



Сессия-конференция СЯФ ОФН РАН

Neutrino oscillations and decoherence in astrophysical environments

Artem Popov, Alexander Studenikin, Alexander Tcvirov
Moscow State University

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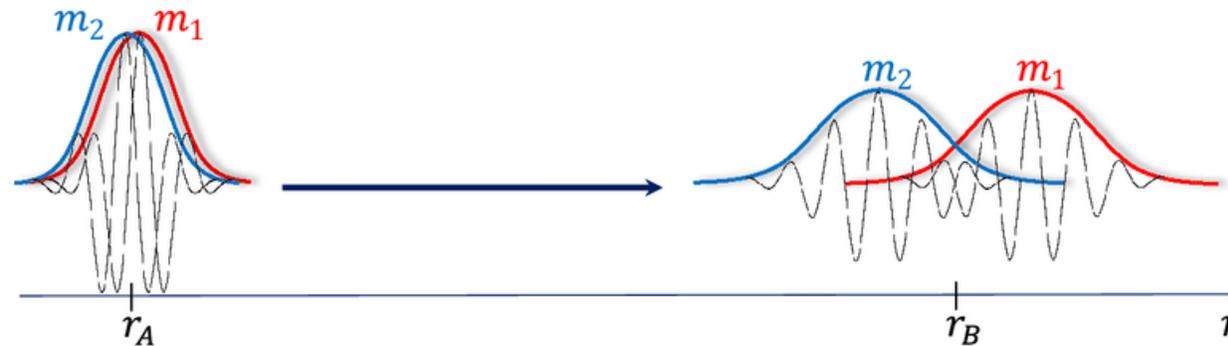
Outline of the talk

- Neutrino electromagnetic interactions
- Neutrino oscillations in a magnetic field
- Wave packets description of neutrino oscillations in external fields



Neutrino oscillations and coherence

- Plane waves description is not applicable for the case of neutrino propagation on at large distances. Instead **wave packet** description must be adopted.



- Massive neutrino states wave packets separation leads to the **exponential damping of neutrino flavour oscillations** at distances exceeding the *coherence length*:

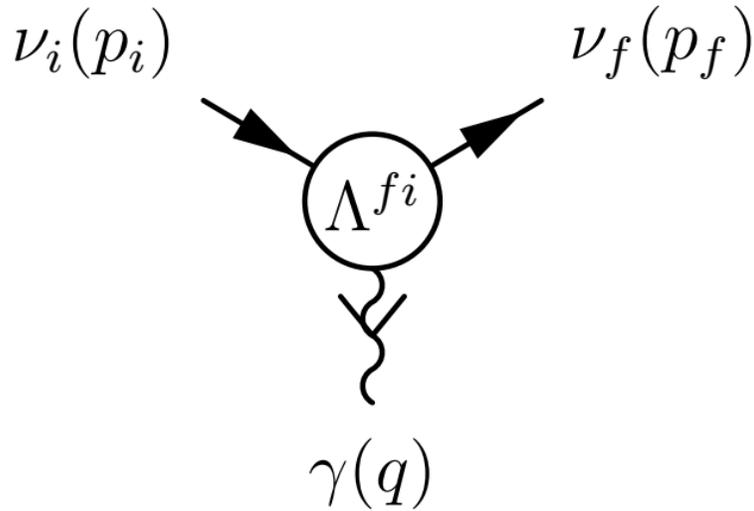
$$P_{osc}(L) \sim \exp\left(-i2\pi\frac{L}{L_{osc}}\right) \exp\left(-\frac{L^2}{L_{coh}^2}\right), \quad L_{coh} \approx \frac{4\sqrt{2}\sigma_x p^2}{\Delta m^2},$$

where σ_x is the wave packet width

- Daya Bay** collaboration: $10^{-11} \text{ cm} \lesssim \sigma_x \lesssim 2 \text{ m}$ (F.P.An et al., Eur.Phys.J.C 77 (2017) 9, 606)
- Predicted **JUNO** sensitivity: $\sigma_x > 10^{-4} \text{ nm}$ (J.Wang et al., JHEP 06 (2022) 062)



Neutrino electromagnetic properties



$$\mathcal{H}_{\text{em}}^{(\nu)}(x) = j_{\mu}^{(\nu)}(x) A^{\mu}(x) = \sum_{k,j=1}^N \bar{\nu}_k(x) \Lambda_{\mu}^{kj} \nu_j(x) A^{\mu}(x),$$

The vertex function is parametrized in terms of **charge, anapole, electric and magnetic form factors**:

$$\Lambda_{\mu}(q) = (\gamma_{\mu} - q_{\mu} \not{q} / q^2) [\mathbb{f}_Q(q^2) + \mathbb{f}_A(q^2) q^2 \gamma_5] - i \sigma_{\mu\nu} q^{\nu} [\mathbb{f}_M(q^2) + i \mathbb{f}_E(q^2) \gamma_5]$$

$$\mathbb{f}_M^{fi}(0) = \mu_{fi} \text{ - neutrino magnetic moments}$$

C.Giunti, A.Studenikin, Rev.Mod.Phys. 87 (2015) 531;

C.Giunti, K.Kouzakov, Y.-F. Li, A.Studenikin, Ann.Rev.Nucl.Part.Sci. 75 (2025) 1, 1-33



Neutrino magnetic moments

Theory (Standard Model):

$$\mu_{ii}^D = \frac{3eG_F m_i}{8\sqrt{2}\pi^2} \approx 3.2 \times 10^{-19} \left(\frac{m_i}{1 \text{ eV}} \right) \mu_B$$

K.Fujikawa, R.Shrock, "*The Magnetic Moment of a Massive Neutrino and Neutrino Spin Rotation*", Phys.Rev.Lett. 45 (1980) 963

Experiment:

$$\mu_\nu < 6.4 \times 10^{-12} \mu_B$$

E.Aprile *et al.* [XENON collaboration], "*Search for New Physics in Electronic Recoil Data from XENONnT*", Phys.Rev.Lett. 129 (2022) 16, 161805

Upper bounds from astrophysical neutrinos:

R.L. Workman *et al.* (Particle Data Group), Prog. Theor. Exp. Phys. 2022, 083C01 (2022)

$$\mu_\nu \lesssim 10^{-12} \mu_B$$



Majorana neutrinos

Dirac fermion

$$\Psi_D = \Psi_L + \Psi_R$$

Majorana fermion

$$\Psi_R = \Psi_L^c$$

A Majorana field can be written as $\Psi_M = \Psi_L + \Psi_L^c$

$\Psi_M^c = \Psi_M$ is satisfied for a Majorana field

Majorana mass term violates total lepton number by 2

$$m_i \bar{\nu}_i \nu_i = m_i \overline{(\nu_i^L)^c} \nu_i^L + m_i \bar{\nu}_i^L (\nu_i^L)^c$$



Neutrino magnetic moments: Dirac and Majorana

CPT-invariance + hermicity:

- Magnetic moments matrix for **Dirac** neutrinos is **real and symmetric**:

$$\mu^D = \begin{pmatrix} \mu_{11} & \mu_{12} & \mu_{13} \\ \mu_{12} & \mu_{22} & \mu_{23} \\ \mu_{13} & \mu_{23} & \mu_{33} \end{pmatrix}$$

- Magnetic moments matrix for **Majorana** neutrinos is **imaginary and asymmetric**:

$$\mu^M = \begin{pmatrix} 0 & i\mu_{12} & i\mu_{13} \\ -i\mu_{12} & 0 & i\mu_{23} \\ -i\mu_{13} & -i\mu_{23} & 0 \end{pmatrix}$$

- Thus, Dirac and Majorana neutrinos can be distinguished by their **electromagnetic properties**.



Flavour transitions in a magnetic field

- Neutrino interactions with a magnetic field modifies the patterns of the neutrino flavour transitions, and even induces neutrino-antineutrino transitions (in the Majorana case.
- The corresponding effects were studied in literature for various classes of astrophysical objects (Solar neutrinos, supernova neutrinos, AGN neutrinos, etc.)
- Decoherence effects due to the neutrino states wave packets separations were not accounted yet.
- The formalism involving wave packets to describe neutrino oscillations in magnetic fields were developed in our papers.



Neutrino evolution in a magnetic field

- Neutrino evolution in a magnetic field is described by the following Dirac equation:

$$(i\gamma^\mu \partial_\mu - m_i)\nu_i(x) - \sum_k \mu_{ik} \boldsymbol{\Sigma} \mathbf{B} \nu_k(x) = 0, \quad (1)$$

A.Popov, A.Studenikin, Phys.Rev.D 103 (2021) 11, 115027

- For the case of wave packet description of neutrino oscillations, and neglecting transition magnetic moments, Equation (1) can be rewritten as

$$i\gamma^0 \partial_t \nu_i(p, t) = (\gamma_3 p + m_i)\nu_i(p, t) - \sum_k \mu_{ik} \boldsymbol{\Sigma} \mathbf{B}(\langle x_i(t) \rangle) \nu_i(p, t) \quad (2)$$

A.Popov, A.Studenikin, Phys.Rev.D 111 (2025) 12, 123001

For ultra-relativistic neutrinos it is safe to assume that $\langle x_i \rangle \approx ct$.

Equation (2) can be solved:

1. Analytically in the adiabatic case (slowly varying magnetic field),
2. Or numerically.



Adiabatic solution (Dirac neutrinos)

- The probabilities of flavour conversions are:

$$P_{\alpha\beta}(L) = \frac{1}{2} \sum_{i=1}^3 U_{\alpha i}^2 U_{\beta i}^2 \left[1 + 2 \cos\left(\frac{2\pi L}{L_i^B}\right) D_i^B(L) \right] + 2 \sum_{i>j} U_{\beta i} U_{\alpha i} U_{\beta j} U_{\alpha j} \cos\left(\frac{2\pi L}{L_{ij}^{vac}}\right) \cos\left(\frac{2\pi L}{L_i^B}\right) \cos\left(\frac{2\pi L}{L_j^B}\right) D_{ij}^{vac}(L),$$

$$D_{ij}^{vac}(L) = \exp\left(-\frac{L^2}{(L_{coh}^{ijss})^2}\right), \quad D_i^B(L) = \exp\left(-\frac{L^2}{(L_{coh}^{iis\sigma})^2}\right)$$

where L_{osc} are oscillations lengths and L_{coh} are **coherence lengths**, $i, j = 1, 2, 3$ and $s, \sigma = \pm 1$.

$$L_{osc}^{ijss} = \frac{4\pi p}{\Delta m_{ij}^2} \quad \text{and} \quad L_{osc}^{ii-+} = \frac{\pi}{\mu_i B_{\perp}}$$

- Oscillations probability is a combination of oscillations on (1) vacuum frequencies

$$\omega_{ik}^{vac} = \frac{\Delta m_{ik}^2}{4p} \quad \text{and} \quad \text{(2) magnetic frequencies } \omega_i^B = \mu_i B_{\perp}.$$

(see A. Popov, A. Studenikin, Eur.Phys.J.C 79 (2019) 2, 144 and references therein)



Coherence lengths (Dirac neutrinos)

- The probabilities of flavour conversions are:

$$P_{\alpha\beta}(L) = \frac{1}{2} \sum_{i=1}^3 U_{\alpha i}^2 U_{\beta i}^2 \left[1 + 2 \cos \left(\frac{2\pi L}{L_i^B} \right) D_i^B(L) \right] + 2 \sum_{i>j} U_{\beta i} U_{\alpha i} U_{\beta j} U_{\alpha j} \cos \left(\frac{2\pi L}{L_{ij}^{vac}} \right) \cos \left(\frac{2\pi L}{L_i^B} \right) \cos \left(\frac{2\pi L}{L_j^B} \right) D_{ij}^{vac}(L),$$

$$D_{ij}^{vac}(L) = \exp \left(- \frac{L^2}{(L_{ij}^{jss})^2} \right), \quad D_i^B(L) = \exp \left(- \frac{L^2}{(L_{coh}^{iis\sigma})^2} \right)$$

where L_{osc} are oscillations lengths and L_{coh} are **coherence lengths**, $i, j = 1, 2, 3$ and $s, \sigma = \pm 1$.

$$L_{coh}^{ijss} \approx \frac{4\sqrt{2}\sigma_x p^2}{\Delta m_{ij}^2},$$

(coincides with the expression for the coherence length for neutrino oscillations in vacuum)

$$L_{coh}^{ii-+} \approx \frac{\sigma_x p^3}{\mu_i B m_i^2}.$$



Adiabatic solution (Majorana neutrinos)

- For illustrative purposes, we present the analytical solution for the case of two neutrino flavours. In the realistic three neutrino case the evolution equation is solved numerically.

$$\begin{aligned} P(\nu_e \rightarrow \nu_\mu; L) &= \frac{\omega_{vac}^2}{2(\omega_{vac}^2 + \omega_B^2)} \sin^2 2\theta \left[1 + \cos \left(\frac{2\pi L}{L_{osc}} \right) \exp \left(-\frac{L^2}{L_{coh}^2} \right) \right], \\ P(\nu_e \rightarrow \bar{\nu}_\mu; L) &= \frac{\omega_B^2}{2(\omega_{vac}^2 + \omega_B^2)} \left[1 + \cos \left(\frac{2\pi L}{L_{osc}} \right) \exp \left(-\frac{L^2}{L_{coh}^2} \right) \right], \\ P(\nu_e \rightarrow \bar{\nu}_e; L) &= 0, \end{aligned}$$

$$L_{osc} = \frac{2\pi}{\sqrt{\omega_B^2 + \omega_{vac}^2}} \quad \omega_{vac} = \Delta m^2 / 4p_0$$
$$\omega_B = \mu B_\perp$$

- In the Majorana case, interaction with strong magnetic field may induce additional **neutrino-antineutrino** transitions.



Coherence length (Majorana neutrinos)

$$L_{coh} \approx \frac{4\sqrt{2}\sigma_x p_0^2}{\Delta m^2} \quad \text{if } \omega_{vac} \gg \omega_B,$$
$$L_{coh} \approx \frac{2\sqrt{2}\sigma_x p_0^3}{(\Delta m^2)^2/\omega_B + 2\omega_B(m_1 + m_2)^2} \quad \text{if } \omega_{vac} \ll \omega_B$$

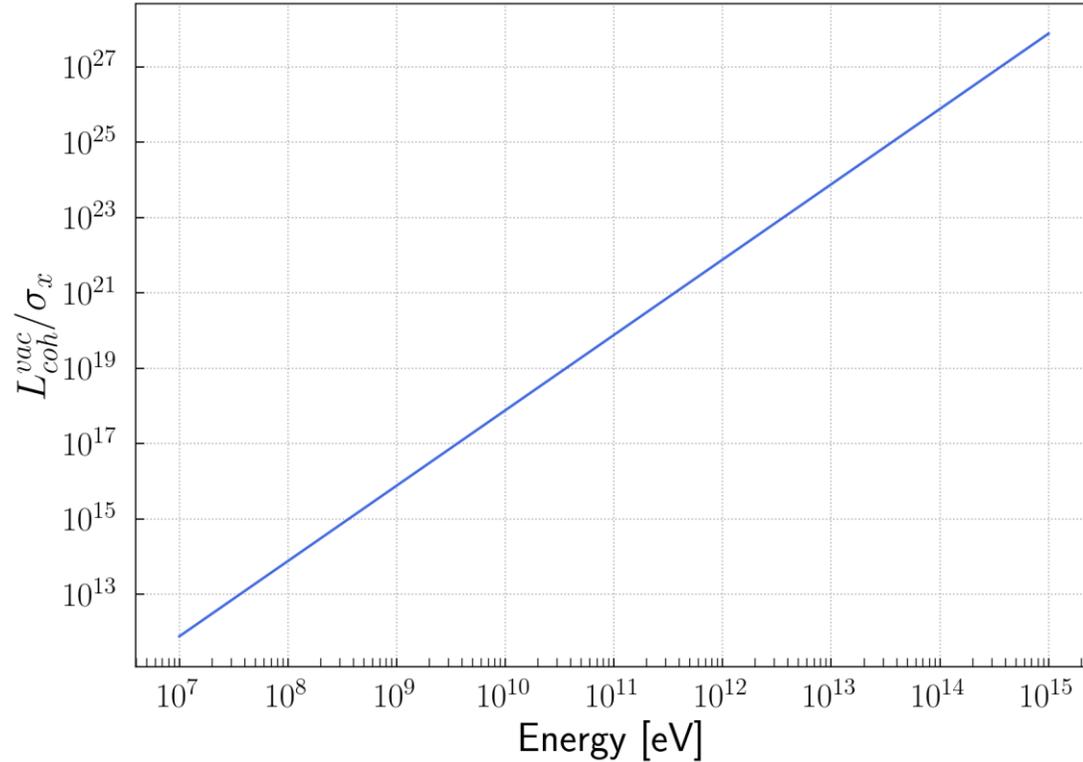
$$\omega_{vac} = \Delta m^2 / 4p_0$$
$$\omega_B = \mu B_{\perp}$$

- In the low energy/low magnetic field limit the expression coincide with the coherence length for neutrino oscillations in vacuum.
- In the transition region there is a more complicated non-power-law dependence on the neutrino energy.
- This pattern remains intact for the realistic three neutrino case, which is confirmed by the numerical solution.

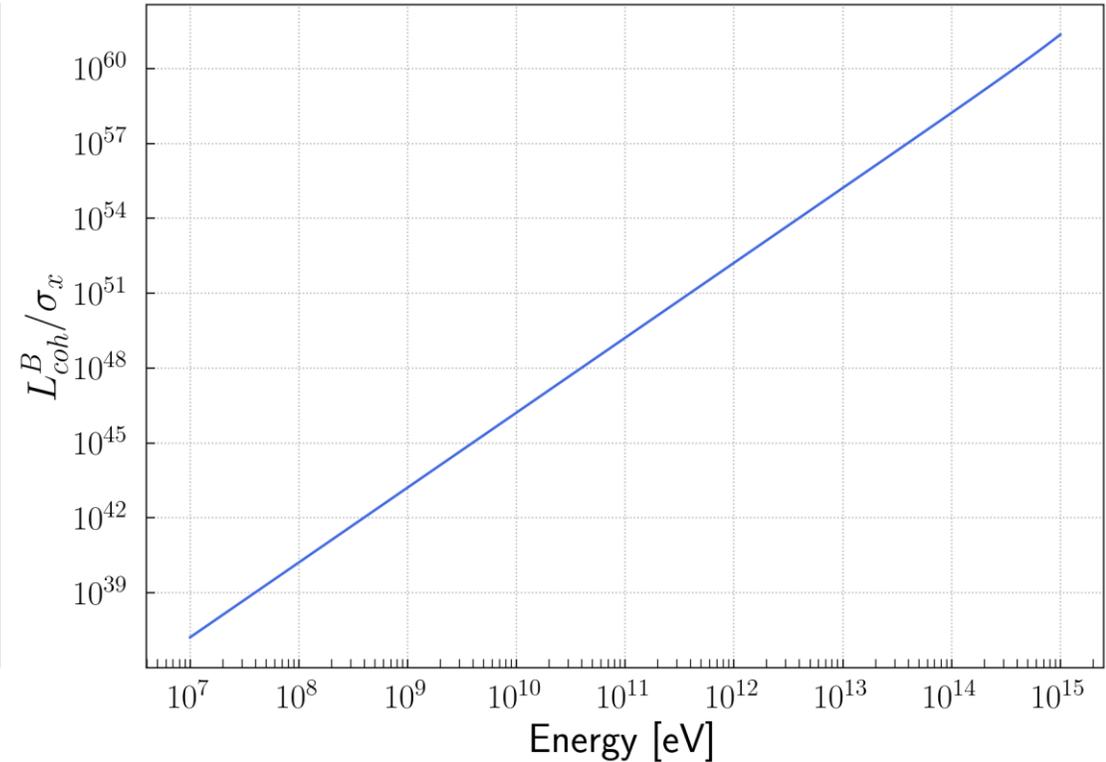


Coherence lengths

$$L_{coh} \sim E^2$$



$$L_{coh} \sim E^3$$



Conclusions

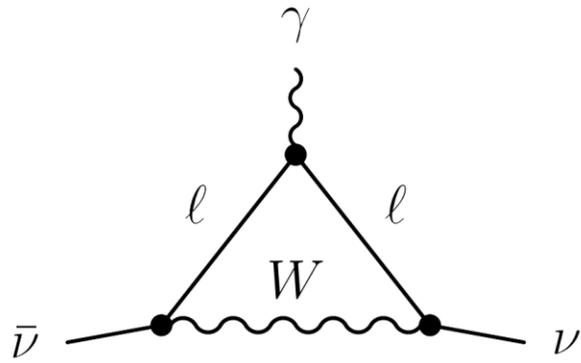
- Neutrino oscillations in a magnetic field are considered accounting for decoherence effects due to the wave packets separation.
- The expressions for coherence lengths are obtained for Dirac and Majorana neutrinos.
- In the Majorana neutrinos case, transitions induced by the interaction with a magnetic field do not disappear
- In the Dirac neutrinos case, partial decoherence appears, i.e. the decoherence between massive, but not helicity states, leading to partial damping of the neutrino flavour transitions.



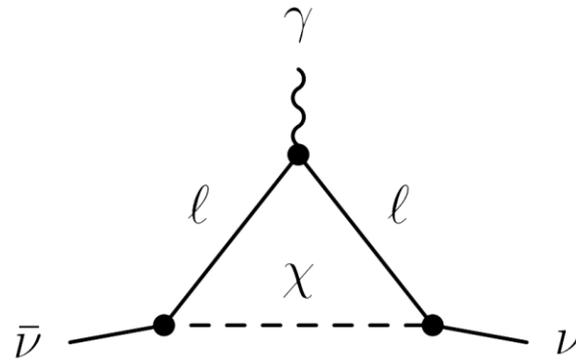
Backup



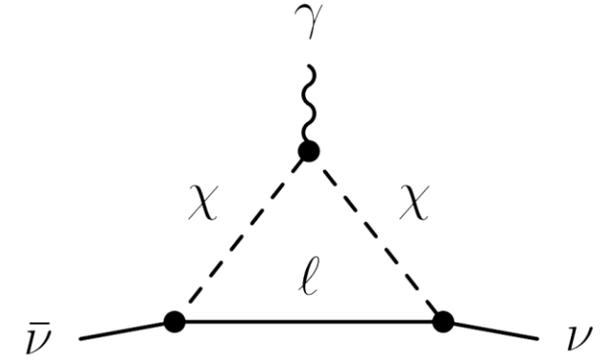
Neutrino magnetic moments



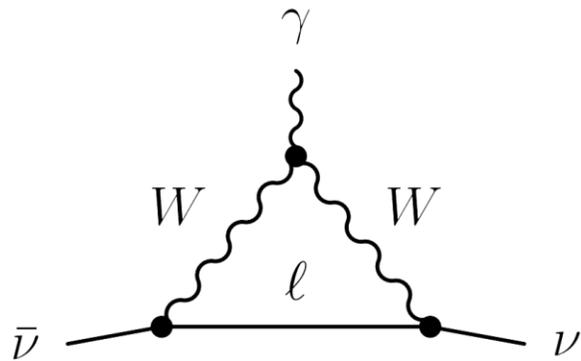
(a)



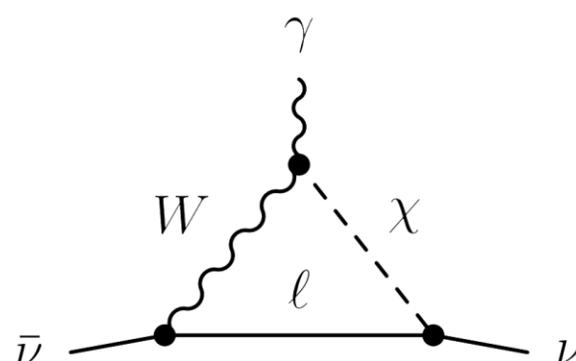
(b)



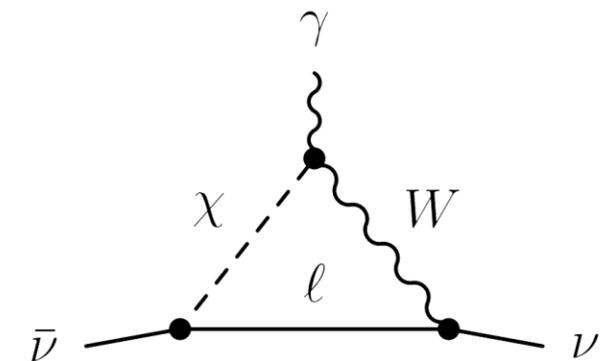
(c)



(d)



(e)



(f)

M.Dvornikov, A.Studenikin, "Electric charge and magnetic moment of massive neutrino", Phys.Rev.D. (2004)



Flavour ratios as a probe of BSM physics

- **Quantum gravity**

IceCube Collaboration, “Searching for Decoherence from Quantum Gravity at the IceCube South Pole Neutrino Observatory”, arXiv 2308.00105

- **Neutrino decay**

P.Baerwald, M.Bustamante, W.Winter, “Neutrino Decays over Cosmological Distances and the Implications for Neutrino Telescopes”, JCAP 10 (2012) 020

- **Lorentz violation**

D.Hooper, D.Morgan, E.Winstanley, “Lorentz and CPT Invariance Violation In High-Energy Neutrinos”, Phys.Rev.D 72 (2005) 065009

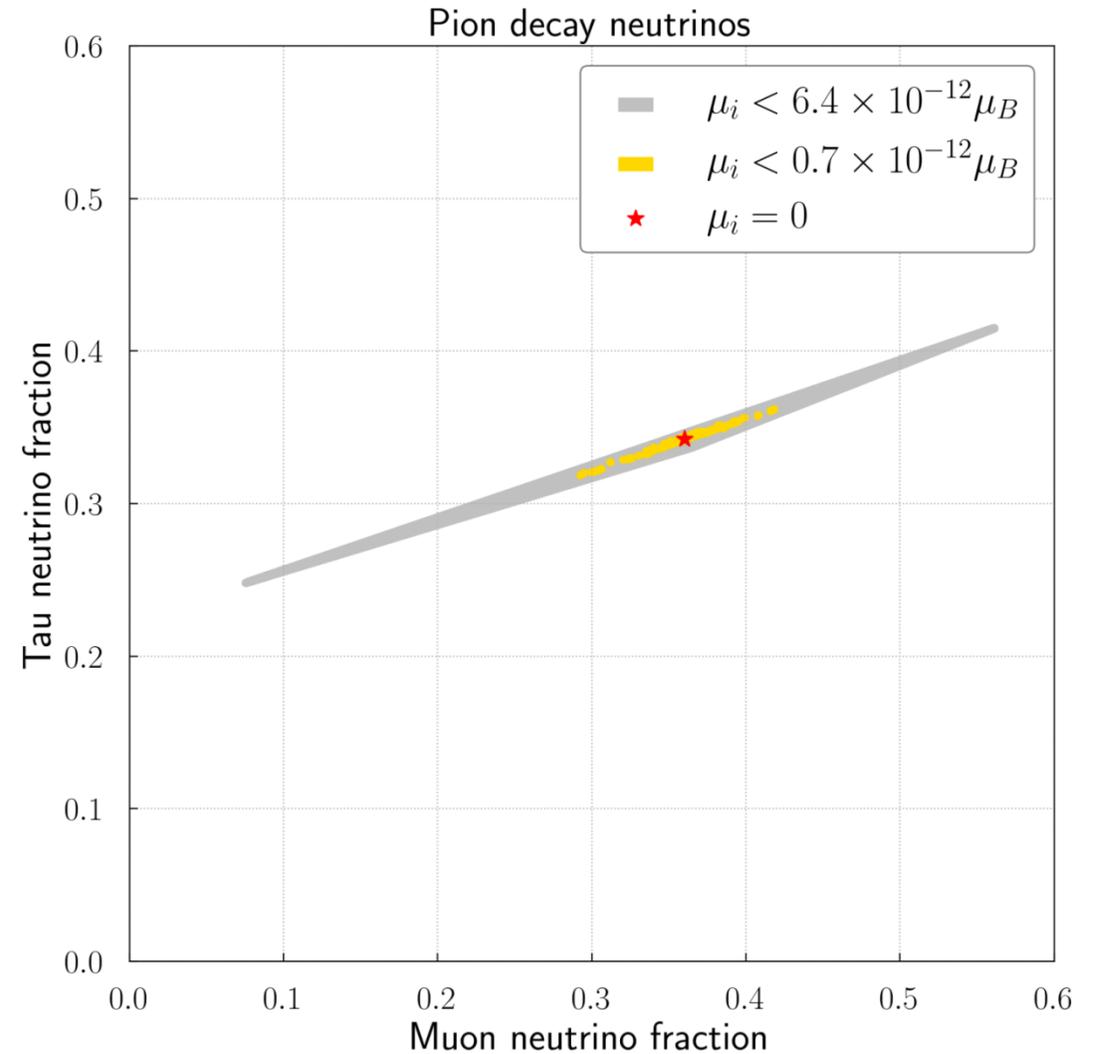
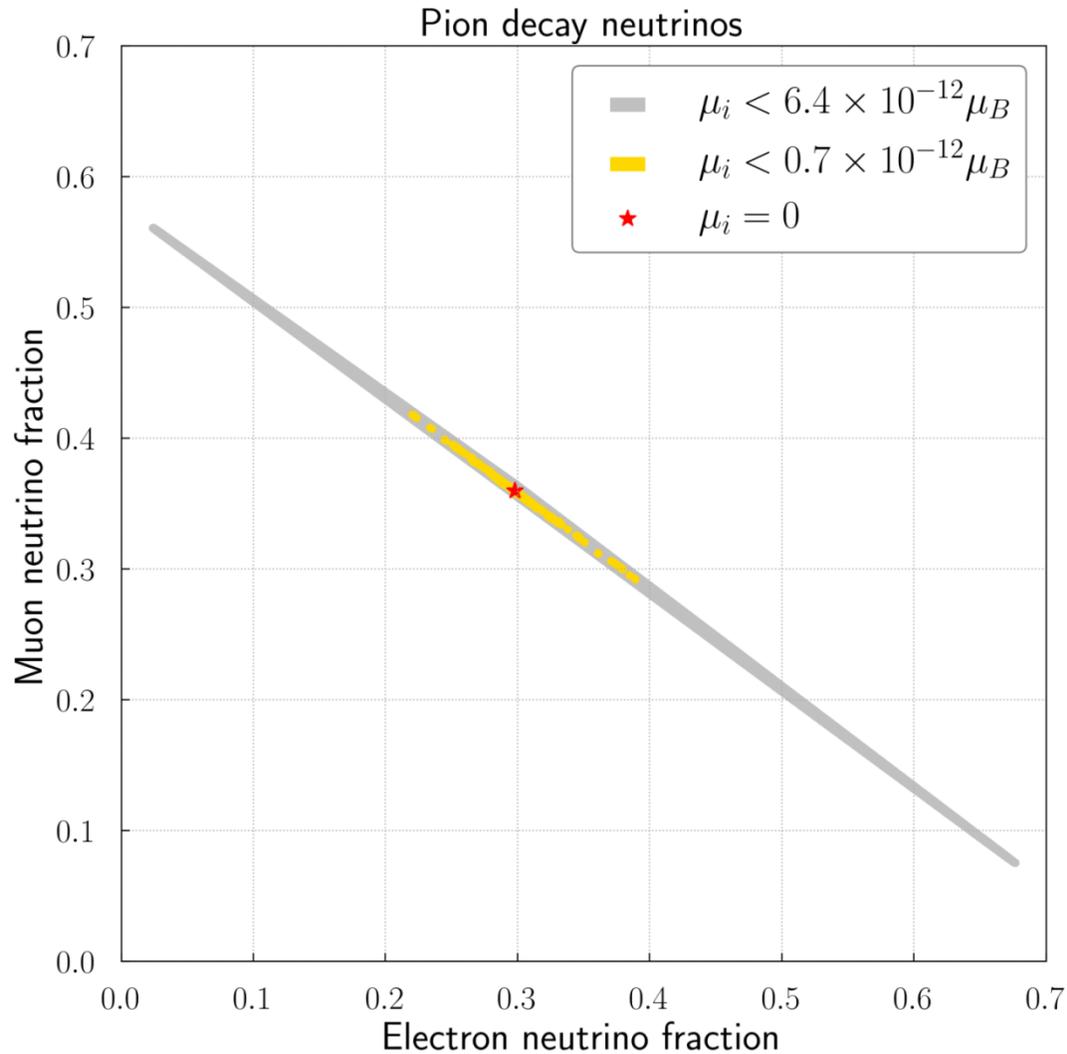
- **Sterile neutrinos**

A.Esmailia, Y.Farzan, “Implications of the Pseudo-Dirac Scenario for Ultra High Energy Neutrinos from GRBs”, JCAP 12 (2012) 014

In this talk we report possible effects of neutrino interaction with a magnetic field on flavour ratios



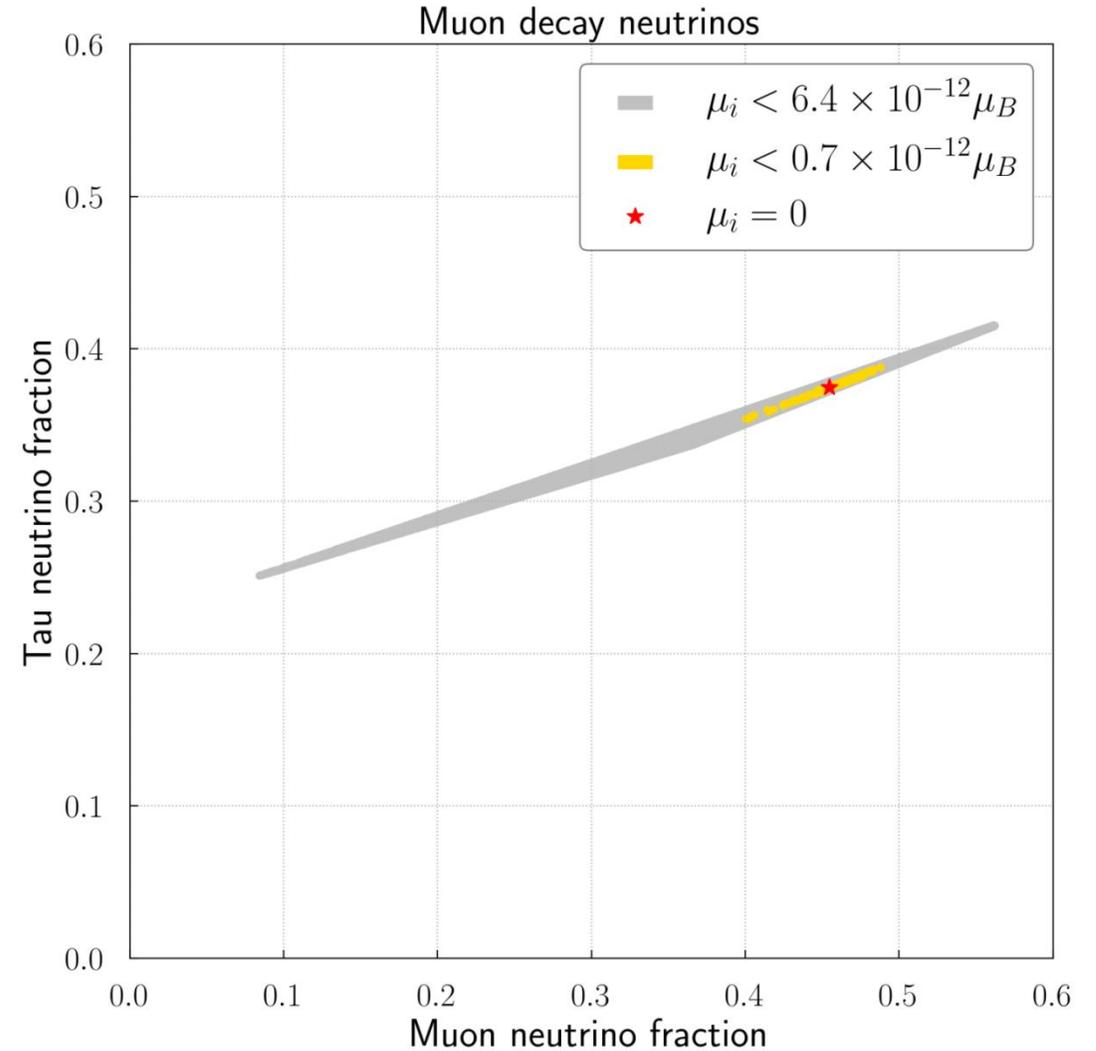
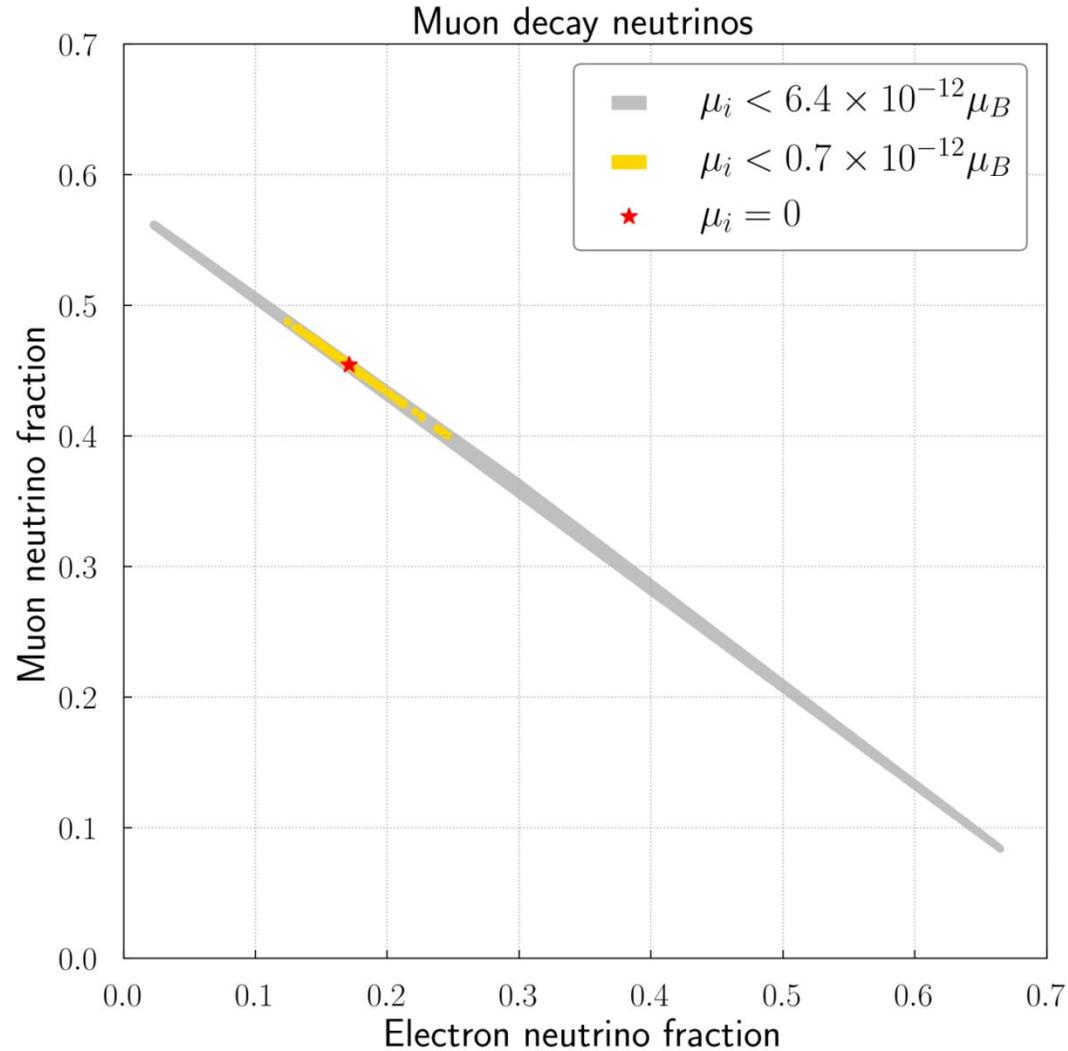
Predicted flavour ratios: π^\pm decay neutrinos



A.Popov, A.Studenikin, "High-energy neutrinos flavour composition as a probe of neutrino magnetic moments", arxiv:2404.02027



Predicted flavour ratios: μ^\pm decay neutrinos



A.Popov, A.Studenikin, "High-energy neutrinos flavour composition as a probe of neutrino magnetic moments", arxiv:2404.02027



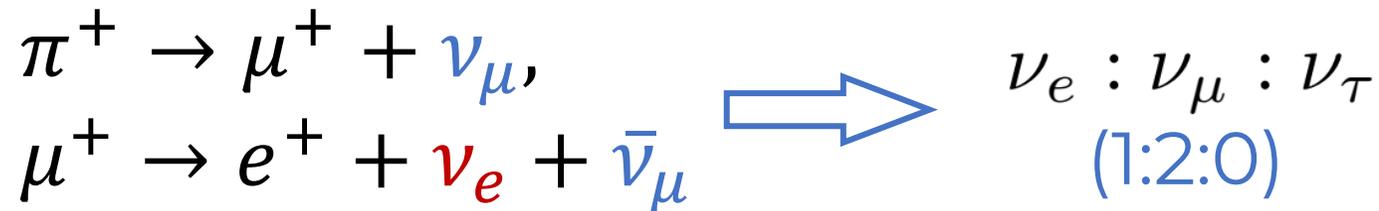
High-energy neutrinos flavour ratios

- Standard neutrino oscillations in vacuum predict the following flavour ratios at the terrestrial neutrino telescope:

$$r_\alpha = \sum_\beta r_\beta^0 \sum_i |U_{\alpha i}|^2 |U_{\beta i}|^2$$

where r_β^0 are **flavour ratios at the neutrino source** ($\alpha, \beta = e, \mu, \tau$).

- Pion decay neutrino production: $r^0 = \left(\frac{1}{2}, \frac{2}{3}, 0\right)$ and $r \approx \left(\frac{1}{3}, \frac{1}{3}, \frac{1}{3}\right)$.



M.Bustamante, J.Beacom, W.Winter, "Theoretically palatable flavor combinations of astrophysical neutrinos", Phys.Rev.Lett. 115 (2015) 16

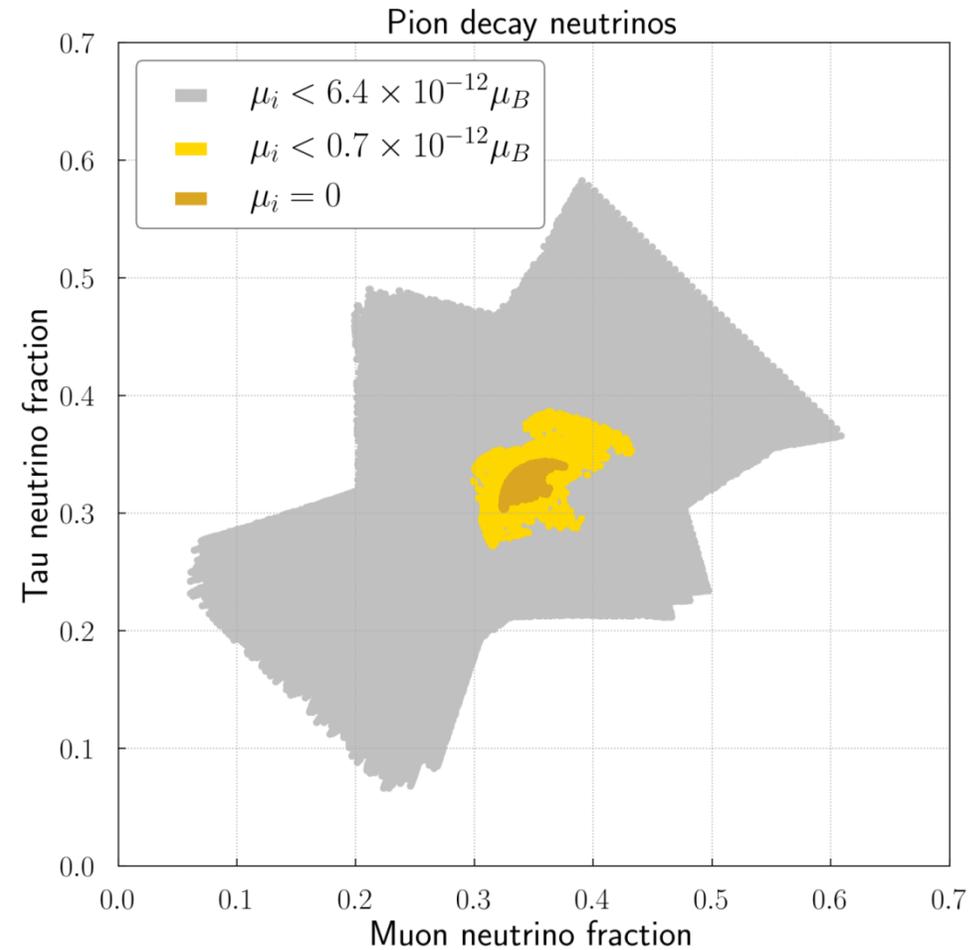
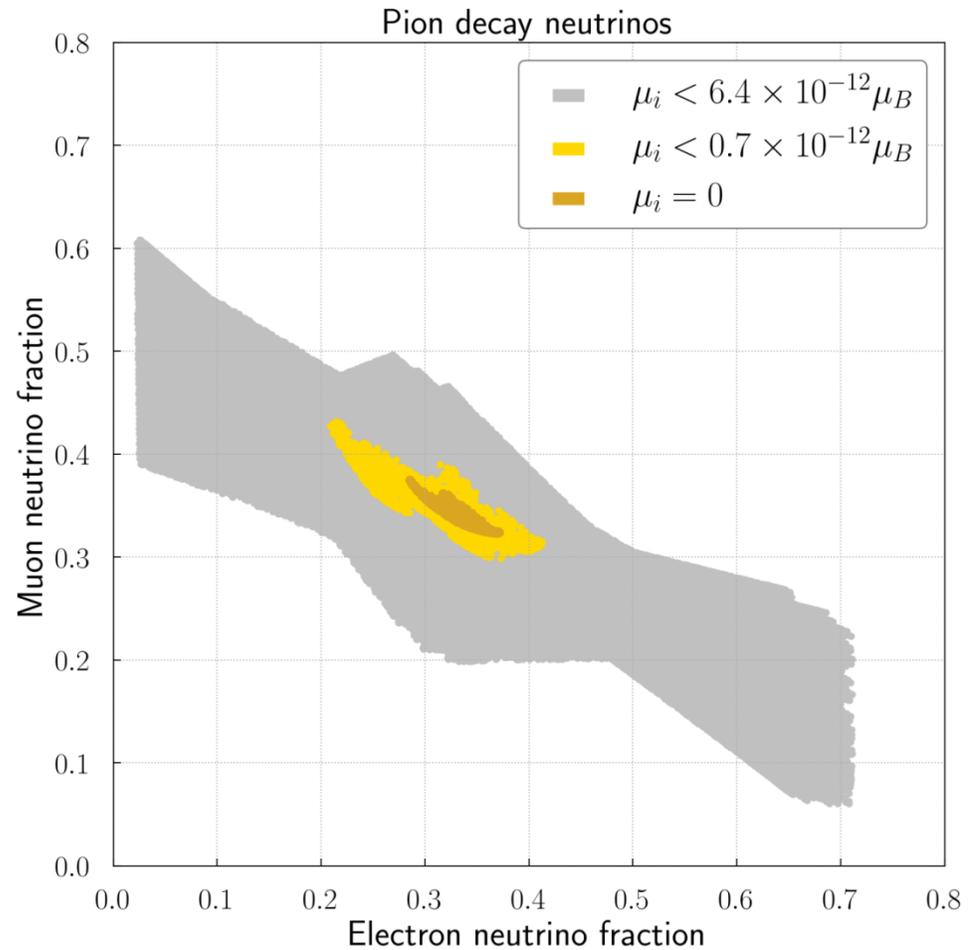


Neutrino oscillations in a Galactic magnetic field

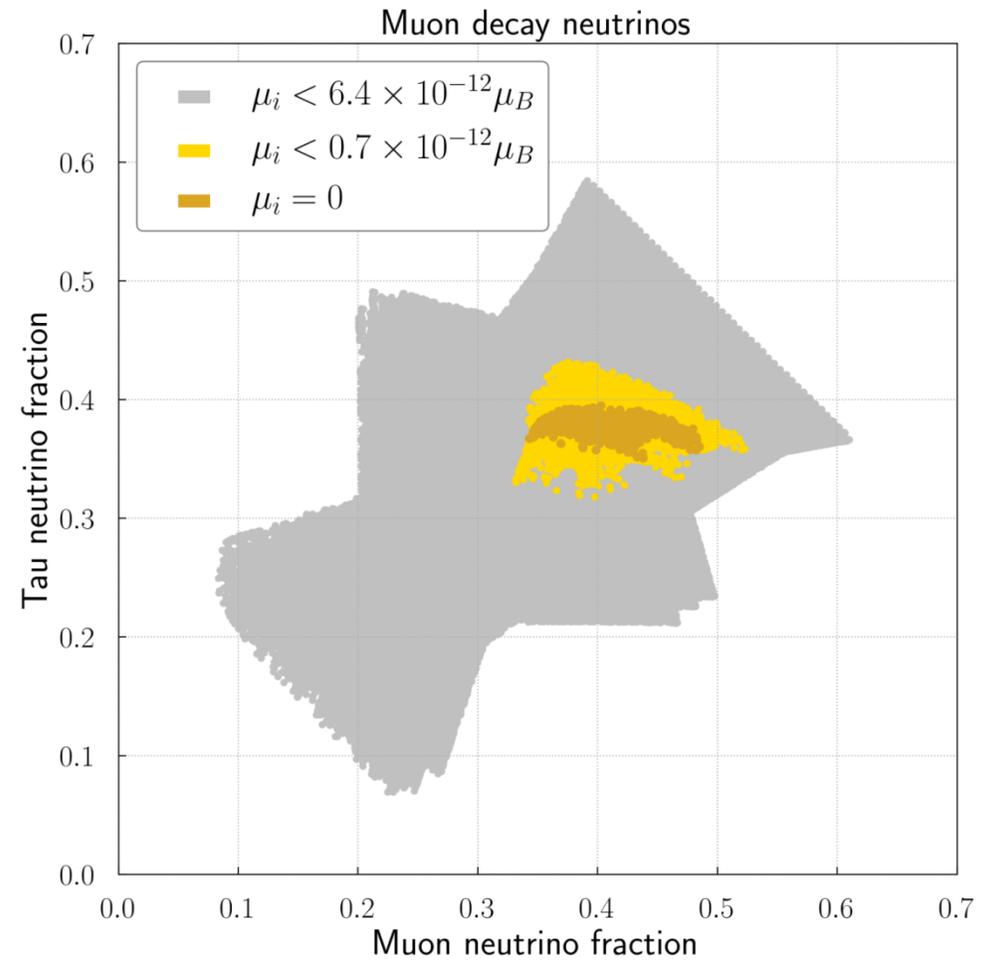
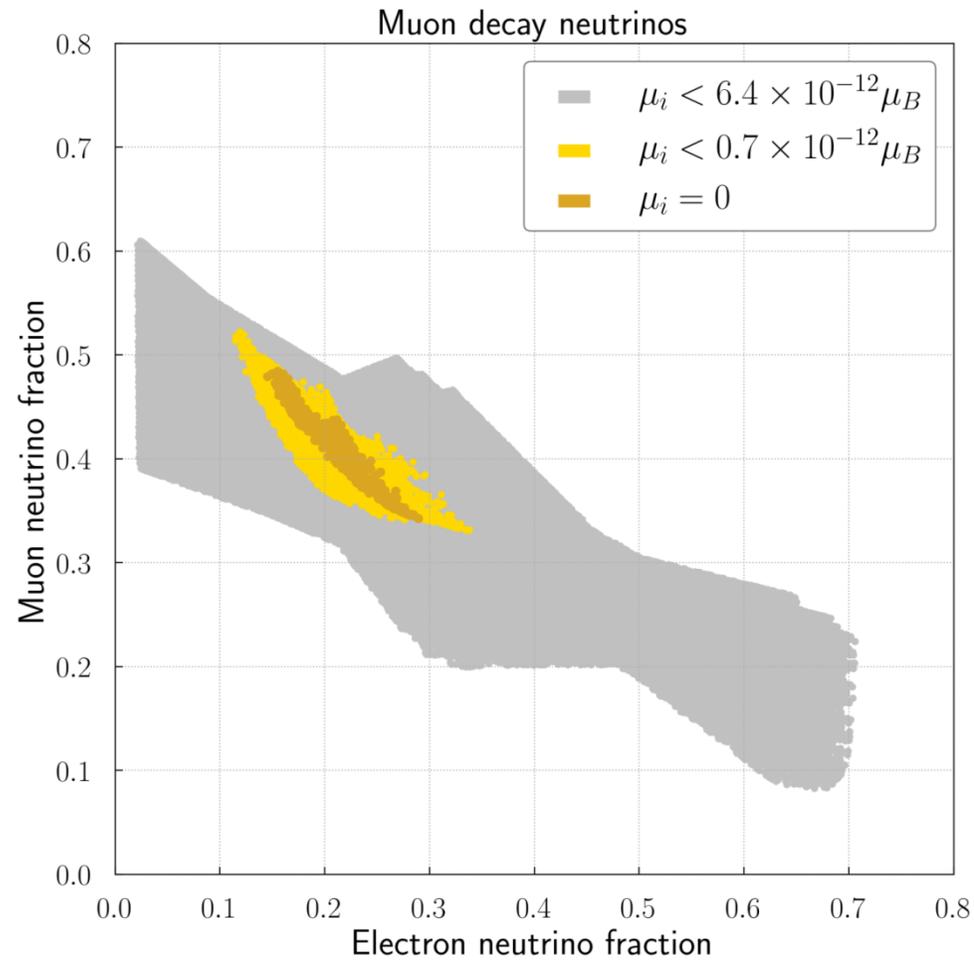
- We use the Galactic magnetic field model provided by R.Jansson, G.Farrar, “*A New Model of the Galactic Magnetic Field*”, *Astrophys.J.* 757 (2012) 14. The field is of order of $O(\mu G)$.
- We consider high-energy neutrinos originating from Galactic center (see IceCube Collaboration, “*Search for Neutrino Emission at the Galactic Center Region with IceCube*”, *PoS ICRC2023* (2023) 1051, and S.Celli, A.Palladino, F.Vissani, “*Neutrinos and γ -rays from the Galactic Center Region After H.E.S.S. Multi-TeV Measurements*”, *Eur.Phys.J.C* 77 (2017) 2, 66).
- Possible flavour ratios are calculated for different values of neutrino magnetic moments μ_1, μ_2 and μ_3 from $(10^{-13}, 6.4 \cdot 10^{-12})$ Bohr magneton range.
- The obtained flavour ratios are compared to ones predicted by standard vacuum neutrino oscillations.



Predicted flavour ratios: π^\pm decay neutrinos



Predicted flavour ratios: μ^\pm decay neutrinos



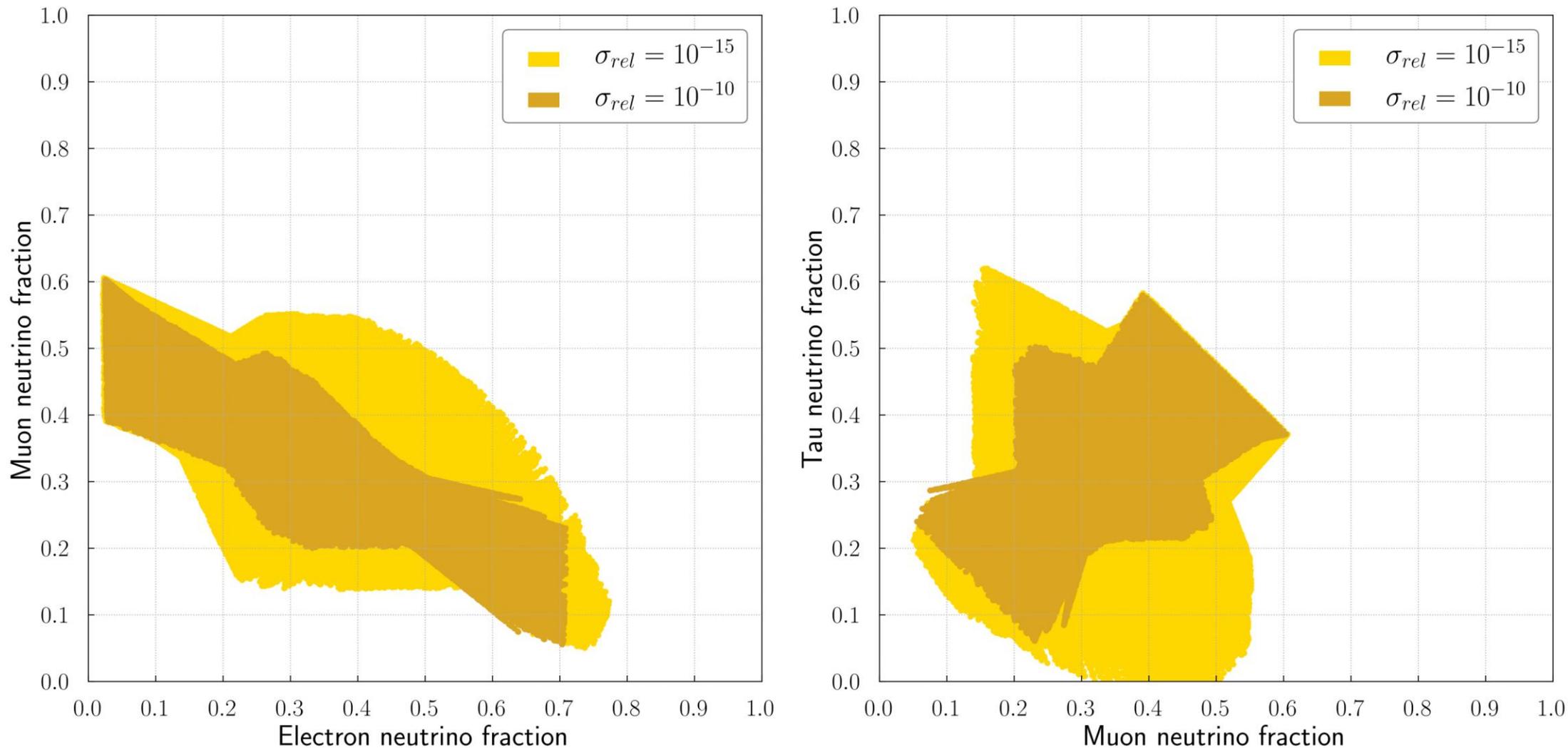


FIG. 8. Flavour compositions of ultra-high energy neutrinos after propagating in the extragalactic magnetic field for different values of the wave packet parameter $\sigma_{rel} = \sigma_p/p$.



Flavour transitions probabilities: an example

$$\mu_1 = \mu_2/2$$

