Light mesons from tau decays

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based on: Escribano, Gonzàlez-Solís, Jamin, Roig JHEP 1409 (2014), Escribano, Gonzàlez-Solís, Roig PRD 94 (2016), 034008, Gonzàlez-Solís, Roig 1902.02273 [hep-ph]

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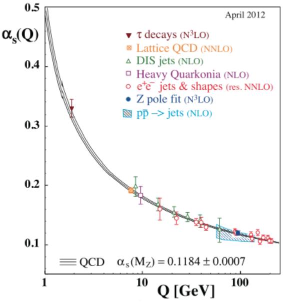
Test of QCD and ElectroWeak Interactions

- Inclusive decays: $\tau^- \to (\bar{u}d, \bar{u}s)\nu_\tau$

  Full hadron spectra (precision physics)

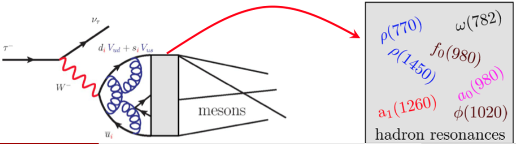
  Fundamental SM parameters: $\alpha_s(m_\tau), m_s, |V_{us}|$



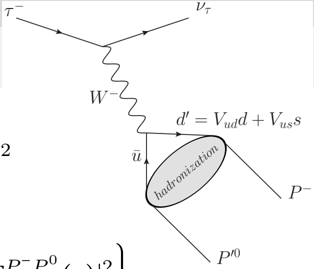
- Exclusive decays: $\tau^- \to (PP, PPP, ...)\nu_\tau$

  specific hadron spectrum (approximate physics)

  Hadronization of QCD currents, study of Form Factors, resonance parameters (M_R, Γ_R)



τ decays into two mesons



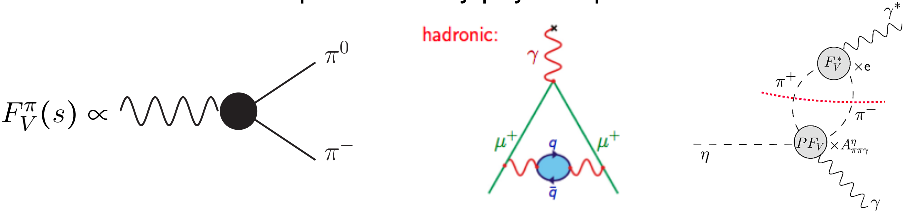
$$\frac{d\Gamma\left(\tau^- \to P^- P^0 \nu_\tau\right)}{ds} = \frac{G_F^2 |V_{ui}|^2 m_\tau^3}{768\pi^3} S_{EW}^{\text{had}} C_{PP'}^2 \left(1 - \frac{s}{M_\tau^2}\right)^2$$

$$\left\{\left(1 + \frac{2s}{m_\tau^2}\right) \lambda_{P^-P^0}^{3/2}(s) |F_V^{P^-P^0}(s)|^2 + 3\frac{\Delta_{P^-P^0}^2}{s^2} \lambda_{P^-P^0}^{1/2}(s) |F_S^{P^-P^0}(s)|^2\right\}$$

- $\tau^- \to \pi^-\pi^0\nu_\tau$: Pion vector form factor, $\rho(770), \rho(1450), \rho(1700)$
- $\tau^- \to K^-K_S\nu_\tau$: Kaon vector form factor, $\rho(770), \rho(1450), \rho(1700)$
- $\tau^- \to K_S\pi^-\nu_\tau$: $K\pi$ form factor, $K^*(892), K^*(1410)$, $K_{\ell 3}$, V_{us} (Passemar)
- $\tau^- \to K^-\eta^{(\prime)}\nu_\tau$: $K^*(1410)$, V_{us}
- $\tau^- \to \pi^-\eta^{(\prime)}\nu_\tau$: isospin-violating decays

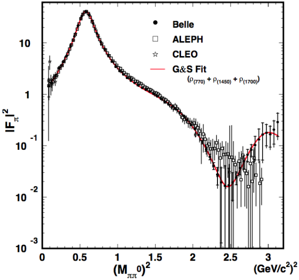
The pion vector form factor: Motivation

- Enters the description of many physical processes



see talk by Colangelo

- Belle measurement of the pion vector form factor (0805.3773)

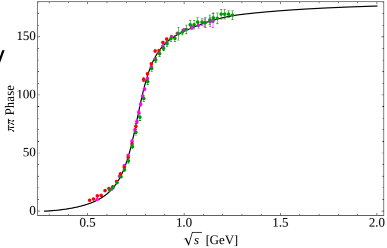


- high-statistics data until de τ mass
- sensitive to $\rho(1450)$ and $\rho(1700)$
- our aim: to improve the description of the $\rho(1450)$ and $\rho(1700)$ region

- Dispersive representation of the pion vector form factor

$$F_V^\pi(s) = \exp\left[\alpha_1 s + \frac{\alpha_2}{2}s^2 + \frac{s^3}{\pi}\int_{4m_\pi^2}^{s_{\rm cut}} ds' \frac{\delta_1^1(s')}{(s')^3(s'-s-i0)}\right],$$

- Form Factor phase $\delta_1^1(s)$
  - $4m_\pi^2 < s < 1$ GeV: $\pi\pi$ phase from Roy (García-Martín et.al PRD 83, 074004 (2011))
  - $1 < s < m_\tau^2$: "Pheno" phase shift
  - $m_\tau^2 < s$: phase guided smoothly to π



- Low-energy observables

$$F_V^\pi(s) = 1 + \frac{1}{6}\langle r^2\rangle_V^\pi s + c_V^\pi s^2 + d_V^\pi s^3 + \cdots .$$

$$\langle r^2\rangle_V^\pi = 6\alpha_1 , \quad c_V^\pi = \frac{1}{2}\left(\alpha_2 + \alpha_1^2\right) .$$

ChPT with resonances + Omnès: Exponential representation

- Get a model for the (Pheno) phase

$$F_V^\pi(s) = P_n(s)\exp\left\{\frac{s^n}{\pi}\int_{4m_\pi^2}^\infty \frac{ds'}{(s')^n}\frac{\delta_1^1(s')}{s'-s-i0}\right\},$$

- $\pi\pi \to \pi\pi$ scattering at $\mathcal{O}(p^2)$

$$T(s) = \frac{s-m_\pi^2}{F_\pi^2} \longrightarrow T_1^1(s) = \frac{s\sigma_\pi^2(s)}{96\pi F_\pi^2} \longrightarrow \delta_1^1(s) = \sigma_\pi(s)T_1^1(s) = \frac{s\sigma_\pi^3(s)}{96\pi F_\pi^2},$$

- Exponential Omnès representation of the form factor

$$F_V^\pi(s) = \frac{M_\rho^2}{M_\rho^2 - s - iM_\rho\Gamma_\rho(s)}\exp\left\{-\frac{s}{96\pi^2 F_\pi^2}\mathrm{Re}\left[A_\pi(s,\mu^2) + \frac{1}{2}A_K(s,\mu^2)\right]\right\}$$

$$\Gamma_\rho(s) = -\frac{M_\rho s}{96\pi^2 F_\pi^2}\mathrm{Im}\left[A_\pi(s) + \frac{1}{2}A_K(s)\right]$$

- Incorporation of the $\rho' \equiv \rho(1450), \rho'' \equiv \rho(1700)$

$$
\begin{aligned}
F_V^\pi(s) \;=\; & \frac{M_\rho^2 + s\left(\gamma e^{i\phi_1} + \delta e^{i\phi_2}\right)}{M_\rho^2 - s - iM_\rho\Gamma_\rho(s)} \exp\left\{\mathrm{Re}\left[-\frac{s}{96\pi^2 F_\pi^2}\left(A_\pi(s) + \frac{1}{2}A_K(s)\right)\right]\right\} \\
& -\gamma \frac{s\, e^{i\phi_1}}{M_{\rho'}^2 - s - iM_{\rho'}\Gamma_{\rho'}(s)} \exp\left\{-\frac{s\Gamma_{\rho'}(M_{\rho'}^2)}{\pi M_{\rho'}^3 \sigma_\pi^3(M_{\rho'}^2)}\mathrm{Re}A_\pi(s)\right\} \\
& -\delta \frac{s\, e^{i\phi_2}}{M_{\rho''}^2 - s - iM_{\rho''}\Gamma_{\rho''}(s)} \exp\left\{-\frac{s\Gamma_{\rho''}(M_{\rho''}^2)}{\pi M_{\rho''}^3 \sigma_\pi^3(M_{\rho''}^2)}\mathrm{Re}A_\pi(s)\right\},
\end{aligned}
$$

$$
\Gamma_{\rho',\rho''}(s) = \Gamma_{\rho',\rho''} \frac{s}{M_{\rho',\rho''}^2} \frac{\sigma_\pi^3(s)}{\sigma_\pi^3(M_{\rho',\rho''}^2)} \theta(s - 4m_\pi^2).
$$

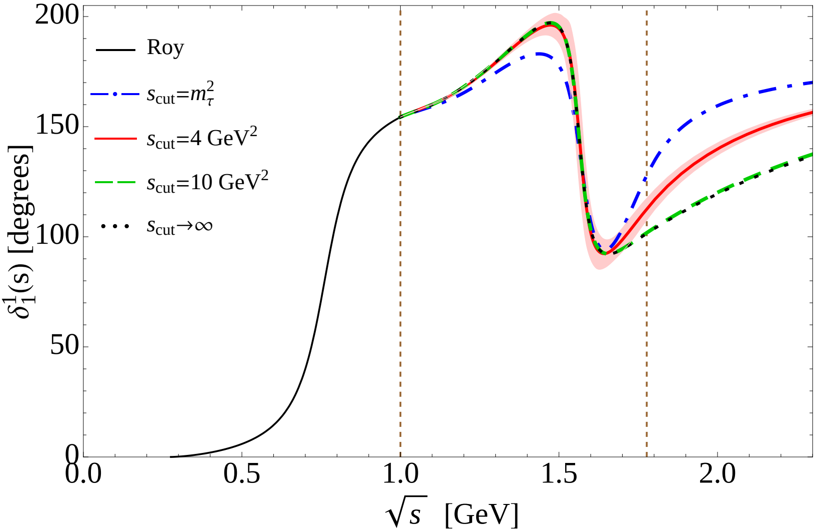
$$
\tan\delta_1^1(s) = \frac{\mathrm{Im}F_V^\pi(s)}{\mathrm{Re}F_V^\pi(s)}
$$

Dispersive Fits to the Pion Vector Form Factor

- Fits for different values of $s_{\rm cut}$ and matching at 1 GeV

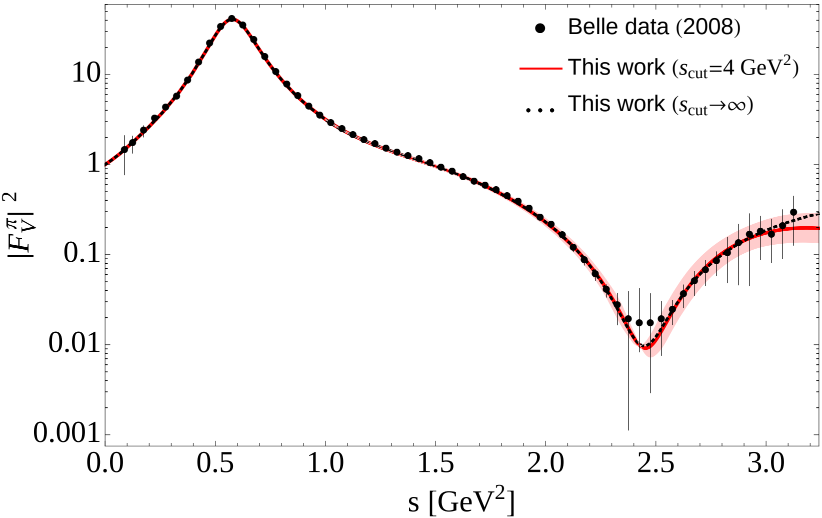
Fits	Parameter	$s_{\rm cut}$ [GeV^2]			
		m_τ^2	4 (reference fit)	10	∞
Fit 1	α_1 [GeV^{-2}]	$1.87(1)$	$1.88(1)$	$1.89(1)$	$1.89(1)$
	α_2 [GeV^{-4}]	$4.40(1)$	$4.34(1)$	$4.32(1)$	$4.32(1)$
	m_ρ [MeV]	$= 773.6(9)$	$= 773.6(9)$	$= 773.6(9)$	$= 773.6(9)$
	M_ρ [MeV]	$= m_\rho$	$= m_\rho$	$= m_\rho$	$= m_\rho$
	$M_{\rho'}$ [MeV]	$1365(15)$	$1376(6)$	$1313(15)$	$1311(5)$
	$\Gamma_{\rho'}$ [MeV]	$562(55)$	$603(22)$	$700(6)$	$701(28)$
	$M_{\rho''}$[MeV]	$1727(12)$	$1718(4)$	$1660(9)$	$1658(1)$
	$\Gamma_{\rho''}$ [MeV]	$278(1)$	$465(9)$	$601(39)$	$602(3)$
	γ	$0.12(2)$	$0.15(1)$	$0.16(1)$	$0.16(1)$
	ϕ_1	$-0.69(1)$	$-0.66(1)$	$-1.36(10)$	$-1.39(1)$
	δ	$-0.09(1)$	$-0.13(1)$	$-0.16(1)$	$-0.17(1)$
	ϕ_2	$-0.17(5)$	$-0.44(3)$	$-1.01(5)$	$-1.03(2)$
	χ^2/d.o.f	1.47	0.70	0.64	0.64

- Form Factor phase shift for different values of s_{cut}



- The results can be found in tables provided as ancillary material in 1902.02273 [hep-ph]

- Modulus squared of the pion vector form factor



- The results can be found in tables provided as ancillary material in 1902.02273 [hep-ph]

- Fits for different matching point and with $s_{\rm cut} = 4$ GeV

Fits	Parameter	Matching point [GeV]			
		0.85	0.9	0.95	1 (reference fit)
Fit I	α_1 [GeV^{-2}]	$1.88(1)$	$1.88(1)$	$1.88(1)$	$1.88(1)$
	α_2 [GeV^{-4}]	$4.35(1)$	$4.35(1)$	$4.34(1)$	$4.34(1)$
	m_ρ [MeV]	$= 773.6(9)$	$= 773.6(9)$	$= 773.6(9)$	$= 773.6(9)$
	M_ρ [MeV]	$= m_\rho$	$= m_\rho$	$= m_\rho$	$= m_\rho$
	$M_{\rho'}$ [MeV]	$1394(6)$	$1374(8)$	$1351(5)$	$1376(6)$
	$\Gamma_{\rho'}$ [MeV]	$592(19)$	$583(27)$	$592(2)$	$603(22)$
	$M_{\rho''}$[MeV]	$1733(9)$	$1715(1)$	$1697(3)$	$1718(4)$
	$\Gamma_{\rho''}$ [MeV]	$562(3)$	$541(45)$	$486(7)$	$465(9)$
	γ	$0.12(1)$	$0.12(1)$	$0.13(1)$	$0.15(1)$
	ϕ_1	$-0.44(3)$	$-0.60(1)$	$-0.80(1)$	$-0.66(1)$
	δ	$-0.13(1)$	$-0.13(1)$	$-0.13(1)$	$-0.13(1)$
	ϕ_2	$-0.38(3)$	$-0.51(2)$	$-0.62(1)$	$-0.44(3)$
	χ^2/d.o.f	0.75	0.74	0.68	0.70

Variant (II): Inclusion of intermediate states other than $\pi\pi$

- Fit A: $\rho' \to K\bar{K}$ and $\rho'' \to K\bar{K}$
- Fit B: $\rho' \to K\bar{K}$ + $\rho' \to \omega\pi$

Parameter	$s_{\rm cut}$ = 4 GeV^2		
	Fit A	Fit B	reference fit
α_1 [GeV^{-2}]	$1.87(1)$	$1.88(1)$	$1.88(1)$
α_2 [GeV^{-4}]	$4.37(1)$	4.35(1)	$4.34(1)$
m_ρ [MeV]	$= 773.6(9)$	$= 773.6(9)$	$= 773.6(9)$
M_ρ [MeV]	$= m_\rho$	$= m_\rho$	$= m_\rho$
$M_{\rho'}$ [MeV]	$1373(5)$	$1441(3)$	$1376(6)$
$\Gamma_{\rho'}$ [MeV]	$462(14)$	$576(33)$	$603(22)$
$M_{\rho''}$[MeV]	$1775(1)$	$1733(9)$	$1718(4)$
$\Gamma_{\rho''}$ [MeV]	$412(27)$	$349(52)$	$465(9)$
γ	$0.13(1)$	$0.15(3)$	$0.15(1)$
ϕ_1	$-0.80(1)$	$-0.53(5)$	$-0.66(1)$
δ	$-0.14(1)$	$-0.14(1)$	$-0.13(1)$
ϕ_2	$-0.44(2)$	$-0.46(3)$	$-0.44(3)$
χ^2/d.o.f	0.93	0.70	0.70

Variant (III)

- Dispersive representation of the pion vector form factor

$$F_V^\pi(s) = \exp\left[\frac{s}{\pi}\int_{4m_\pi^2}^{s_{\rm cut}} ds' \frac{\delta_1^1(s')}{(s')(s'-s-i0)} + \frac{s}{\pi}\int_{s_{\rm cut}}^{\infty} ds' \frac{\delta_{\rm eff}(s')}{(s')(s'-s-i0)}\right]\Sigma(s)$$

- Properties for $\delta_{\rm eff}(s)$
  - $\delta_{\rm eff}(s_{\rm cut}) = \delta_1^1(s_{\rm cut})$ and $\delta_{\rm eff}(s) \to \pi$ for large s to recover $1/s$ fall-off

$$\delta_{\rm eff}(s) = \pi + \left(\delta_1^1(s_{\rm cut}) - \pi\right)\frac{s_{\rm cut}}{s}$$

  - Integrating the piece with $\delta_{\rm eff}(s)$

$$\begin{aligned} F_V^\pi(s) &= e^{1-\frac{\delta_1^1(s_{\rm cut})}{\pi}}\left(1-\frac{s}{s_{\rm cut}}\right)^{\left(1-\frac{\delta_1^1(s_{\rm cut})}{\pi}\right)\frac{s_{\rm cut}}{s}}\left(1-\frac{s}{s_{\rm cut}}\right)^{-1} \\ &\times \exp\left[\frac{s}{\pi}\int_{4m_\pi^2}^{s_{\rm cut}} ds' \frac{\delta_1^1(s')}{(s')(s'-s-i0)}\right]\Sigma(s) \\ \Sigma(s) &= \sum_{i=0}^{\infty} a_i \omega^i(s)\,, \quad \omega(s) = \frac{\sqrt{s_{\rm cut}}-\sqrt{s_{\rm cut}-s}}{\sqrt{s_{\rm cut}}+\sqrt{s_{\rm cut}-s}} \end{aligned}$$

The resulting fit parameters are found to be

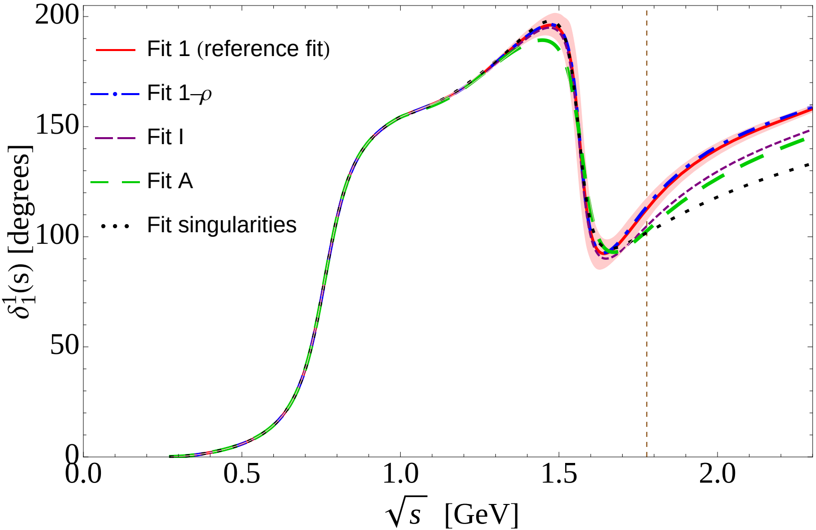
$$a_1 = 2.99(12)\,,$$
$$M_{\rho'} = 1261(7)\,\mathrm{MeV}\,, \quad \Gamma_{\rho'} = 855(15)\,\mathrm{MeV}\,,$$
$$M_{\rho''} = 1600(1)\,\mathrm{MeV}\,, \quad \Gamma_{\rho''} = 486(26)\,\mathrm{MeV}\,,$$
$$\gamma = 0.25(2)\,, \quad \phi_1 = -1.90(6)\,,$$
$$\delta = -0.15(1)\,, \quad \phi_2 = -1.60(4)\,,$$

with a χ^2/d.o.f $= 32.3/53 \sim 0.61$ for the one-parameter fit, and

$$a_1 = 3.03(20)\,, \quad a_2 = 1.04(2.10)\,,$$
$$M_{\rho'} = 1303(19)\,\mathrm{MeV}\,, \quad \Gamma_{\rho'} = 839(102)\,\mathrm{MeV}\,,$$
$$M_{\rho''} = 1624(1)\,\mathrm{MeV}\,, \quad \Gamma_{\rho''} = 570(99)\,\mathrm{MeV}$$
$$\gamma = 0.22(10)\,, \quad \phi_1 = -1.65(4)\,,$$
$$\delta = -0.18(1)\,, \quad \phi_2 = -1.34(14)\,,$$

with a χ^2/d.o.f $= 35.6/52 \sim 0.63$ for the two-parameter fit.

- Form Factor phase shift for different parametrizations



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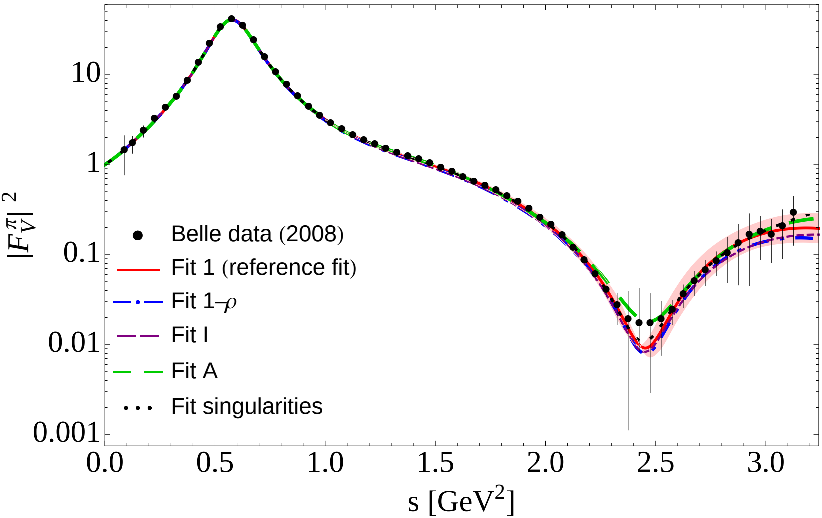
- Modulus squared of the pion vector form factor



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Central results

- Fit results (central value ± statistical fit error ± systematic th. error)

$$\alpha_1 = 1.88 \pm 0.01 \pm 0.01 \ \mathrm{GeV}^{-2}, \quad \alpha_2 = 4.34 \pm 0.01 \pm 0.03 \ \mathrm{GeV}^{-4},$$

$$M_\rho \doteq 773.6 \pm 0.9 \pm 0.3 \ \mathrm{MeV},$$

$$M_{\rho'} = 1376 \pm 6^{+18}_{-73} \ \mathrm{MeV}, \quad \Gamma_{\rho'} = 603 \pm 22^{+236}_{-141} \ \mathrm{MeV},$$

$$M_{\rho''} = 1718 \pm 4^{+57}_{-94} \ \mathrm{MeV}, \quad \Gamma_{\rho''} = 465 \pm 9^{+137}_{-53} \ \mathrm{MeV},$$

$$\gamma = 0.15 \pm 0.01^{+0.07}_{-0.03}, \quad \phi_1 = -0.66 \pm 0.01^{+0.22}_{-0.99},$$

$$\delta = -0.13 \pm 0.01^{+0.00}_{-0.05}, \quad \phi_2 = -0.44 \pm 0.03^{+0.06}_{-0.90},$$

- Physical pole mass and width

$$M_\rho^{\mathrm{pole}} = 760.6 \pm 0.8 \ \mathrm{MeV}, \quad \Gamma_\rho^{\mathrm{pole}} = 142.0 \pm 0.4 \ \mathrm{MeV},$$

$$M_{\rho'}^{\mathrm{pole}} = 1289 \pm 8^{+52}_{-143} \ \mathrm{MeV}, \quad \Gamma_{\rho'}^{\mathrm{pole}} = 540 \pm 16^{+151}_{-111} \ \mathrm{MeV},$$

$$M_{\rho''}^{\mathrm{pole}} = 1673 \pm 4^{+68}_{-125} \ \mathrm{MeV}, \quad \Gamma_{\rho''}^{\mathrm{pole}} = 445 \pm 8^{+117}_{-49} \ \mathrm{MeV},$$

Determination of the $\rho(1450)$ and $\rho(1700)$ resonance parameters

Reference	Model parameters $M_{\rho'}, \Gamma_{\rho'}$ [MeV]	Pole parameters $M_{\rho'}^{\rm pole}, \Gamma_{\rho'}^{\rm pole}$ [MeV]	Data
ALEPH	$1328 \pm 15, 468 \pm 41$	$1268 \pm 19, 429 \pm 31$	τ
ALEPH	$1409 \pm 12, 501 \pm 37$	$1345 \pm 15, 459 \pm 28$	τ & e^+e^-
Belle (fixed $\lvert F_V^\pi(0)\rvert^2$)	$1446 \pm 7 \pm 28, 434 \pm 16 \pm 60$	$1398 \pm 8 \pm 31, 408 \pm 13 \pm 50$	τ
Belle (all free)	$1428 \pm 15 \pm 26, 413 \pm 12 \pm 57$	$1384 \pm 16 \pm 29, 390 \pm 10 \pm 48$	τ
Dumm et. al.	—	$1440 \pm 80, 320 \pm 80$	τ
Celis et. al.	$1497 \pm 7, 785 \pm 51$	$1278 \pm 18, 525 \pm 16$	τ
Bartos et. al.	—	$1342 \pm 47, 492 \pm 138$	e^+e^-
Bartos et. al.	—	$1374 \pm 11, 341 \pm 24$	τ
This work	$1376 \pm 6^{+18}_{-73}, 603 \pm 22^{+236}_{-141}$	$1289 \pm 8^{+52}_{-143}, 540 \pm 16^{+151}_{-111}$	τ

Reference	Model parameters $(M_{\rho''}, \Gamma_{\rho''})$ [MeV]	Pole parameters $(M_{\rho''}^{\rm pole}, \Gamma_{\rho''}^{\rm pole})$ [MeV]	Data
ALEPH	$= 1713, = 235$	$1700, 232$	τ
ALEPH	$1740 \pm 20, = 235$	$1728 \pm 20, 232$	τ & e^+e^-
Belle (fixed $\lvert F_V^\pi(0)\rvert^2$)	$1728 \pm 17 \pm 89, 164 \pm 21^{+89}_{-26}$	$1722 \pm 18, 163 \pm 21^{+88}_{-27}$	τ
Belle (all free)	$1694 \pm 41, 135 \pm 36^{+50}_{-26}$	$1690 \pm 94, 134 \pm 36^{+49}_{-28}$	τ
Dumm et. al.	—	$1720 \pm 90, 180 \pm 90$	τ
Celis et. al.	$1685 \pm 30, 800 \pm 31$	$1494 \pm 37, 600 \pm 17$	τ
Bartos et. al.	—	$1719 \pm 65, 490 \pm 17$	e^+e^-
Bartos et. al.	—	$1767 \pm 52, 415 \pm 120$	τ
This work	$1718 \pm 4^{+57}_{-94}, 465 \pm 9^{+137}_{-53}$	$1673 \pm 4^{+68}_{-125}, 445 \pm 8^{+117}_{-49}$	τ

Determination of resonance parameters

- To look for a zero of the propagator in the complex plane

$$M_\rho^{\rm pole} = 760.6 \pm 0.8\ {\rm MeV}\,, \quad \Gamma_\rho^{\rm pole} = 142.0 \pm 0.4\ {\rm MeV}\,,$$

$$M_{\rho'}^{\rm pole} = 1289 \pm 8^{+52}_{-143}\ {\rm MeV}\,, \quad \Gamma_{\rho'}^{\rm pole} = 540 \pm 16^{+151}_{-111}\ {\rm MeV}\,,$$

$$M_{\rho''}^{\rm pole} = 1673 \pm 4^{+68}_{-125}\ {\rm MeV}\,, \quad \Gamma_{\rho''}^{\rm pole} = 445 \pm 8^{+117}_{-49}\ {\rm MeV}\,,$$

$\rho(1450)$ $\quad I^G(J^{PC}) = 1^+(1^{--})$

See our mini-review under the $\rho(1700)$.

$\rho(1450)$ MASS

$\pi\pi$ MODE

• • • We do not use the following data for averages, fits, limits, etc. • • •

VALUE (MeV)	EVTS	DOCUMENT ID		TECN	COMMENT
$1350 \pm 20\ ^{+20}_{-30}$	63.5k	[1] ABRAMOWICZ	12	ZEUS	$ep \to e\pi^+\pi^- p$
1493 ± 15		[2] LEES	12G	BABR	$e^+e^- \to \pi^+\pi^-\gamma$
$1446 \pm 7 \pm 28$	5.4M	[3,4] FUJIKAWA	08	BELL	$\tau^- \to \pi^-\pi^0\nu_\tau$
1328 ± 15		[5] SCHAEL	05C	ALEP	$\tau^- \to \pi^-\pi^0\nu_\tau$
1406 ± 15	87k	[3,6] ANDERSON	00A	CLE2	$\tau^- \to \pi^-\pi^0\nu_\tau$
~ 1368		[7] ABELE	99C	CBAR	$0.0\ \bar{p}d \to \pi^+\pi^-\pi^- p$
1348 ± 33		BERTIN	98	OBLX	0.05–$0.405\ \bar{n}p \to 2\pi^+\pi^-$
1411 ± 14		[8] ABELE	97	CBAR	$\bar{p}n \to \pi^-\pi^0\pi^0$
$1370\ ^{+90}_{-70}$		ACHASOV	97	RVUE	$e^+e^- \to \pi^+\pi^-$
1359 ± 40		[6] BERTIN	97C	OBLX	$0.0\ \bar{p}p \to \pi^+\pi^-\pi^0$
1282 ± 37		BERTIN	97D	OBLX	$0.05\ \bar{p}p \to 2\pi^+2\pi^-$
1424 ± 25		BISELLO	89	DM2	$e^+e^- \to \pi^+\pi^-$
1265.5 ± 75.3		DUBNICKA	89	RVUE	$e^+e^- \to \pi^+\pi^-$
1292 ± 17		[9] KURDADZE	83	OLYA	0.64–$1.4\ e^+e^- \to \pi^+\pi^-$

$\rho(1450)$ WIDTH

VALUE (MeV)	DOCUMENT ID
400±60 OUR ESTIMATE This is only an educated guess; the e the error on the average of the published values.	

$\pi\pi$ MODE

• • • We do not use the following data for averages, fits, limits, etc. • • •

VALUE (MeV)	EVTS	DOCUMENT ID		TECN	COMMENT
$460 \pm 30\ ^{+40}_{-45}$	63.5k	[1] ABRAMOWICZ	12	ZEUS	$ep \to e\pi^+\pi^- p$
427 ± 31		[2] LEES	12G	BABR	$e^+e^- \to \pi^+\pi^-\gamma$
$434 \pm 16 \pm 60$	5.4M	[3,4] FUJIKAWA	08	BELL	$\tau^- \to \pi^-\pi^0\nu_\tau$
468 ± 41		[5] SCHAEL	05C	ALEP	$\tau^- \to \pi^-\pi^0\nu_\tau$
455 ± 41	87k	[3,6] ANDERSON	00A	CLE2	$\tau^- \to \pi^-\pi^0\nu_\tau$
~ 374		[7] ABELE	99C	CBAR	$0.0\ \bar{p}d \to \pi^+\pi^-\pi^- p$
275 ± 10		BERTIN	98	OBLX	0.05–$0.405\ \bar{n}p \to \pi^+\pi^+\pi^-$
343 ± 20		[8] ABELE	97	CBAR	$\bar{p}n \to \pi^-\pi^0\pi^0$
310 ± 40		[6] BERTIN	97C	OBLX	$0.0\ \bar{p}p \to \pi^+\pi^-\pi^0$
236 ± 36		BERTIN	97D	OBLX	$0.05\ \bar{p}p \to 2\pi^+2\pi^-$
269 ± 31		BISELLO	89	DM2	$e^+e^- \to \pi^+\pi^-$
391 ± 70		DUBNICKA	89	RVUE	$e^+e^- \to \pi^+\pi^-$
218 ± 46		[9] KURDADZE	83	OLYA	0.64–$1.4\ e^+e^- \to \pi^+\pi^-$

Kaon vector Form Factor

$$\frac{d\Gamma(\tau^- \to K^- K^0 \nu_\tau)}{d\sqrt{s}} = \frac{G_F^2 |V_{ud}|^2}{768\pi^3} M_\tau^3 \left(1 - \frac{s}{M_\tau^2}\right)^2 \left(1 + \frac{2s}{M_\tau^2}\right) \sigma_K^3(s) |F_V^K(s)|^2 ,$$

- Chiral Perturbation Theory $\mathcal{O}(p^4)$

$$F_{K^+K^-}(s) = 1 + \frac{2L_9^r}{F_\pi^2} - \frac{s}{192\pi^2 F_\pi^2} \left[A_\pi(s,\mu^2) + 2A_K(s,\mu^2) \right] ,$$

$$F_{K^0\bar{K}^0}(s) = -\frac{s}{192\pi^2 F_\pi^2} \left[A_\pi(s,\mu^2) - A_K(s,\mu^2) \right] .$$

- Extract the $I = 1$ component

$$F_V^K(s) = 1 + \frac{2L_9^r}{F_\pi^2} - \frac{s}{96\pi^2 F_\pi^2} \left[A_\pi(s,\mu^2) + \frac{1}{2} A_K(s,\mu^2) \right] .$$

- At $\mathcal{O}(p^4)$, the pion and kaon vector form factor are the same
- Assumption: we consider that both are also the same at higher energies

Kaon vector form factor Omnès exponential representation

- Different resonance mixing contribution than $F_V^{\pi}(s)$

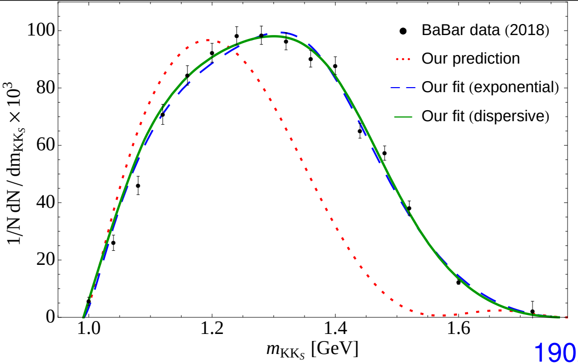
$$F_V^K(s) = \frac{M_\rho^2 + s\left(\tilde{\gamma}e^{i\tilde{\phi}_1} + \tilde{\delta}e^{i\tilde{\phi}_2}\right)}{M_\rho^2 - s - iM_\rho\Gamma_\rho(s)} \exp\left\{\mathrm{Re}\left[-\frac{s}{96\pi^2F_\pi^2}\left(A_\pi(s) + \frac{1}{2}A_K(s)\right)\right]\right\}$$
$$-\tilde{\gamma}\frac{s\,e^{i\tilde{\phi}_1}}{M_{\rho'}^2 - s - iM_{\rho'}\Gamma_{\rho'}(s)} \exp\left\{-\frac{s\Gamma_{\rho'}(M_{\rho'}^2)}{\pi M_{\rho'}^3\sigma_\pi^3(M_{\rho'}^2)}\mathrm{Re}A_\pi(s)\right\}$$
$$-\tilde{\delta}\frac{s\,e^{i\tilde{\phi}_2}}{M_{\rho''}^2 - s - iM_{\rho''}\Gamma_{\rho''}(s)} \exp\left\{-\frac{s\Gamma_{\rho''}(M_{\rho''}^2)}{\pi M_{\rho''}^3\sigma_\pi^3(M_{\rho''}^2)}\mathrm{Re}A_\pi(s)\right\},$$

$$\Gamma_{\rho',\rho''}(s) = \Gamma_{\rho',\rho''}\frac{s}{M_{\rho',\rho''}^2}\frac{\sigma_\pi^3(s)}{\sigma_\pi^3(M_{\rho',\rho''}^2)}\theta(s - 4m_\pi^2)\,.$$

- Extract the phase $\tan\phi_{KK}(s) = \mathrm{Im}F_V^K(s)/\mathrm{Re}F_V^K(s)$
- Use a three-times subtracted dispersion relation

Fit results to BaBar $\tau^- \to K^- K_S \nu_\tau$ data

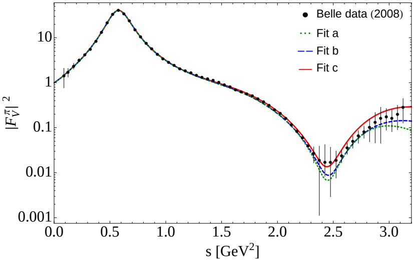
Parameter	$s_{\rm cut}$ = 4 [GeV^2]			
	Fit $i)$	Fit $ii)$	Fit $iii)$	Fit $iv)$
$\tilde{\alpha}_1$	$= 1.88(1)$	$= 1.84$	$= 1.88(1)$	—
$\tilde{\alpha}_2$	$= 4.34(1)$	$= 4.34$	$= 4.34(1)$	—
$M_{\rho'}$ [MeV]	$1467(24)$	$1538(32)$	$1489(25)$	1411(12)
$\Gamma_{\rho'}$ [MeV]	$415(48)$	$604(83)$	$297(36)$	394(35)
$\tilde{\gamma}$	$0.10(2)$	$0.36(11)$	$0.10(2)$	0.09(1)
$\tilde{\phi}_1$	$-1.19(16)$	$-1.48(13)$	$-1.10(15)$	$-1.88(9)$
χ^2/d.o.f.	2.9	1.9	2.9	3.3

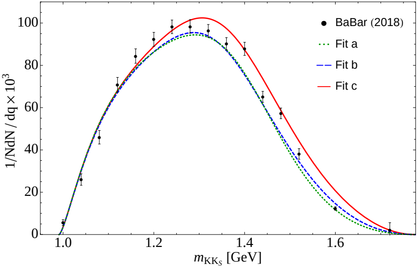


1902.02273 [hep-ph]

Combined analysis of $F_V^\pi(s)$ and $\tau^- \to K^- K_S \nu_\tau$

Parameter	$s_{\text{cut}} = 4$ [GeV^2]		
	Fit a	Fit b	Fit c
α_1	$1.88(1)$	$1.89(1)$	$1.87(1)$
α_2	$4.34(2)$	$4.31(2)$	$4.38(3)$
$\tilde{\alpha}_1$	$= \alpha_1$	$= \alpha_1$	$1.88(24)$
$\tilde{\alpha}_2$	$= \alpha_2$	$= \alpha_2$	$4.38(29)$
m_ρ [MeV]	$= 773.6(9)$	$= 773.6(9)$	$= 773.6(9)$
M_ρ [MeV]	$= m_\rho$	$= m_\rho$	$= m_\rho$
$M_{\rho'}$ [MeV]	$1396(19)$	$1453(19)$	$1406(61)$
$\Gamma_{\rho'}$ [MeV]	$507(31)$	$499(51)$	$524(149)$
$M_{\rho''}$[MeV]	$1724(41)$	$1712(32)$	$1746(1)$
$\Gamma_{\rho''}$ [MeV]	$399(126)$	$284(72)$	$413(362)$
γ	$0.12(3)$	$0.15(3)$	$0.11(11)$
$\tilde{\gamma}$	$0.11(2)$	$= \gamma$	$0.11(5)$
ϕ_1	$-0.23(26)$	$0.29(21)$	$-0.27(42)$
$\tilde{\phi}_1$	$-1.83(14)$	$-1.48(13)$	$-1.90(67)$
δ	$-0.09(2)$	$-0.07(2)$	$-0.10(5)$
$\tilde{\delta}$	$= 0$	$= 0$	$-0.01(4)$
ϕ_2	$-0.20(31)$	$0.27(29)$	$-1.15(71)$
$\tilde{\phi}_2$	$= 0$	$= 0$	$0.40(3)$
χ^2/d.o.f	1.52	1.19	1.25





- RχT with two resonances: $K^*(892)$ and $K^*(1410)$

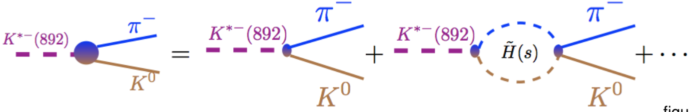


figure courtesy of D. Boito

$$\widetilde{F}_V^{K\pi}(s) = \frac{m_{K^\star}^2 - \kappa_{K^\star}\widetilde{H}_{K\pi}(0) + \gamma s}{D(m_{K^\star}, \gamma_{K^\star})} - \frac{\gamma s}{D(m_{K^{\star\prime}}, \gamma_{K^{\star\prime}})},$$

$$D(m_n, \gamma_n) \equiv m_n^2 - s - \kappa_n \mathrm{Re}\left[H_{K\pi}(s)\right] - i m_n \Gamma_n(s),$$

$$\kappa_n = \frac{192\pi F_K F_\pi}{\sigma_{K\pi}(m_{K^*}^2)} \frac{\gamma_{K^*}}{m_{K^*}}, \quad \Gamma_n(s) = \Gamma_n \frac{s}{m_\pi^2} \frac{\sigma_{K\pi}^3(s)}{\sigma_{K\pi}^3(m_n^2)}$$

- We then have a phase with two resonances

$$\delta^{K\pi}(s) = \tan^{-1}\left[\frac{\mathrm{Im}F_V^{K\pi}(s)}{\mathrm{Re}F_V^{K\pi}(s)}\right]$$

Vector Form Factor: Dispersive representation

- Three subtractions: helps the convergence of the form factor and suppresses the the high-energy region of the integral

$$F_V^{K\pi}(s) = P(s)\exp\left[\alpha_1 \frac{s}{m_{\pi^-}^2} + \frac{1}{2}\alpha_2 \frac{s^2}{m_{\pi^-}^4} + \frac{s^3}{\pi}\int_{s_{K\pi}}^{s_{\rm cut}} ds' \frac{\delta^{K\pi}(s')}{(s')^3(s'-s-i0)}\right]$$

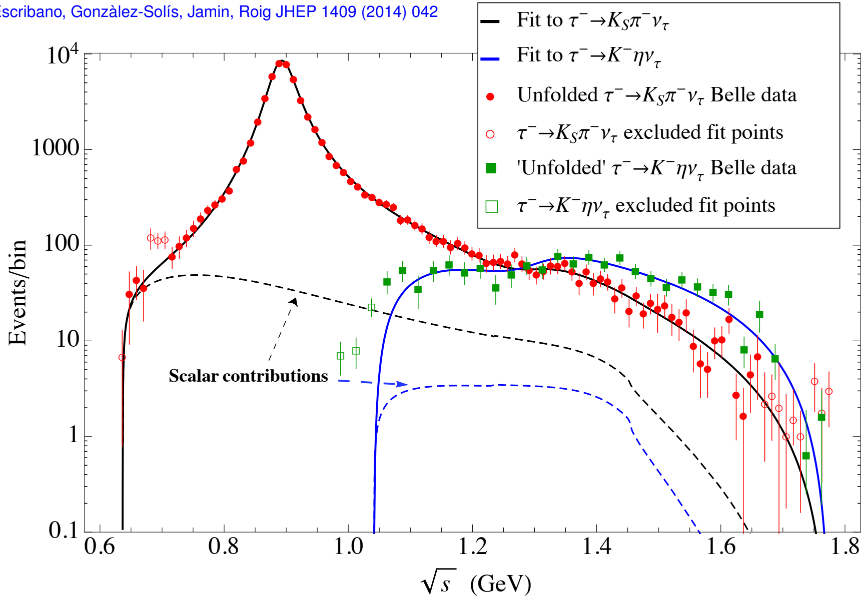
- $\alpha_1 = \lambda_+^{'}$ and $\alpha_1^2 + \alpha_2 = \lambda_+^{''}$ low energies parameters

$$F_V^{K\pi}(t) = 1 + \frac{\lambda_+^{'}}{M_{\pi^-}^2}t + \frac{1}{2}\frac{\lambda_+^{''}}{M_{\pi^-}^4}t^2$$

- $s_{\rm cut}$: cut-off to check stability
- Parameters to Fit: λ_+' , λ_+'', $m_{K^*}, \gamma_{K^*}, m_{K^{*\prime}}, \gamma_{K^{*\prime}}$

Results of the combined $\tau^- \to K_S \pi^- \nu_\tau$ and $\tau^- \to K^- \eta \nu_\tau$ analysis

Escribano, Gonzàlez-Solís, Jamin, Roig JHEP 1409 (2014) 042



Results of the combined $\tau^- \to K_S\pi^-\nu_\tau$ and $\tau^- \to K^-\eta\nu_\tau$ analysis

- Different choices regarding linear slopes and resonance mixing parameters (s_{cut} = 4 GeV^2)

Fitted value	Reference Fit	Fit A	Fit B	Fit C
$\bar{B}_{K\pi}(\%)$	0.404 ± 0.012	0.400 ± 0.012	0.404 ± 0.012	0.397 ± 0.012
$(B^{th}_{K\pi})(\%)$	(0.402)	(0.394)	(0.400)	(0.394)
M_{K^*}	892.03 ± 0.19	892.04 ± 0.19	892.03 ± 0.19	892.07 ± 0.19
Γ_{K^*}	46.18 ± 0.42	46.11 ± 0.42	46.15 ± 0.42	46.13 ± 0.42
$M_{K^{*\prime}}$	1305^{+15}_{-18}	1308^{+16}_{-19}	1305^{+15}_{-18}	1310^{+14}_{-17}
$\Gamma_{K^{*\prime}}$	168^{+52}_{-44}	212^{+66}_{-54}	174^{+58}_{-47}	184^{+56}_{-46}
$\gamma_{K\pi} \times 10^2$	$= \gamma_{K\eta}$	$-3.6^{+1.1}_{-1.5}$	$-3.3^{+1.0}_{-1.3}$	$= \gamma_{K\eta}$
$\lambda'_{K\pi} \times 10^3$	23.9 ± 0.7	23.6 ± 0.7	23.8 ± 0.7	23.6 ± 0.7
$\lambda''_{K\pi} \times 10^4$	11.8 ± 0.2	11.7 ± 0.2	11.7 ± 0.2	11.6 ± 0.2
$\bar{B}_{K\eta} \times 10^4$	1.58 ± 0.10	1.62 ± 0.10	1.57 ± 0.10	1.66 ± 0.09
$(B^{th}_{K\eta}) \times 10^4$	(1.45)	(1.51)	(1.44)	(1.58)
$\gamma_{K\eta} \times 10^2$	$-3.4^{+1.0}_{-1.3}$	$-5.4^{+1.8}_{-2.6}$	$-3.9^{+1.4}_{-2.1}$	$-3.7^{+1.0}_{-1.4}$
$\lambda'_{K\eta} \times 10^3$	20.9 ± 1.5	$= \lambda'_{K\pi}$	21.2 ± 1.7	$= \lambda'_{K\pi}$
$\lambda''_{K\eta} \times 10^4$	11.1 ± 0.4	11.7 ± 0.2	11.1 ± 0.4	11.8 ± 0.2
χ^2/n.d.f.	$108.1/105 \sim 1.03$	$109.9/105 \sim 1.05$	$107.8/104 \sim 1.04$	$111.9/106 \sim 1.06$

Results of the combined $\tau^- \to K_S\pi^-\nu_\tau$ and $\tau^- \to K^-\eta\nu_\tau$ analysis

- Reference fit results obtained for different values of s_{cut}

Parameter \ s_{cut} (GeV^2)	3.24	4	9	∞
$\bar{B}_{K\pi}(\%)$	0.402 ± 0.013	0.404 ± 0.012	0.405 ± 0.012	0.405 ± 0.012
$(B^{th}_{K\pi})(\%)$	(0.399)	(0.402)	(0.403)	(0.403)
M_{K^*}	892.01 ± 0.19	892.03 ± 0.19	892.05 ± 0.19	892.05 ± 0.19
Γ_{K^*}	46.04 ± 0.43	46.18 ± 0.42	46.27 ± 0.42	46.27 ± 0.41
$M_{K^{*\prime}}$	1301^{+17}_{-22}	1305^{+15}_{-18}	1306^{+14}_{-17}	1306^{+14}_{-17}
$\Gamma_{K^{*\prime}}$	207^{+73}_{-58}	168^{+52}_{-44}	155^{+48}_{-41}	155^{+47}_{-40}
$\gamma_{K\pi}$	$= \gamma_{K\eta}$	$= \gamma_{K\eta}$	$= \gamma_{K\eta}$	$= \gamma_{K\eta}$
$\lambda'_{K\pi} \times 10^3$	23.3 ± 0.8	23.9 ± 0.7	24.3 ± 0.7	24.3 ± 0.7
$\lambda''_{K\pi} \times 10^4$	11.8 ± 0.2	11.8 ± 0.2	11.7 ± 0.2	11.7 ± 0.2
$\bar{B}_{K\eta} \times 10^4$	1.57 ± 0.10	1.58 ± 0.10	1.58 ± 0.10	1.58 ± 0.10
$(B^{th}_{K\eta}) \times 10^4$	(1.43)	(1.45)	(1.46)	(1.46)
$\gamma_{K\eta} \times 10^2$	$-4.0^{+1.3}_{-1.9}$	$-3.4^{+1.0}_{-1.3}$	$-3.2^{+0.9}_{-1.1}$	$-3.2^{+0.9}_{-1.1}$
$\lambda'_{K\eta} \times 10^3$	18.6 ± 1.7	20.9 ± 1.5	22.1 ± 1.4	22.1 ± 1.4
$\lambda''_{K\eta} \times 10^4$	10.8 ± 0.3	11.1 ± 0.4	11.2 ± 0.4	11.2 ± 0.4
χ^2/n.d.f.	105.8/105	108.1/105	111.0/105	111.1/105

Results of the combined $\tau^- \to K_S \pi^- \nu_\tau$ and $\tau^- \to K^- \eta \nu_\tau$ analysis

- Central results including the largest variation of s_{cut}

$M_{K^{*-}(892)} = 892.03 \pm 0.19$ MeV
$\Gamma_{K^{*-}(892)} = 46.18 \pm 0.44$ MeV
} no gain

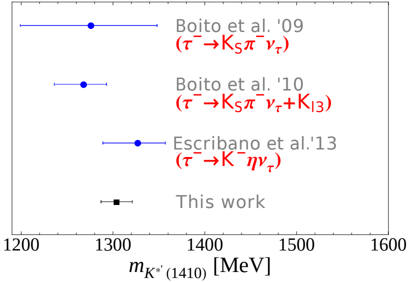
$M_{K^{*-}(1410)} = 1305^{+16}_{-18}$ MeV
$\Gamma_{K^{*-}(1410)} = 168^{+65}_{-59}$ MeV
} improvement

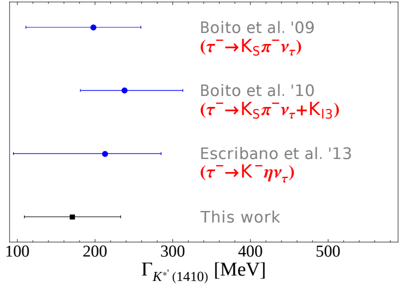
$\gamma_{K\pi} = \gamma_{K\eta} = -3.4^{+1.2}_{-1.4} \cdot 10^{-2}$

$\bar{B}_{K\pi} = (0.0404 \pm 0.012)\%$

$\bar{B}_{K\eta} = (1.58 \pm 0.10) \cdot 10^{-4}$

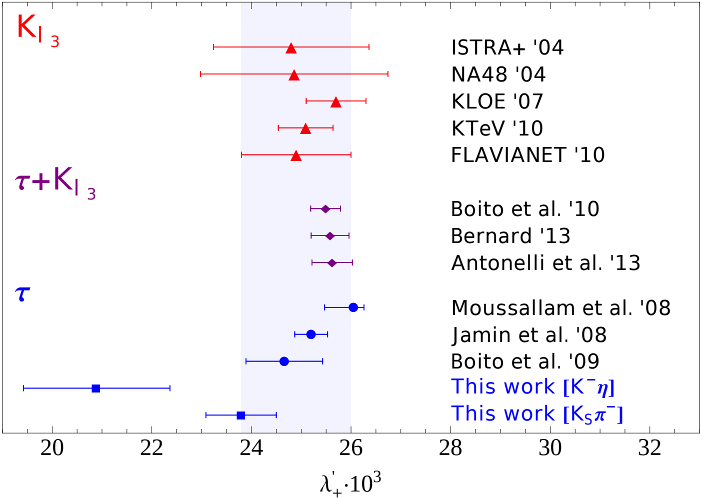
$\chi^2/d.o.f = 108.1/105 = 1.03$





Results of the combined $\tau^- \to K_S \pi^- \nu_\tau$ and $\tau^- \to K^- \eta \nu_\tau$ analysis

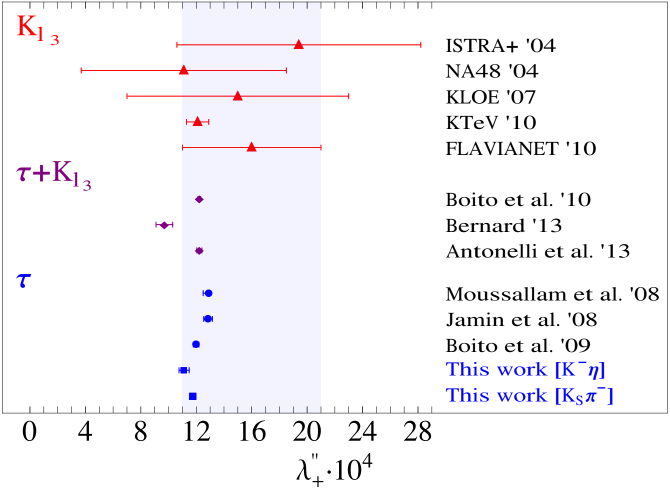
$$\left.\begin{array}{l}\lambda'_{K\pi} = (23.9 \pm 0.9)\cdot 10^{-3} \\ \lambda'_{K\eta} = (20.9 \pm 2.7)\cdot 10^{-3}\end{array}\right\}$$ isospin violation? spectra for $\tau^- \to K^- \pi^0 \nu_\tau$ needed



Results of the combined $\tau^- \to K_S\pi^-\nu_\tau$ and $\tau^- \to K^-\eta\nu_\tau$ analysis

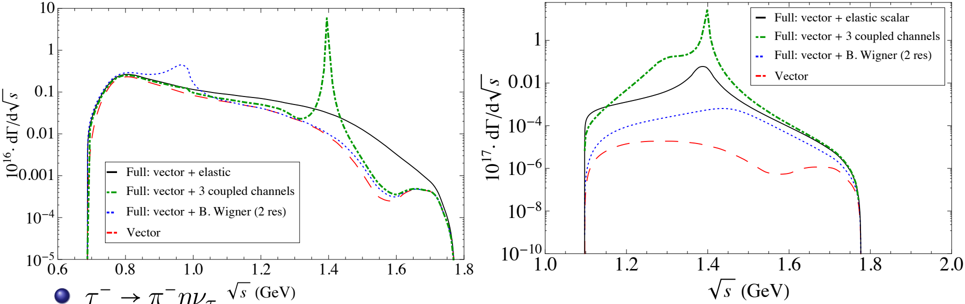
$\lambda''_{K\pi} = (11.8 \pm 0.2) \cdot 10^{-4}$

$\lambda''_{K\eta} = (11.1 \pm 0.5) \cdot 10^{-4}$



$\tau^- \to \pi^-\eta^{(\prime)}\nu_\tau$: Invariant mass distribution and Branching Ratio

Escribano, Gonzàlez-Solís, Roig PRD 94 (2016)



- $\tau^- \to \pi^-\eta\nu_\tau$
  - Theory predictions: $BR \sim 1 \times 10^{-5}$ (Escribano'16, Moussallam'14)
  - BaBar: $BR < 9.9 \cdot 10^{-5}$ 95% CL , Belle: $BR < 7.3 \cdot 10^{-5}$ 90% CL
- $\tau^- \to \pi^-\eta'\nu_\tau$
  - Theory predictions: $BR \sim [10^{-7}, 10^{-6}]$ (Escribano'16)
  - BaBar: $BR < 4 \cdot 10^{-6}$ 90% CL

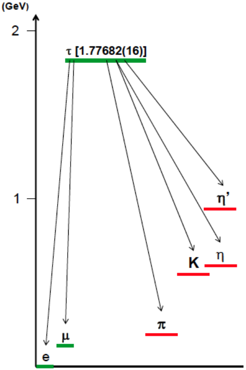
see talk by P. Rados

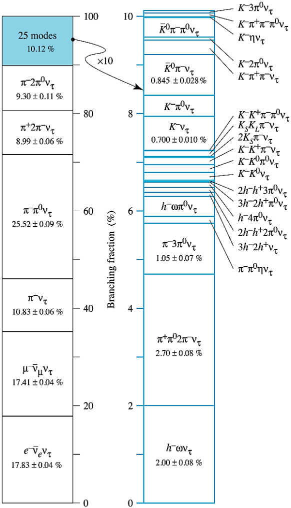
Challenging for Belle II

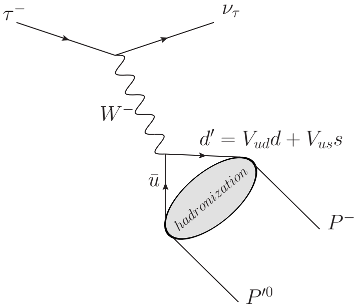
Outlook

- Tau physics is a very rich field to test QCD and EW
- Important experimental activities: Belle (II), BaBar, LHCb, BESIII
- τ decays into two mesons are a privileged laboratory to access the non-perturbative regime of QCD
- Form Factors from dispersion relations with subtractions
  - Extraction of the $K^*(892)$ parameters from a fit to $\tau \to K_S \pi^- \nu_\tau$
  - Extraction of the $K^*(1410)$ from $\tau^- \to K_S \pi^- \nu_\tau$ and $\tau^- \to K^- \eta \nu_\tau$
  - $F_V^\pi(s)$: important for testing QCD dynamics and the SM and NP
  - $\tau^- \to K_S K^- \nu_\tau$: extraction of the $\rho(1450)$ and $\rho(1700)$ parameters
- A lot of interesting physics to be done in the tau sector

Hadronic Tau decays

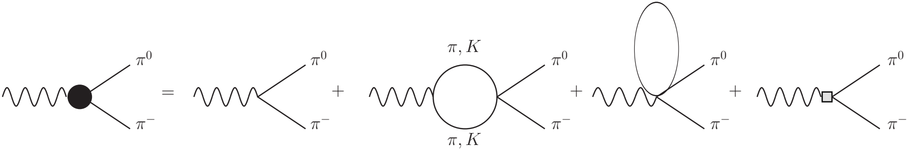






Test of QCD and ElectroWeak Interactions

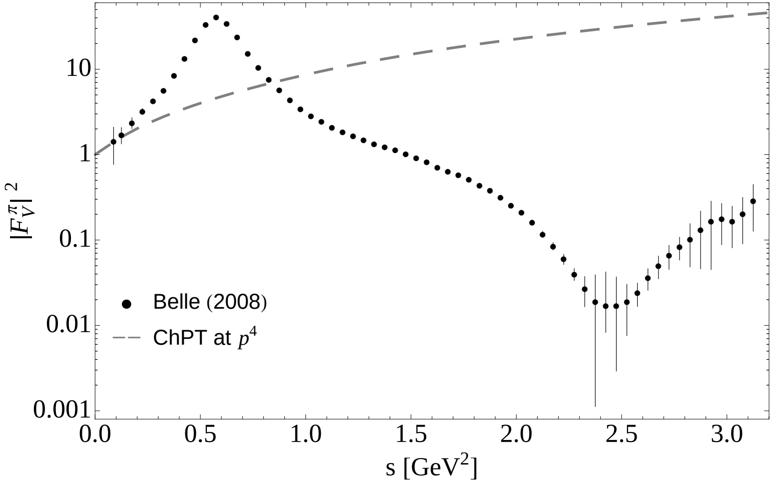
Pion vector form factor: Chiral Perturbation Theory $\mathcal{O}(p^4)$



$$F_\pi^V(s) = 1 + \frac{2L_9^r(\mu)}{F_\pi^2} s - \frac{s}{96\pi^2 F_\pi^2}\left(A_\pi(s,\mu^2) + \frac{1}{2}A_K(s,\mu^2)\right)$$

$$A_P(s,\mu^2) = \log\frac{m_P^2}{\mu^2} + 8\frac{m_P^2}{s} - \frac{5}{3} + \sigma_P^3(s)\log\left(\frac{\sigma_P(s)+1}{\sigma_P(s)-1}\right), \sigma_P(s) = \sqrt{1-4\frac{m_P^2}{s}}$$

Pion vector form factor: Chiral Perturbation Theory $\mathcal{O}(p^4)$



Pion vector Form Factor: ChPT with resonances

- Resonace Chiral Theory

$$F_V^\pi(s) = 1 + \frac{F_V G_V}{F_\pi^2}\frac{s}{M_\rho^2 - s} \stackrel{F_V G_V = F_\pi^2}{\Longrightarrow} \frac{M_\rho^2}{M_\rho^2 - s},$$

- Expansion in s and comparing ChPT and RχT

$$F_\pi^V(s) = 1 + \frac{2L_9^r(\mu)}{F_\pi^2}s - \frac{s}{96\pi^2 F_\pi^2}\left(A_\pi(s,\mu^2) + \frac{1}{2}A_K(s,\mu^2)\right)$$

$$F_\pi^V(s) = 1 + \left(\frac{s}{M_\rho^2}\right) + \left(\frac{s}{M_\rho^2}\right)^2 + \cdot$$

- Chiral coupling estimate: $L_9^r(M_\rho) = \frac{F_V G_V}{2M_\rho^2} = \frac{F_\pi^2}{2M_\rho^2} \sim 7.2 \times 10^{-3}$
- Combining ChPT and RχT

$$F_V^\pi(s) = \frac{M_\rho^2}{M_\rho^2 - s} - \frac{s}{96\pi^2 F_\pi^2}\left[A_\pi(s,\mu^2) + \frac{1}{2}A_K(s,\mu^2)\right],$$

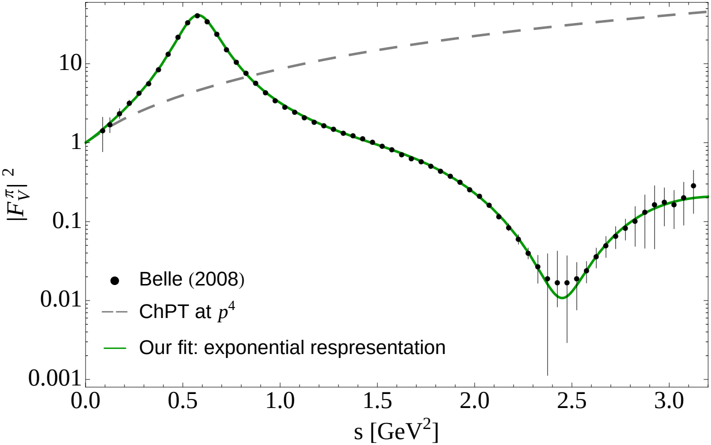
ChPT with resonances + Omnès: Exponential representation

- Incorporate the (off-shell) ρ width

$$\Gamma_\rho(s) = -\frac{M_\rho s}{96\pi^2 F_\pi^2} \mathrm{Im}\left[A_\pi(s) + \frac{1}{2}A_K(s)\right]$$
$$= \frac{M_\rho s}{96\pi F_\pi^2}\left[\sigma_\pi(s)^3\theta(s-4m_\pi^2) + \sigma_K(s)^3\theta(s-4m_K^2)\right].$$

$$F_V^\pi(s) = \frac{M_\rho^2}{M_\rho^2 - s - iM_\rho\Gamma_\rho(s)} \exp\left\{-\frac{s}{96\pi^2 F_\pi^2}\mathrm{Re}\left[A_\pi(s,\mu^2) + \frac{1}{2}A_K(s,\mu^2)\right]\right\}.$$

ChPT with resonances + Omnès+ρ', ρ'': Exponential representation

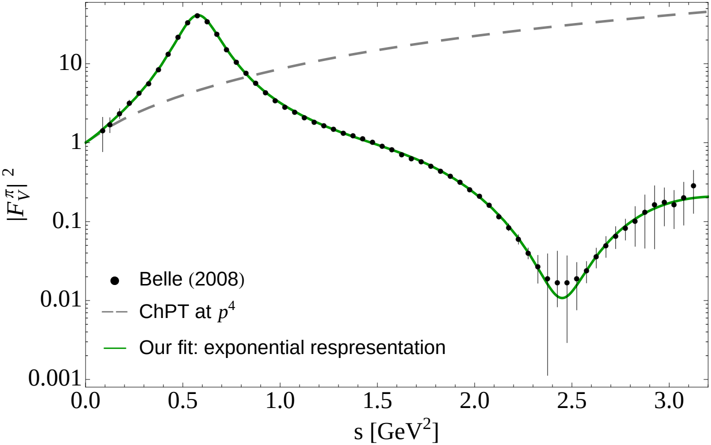


$$M_\rho = 775.2(4)\,\text{MeV}\,, \quad \gamma = 0.15(4)\,, \quad \phi_1 = -0.36(24)\,,$$

$$M_{\rho'} = 1438(39)\,\text{MeV}\,, \quad \Gamma_{\rho'} = 535(63)\,\text{MeV}\,, \quad \delta = -0.12(4)\,, \quad \phi_2\,, = -0.02(45)\,,$$

$$M_{\rho''} = 1754(91)\,\text{MeV}\,, \quad \Gamma_{\rho''} = 412(102)\,\text{MeV}\,, \quad \chi^2_{\text{dof}} = 0.92$$

ChPT with resonances + Omnès+ρ', ρ'': Exponential representation



$$M_\rho^{\text{pole}} = 762.0(3)\,\text{MeV}\,,\quad \Gamma_\rho^{\text{pole}} = 143.0(2)\,\text{MeV}\,,$$

$$M_{\rho'}^{\text{pole}} = 1366(38)\,\text{MeV}\,,\quad \Gamma_{\rho'}^{\text{pole}} = 488(48)\,\text{MeV}\,,$$

$$M_{\rho''}^{\text{pole}} = 1718(82)\,\text{MeV}\,,\quad \Gamma_{\rho''}^{\text{pole}} = 397(88)\,\text{MeV}\,,$$

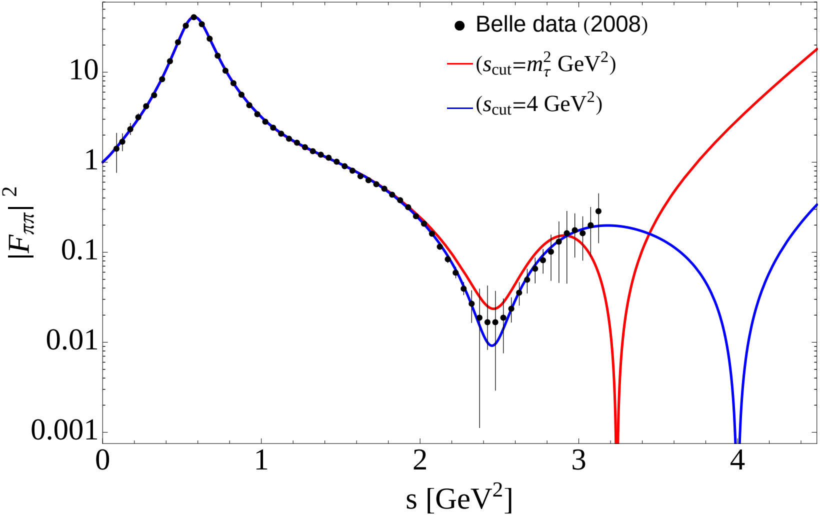
Variant (I)

- Fits for different values of $s_{\rm cut}$ and allowing the ρ-mass to float

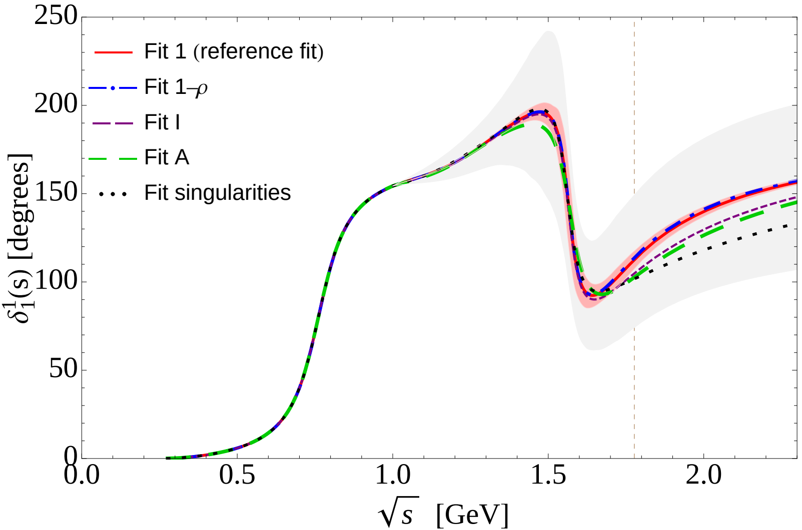
Fits	Parameter	$s_{\rm cut}$ [GeV^2]			
		m_τ^2	4 (reference fit)	10	∞
Fit 1-ρ	α_1 [GeV^{-2}]	$1.88(1)$	$1.88(1)$	$1.89(1)$	$1.88(1)$
	α_2 [GeV^{-4}]	$4.37(3)$	$4.34(1)$	$4.31(3)$	$4.34(1)$
	m_ρ [MeV]	$773.9(3)$	$773.8(3)$	$773.9(3)$	$773.9(3)$
	M_ρ [MeV]	$= m_\rho$	$= m_\rho$	$= m_\rho$	$= m_\rho$
	$M_{\rho'}$ [MeV]	$1382(71)$	$1375(11)$	$1316(9)$	$1312(8)$
	$\Gamma_{\rho'}$ [MeV]	$516(165)$	$608(35)$	$728(92)$	$726(26)$
	$M_{\rho''}$[MeV]	$1723(1)$	$1715(22)$	$1655(1)$	$1656(8)$
	$\Gamma_{\rho''}$ [MeV]	$315(271)$	$455(16)$	$569(160)$	$571(13)$
	γ	$0.12(13)$	$0.16(1)$	$0.18(2)$	$0.17(1)$
	ϕ_1	$-0.56(35)$	$-0.69(1)$	$-1.40(19)$	$-1.41(8)$
	δ	$-0.09(3)$	$-0.13(1)$	$-0.17(4)$	$-0.17(3)$
	ϕ_2	$-0.19(69)$	$-0.45(12)$	$-1.06(10)$	$-1.05(11)$
	χ^2/d.o.f	1.09	0.70	0.63	0.66

Dispersive representation: on the singularities at $s = s_{\rm cut}$

- Modulus squared of the pion form factor for $s_{\rm cut} = m_\tau, 4$ GeV^2

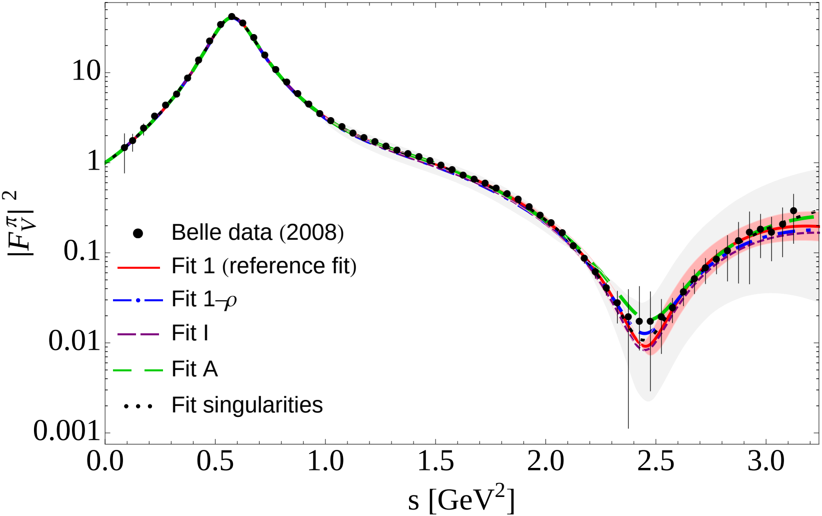


- Form Factor phase shift including systematic th. uncertainties



- The results can be found as ancillary material in 1902.02273 [hep-ph]

- Form Factor including systematic th. uncertainties



- The results can be found as ancillary material in 1902.02273 [hep-ph]

$\tau^- \to \nu_\tau$+strange

- Tau partial width to strange $\sim 3\%$

Branching fraction	HFAG Winter 2012 fit
$\Gamma_{10} = K^-\nu_\tau$	$(0.6955 \pm 0.0096) \cdot 10^{-2}$
$\Gamma_{16} = K^-\pi^0\nu_\tau$	$(0.4322 \pm 0.0149) \cdot 10^{-2}$
$\Gamma_{23} = K^-2\pi^0\nu_\tau$ (ex. K^0)	$(0.0630 \pm 0.0222) \cdot 10^{-2}$
$\Gamma_{28} = K^-3\pi^0\nu_\tau$ (ex. K^0, η)	$(0.0419 \pm 0.0218) \cdot 10^{-2}$
$\Gamma_{35} = \pi^-\overline{K}^0\nu_\tau$	$(0.8206 \pm 0.0182) \cdot 10^{-2}$
$\Gamma_{40} = \pi^-\overline{K}^0\pi^0\nu_\tau$	$(0.3649 \pm 0.0108) \cdot 10^{-2}$
$\Gamma_{44} = \pi^-\overline{K}^0\pi^0\pi^0\nu_\tau$	$(0.0269 \pm 0.0230) \cdot 10^{-2}$
$\Gamma_{53} = \overline{K}^0h^-h^-h^+\nu_\tau$	$(0.0222 \pm 0.0202) \cdot 10^{-2}$
$\Gamma_{128} = K^-\eta\nu_\tau$	$(0.0153 \pm 0.0008) \cdot 10^{-2}$
$\Gamma_{130} = K^-\pi^0\eta\nu_\tau$	$(0.0048 \pm 0.0012) \cdot 10^{-2}$
$\Gamma_{132} = \pi^-\overline{K}^0\eta\nu_\tau$	$(0.0094 \pm 0.0015) \cdot 10^{-2}$
$\Gamma_{151} = K^-\omega\nu_\tau$	$(0.0410 \pm 0.0092) \cdot 10^{-2}$
$\Gamma_{801} = K^-\phi\nu_\tau(\phi \to KK)$	$(0.0037 \pm 0.0014) \cdot 10^{-2}$
$\Gamma_{802} = K^-\pi^-\pi^+\nu_\tau$ (ex. K^0, ω)	$(0.2923 \pm 0.0068) \cdot 10^{-2}$
$\Gamma_{803} = K^-\pi^-\pi^+\pi^0\nu_\tau$ (ex. K^0, ω, η)	$(0.0411 \pm 0.0143) \cdot 10^{-2}$
$\Gamma_{110} = X_s^-\nu_\tau$	$(2.8746 \pm 0.0498) \cdot 10^{-2}$

- $\tau \to (K\pi)^-\nu_\tau$ and $\tau \to K^-\eta^{(\prime)}\nu_\tau \longrightarrow$ this talk