K}\left(\frac{\gamma^2}{1+\gamma^2}\right) - \mathbf{E}\left(\frac{\gamma^2}{1+\gamma^2}\right)\right]$$

$$\sim \begin{cases} 1-\frac{1}{8}\gamma^2 \quad , \quad \gamma \ll 1 \\ \frac{4}{\pi\gamma}\ln\gamma \quad , \quad \gamma \gg 1 \end{cases}$$

Possible interpolation between two regimes

$$P \sim \begin{cases} \exp\left[-\pi\, \frac{m^2 c^3}{eE\hbar}\right] \quad , \quad \gamma \ll 1 \quad \text{(nonperturbative)} \\ \left(\frac{eE}{\omega m c}\right)^{4mc^2/\hbar\omega} \quad , \quad \gamma \gg 1 \quad \text{(perturbative)} \end{cases}$$

Laser assisted pair production

$$\mathcal{E}_{\rm c} = \frac{m^2 c^3}{e\,\hbar} \approx 10^{16}\ \mathrm{V/cm}$$

$$\mathcal{E}_{\rm slow}(t) = \mathcal{E}\,\mathrm{sech}^2(\Omega t) \quad ; \quad \mathcal{E}_{\rm fast}(t) = \epsilon\,\mathrm{sech}^2(\omega t)$$

$$0 < \epsilon \ll \mathcal{E} \ll \mathcal{E}_c \quad ; \quad 0 < \Omega \ll \omega \ll m$$

Two laser beams with very different parameters. Surprizingly strong enhancement of the pair production!

Schutzhold, Gies, Dunne, Alekofer,Schubert
Monin-Voloshin, Popruzhenko, Narozhnyi

2008- now

Limit case — laser beam + constant field - Induced tunneling

Laser Induced process

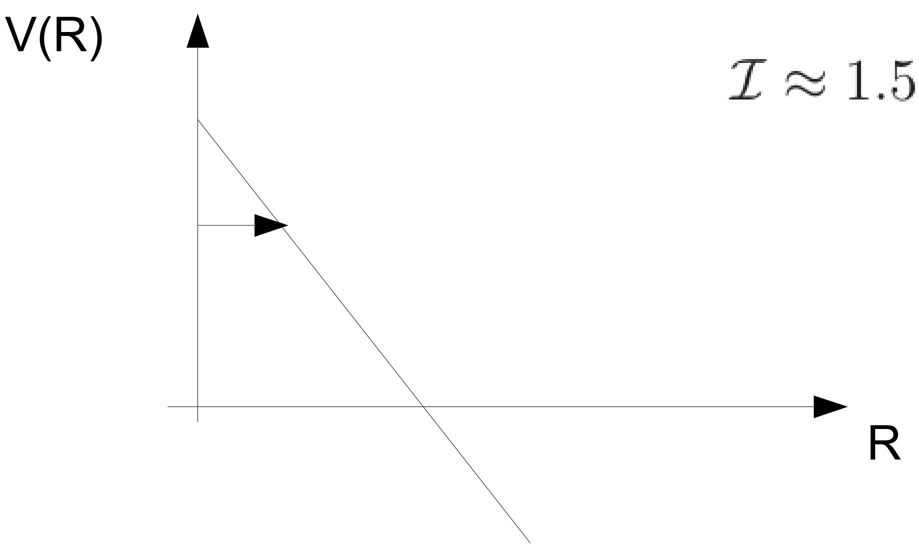
Review Fedotov, Idlerton et al (2023), Physics.Reports.

$$\mathcal{E}_{\rm cr} = \frac{m_{\rm e}^2 c^3}{e\hbar} = 1{,}32 \times 10^{16}\ \text{В см}^{-1}$$

Critical electric field for the Schwinger process

$$\mathcal{E}_0 \approx 10^{12}\ \ \text{В}\ \ \text{см}^{-1}$$

Electric field for modern lasers



$$\mathcal{I} \approx 1.5 \cdot 10^{27}\ \text{Вт}/\text{см}^2$$

At such intensity:
one shot-one pair

$$N_{e^+e^-} \approx \frac{R^2 L\tau}{\pi^3 l_C^4} \left(\frac{\mathcal{E}_0}{\mathcal{E}_{cr}}\right)^2 \exp\left(-\frac{\pi \mathcal{E}_{cr}}{\mathcal{E}_0}\right).$$

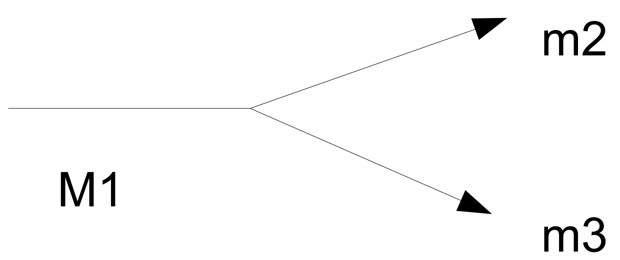
The preexponential factor can be huge.
Drastically depends on the effects
of the laser beam focusing .

Potentially can compensate exponent.

Decay of stable particles

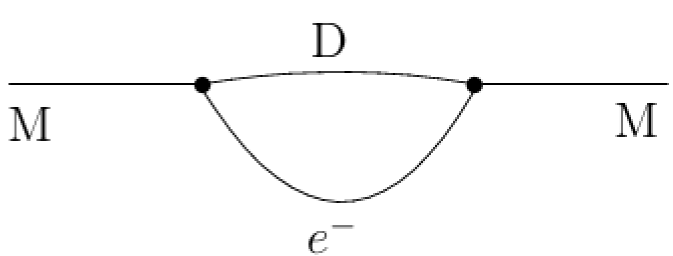
Idea — Ginsburg - 65 ?

Calculation of probability via the overlap of three exact wave functions in the external field Nikishov-Ritus 88'



M1< m2+m3

Particle is stable without the external electric field

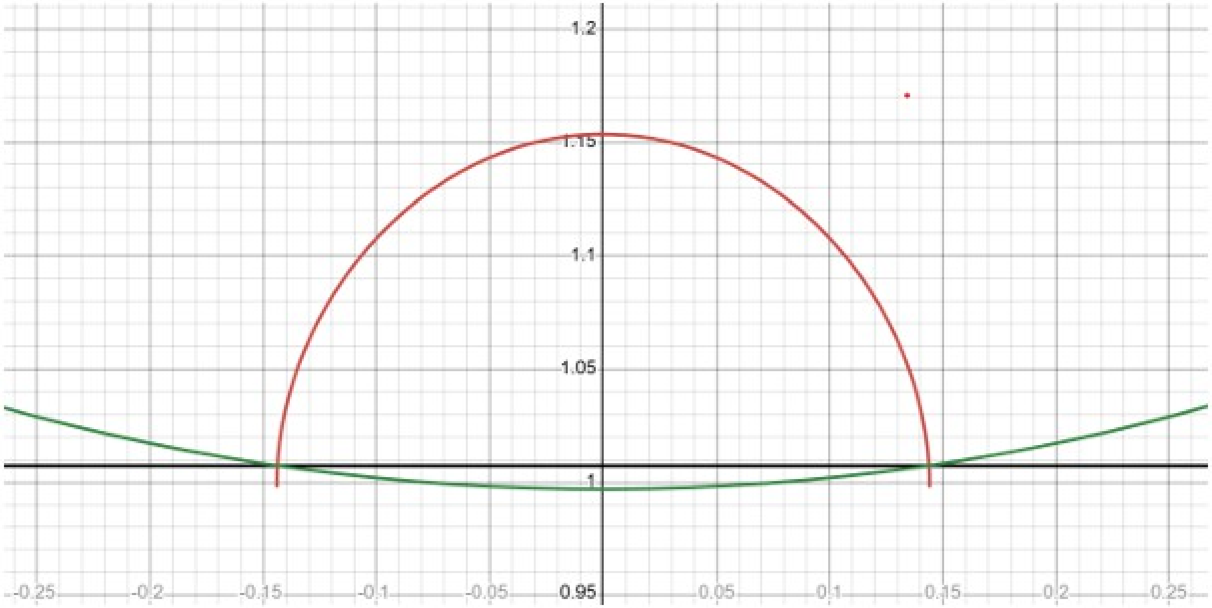


Composite worldline instantons.
Saraikin,Selivanov, A.G. 2001

Gluing several classical tunneling trajectories in the Euclidean space-time

Proton decay in electric field.

$p \to n\pi^+$



'igure 1: Euclidean time θ flows from left to right; ρ is plotted upward. The lack line corresponds to the trajectory of the initial particle, the red line to the harged daughter particle, and the green line to the neutral daughter particle.

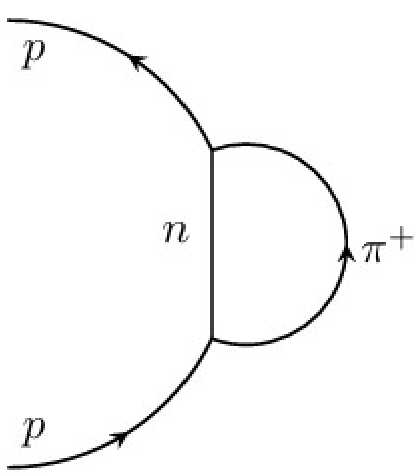
Composite wordline instanton   Poluboyarinov, A.G. To appear

Rindler coordinates. Proton is at rest before tunneling .

Proton decay in the electric field. Strong vertex

$$S_{\rm inst} = \frac{m_\pi^2}{eE}\arccos\left(\frac{m_p^2 - m_\pi^2 - m_n^2}{2m_\pi m_n}\right) - \frac{m_p^2}{eE}\arccos\left(\frac{m_p^2 + m_n^2 - m_\pi^2}{2m_p m_n}\right) + \\ + \frac{m_\pi m_n}{eE}\sqrt{1 - \left(\frac{m_\pi^2 + m_n^2 - m_p^2}{2m_\pi m_m}\right)^2}.$$

Leading exponential factor for the proton decay via strong vertex. The pion is heavier then electron hence much more stronger suppression in comparison with the conventional Schwinger process



Composite worldline instanton
In Euclidean space-time.

The results in Rindler and Euclid spacetime are the same

Proton decay in the electric field. Weak vertex

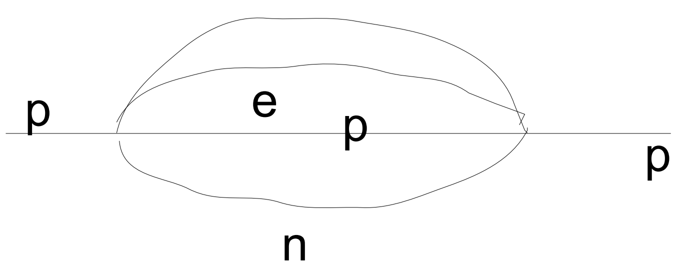
Idea has been suggested by R.Muller 97' with some model calculation

$$\mathbf{p} \rightarrow \mathbf{n} \mathbf{e}^+ \nu_e \text{ and } p\bar{\nu}_e \rightarrow n e^+$$

We performed two new evaluations of probability

- via composite worldline instanton similar to the previous slides

- imaginary part of dressed propagator of proton in the electric field



Two different calculations yield the same answer.
Poluboyarinov, A.G. To appear

Weak proton decay

$$P \propto \exp(-(\frac{\pi m_e^2}{eE} + \frac{2m_e(m_n - m_p)}{eE}))$$

Small preexponent via the
weak coupling constant

The exponential suppression is of the
same
order as in e+ e- pair production due to
small mass difference of proton and
neutron

The particles are entangled in the final state! They carry the information about the spin and momenta correlations at the tunneling time

Non-perturbative creation of specific states in electric fields

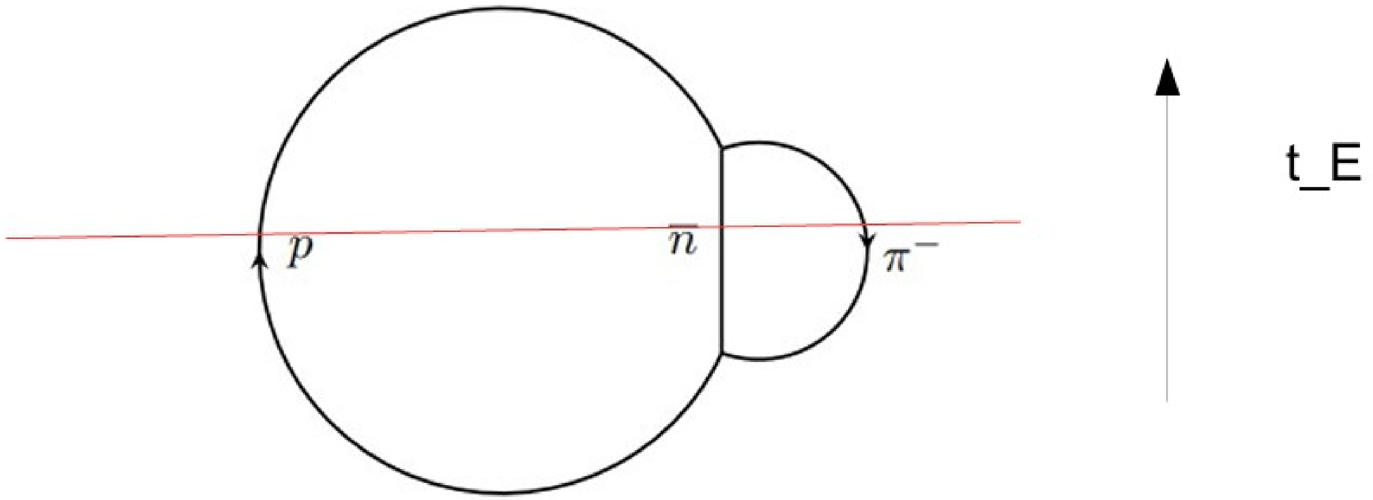
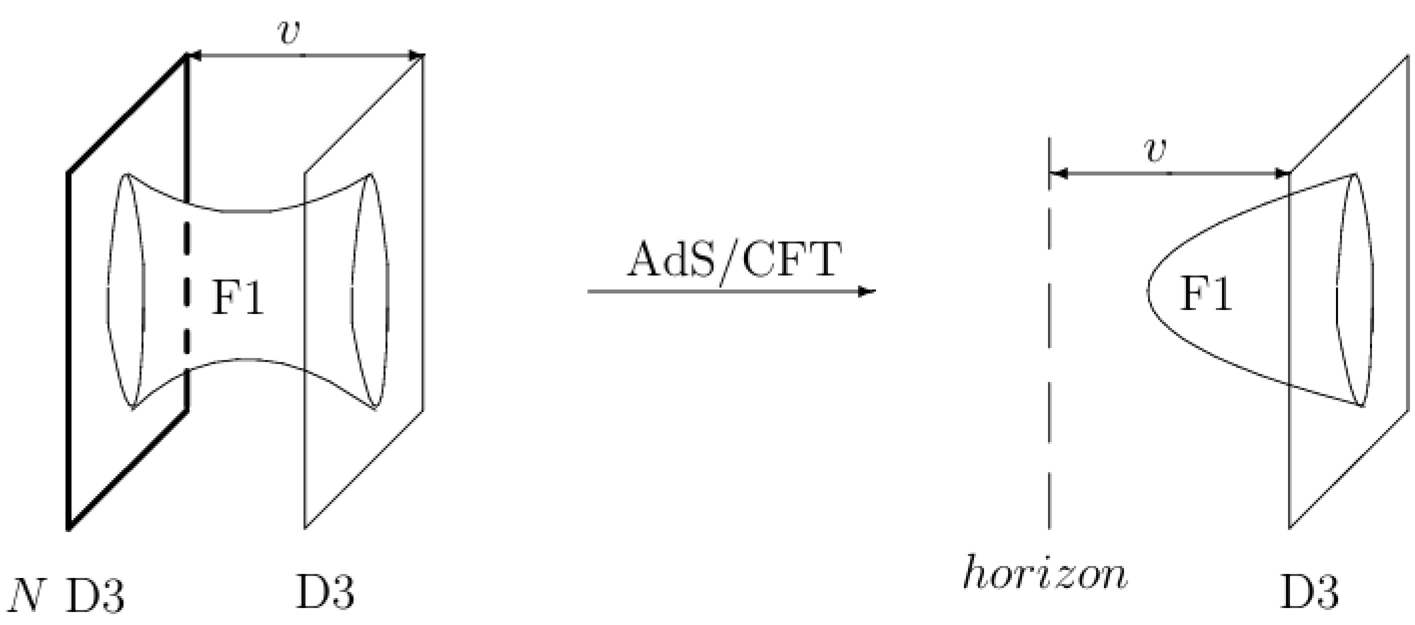


Figure 5: The Euclidean trajectory for $p\bar{n}\pi^-$ creation

$$\omega \propto \exp\left(-\frac{\pi(M_p^2+m_\rho^2)}{eE} - \frac{M_pM_n}{eE}\sqrt{1-\left(\frac{M_p^2-m_\rho^2+M_n^2}{2M_pM_n}\right)^2} + \right.$$
$$\left. + \frac{M_p^2}{eE}\arccos\frac{M_p^2-m_\rho^2+M_n^2}{2M_pM_n} + \frac{m_\rho^2}{eE}\arccos\frac{m_\rho^2-M_p^2+M_n^2}{2m_\rho M_n}\right).$$

Pikalov, A.G. 2024

Schwinger process in holography SYM



Saraikin, Selivanov, A.G, 2002
Semenoff, Zarembo 2011

Production of W-bosons in SYM at strong coupling

Ambjorn Makeenko 2012

Evaluation of the Wilson loop at strong coupling

$$S_{eff} = T_{F1} Area(\Sigma) + E Area(\partial\Sigma)$$

Pair creation at strong coupling in SYM

$$S_{\mathrm{cl}(2)} = n\left[\sqrt{(2\pi m R)^2+\lambda}-\sqrt{\lambda}\right]-\frac{1}{2}(2\pi n)ER^2$$

Effective resummation of the higher loop corrections

$$R=\frac{1}{2\pi m}\sqrt{\left(\frac{2\pi m^2}{E}\right)^2-\lambda}$$

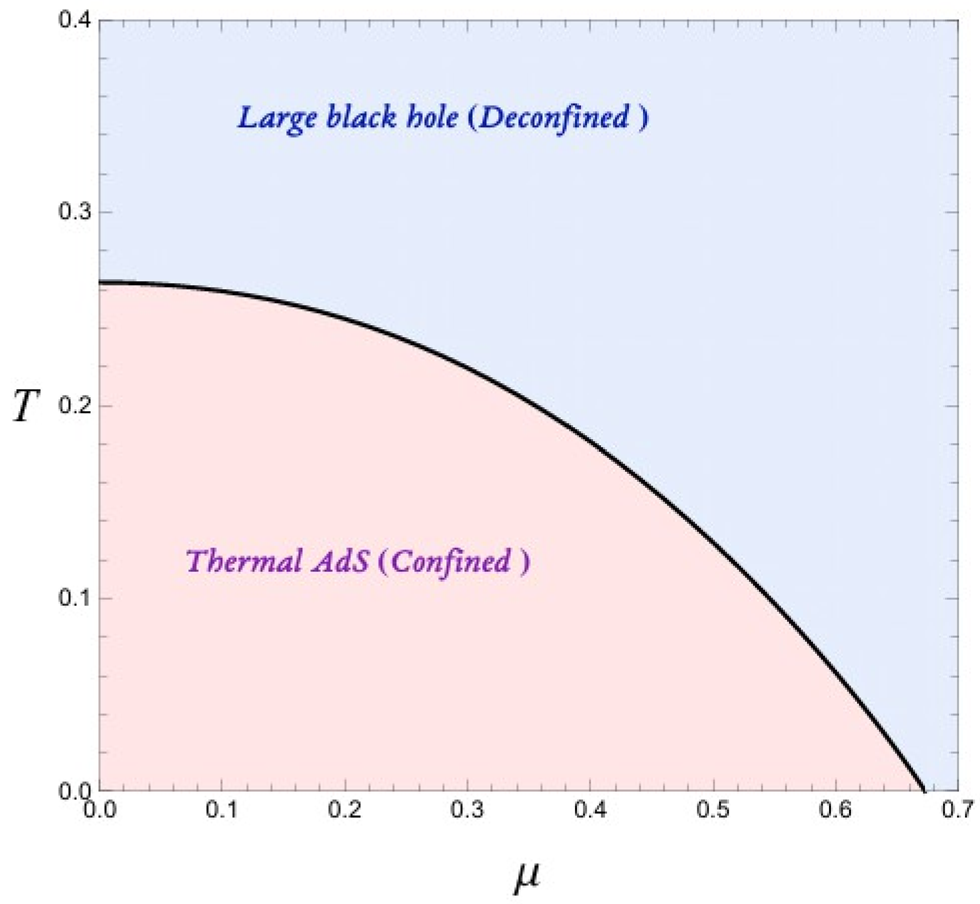
$$S_{\mathrm{cl}(2)}=\frac{n\sqrt{\lambda}}{2}\left(\sqrt{\frac{E_c}{E}}-\sqrt{\frac{E}{E_c}}\right)^2$$

Semenoff,Zarembo

$$E_{\mathrm{c}}=\frac{2\pi m^2}{\sqrt{\lambda}}$$

Critical electric field when the exponential suppression disappear at strong coupling

Pair creation in holographic QCD



Holography — our 4d + additional coordinate-scale in our 4d

Geometry — IR wall at small temperature

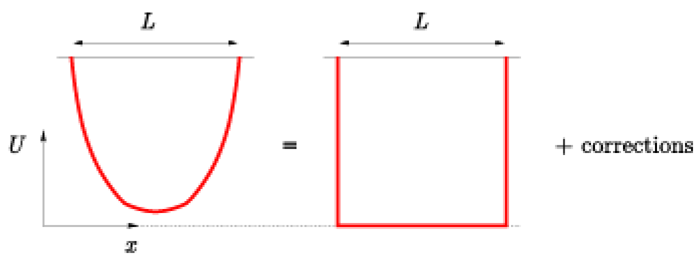
Black hole horizon in the deconfined phase

Sato, Kawai, Yoshida 2015-2017

Pair creation in holographic QCD

The quark Wilson loop in QCD in electric field has to be evaluated in the particulat geometry

- IR wall in the confinement phase
-BH in the deconfined phase



In confined phase the string between the quark-antiquark pair
is taken into account via geometry

The direct creation of the baryon-antibaryon pair is possible and the production rate was evaluated holographically - strongly suppressed.

Pair creation and entanglement

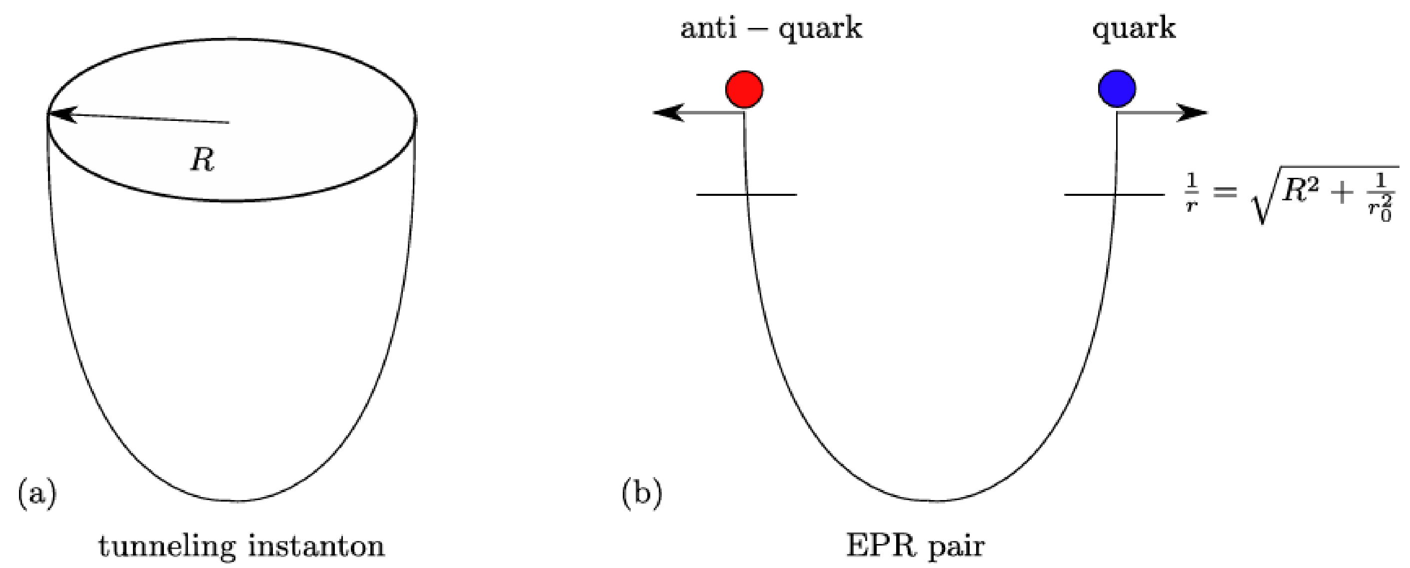


FIG. 1 (color online). The Euclidean solution describing Schwinger pair production is an instanton ending at $r = r_0$ in a circle of radius R. Its Lorentzian continuation is the EPR geometry corresponding to an entangled pair of (anti) quarks with two world sheet horizons at $(1/r) = \sqrt{R^2 + (1/r_0^2)}$ and a wormhole connecting them. The instanton for $n > 1$ is the n cover of the instanton for $n = 1$ and gives the contribution to the production probability due to n pairs. The figure has been analytically continued to Lorentzian time from the left to the right panel and the circle $\tau^2 + x^2$ has been transformed in to the hyperbola $-t^2 + x^2$.

Karch-Jensen, Sonner 2013

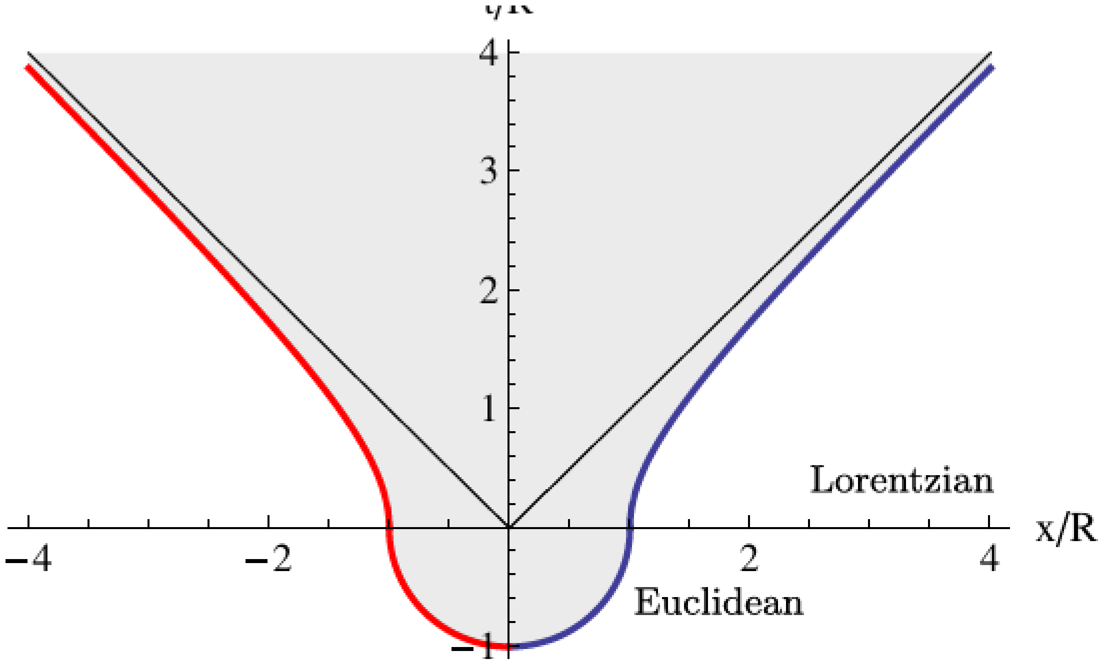


FIG. 2 (color online). In the upper half plane, the end points of the string follow the hyperbola [Eq. (5)] and the world sheet horizon is induced from the Rindler horizon of the pair shown as solid black lines. The world sheet of the string extends in the direction perpendicular to the $x - t$ plane, filling in the space between the blue and red curves (shaded in gray). The Rindler horizons project to the world sheet horizons at $r = (R^2 + 1/r_0^2)^{-1/2}$. The full solution arises by analytically continuing the Euclidean instanton through $t = 0$, which we depict by drawing the instanton-circle trajectory in the lower half plane. The causal structure is, thus, the same as the upper half of the Penrose diagram of the eternal AdS black hole and inherits its ER bridge.

Example of ER bridge

There is wormhole on the string worldsheet

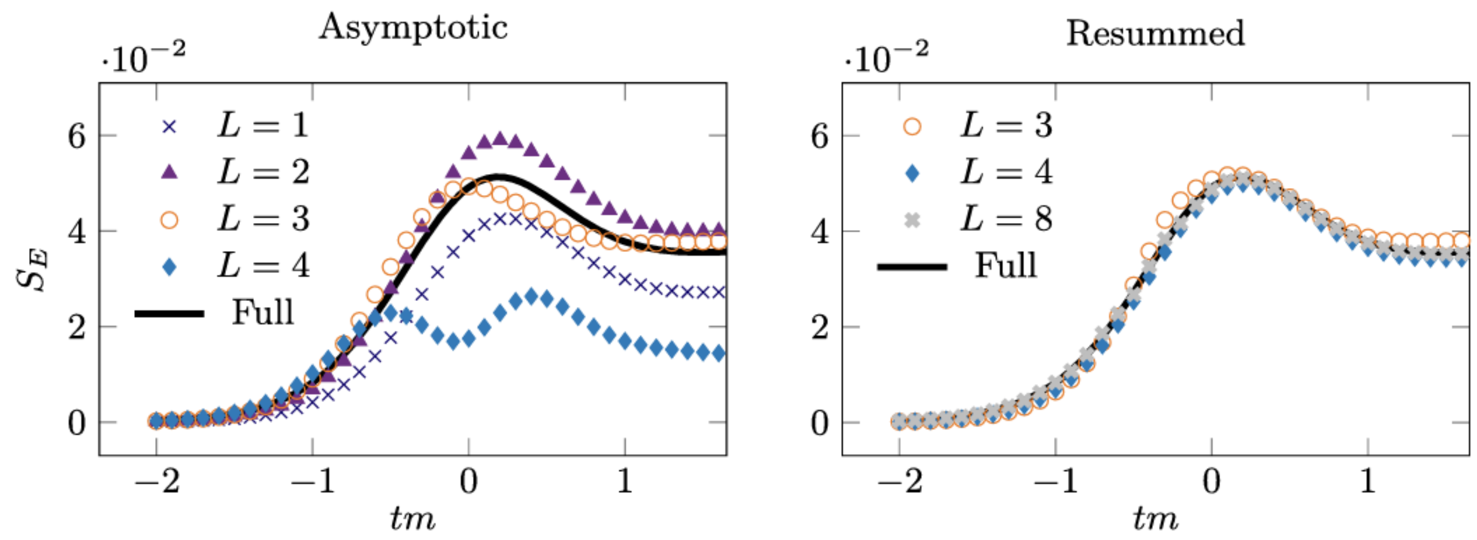


FIG. 4. Entanglement entropy as a function of time obtained from the moments of the distribution of created particles. On the l.h.s., we show the entropy obtained from the asymptotic expansion (63) truncated after L terms. On the rhs, we show the results obtained with the resummed expression (89).

Entanglement entropy of the left and right movers of the created pair coincides with the Gibbs entropy of the created particles

The entabglement entropy is correlated with the multiplicities of the created jets

$$S_E^C = \beta^2 \frac{\partial F_{OS}}{\partial \beta} \qquad \beta = \frac{2\pi}{\bar{a}} \equiv \frac{2\pi}{a\sqrt{1+\gamma^2}} \qquad a = E/M \text{ and } \gamma = \omega/a.$$

Kharzeev, Flavio, Guo, Zahed, Grieniger - 2021-23

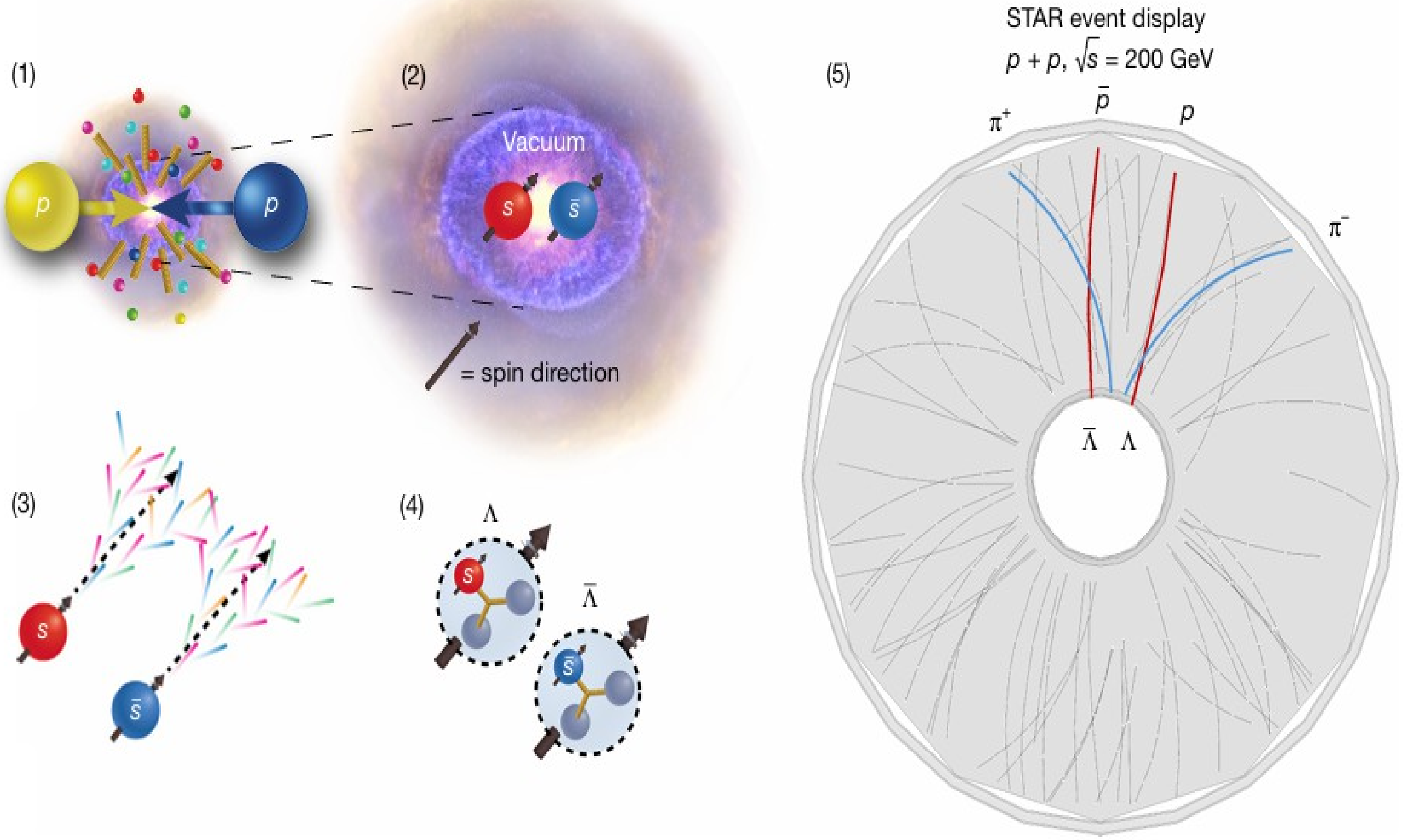


Fig. 1 | Exciting the vacuum in high-energy proton–proton collisions. Illustration of tracing the QCD evolution of the spin of a strange quark–antiquark pair to a $\Lambda\bar{\Lambda}$ hyperon pair and how it can be measured by the STAR experiment at RHIC. See (1)–(5) in the text for details.

Measuring of the spin correlation between quarks
due to the QCD confinement

Nature, February 2026

Star Collaboration

Correlation through entanglement between final particles.

Schwinger near Black Hole

$$\breve{E} \sim Q_{BH}/r_g^2,$$

Electric field near BH horizon

$$E(R) = \frac{\alpha Q}{R^2} \sim E_c = \frac{m_e^2}{\sqrt{4\pi\alpha}},$$

Assumption
Dolgov-Rudenko — 24'

$$\alpha Q < \frac{M m_p}{m_{Pl}^2},$$

Gravity is stronger then EM for proton

$$M \lesssim \frac{\sqrt{\alpha} m_{Pl}^2 m_p}{m_e^2} \left(\frac{r_g}{R}\right)^2 \sim 10^{20} \left(\frac{r_g}{R}\right)^2 \text{ g}.$$

Condition for BH mass

511 kev line?

BH with masses ~ 10^{20} g

Dolgov-Rudenko

Can partially explain the production of positrons via Schwinger mechanism necessary for the electron-positron annihilation in the interstellar medium → 511 kev photons

The flows of protons via gravity and electrons by EM attraction equalize hence BH charge stabilizes via combination of proton accretion and the Schwinger pair production

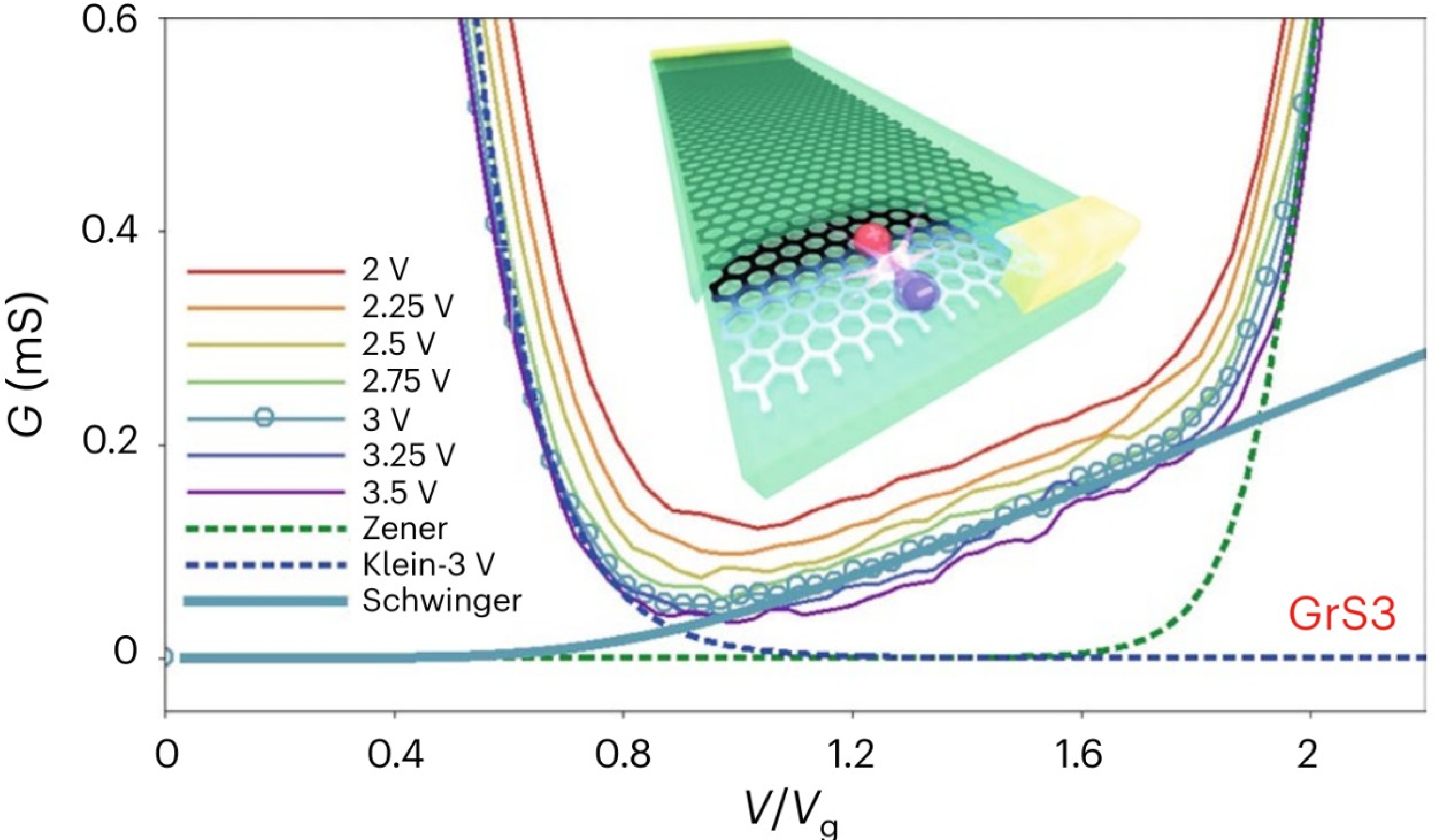
There is the additional mechanism of the positron production via Non-perturbative decay of the proton in the electric field! Has to be taken into account!

Schwinger process in de Sitter

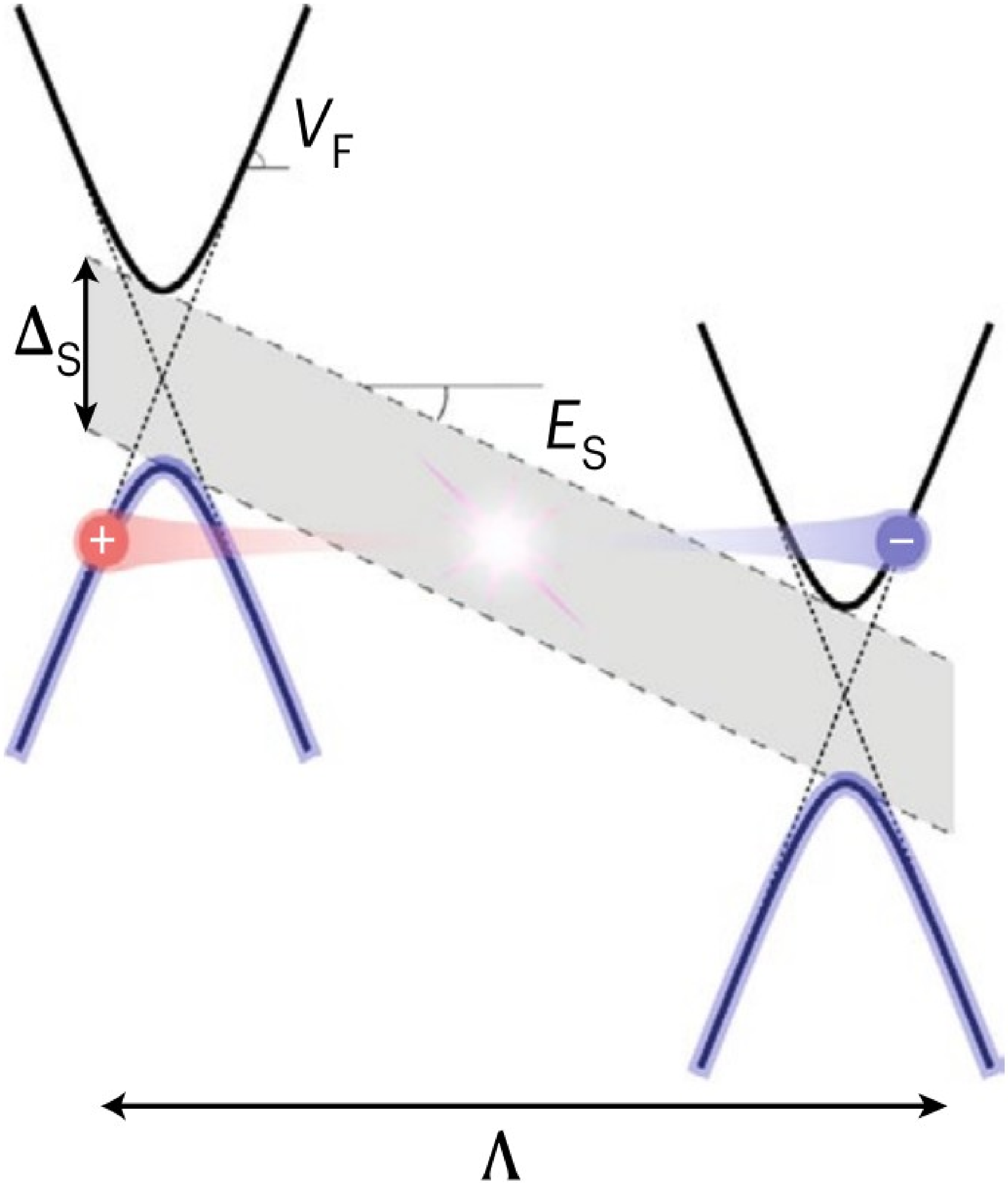
- De Sitter is unstable with respect to the particle creation
- The current is generated by the tunneling mechanism
- Combination of the electric field and the cosmological constant yields the several regimes for the current

Mottola (85), Bousso,Maloney,Strominger (2002), Polyakov(2008).......
Some recent progress in the proper definition of the particle current
Dabrowski,Dunne—2014(special basis)
Tunneling Universe : since Rubakov 84, different approches for pair creation
In the expanding Universe

Observation of pair creation in graphene



A. Schmitt et al. / Nature Physics, 2023



Theoretically predicted phenomenon since graphene has the proper effective QED description

False vacuum decay on the table

In 2d Schwinger process = false vacuum decay

False vacuum decay — Voloshin,Kobzarev,Okun -75
Coleman -77'

arxiv.org/pdf/2512.04637

Rydberg atoms array - 2025-26

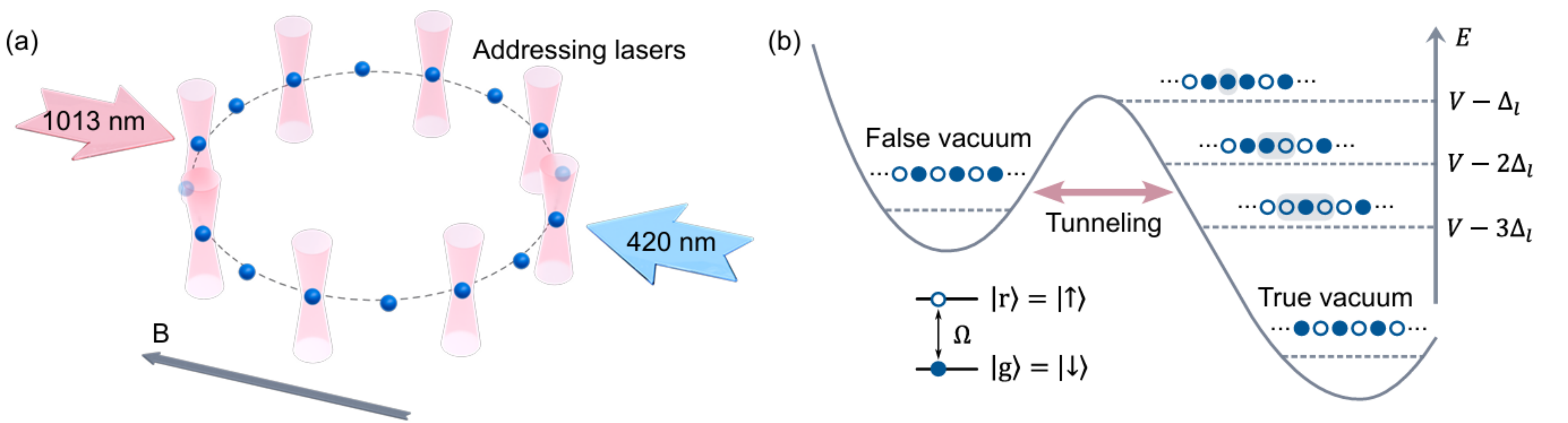


FIG. 1: Simulating false vacuum decay with a programmable Rydberg atom array. (a) Schematic of the experimental setup. Neutral atoms are trapped in a ring geometry and illuminated globally by the 420-nm and 1013-nm lasers, coupling the ground state to a high-lying Rydberg state (70S). A set of far-detuned, site-selective addressing lasers (pink beams) illuminates every other atom, generating a staggered detuning. A magnetic field B parallel to the 420-nm laser defines the quantization axis. (b) Energy landscape for false vacuum decay in an antiferromagnetic Ising model. The staggered longitudinal field breaks the degeneracy of the two Néel-ordered ground states, creating a metastable false vacuum and a stable true vacuum. Quantum tunneling from the false vacuum towards the true vacuum proceeds via the nucleation of true vacuum bubbles (illustrated as gray domains of flipped spins). The dashed lines denote static energies ($\Omega = 0$) of representative spin configurations along the unfolded ring.

Conclusions

- The Schwinger pair production is expected to be observed in laser experiments in the near future
- Interesting interplay with the condmat experiments
- Interesting noperturbative modes of the «stable» particles decays. Proton decay

Спасибо за внимание!