Nonlinear Compton scattering in the field of two strong laser waves

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Outline

- Backward Compton scattering – the source of high energy real photons.
- Strong laser fields lead to nonlinear effects.
  Volkov solution [1937] of the Dirac equation.
  Nikishov, Ritus [1964] results for nonlinear Compton process.
  my background:
  D.I., G. Kotkin and V. Serbo [2004]
  full results for polarization effects (the case when all particles polarized) in

$$e + n\gamma_L \to e + \gamma \ , \quad \gamma + n\gamma_L \to e^+ + e^-$$

- My topic:
  Compton scattering in the field of laser two circularly polarized plane waves: $A = A_\omega + A_{2\omega}$.
  Experiment hopefully possible thanks to well known [since 1961] experimental technique of (SHG) Second-Harmonic-Generation.
- Narozhny and Fofanov [2000]: case of two linearly polarized waves.
- Amplitude and probability.
- Results for photon spectra.
- Summary.

Exact Volkov solution

Circularly polarized plane wave:

$$A = a_1 \cos\phi + a_2 \sin\phi \ , \quad \phi = kx \ , \quad a_{1,2}\, k = a_1 a_2 = 0 \ , \quad a_{1,2}^2 \equiv a^2$$

Solution of Dirac equation in this field

$$\Psi_p(x) = \left[1 + \frac{e}{2kp}\hat{k}\hat{A}\right] \frac{u_p}{\sqrt{2p_0}} e^{iS}$$

where

$$S = -px - \int_0^{kx} \frac{e}{kp}\left[pA - \frac{e}{2}A^2\right] d\phi$$

Note that if Ψ^{A_1} and Ψ^{A_2} are solutions of DEq. for fields A_1 and A_2 it is not true that $\Psi^{A_1+A_2} = \Psi^{A_1} + \Psi^{A_2}$ – solution for $A = A_1 + A_2$.

Separate consideration of the circular and linear polarizations of the laser field.

Second-Harmonic-Generation in laser physics

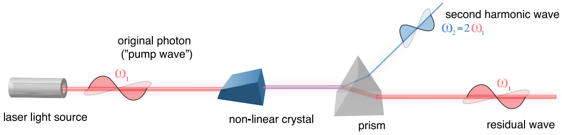

Figure: Setup of the second harmonic generation experiment.

Laser photons interacting with a nonlinear material,

$$P = \chi^{(1)} E + \chi^{(2)} E^2 + \dots$$

P- polarization, $\chi^{(2)}$- non-linear part of electric susceptibility, are effectively "combined" to generate new photons with twice the frequency,

$$\chi^{(2)} E^2 \sim \cos^2(\omega t) \sim (1 + \cos(2\omega t))/2$$

Effective electron mass

The method works also for more general case of superposition of the two plane waves, $A = A_\omega + A_{2\omega}$:

$$A = a_1 \left[\cos\phi + \alpha\cos(2\phi + \phi_0)\right] + a_2 \left[\sin\phi + \alpha\sin(2\phi + \phi_0)\right]$$

where $\alpha = |A_{2\omega}|/|A_\omega|$, and ϕ_0- relative phase

$$A^2 = a^2 \left[1 + \alpha^2 + 2\alpha\cos(\phi + \phi_0)\right]$$

The current $j_\mu = \bar{\Psi}_p \gamma_\mu \Psi_p$ calculated with Volkov solution reads

$$\langle j_\mu \rangle = \frac{1}{p_0}\left(p_\mu - k_\mu \frac{e^2\langle A^2\rangle}{2pk}\right)$$

therefore electron acquires

$$p \to q = p - k\frac{e^2\langle A^2\rangle}{2pk} \quad \text{with} \quad q^2 \equiv m_*^2$$

effective mass (analogy with Bloch theorem in QM), ξ- non-linearity parameter

$$m_*^2 = m^2\left[1 + \xi^2(1 + \alpha^2)\right] \ , \quad \xi^2 = \frac{e^2(-a^2)}{m^2}$$

Photon radiation, Furry picture

$S-$ matrix for radiation of photon with momentum k' and polarization vector e'

$$S_{fi} = -ie \int \bar{\Psi}_{p'}(x) \hat{e}'^{*} \Psi_p(x) \frac{e^{ik'x}}{\sqrt{2\omega'}} d^4x$$

Volkov solutions here are not plane waves! $\Psi_p(x) \sim e^{iS}$, where

$$S = -qx - \frac{\alpha m^2 \xi^2}{kp} \sin(\phi + \phi_0)$$

$$-e\frac{a_1 p}{kp}\left[\sin\phi + \frac{\alpha}{2}\sin(2\phi + \phi_0)\right] + e\frac{a_2 p}{kp}\left[\cos\phi + \frac{\alpha}{2}\cos(2\phi + \phi_0)\right]$$

remind that $\phi = kx$. Similar eq. for $\bar{\Psi}_{p'} = \dots$
ϕ dependence is periodic, therefore we need to separate harmonics!

$$B(\phi) = \sum_{n=-\infty}^{+\infty} B_n e^{-in\phi}$$

where n- number of quanta absorbed from the laser wave.

Amplitude

After harmonic separation and x- integration

$$S_{fi} = \sum_{n=1}^{\infty} \frac{iM_n}{\sqrt{2\omega' 2q_0 2q_0'}} (2\pi)^4 \delta^4(q' + k' - nk - q)$$

M_n - amplitude with absorption of n laser quanta. Since

$$\bar{\Psi}_{p'}(x)\hat{e}'^* \Psi_p(x) \sim \bar{u}_{p'} \left[\hat{e}'^* + \frac{e}{2p'k} \hat{A}\hat{k}\hat{e}'^* + \frac{e}{2pk} \hat{e}'^* \hat{k}\hat{A} - \frac{(e'^* k) e^2 A^2}{2(p'k)(pk)} \right] u_p$$

we will need 4 different Fourier projections: $\mathcal{D}_n^0, \mathcal{D}_n^2, \mathcal{D}_n^i$, where $i = 1, 2$,

$$M_n \sim \bar{u}_{p'} \left[\left(\hat{e}'^* + \frac{(e'^* k) m^2 \xi^2 (1 + \alpha^2)}{2(p'k)(pk)} \hat{k} \right) \mathcal{D}_n^0 + \right.$$

$$\left. \left(\frac{e}{2p'k} \hat{a}_i \hat{k} \hat{e}'^* + \frac{e}{2pk} \hat{e}'^* \hat{k} \hat{a}_i \right) \mathcal{D}_n^i + \frac{(e'^* k) m^2 \xi^2 \alpha}{(p'k)(pk)} \hat{k}\, \mathcal{D}_n^2 \right] u_p$$

Kinematic notations

$$q + nk = q' + k' \ , \quad q^2 = q'^2 = m_*^2 \ , \quad q = p + k\frac{m_*^2 - m^2}{2pk}$$

we define

$$x = \frac{2pk}{m^2} \ , \quad y = \frac{kk'}{kp} \ , \quad 0 \le y \le y_n = \frac{nx}{nx + m_*^2/m^2}$$

for our final state we have

$$d\Gamma_n = \delta^4(q + nk - q' - k')\frac{d^3k'}{\omega'}\frac{d^3q'}{q_0'} = dy\, d\varphi$$

φ- final electron azimuthal angle, $(\varphi + \pi)$- angle of the final photon. For initial electron $p = (E, \vec{p})$ and $|\vec{p}| \approx E$ for ultra relativistic electron

$$\omega' = (|\vec{p}| - n\omega)y + n\omega$$

final photon polar angle is θ

$$y = \frac{(1 + \cos\theta)n\omega}{(1 + \cos\theta)n\omega + E - |\vec{p}|\cos\theta}$$

Gauge invariance

we will need also

$$\alpha_i \equiv e\left(\frac{a_i p}{kp} - \frac{a_i p'}{kp'}\right) \ , \quad z^2 \equiv \alpha_1^2 + \alpha_2^2 \ , \quad \alpha_1 = z\cos\varphi \ , \alpha_2 = z\sin\varphi$$

and

$$z = \frac{2n\,\xi}{m_*/m}\sqrt{\frac{u}{u_n}\left(1 - \frac{u}{u_n}\right)} \ , \quad u \equiv \frac{y}{1-y} \ , \quad 0 \le u \le u_n = \frac{nx}{m_*^2/m^2}$$

Gauge invariance: $M_n \to 0$ if we substitute polarization vector, $e' \to k'$

$$\alpha_i \mathcal{D}_n^i = \alpha_1 \mathcal{D}_n^{i=1} + \alpha_2 \mathcal{D}_n^{i=2} = n\mathcal{D}_n^0 + \frac{2\alpha y \xi^2}{x(1-y)}\mathcal{D}_n^2$$

non-trivial check of our calculation, and allows to simplify our final result.

Fourier projections

we introduce notation $\psi = \varphi + \phi_0$ then

$$\mathcal{D}_n^0 = e^{in\varphi} \int_0^{2\pi} \frac{d\phi}{2\pi} e^{in\phi - iz\left(\sin\phi + \frac{\alpha}{2}\sin(2\phi+\psi)\right) + 2i\alpha\xi^2 \frac{u}{x}\sin(\phi+\psi)}$$

$$\mathcal{D}_n^2 = e^{in\varphi} \int_0^{2\pi} \frac{d\phi}{2\pi} \cos(\phi+\psi) e^{[\cdots]}$$

$$\mathcal{D}_n^{i=1} = e^{in\varphi} \int_0^{2\pi} \frac{d\phi}{2\pi} \left[\cos(\phi+\varphi) + \alpha\cos(2\phi+\psi+\varphi)\right] e^{[\cdots]}$$

$$\mathcal{D}_n^{i=2} = e^{in\varphi} \int_0^{2\pi} \frac{d\phi}{2\pi} \left[\sin(\phi+\varphi) + \alpha\sin(2\phi+\psi+\varphi)\right] e^{[\cdots]}$$

Emission probability

So, we got for probability of emission per unit time ($\bar{y} \equiv 1 - y$):

$$\frac{d\dot{W}_n}{dy d\varphi} = \frac{\alpha_{em} m^2}{4\pi q_0} \times$$

$$\left[\xi^2 \left(\frac{1+\bar{y}^2}{\bar{y}} \right) \left\{ \mathcal{D}_n^i \mathcal{D}_n^{i*} - (1+\alpha^2) \mathcal{D}_n^0 \mathcal{D}_n^{0*} - \alpha \left(\mathcal{D}_n^0 \mathcal{D}_n^{2*} + \mathcal{D}_n^2 \mathcal{D}_n^{0*} \right) \right\} - 2\mathcal{D}_n^0 \mathcal{D}_n^{0*} \right]$$

Our probability depends on $\psi = \varphi + \phi_0$ angle only! Changing ϕ_0 one can "rotate" azimuthal angle distribution.

At $\alpha \to 0$, our $\mathcal{D}$'s are expressed through Bessel $J_n(z)$ functions. The gauge invariance condition then is reduced to the known relation for Bessel functions

$$J_{n+1}(z) + J_{n-1}(z) = \frac{2n}{z} J_n(z)$$

In our case we use numerical approach to $\mathcal{D}$'s.

In the limit $\alpha = 0$, Nikishov-Ritus result is restored (being independent on φ)

$$\frac{d\dot{W}_n^{NR}}{dy d\varphi} \sim \left[\xi^2 \left(\frac{1+\bar{y}^2}{2\bar{y}} \right) \left\{ J_{n+1}^2(z) + J_{n-1}^2(z) - 2J_n^2(z) \right\} - 2J_n^2(z) \right]$$

Azimuthal angle dependence - new nonlinear effect

- We got probability dependence on the azimuthal angle! How it could be?

Our $A = A_\omega + A_{2\omega}$, electric field $\vec{E} = -\frac{\partial \vec{A}}{\partial t}$.

$$\vec{E} = \omega \{\vec{a}_1 [\sin\phi + 2\alpha \sin(2\phi + \phi_0)] - \vec{a}_2 [\cos\phi + 2\alpha \cos(2\phi + \phi_0)]\}$$

- We introduce vector in the transverse plane $\vec{n} \equiv \vec{a}_1 \cos\varphi + \vec{a}_2 \sin\varphi$, then

$$(\vec{n}\vec{E}) = \omega(-a^2) [\sin(\phi - \varphi) + 2\alpha \sin(2(\phi - \varphi) + \phi_0 + \varphi)]$$

and consider correlators $\langle P^n \rangle = \langle (\vec{n}\vec{E})^n \rangle / (\omega(-a^2)^n)$ $(\psi \equiv \phi_0 + \varphi)$

$$\langle P^1 \rangle = 0 \ , \quad \langle P^2 \rangle = \frac{1}{2} + 2\alpha^2 \ , \quad \langle P^3 \rangle = -\frac{3}{2}\alpha \sin\psi$$

$$\langle P^4 \rangle = \frac{3}{8} + 6\alpha^2 + 6\alpha^4 \ , \quad \langle P^5 \rangle = -\frac{5}{2}\alpha(1 + 6\alpha^2) \sin\psi$$

- In the presence of second wave there is preferable direction in the transverse plane. It is a non-linear effect (starts from $\langle P^3 \rangle$)!

Typical energy range for Compton photons, when $\omega = 1$ eV

- Particle physics I:

$$\omega' \sim 260 \text{ GeV} , \quad \text{x} = 4.8 , \quad \text{E} = 312 \text{ GeV}$$

- Particle physics II:

$$\omega' \sim 11 \text{ GeV} , \quad \text{x} = 0.5 , \quad \text{E} = 32.5 \text{ GeV}$$

- Nuclear physics:

$$\omega' \sim 6 \text{ MeV} , \quad \text{x} = 0.01 , \quad \text{E} = 650 \text{ MeV}$$

- "Solid state" physics:

$$\omega' \sim 16 \text{ KeV} , \quad \text{x} = 0.5\, 10^{-3} , \quad \text{E} = 32.5 \text{ MeV}$$

The last two cases correspond to the low energy Thomson limit $(p+k)^2 - m^2 \ll m^2$, or $x \ll 1$.

Strategy to choose $\alpha = A_{2\omega}/A_{\omega}$

For each ξ :

$$\xi^2 \sim \left(\frac{J}{10^{18}[W/cm^2]}\right)\left(\frac{1\ \mathrm{eV}}{\hbar\omega}\right)^2$$

to find such $\alpha = A_{2\omega}/A_{\omega}$ where there is order of 100% interference between the processes of single absorption from $A_{2\omega}$ and non-linear double absorption from A_{ω}.

In this way we will consider the following pairs of parameters:

$$\xi = 0.2 \quad \alpha = 0.2$$

$$\xi = 0.3 \quad \alpha = 0.267$$

$$\xi = 0.4 \quad \alpha = 0.325$$

Also we will consider $\xi = 1$, and $\alpha = 1$ cases.

Results: "collider" energies

After conversion $e + \gamma_L \to e' + \gamma'$ the process of e^+e^- pair production, $\gamma' + \gamma_L \to e^+ + e^-$, may occur if energy is above threshold. The condition for that

$$x^2 = 4(1+\xi^2)(1+x+\xi^2)$$

for $\xi = 0$, the solution is $x = 2 + \sqrt{8} = 4.83$. For $\omega = 1\ \mathrm{eV}$ it corresponds to $E \approx 300\ \mathrm{GeV}$. It can be realized at the future Electron Linear Colliders.

Just single strong wave with $\xi = 1$

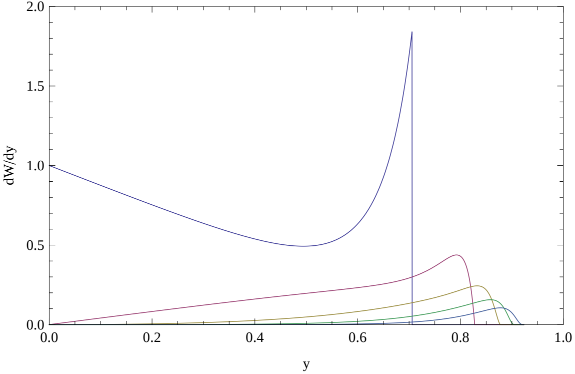

Figure: Single wave: $x = 4.8$, $\xi = 1$.

Results: "collider" energies

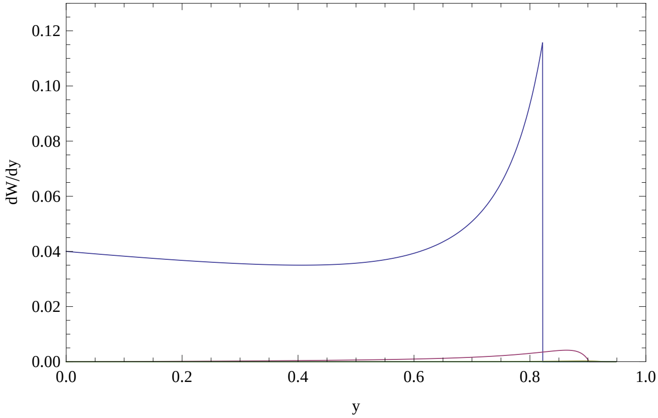

Figure: Single wave: $x = 4.8$, $\xi = 0.2$.

Results: "collider" energies

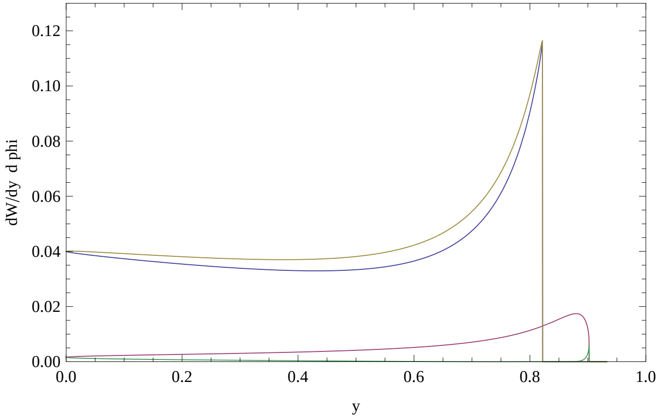

Figure: 2waves: $x = 4.8$, $\xi = 0.2$, $\alpha = 0.2$, $\psi = 0$ and π.

Results: "collider" energies

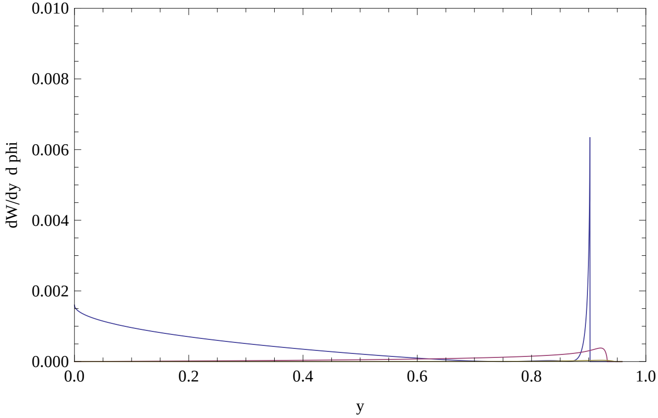

Figure: 2waves: $x = 4.8$, $\xi = 0.2$, $\alpha = 0.2$, $\psi = 0$.

Results: "collider" energies

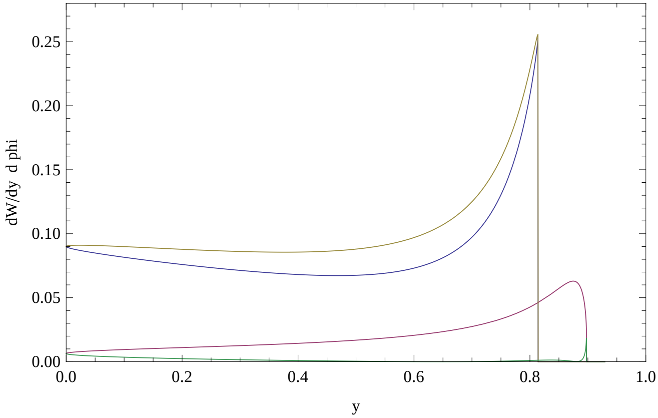

Figure: 2waves: $x = 4.8$, $\xi = 0.3$, $\alpha = 0.27$, $\psi = 0$ and π.

Results: "collider" energies

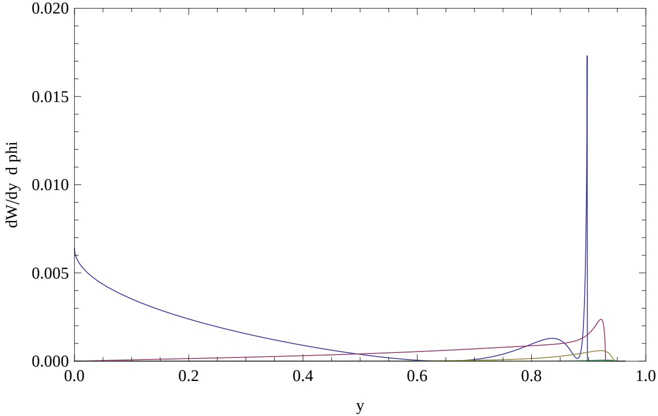

Figure: 2waves: $x = 4.8$, $\xi = 0.3$, $\alpha = 0.27$, $\psi = 0$.

Results: "collider" energies

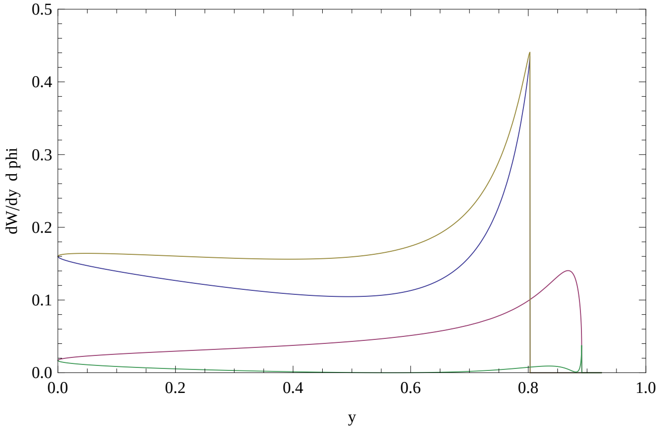

Figure: 2waves: $x = 4.8$, $\xi = 0.4$, $\alpha = 0.33$, $\psi = 0$ and π.

Results: "collider" energies

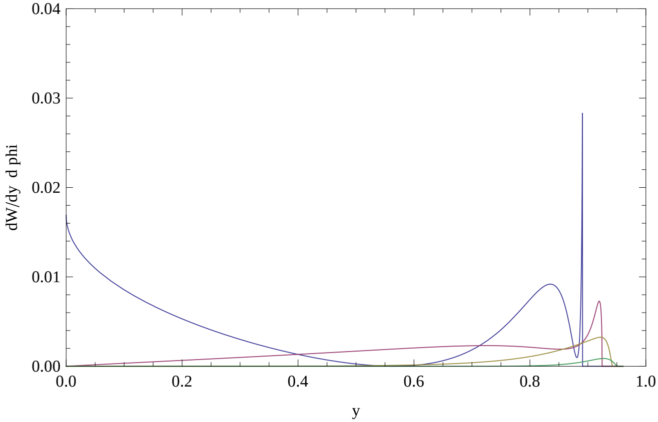

Figure: 2waves: $x = 4.8$, $\xi = 0.4$, $\alpha = 0.33$, $\psi = 0$.

Results: "small" energy, 32.5 MeV

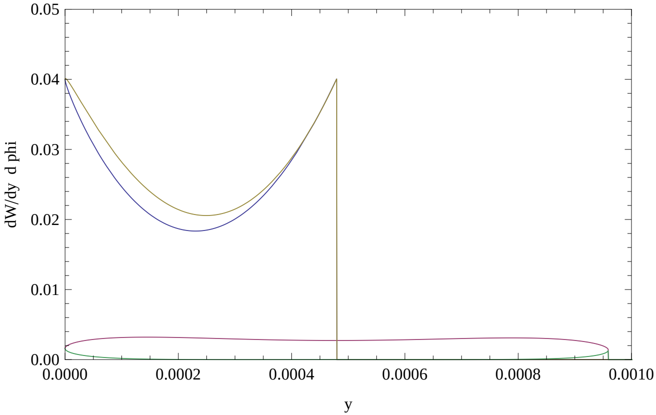

Figure: 2waves: $x = 0.510^{-3}$, $\xi = 0.2$, $\alpha = 0.2$, $\psi = 0$ and π.

Results: "small" energy, 32.5 MeV

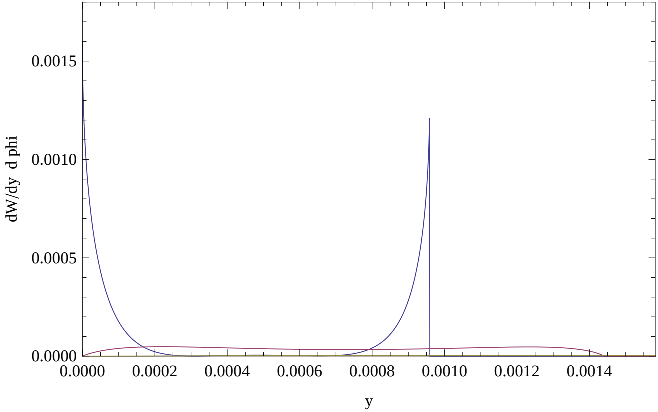

Figure: 2waves: $x = 0.510^{-3}$, $\xi = 0.2$, $\alpha = 0.2$, $\psi = 0$.

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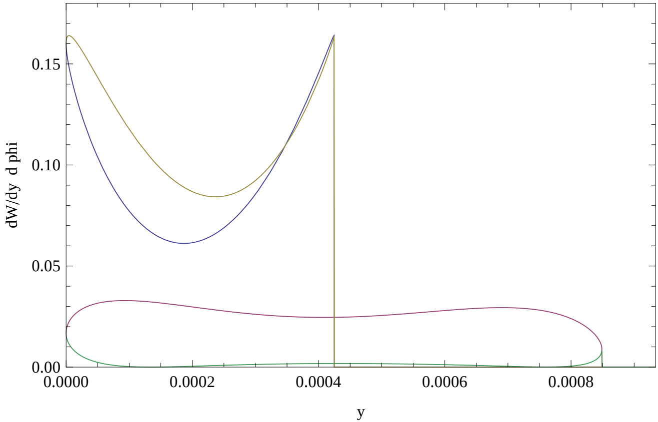

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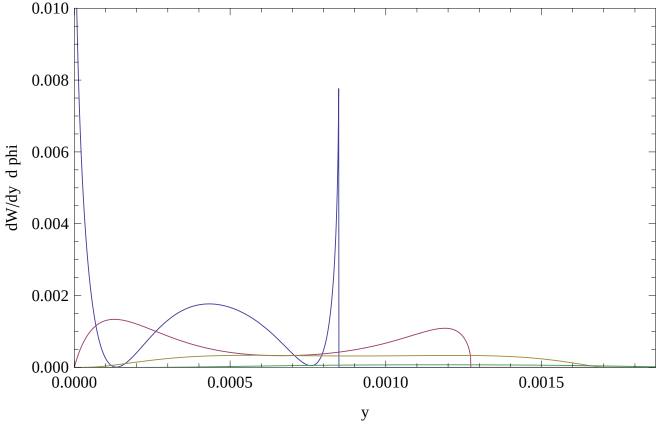

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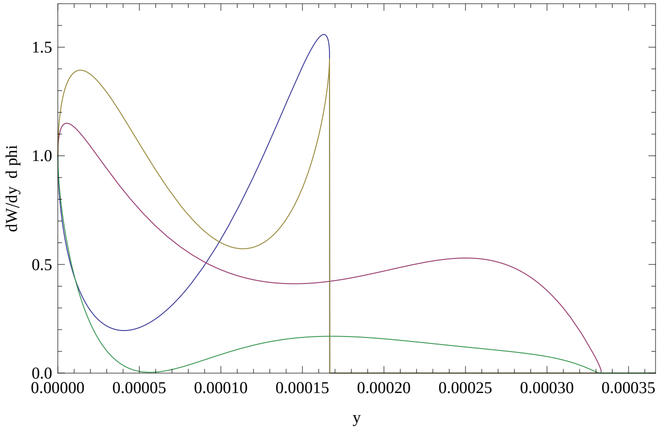

Figure: 2waves: $x = 0.510^{-3}$, $\xi = 1.0$, $\alpha = 1.0$, $\psi = 0$ and π.

Summary

- We derived results for the non-linear Compton scattering in the field of two waves, $A_\omega + A_{2\omega}$, both circularly polarized.
- Non-linearity in such field induces azimuthal angle dependence of the Compton spectra:
  We found at some values of ξ and α strong influence (interference) between the processes of absorption of one quanta from $A_{2\omega}$ and two quanta from A_ω. This leads to very strong azimuthal angle dependence for the second harmonic.
  At $\psi = 0$ It leads to almost full cancellation of the second harmonic, with the "quasy monochomatic" peak at its threshold.
  At $\psi = \pi$ we found instead large enhancement of the second harmonic.
- These effects are special for circular polarization and are related with angular momentum conservation low.
- Relative phase ϕ_0 parameter between $A_{2\omega}$ and A_ω may be used to "rotate" azimuthal angle distribution of the Compton photons.
- Study of polarization effects is in progress.