Emergent Transport and Dissipation from Horizonless Lin-Lunin-Maldacena Microstates

Aleksandr I. Belokon
National University of Science and Technology MISIS

work in progress with
Mihailo Čubrović, Institute of Physics, Belgrade

Session-conference "Physics of Fundamental Interactions"
Nuclear Physics Section, Division of Physical Sciences, Russian Academy of Sciences
Budker Institute of Nuclear Physic, Novosibirsk
March 13, 2026

Black Hole Microstates

Black holes as thermodynamic systems

- BHs possess entropy [Bekenstein'73]:

$$S_{\text{BH}} = \frac{\text{Area(horizon)}}{4G_N}. \tag{1}$$

- BHs radiate with the temperature [Hawking'75]:

$$T_{\text{H}} = \frac{\kappa_h}{2\pi}. \tag{2}$$

Holographic principle

- BH entropy scales with the area of horizon, not the interior volume. This suggests that gravity in a bulk spacetime may admit an equivalent description in terms of a theory on the boundary [t Hooft'93; Susskind'95].
- Concrete realization: AdS/CFT correspondence [Maldacena'97; Gubser et al.'98; Witten'98].

BLACK HOLE MICROSTATES

Microscopic description

- "Central dogma": BHs are quantum systems with $e^{S_{\text{BH}}}$ *microstates* [Mathur'05; '09].
- Quantum description of BH microstates is an open problem.

Microstate counting in SUSY

- BPS sectors are protected against quantum corrections [Witten-Olive'78].
- Bekenstein-Hawking entropy of 5*d* BPS BHs can be reproduced by counting D-brane microstates at weak coupling and relating the result to the strong coupling regime due to SUSY protection [Strominger-Vafa'96] .
- Conceptual message: BH entropy can really be understood as counting quantum microstates.

$\mathcal{N} = 4$ SUPER-YANG–MILLS

- The $4d$ boundary dual of type IIB string theory on $AdS_5 \times S^5$.
- $SU(N)$ gauge group.
- Field content (all in the adjoint representation of $SU(N)$):
  - gauge field A_μ, $\quad \mu = 0, 1, 2, 3$,
  - six real scalars ϕ^I, $\quad I = 1, \ldots, 6$,
  - four Weyl fermions λ^A, $\quad A = 1, \ldots, 4$.
- Global $SO(6)_R$ R-symmetry (matches the isometry of the internal S^5 on the string side).
- Lagrangian:

$$\mathcal{L} = \frac{1}{g_{\mathrm{YM}}^2} \mathrm{Tr} \left(-\frac{1}{4} F_{\mu\nu} F^{\mu\nu} - \frac{1}{2} \sum_{I=1}^{6} D_\mu \phi^I D^\mu \phi^I + \frac{1}{4} \sum_{I,J=1}^{6} [\phi^I, \phi^J][\phi^I, \phi^J] + i\, \bar{\lambda} \gamma^\mu D_\mu \lambda + \bar{\lambda}\, \Gamma^I [\phi^I, \lambda] \right) .$$

1/2-BPS SECTOR OF $\mathcal{N} = 4$ SYM

- ϕ^I can be grouped into three complex scalars:

$$Z = \phi^1 + i\phi^2, \quad Y = \phi^3 + i\phi^4, \quad X = \phi^5 + i\phi^6.$$

- Choose a $U(1)_R \subset SO(6)_R$ that rotates the (ϕ^1, ϕ^2) plane:

$$Z \to e^{i\alpha} Z, \qquad \bar{Z} \to e^{-i\alpha} \bar{Z}, \qquad X, Y \text{ neutral}.$$

- 1/2-BPS sector:
  - The 1/2-BPS local operators are gauge-invariant holomorphic polynomials in one complex scalar Z, with no derivatives and no other fields [Minwalla'97; Corley et al.'02].
  - $\Delta = J$, where Δ = conformal dimension, $J = U(1)_R$ charge [Witten-Olive'78; Minwalla'97].
  - Examples: $\mathrm{Tr}\,(Z^k)$, $\mathrm{Tr}\,(Z^k)\,\mathrm{Tr}\,(Z^n)$, ..., Schur polynomials $\chi_R(Z)$.

1/2-BPS SECTOR OF $\mathcal{N} = 4$ SYM

Closure of 1/2-BPS sector

- For the chosen $U(1)_R$, the field content has the following values of $\Delta - J$:

Field	$\Delta - J$
Z	0
X, Y	1
$\bar{Z}$	2
D_μ	1
$\lambda, F_{\mu\nu}$	> 0

- Therefore, operators with $\Delta = J$ contain only Z.

SUSY protection of 1/2-BPS sector

- In general, $\Delta = \Delta_{\text{classical}} + \gamma(g_{\text{YM}})$, where γ is the anomalous dimension.
- For 1/2-BPS operators, $\gamma = 0$ [Witten-Olive'78; Minwalla'97].
- Hence $\Delta = J$ holds non-perturbatively.

1/2-BPS SECTOR AS A MATRIX MODEL

- In radial quantization on $\mathbb{R} \times S^3$:

$$H = D, \qquad E = \Delta.$$

- $Z(t, \Omega) = \sum_{l,m} Z_{lm}(t)\, Y_{lm}(\Omega)$, with $E_l = l + 1$.
- For 1/2-BPS states: $\Delta = J \Rightarrow E = J$.
- Hence, for a single Z with $J(Z) = 1$:

$$l + 1 = 1 \quad \Rightarrow \quad l = 0.$$

- Therefore, 1/2-BPS sector reduces to a matrix quantum mechanics for a single mode $Z_{00}(t) \equiv Z$:

$$S = \int dt\ \mathrm{Tr}\left(D_t Z^\dagger D_t Z - Z^\dagger Z\right), \qquad D_t Z = \dot{Z} + i[A_0, Z].$$

1/2-BPS SECTOR AS A MATRIX MODEL

- Physical states are gauge singlets, $Z \sim UZU^\dagger$, hence, wavefunctions $\Psi[Z,t]$ depend only on the eigenvalues λ_i of Z.
- In the eigenvalue basis, the Hamiltonian becomes:

$$H\psi = \frac{1}{2}\sum_{i=1}^{N}\left[-\mu^{-1}\partial_{\lambda_i}(\mu\,\partial_{\lambda_i}) + \lambda_i^2\right]\psi, \quad \mu = \Delta(\lambda)^2, \quad \Delta(\lambda) = \prod_{i<j}(\lambda_i - \lambda_j).$$

- After redefinition: $\psi(\lambda) = \Delta(\lambda)^{-1}\widetilde{\psi}(\lambda)$, the Hamiltonian becomes:

$$\widetilde{H}\widetilde{\psi} = \frac{1}{2}\sum_i \left(-\partial_{\lambda_i}^2 + \lambda_i^2\right)\widetilde{\psi},$$

  while $\widetilde{\psi}(\lambda)$ becomes antisymmetric under exchange of eigenvalues λ_i.
- The eigenvalues λ_i behave as N free fermions in a harmonic oscillator potential [Berenstein'04].

Fermion occupation picture and droplets

Ground & Excited 1/2-BPS states

- The lowest-energy gauge-singlet state is exactly the N-fermion ground state:

$$\underbrace{0, 1, 2, \ldots, N-1}_{\text{occupied}}, \; \underbrace{N, N+1, \ldots}_{\text{empty}}$$

- Excited $1/2$-BPS states are obtained by changing fermion occupation numbers.

Droplet picture ($N \gg 1$)

- Each one-particle state occupies a cell of area $2\pi\hbar$ in phase space (x, p).
- N filled fermion levels are represented by a droplet in phase space.
- Vacuum state: occupied levels form a circular droplet in phase space.
- Excited states: changing the fermion occupation numbers deforms the droplet shape.

LIN-LUNIN-MALDACENA GEOMETRIES

LLM geometries are general smooth horizonless 1/2-BPS solutions of type IIB supergravity with asymptotics $\mathrm{AdS}_5 \times S^5$ and symmetry $\mathbb{R} \times SO(4) \times SO(4)$.

	1/2-BPS sector of $\mathcal{N} = 4$ SYM	$\mathbf{AdS_5} \times S^5$
$\mathbb{R}$	Time translations on $\mathbb{R} \times S^3$ Generated by D in radial quantization	Stationary geometry Killing vector ∂_t
$SO(4)$	Spatial rotations of the boundary S^3	Isometry of $S^3 \subset AdS_5$
$SO(4)$	Residual R-symmetry $$SO(6)_R \to SO(4) \times U(1)_R$$ after choosing $Z = \phi^1 + i\phi^2$	Isometry of $S^3 \subset S^5$
$U(1)_R$	Phase rotation of Z, with charge J: $$Z \to e^{i\alpha} Z$$	Isometry is a combination of time translation ∂_t and internal rotation ∂_ϕ on S^5

Lin-Lunin-Maldacena geometries

The explicit form of Lin-Lunin-Maldacena metric is given by:

$$ds^2 = -h^{-2}(dt + V_i dx^i)^2 + h^2(dy^2 + dx_i dx_i) + ye^G d\Omega_3^2 + ye^{-G} d\tilde{\Omega}_3^2,$$

$$h^{-2} = 2y\cosh G, \quad z = \frac{1}{2}\tanh G, \quad y\,\partial_y V_i = \epsilon_{ij}\partial_j z, \quad y(\partial_i V_j - \partial_j V_i) = \epsilon_{ij}\partial_y z.$$

- The whole solution is determined by the Laplace-type equation:

$$\partial_i\partial_i z + y\,\partial_y\left(\frac{1}{y}\partial_y z\right) = 0.$$

- Regularity at $y = 0$ requires:

$$z_0(x_1, x_2) \equiv z(y = 0, x_1, x_2) = \pm\frac{1}{2} \text{ (black/white droplet picture).}$$

- Given $z_0(x_1, x_2)$, the bulk solution is:

$$z(y, x) = \frac{y^2}{\pi}\int d^2 s\,\frac{z_0(s)}{\left(y^2 + |x - s|^2\right)^2}.$$

LIN-LUNIN-MALDACENA GEOMETRIES

Droplet $\leftrightarrow$ Geometry dictionary

- At $y = 0$, one of the two three-spheres shrinks smoothly:

$$z_0 = -\frac{1}{2} \;\Rightarrow\; S^3 \text{ shrinks}, \qquad z_0 = +\frac{1}{2} \;\Rightarrow\; \tilde{S}^3 \text{ shrinks}.$$

- The boundary between black and white regions is where both spheres degenerate in a smooth way; this is what makes the full geometry horizonless and regular [Lin-Lunin-Maldacena'04].
- Droplet pattern on $y = 0$ plane is the complete boundary data that determines the ten-dimensional geometry.

$\mathbf{AdS_5} \times S^5$ as the simplest droplet

- The vacuum of the dual free-fermion system is a single circular droplet.
- The corresponding LLM geometry is precisely global $AdS_5 \times S^5$ [Lin-Lunin-Maldacena'04].
- Small deformations of the circular droplet correspond to $1/2$-BPS excitations around the vacuum.
- More complicated droplets give bubbling, yet still smooth and horizonless, geometries.

Coarse-graining in LLM

- Exact LLM microstates are smooth black/white droplet geometries. After coarse-graining over Planck-scale black/white rings, the boundary data become grayscale, i.e.
$$-\frac{1}{2} < z_0(x_1, x_2) < \frac{1}{2},$$
instead of $z_0 = \pm\frac{1}{2}$ [Balasubramanian et al.'18; Berenstein et al.'25].
- In regular LLM solutions, at $y = 0$ only one of the two S^3's shrinks. In the coarse-grained grayscale solution, the naked singularity is located on $y = 0$ in the region where $-\frac{1}{2} < z_0(x_1, x_2) < \frac{1}{2}$.
- The naked singularity is the coarse-grained image of many underlying microstates [Balasubramanian et al.'18; Berenstein et al.'25].
- The singularity should therefore be viewed not as a fundamental microscopic pathology, but as a signal that the classical long-distance description breaks down near the singularity locus.

LLM AS INCIPIENT BLACK HOLES

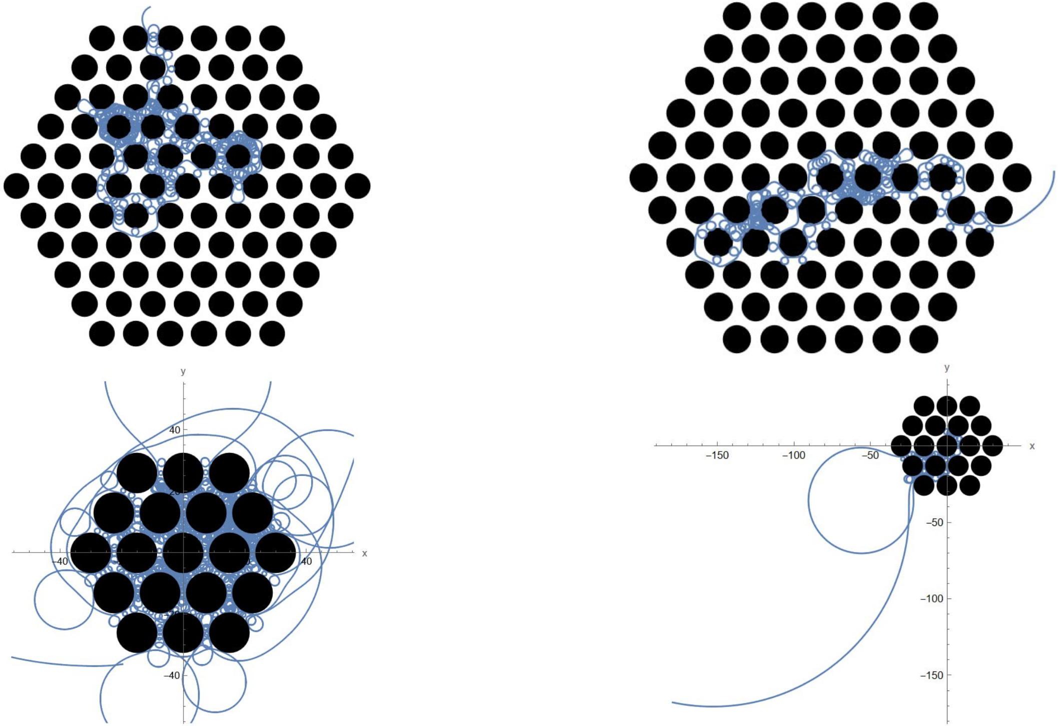
1/2-BPS coarse-grained LLM is an *incipient black hole*: a horizonless, singular geometry that is “almost” a black hole, in the sense that adding some extra energy would produce a finite-area horizon [Balasubramanian et al.’18].

Feature	**Coarse-grained LLM**	**Extremal black hole**
Throat	Strongly redshifted region near the grayscale singular core on the LLM plane $y = 0$	Near-horizon region outside the extremal horizon
Endpoint	Naked singularity, no horizon	True horizon, then interior
Redshift	$g_{tt} = h^{-2} \to 0$ near the singularity	$g_{tt} \to 0$ at the horizon
Potential	Attractive effective potential; averaging creates potential wells	Near-horizon region traps infalling probes
Geodesics	Long-lived trapping; infall freezes as seen from infinity	Infall freezes near the horizon in coordinate time
Interpretation	Incipient black hole	Genuine black hole

RECENT RESULTS ON NULL GEODESIC SCATTERING IN LLM

- In-plane dynamics of null geodesics in LLM is described by a low-dimensional billiard-like Hamiltonian system with an effective magnetic field [Berenstein et al.'24].
- Droplets act as obstacles, producing chaotic scattering, long-lived trapped trajectories, positive Lyapunov exponents [Berenstein et al.'24; Berenstein et al.'25].
- In dense droplet regions, escape can look diffusive, suggesting a mechanism for effective signal loss without a true horizon [Berenstein et al.'24].
- In coarse-grained grayscale geometries, the singularity produces an attractive potential and strong redshift, while chaos is strongly suppressed; the averaged dynamics becomes more black-hole-like, but only up to a finite self-averaging timescale [Berenstein et al.'25].

NUMERICAL SIMULATIONS



In-plane chaotic null geodesic scattering in different multi-droplet LLM backgrounds

Membrane paradigm of black holes

- For an external observer, the black-hole horizon can be replaced by a stretched horizon located slightly outside the true horizon.
- This fictitious surface behaves as a dissipative medium carrying effective physical properties:
  - electrical conductivity;
  - shear viscosity;
  - momentum and charge densities.
- The horizon dynamics can be written in fluid-like form (Ohm's law, Navier–Stokes equations) [Price-Thorne'86; Parikh-Wilczek'98].
- In AdS/CFT in the hydrodynamic limit, transport coefficients of the boundary theory are determined entirely by horizon data [Iqbal-Liu'08].

Why test the membrane paradigm in LLM geometries?

Key difference with black holes

- LLM microstate geometries are smooth and horizonless.
- The standard membrane paradigm has no obvious horizon to define the effective fluid.

However: geodesic dynamics suggests transport-like behavior

- Null geodesics in multi-droplet LLM geometries exhibit chaotic billiard-like scattering.
- In dense droplet regions, escape resembles *diffusion*, with long trapping times.
- After coarse-graining, grayscale LLM geometries develop:
  - strong redshift,
  - attractive effective potential,
  - long throat-like region near the singular core.

Main question

Which aspects of black-hole transport physics require a true horizon, and which can emerge already from coarse-graining over horizonless LLM microstates with chaotic probe dynamics?